

Scattering of Polarized Light on Magnetically
Aligned Particles in Multipole Magnetic Fields*

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ABSTRACT

The theory of polarized light scattering on aligned colloidal magnetic particles (Fe_3O_4) in multipole magnetic fields is given. With the assumption of an anisotropic light scattering process, the azimuthal location of the observed scattering pattern can be explained. The observed eight-fold symmetry in the scattering pattern of a sextupole magnetic field and the twelve-fold symmetry of an octupole field can be interpreted as consequences of the field configuration symmetry in a sextupole and an octupole, respectively. The temperature dependence of the scattering pattern intensity can be explained as a change in the number of scattering centers. It is assumed that the number of aligned scattering centers is given by the Langevin function as a function of the temperature of the colloidal solution. Some applications of this theory, such as the magnetic center location in multipoles, are discussed.

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I. INTRODUCTION

The simplest case of light scattering is that by randomly distributed small particles which are nonconducting, optically isotropic, and transparent. Each particle scatters the light as a linear electrical dipole.

In dilute solutions of isotropic molecules the light scattering problem is simple when the dimensions of the molecules are considerably less than the wavelength of the light. In this case one can eliminate the scattering on the solvent and there remains only the problem of light scattering by a perfect gas, which has been known since the work of Lord Rayleigh¹ (1899). If the number of scattering centers is N , the total intensity of scattered unpolarized light is the sum of the scattered intensities of each of the N molecules, which can be written as

$$I = \frac{8\pi^4}{\lambda^4} N \alpha^2 E^2 (1 + \cos^2 \phi)$$

where α is the polarizability ($\vec{P} = \alpha \vec{E}$), λ is the wavelength of the light, ϕ is the angle between the incident and scattered beams and E is the electric field vector.

When the size of the scattering centers is comparable in magnitude to the wavelength of the light, the scattering centers cannot be treated as point dipoles. The emitted radiation from different parts of the scattering centers may not have the same phase, which will cause a dissymmetry of the scattering pattern; thus the scattered intensity in this case will be a function of the geometry of the scattering centers.

The Mie² theory gives a rigorous solution for this scattering problem when the scattering centers are spheres. The scattering theory must be

modified when the scattering centers are optically anisotropic, or when there is some interaction between the scattering particles, or when the scattering particles interact with an applied electric or magnetic field. A very interesting consequence of this type of anisotropic scattering is the polarization of starlight by oriented nonspherical particles.³ The interstellar grains aligned with the direction of the magnetic field are the scattering centers of this anisotropic scattering process.⁴

In this paper the measurement and the theory of polarized light scattering on aligned colloidal magnetic particles in multipole magnetic fields will be discussed. The symmetry properties of the magnetic multipoles allow a large number of simplifications in the calculations and make it possible to use this effect for practical applications, which will be discussed later.

II. EXPERIMENTAL PROCEDURE

The experimental setup to study the light scattering patterns is shown in Fig. 1. This technique has been used to locate the magnetic center of quadrupole fields⁵; similar experimental setup is used to study light scattering by thin polymer films.⁶ The polarized light beam passes through a colloidal solution, which is located in the multipole magnetic field, and is analyzed by the analyzer. When there is no magnetic field, one rotates the analyzer until the light is extinguished. If the multipole magnetic field is turned on, the scattering pattern can be seen or photographed. Typical scattering patterns in multipole fields are shown in Fig. 2 for a quadrupole field, in Fig. 3 for a sextupole field, and in Fig. 4 for an octupole field. The angle Θ is the angle between the direction of polarization and the Y axis. The asymmetry of the pictures is caused by the misalignment of the camera with the Z axis.

In this experiment the scattering centers are Fe_3O_4 crystallites in a colloidal suspension. The preparation of such a colloidal solution is described by D. J. Craik and P. M. Griffiths.⁷ The individual crystallites of the magnetite were measured using the electron microscope method of Craik.⁸ It was found that the particles are of the order of 100\AA .

III. ALIGNMENT OF THE MAGNETIC PARTICLES IN A QUADRUPOLE FIELD

To study the alignment mechanisms of the Fe_3O_4 particles in multipole magnetic fields, a simple light intensity measurement was conducted at different solution temperatures. The experimental setup is shown in Fig. 5. The magnetic field was measured with a gauss meter at the quadrupole pole face, and light intensity was measured with a linear light detector.

It was found that transmitted light intensity at low field values is proportional to the magnetic field; at higher field values, the transmitted intensity shows some saturation effect which is temperature dependent (see Fig. 6).

This result might be explained using the theory of paramagnetism. The magnetite crystallites, having magnetic moment, align themselves preferentially with the direction of the magnetic field in the multipoles. If N_0 is the number of crystallites per unit volume, the number of aligned scattering centers is given by the following formula

$$N = N_0 L(a) = N_0 \left[\cosh \frac{mH}{kT} - \frac{kT}{mH} \right] ; \quad a = \frac{mH}{kT}$$

where $L(a)$ is the well known Langevin function used in the classical theory of paramagnetism.

In the case of very strong field or very low temperature the Langevin function becomes unity; thus

$$N = N_0 = N_{\text{sat}}$$

If all of the dipoles are aligned with the field, the number of scattering centers is independent of the applied field, that is, the number of scattering centers is saturated.

It is interesting to note that if the colloid solution is frozen, it acts as a complete depolarizer. In this case the randomly oriented dipoles scatter the incoming polarized light into any direction with equal probability.

IV. THEORY OF ANISOTROPIC SCATTERING

In order to explain the intensity distribution of the scattered polarized light on the aligned magnetite crystallites, one can assume anisotropy in the scattering process. One of the simplest assumptions is that the aligned magnetite has a different polarizability along the magnetic field than it does perpendicular to the field. The polarizability tensor in the coordinate system of the aligned particle ($X'-Y'$) can then be written as

$$\alpha_{ik} = \begin{vmatrix} \alpha_{\perp} & 0 \\ 0 & \alpha_{\parallel} \end{vmatrix}$$

In calculating the polarizability tensor in the $X-Y$ coordinate system, it is desirable to use the symmetry properties of the multipole fields. In a quadrupole field, any line passing through the center of symmetry with an angle φ with respect to the Y axis will cross the field at an angle β , where

$$\beta = \frac{\pi}{2} - 2\varphi$$

Therefore, if $\varphi = \theta$ is the angle of polarization, the angle between the electrical vector \vec{E} and the long axis of the aligned crystallite, or the magnetic field vector \vec{H} , is β . (See Fig. 7.) One can find similar relationships for

other multipole fields, as shown in Appendix A. In a sextupole field, a line passing through the center of symmetry with an angle ϕ with respect to the Y axis will cross the field at an angle β , which is given as

$$\beta = -3\phi$$

and, similarly, for an octupole field the angle between a line passing through the center of symmetry and the direction of the magnetic field vector can be written

$$\beta = -4\phi$$

With these relationships, the polarizability tensor in the X-Y system can be expressed using a rotational transformation:

$$\left| \alpha_{ik} \right|_{XY} = S(-\phi-\beta) \begin{vmatrix} \alpha_{\perp} & 0 \\ 0 & \alpha_{\parallel} \end{vmatrix}$$

where $S(-\phi-\beta)$ is the transformation matrix, i.e.,

$$S(-\phi-\beta) = \begin{pmatrix} \cos(-\phi-\beta) & -\sin(-\phi-\beta) \\ \sin(-\phi-\beta) & \cos(-\phi-\beta) \end{pmatrix}$$

Because all the quantities can be expressed in the X-Y coordinate system, the scattering amplitude can be calculated easily. The size of the scattering centers are small as compared to the wavelength of the polarized light, so the Rayleigh approximation can be used. The scattering amplitude by the i -th volume element of the system at the location of the observer is given⁹ as

$$A_i = K(\vec{P}_i \cdot \vec{O}) \cos k(\vec{r}_i \cdot \vec{s})$$

\vec{P}_i is the induced dipole in the i -th volume element, located a distance r_i from the origin $k = \frac{2\pi}{\lambda}$ (λ = wavelength in the medium); $\vec{s} = \vec{s}' - \vec{s}_0$ where \vec{s}' and \vec{s}_0 are unit vectors along the scattered and incident beams; \vec{O} is the unit vector perpendicular to the scattered light beam and along the polarization direction of the scattered light beam. K is a proportionality constant.

The dipole moment P_i is given by

$$P_i = \alpha_{ik} E_k$$

In the X-Y coordinate system the components of \vec{E} are given as

$$\vec{E} = E_0 \left[(\sin \Theta) \vec{i} + (\cos \Theta) \vec{j} \right]$$

where Θ is the angle of polarization before the scattering. The components of \vec{O} can be expressed as

$$\vec{O} = \left[(\cos \Theta) \vec{i} - (\sin \Theta) \vec{j} \right]$$

when observation is perpendicular to the X-Y plane and along the symmetry axis of the multipoles. In this case

$$\vec{s}' = \vec{s}_0 \quad \text{and} \quad \cos k(\vec{r}_i \cdot \vec{s}) = 1$$

The total amplitude of the scattered light from the X-Y plane can be written

$$A = \Sigma A_i = K \int_{r=0}^R \int_{\phi=0}^{2\pi} (\vec{O} \cdot \vec{P}) r dr d\phi$$

By squaring the total amplitude, the intensity is obtained.

We would now like to calculate the angle φ relative to the Y axis at which the intensity is zero for a given polarization angle Θ . This condition is given by the following expression:

$$(\vec{P} \cdot \vec{O}) = P_X \cos \Theta - P_Y \sin \Theta = 0$$

Scattering processes in different multipoles will now be considered.

A. Light Scattering on Aligned Particles in a Quadrupole Field

In a quadrupole field, the dielectric tensor in the X-Y system can be written as

$$\alpha_{|XY} = \begin{pmatrix} \cos(\varphi - \frac{\pi}{2}) & -\sin(\varphi - \frac{\pi}{2}) \\ \sin(\varphi - \frac{\pi}{2}) & \cos(\varphi - \frac{\pi}{2}) \end{pmatrix} \begin{pmatrix} \alpha_{\perp} & 0 \\ 0 & \alpha_{\parallel} \end{pmatrix}$$

and the induced dipole moment as

$$\vec{P} = |\alpha_{|XY} \vec{E} = \begin{pmatrix} \alpha_{\perp} \sin \varphi \sin \Theta + \alpha_{\parallel} \cos \varphi \cos \Theta \\ -\alpha_{\perp} \cos \varphi \sin \Theta + \alpha_{\parallel} \sin \varphi \cos \Theta \end{pmatrix}$$

With the above, and using the condition for zero intensity,

$$(\vec{P} \cdot \vec{O}) = P_X \cos \Theta - P_Y \sin \Theta = 0$$

one obtains φ in terms of Θ :

$\begin{aligned} \text{and } \alpha_{\perp} \tan \Theta &= -\alpha_{\perp} \tan \varphi \\ \alpha_{\parallel} \tan \Theta &= \alpha_{\parallel} \cot \varphi \end{aligned}$

From the experimental observation, the locations of the two dark lines as functions of the polarization angle are found consistent within the experimental error

with the following equations:

$$\tan \theta = -\tan \varphi$$

$$\tan \theta = \cot \varphi$$

The scattering intensity is proportional to the square of the amplitude; consequently,

$$I_1 \propto A_1^2 = K^2 (\vec{O} \cdot \vec{P})^2$$

The numerical value for the constant K might be obtained from the Rayleigh formula, from which

$$K^2 = \frac{8\pi^4}{\lambda^4} N_1 E^2$$

where N_1 is the density of scattering centers in volume element V_1 , and

$$N_1 = N_0 \left[\cosh \frac{mH}{kT} - \frac{kT}{mH} \right]$$

Along the Z axis, the scattered light intensity from volume element V_1 can be expressed as

$$I = \frac{8\pi^4 N_0 E^2}{\lambda^4} \left[\cosh \frac{mH}{kT} - \frac{kT}{mH} \right] \left[\vec{O} \cdot \vec{P} \right]^2$$

B. Light Scattering on Aligned Particles in Sextupole and Octupole Fields

In the sextupole and octupole fields, the magnetic field intensity changes as $(B_0/R_0^2)r^2$ and $(B_0/R_0^3)r^3$, respectively, where B_0 is the field at the pole faces, R_0 is the half aperture, and $r^2 = X^2 + Y^2$.

Therefore, the magnetic field intensity is very low near the Z axis and is not sufficient to align the scattering centers in the field direction. This might be the reason for the unclear scattering picture near the Z axis as seen in Figs. 3 and 4.

In a sextupole field the dipole moment can be written as

$$\begin{aligned} \vec{P} &= |\alpha|_{XY} \vec{E} = \begin{pmatrix} \cos 2\varphi & -\sin 2\varphi \\ \sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} \alpha_{\perp} & 0 \\ 0 & \alpha_{\parallel} \end{pmatrix} \begin{pmatrix} \sin \Theta \\ \cos \Theta \end{pmatrix} E_0 \\ &= E_0 \begin{pmatrix} \alpha_{\perp} \cos 2\varphi \sin \Theta - \alpha_{\parallel} \sin 2\varphi \cos \Theta \\ \alpha_{\perp} \sin 2\varphi \sin \Theta + \alpha_{\parallel} \cos 2\varphi \cos \Theta \end{pmatrix} \end{aligned}$$

when the angle of polarization is Θ . The azimuth angle φ for zero intensity lines was obtained from

$$(\vec{O} \cdot \vec{P}) = P_X \cos \Theta - P_Y \sin \Theta = 0,$$

and with this one finds that

$\alpha_{\perp} \tan \Theta = \alpha_{\parallel} \cot 2\varphi$
$\alpha_{\parallel} \tan \Theta = -\alpha_{\perp} \tan 2\varphi$

Quite similarly, for an octupole field the dipole moment of the aligned colloidal particles can be written as:

$$\vec{P} = |\alpha|_{XY} \vec{E} = \begin{pmatrix} \alpha_{\perp} \cos 3\varphi \sin \Theta - \alpha_{\parallel} \sin 3\varphi \cos \Theta \\ \alpha_{\perp} \sin 3\varphi \sin \Theta + \alpha_{\parallel} \cos 3\varphi \cos \Theta \end{pmatrix}$$

from which, using $(\vec{O} \cdot \vec{P}) = 0$, one obtains

$$\begin{array}{l} \alpha_{\perp} \tan \Theta = \alpha_{\perp} \cot 3\varphi \\ \alpha_{\parallel} \tan \Theta = -\alpha_{\parallel} \tan 3\varphi \end{array}$$

In both cases the observed locations of dark lines characterized by the azimuth angle φ agree with the calculated values for a given polarization angle Θ . At zero polarization angles, as shown in Figs. 3 and 4, the dark lines passing through the center are located at

$$\varphi = 0^{\circ}, 45^{\circ}, 90^{\circ}, \text{ and } 135^{\circ}$$

for the sextupole field, and at

$$\varphi = 0^{\circ}, 30^{\circ}, 60^{\circ}, 120^{\circ}, \text{ and } 150^{\circ}$$

for octupole fields. It is interesting to note that the angular separation of the dark lines is 45° in a sextupole field and 30° in the octupole field (see Figs. 3 and 4).

The calculated azimuthal location of the dark lines as a function of the polarization angle Θ ($0^{\circ} \leq \Theta \leq 60^{\circ}$) is tabulated in Fig. 8.

V. APPLICATIONS

One of the most interesting applications of this light scattering effect was proposed by R. M. Johson,⁵ who used the scattering pattern to locate the magnetic center of a quadrupole. In this experimental setup the polarized light was directed through the vial of colloidal solution from one end of the quadrupole magnet. The observer at the opposite end of the magnet then looked at the vial through a plane-polarizing analyzer so aligned with the polarizer of the incoming light that complete cancellation of light should

occur. When the magnetic field was turned on, the center of the scattering pattern coincided with the magnetic center of the quadrupole. The accuracy of this type of center determination is of the order of ± 0.001 inch. The vial with the polarizer and analyzer can be mounted in a small carriage which could be moved along the Z axis of the magnet. With this device the "average magnetic center line" can be measured.

A typical measuring setup in a quadrupole magnet is shown in Fig. 9. Using the orientation of the dark cross, one can use this device to find the relation between the magnetic and mechanical axes in a quadrupole. Because of the unclear center portion, this method probably cannot be used for center location in higher poles.

It might be interesting to try light scattering in electrical multipole fields, using electrical rather than magnetic alignment for the scattering centers. If the relaxation time of orientation of the scattering centers in the field direction is short, this effect might be useful for light modulation.

LIST OF REFERENCES

1. J.W.S. Rayleigh, *Phil. Mag.* 47, 375 (1899).
2. G. Mie, *Ann. Phys.* 33, 1275 (1910).
3. J. M. Greenberg, N. E. Pedersen, and J. C. Pedersen, *J. Appl. Phys.* 32, 233 (1961).
4. L. Davis, Jr. and J. L. Greenstein, *J. Appl. Phys.* 114, 206 (1951).
5. R. M. Johson, Internal Report BEV-687, Lawrence Radiation Laboratory, Berkeley, California (October 1961).
6. R. S. Stein and M. B. Rhodes, *J. Appl. Phys.* 31, 1873 (1960).
7. D. J. Craik and P. M. Griffiths, *Proc. Phys. Soc. (Vol. B)* 70, 1000 (1957).
8. D. J. Craik, *Proc. Phys. Soc. (Vol. B)* 69, 647 (1956).
9. H. C. van de Hulst, Light Scattering by Small Particles (Wiley and Sons, New York, 1957).

APPENDIX A

Table A-I lists the symmetry properties and the magnetic scalar potentials for quadrupole, sextupole and octupole fields.

The magnetic field intensity is given as a gradient of the scalar potential u ,

$$\vec{H} = -\vec{\nabla}u$$

and the field components can be written as

$$H_X = -\frac{\partial u}{\partial X}$$

$$H_Y = -\frac{\partial u}{\partial Y}$$

In order to calculate the angle β between the direction of the magnetic field and any line passing through the center [with direction cosines $\cos \varphi = \frac{Y}{r}$, $\cos (\frac{\pi}{2} - \varphi) = \frac{X}{r} = \sin \varphi$], one can use the definition of the scalar product:

$$\cos \beta = \frac{H_X \sin \varphi + H_Y \cos \varphi}{|H| |I|}$$

With this formula, β can be calculated in terms of φ .

A. Calculation of β for a Quadrupole Field

Using

$$H_X = -\frac{\partial u}{\partial X} = -B_2 r \cos \varphi$$

$$H_Y = -\frac{\partial u}{\partial Y} = -B_2 r \sin \varphi$$

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$$H_Y = -\frac{\partial u}{\partial Y} = -B_2 r \sin \varphi$$

and

$$H = \sqrt{H_X^2 + H_Y^2} = B_2 r$$

one obtains

$$\cos \beta = \frac{B_2 r (\cos \varphi \sin \varphi + \sin \varphi \cos \varphi)}{B_2 r} = \sin 2\varphi = \cos \left(\frac{\pi}{2} - 2\varphi \right)$$

and

$$\beta = \frac{\pi}{2} - 2\varphi$$

When the particles are aligned opposite to the field direction,

$$\beta = \frac{3\pi}{2} - 2\varphi$$

This particle alignment pattern can be realized by changing the polarity of the poles in relation to the coordinate axis, as is shown in Figs. A-1 and A-2.

For quadrupole fields, both expressions for β result in the same intensity distribution in the scattering pattern. The only effect of the choice of β is that it changes the sign of the dipole moment \vec{P} , where

$$\vec{P} = \left| \alpha_{XY} \right| \vec{E}$$

but because the intensity is proportional to P^2 , the sign of \vec{P} is irrelevant. In the case of higher order poles, the different β values result in different intensity distributions for the scattering pattern in a given coordinate system. However, because the scattering pattern does not change

with a change in polarity, it would seem that a particle aligned parallel with the magnetic field scatters the same way in the scattering process as does a particle that is aligned opposite to the field. Particles with induced magnetic moments are aligned along the field lines irrespective of the relative directions of the magnetic field \vec{H} and the moment \vec{m} . Therefore, the relative orientations of \vec{m} and \vec{H} are not taken into account in further calculations.

B. Calculation of β for a Sextupole Field

This calculation is similar to that for a quadrupole field. Using the following relations:

$$H_X = - \frac{\partial u}{\partial X} = 2B_3 XY = 2B_3 r^2 \cos \varphi \sin \varphi$$

$$H_Y = - \frac{\partial u}{\partial Y} = - B_3 (Y^2 - X^2) = - B_3 r^2 (\cos^2 \varphi - \sin^2 \varphi)$$

and

$$H = \sqrt{H_X^2 + H_Y^2} = B_3 r^2$$

yields

$$\begin{aligned} \cos \beta &= \frac{B_3 r^2 [3 \cos \varphi \sin^2 \varphi - \cos^3 \varphi]}{B_3 r^2} = 3 \cos \varphi - 4 \cos^3 \varphi \\ &= - \cos 3\varphi = \cos (\pi \pm 3\varphi) \end{aligned}$$

Then

$$\beta = \pi - 3\varphi$$

and neglecting the relative orientation of \vec{m} and \vec{H} , one can write that

$$\beta = - 3\varphi$$

C. Calculation of β for an Octupole Field

In the case of an octupole field, with

$$H_X = - \frac{\partial u}{\partial X} = - B_4 (4X^3 - 12 XY^2)$$

$$H_Y = - \frac{\partial u}{\partial Y} = - B_4 (4Y^3 - 12 X^2 Y)$$

and

$$|H| = 4B_4 r^3$$

one can write after a simple calculation that

$$\cos \beta = - [8 \cos^4 \varphi - 8 \cos^2 \varphi + 1] = - \cos^4 \varphi = \cos (\pi \pm 4\varphi)$$

Then β can be expressed as

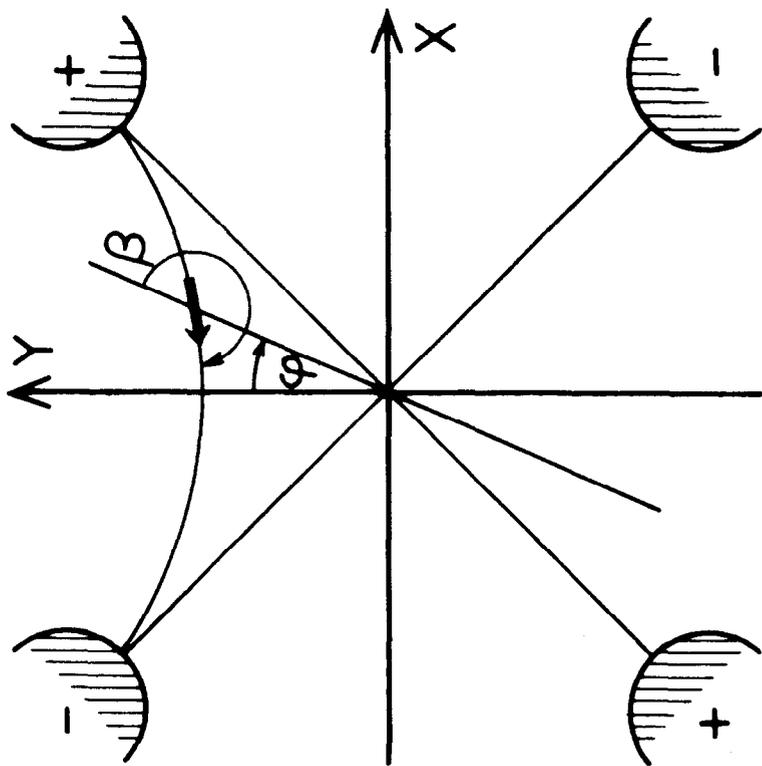
$$\beta = \pi - 4\varphi$$

and again neglecting the relative orientations of \vec{m} and \vec{H} , one finds that

$$\beta = - 4\varphi$$

TABLE A-I

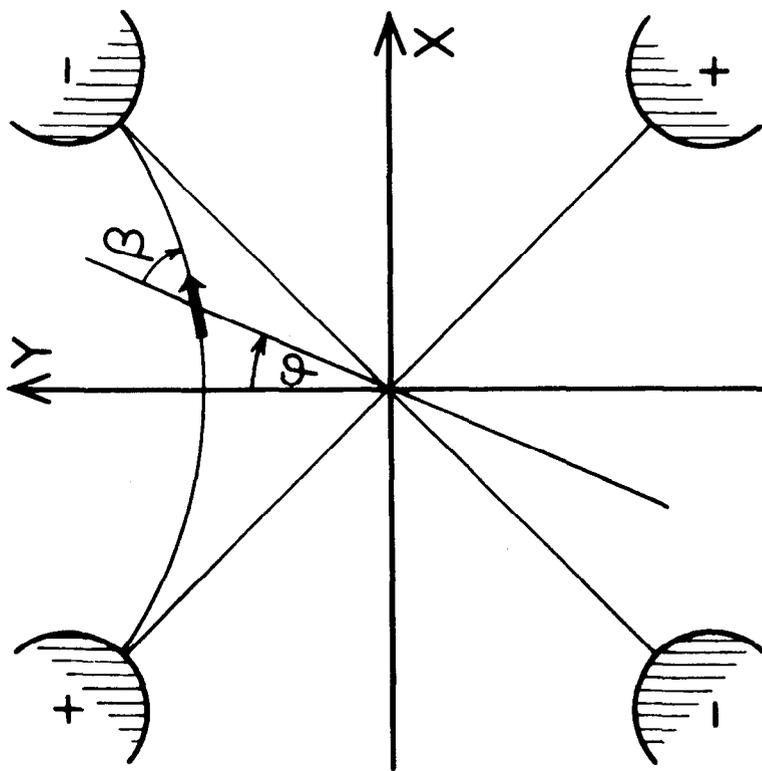
Multipoles	Symmetry Properties	Magnetic Scalar Potential $u(X_1 Y)$ and $u(r_1 \varphi)$
	$u(r_1 \varphi) = -u(r_1 \varphi + \frac{\pi}{2})$ $= -u(r_1 - \varphi)$	$u = B_1^2 r^2 (\cos 2\varphi)$ $= 2B_2^1 XY = B_2 XY$
	$u(r_1 \varphi) = -u(r_1 \varphi + \frac{\pi}{3})$ $= -u(r_1 - \varphi)$	$u = B_3^1 r^3 (\cos 3\varphi)$ $= 3B_3^1 (\frac{1}{3}Y^3 - X^2 Y)$ $= B_3 (\frac{1}{3}Y^3 - X^2 Y)$
	$u(r_1 \varphi) = -(r_1 \varphi + \frac{\pi}{4})$ $= -u(r_1 - \varphi)$	$u = B_4 r^4 (\cos^4 \varphi)$ $= B_4 (X^4 + Y^4 - 6X^2 Y^2)$



$$\beta = \frac{3\pi}{2} - 2\varphi$$

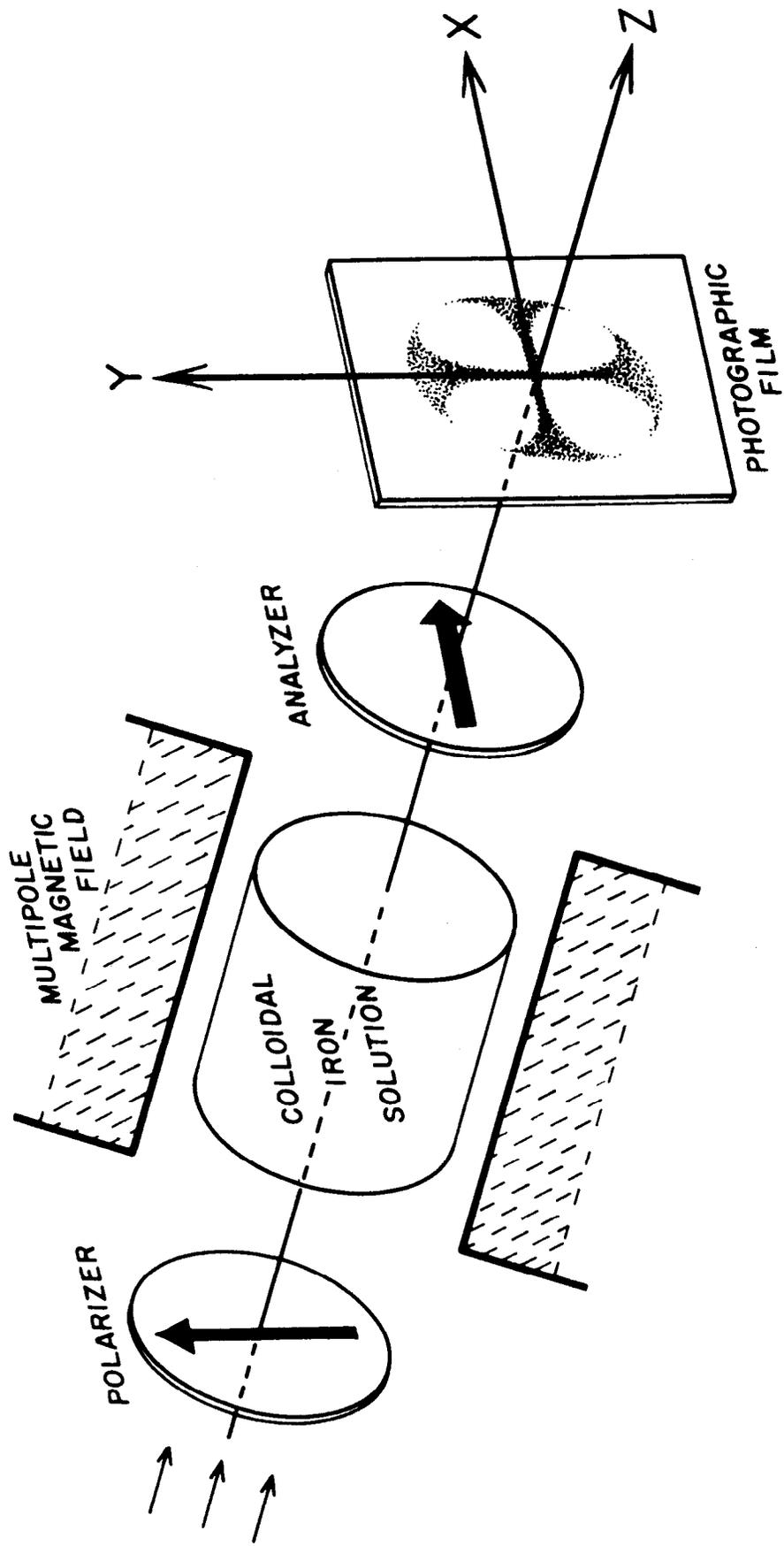
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FIGURE A-2



$$\beta = \frac{\pi}{2} - 2\varphi$$

FIGURE A-1



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FIGURE 1

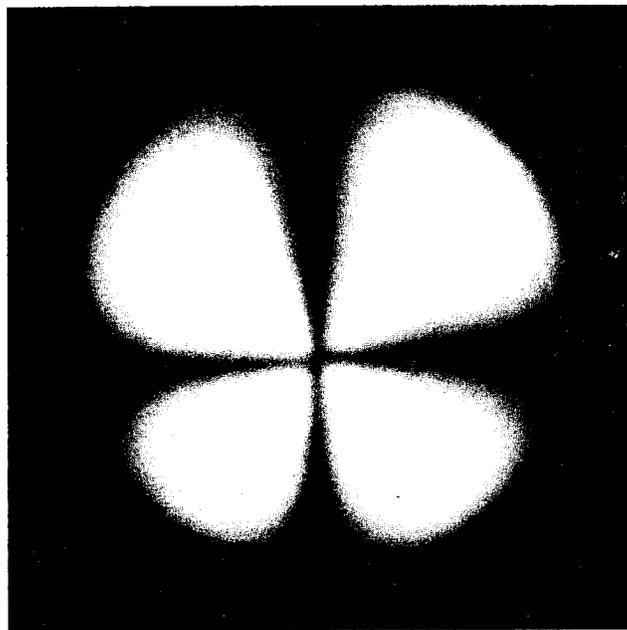


Figure 2

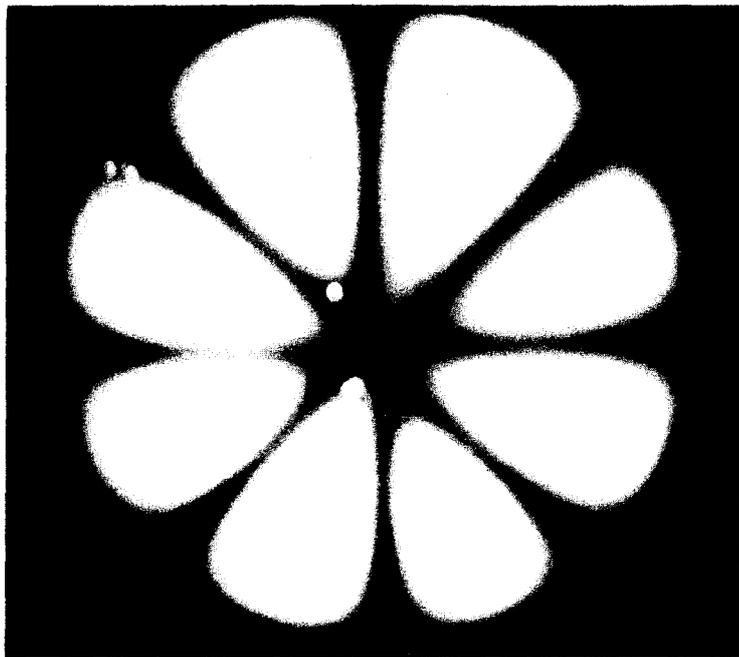


Figure 3

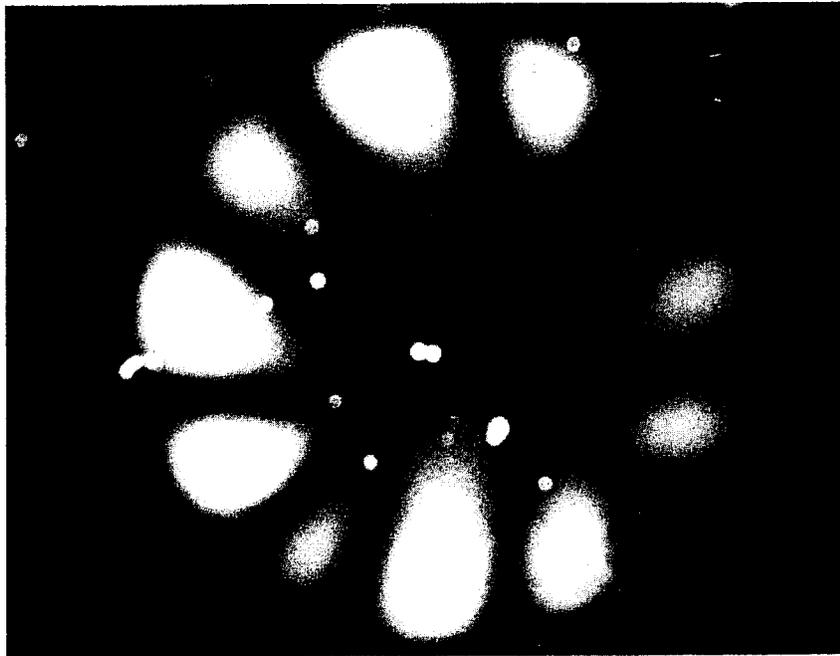
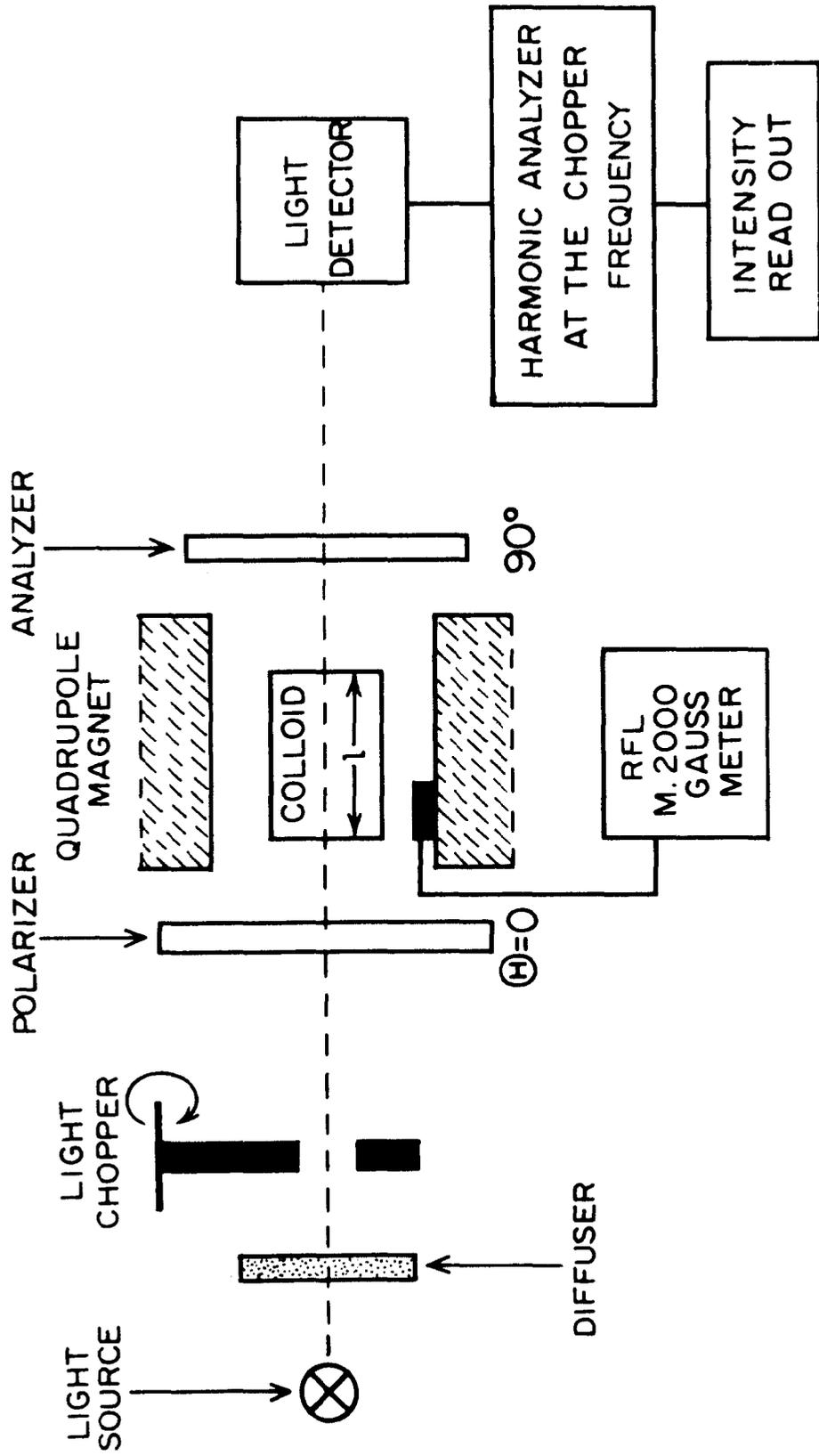


Figure 4



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FIGURE 5

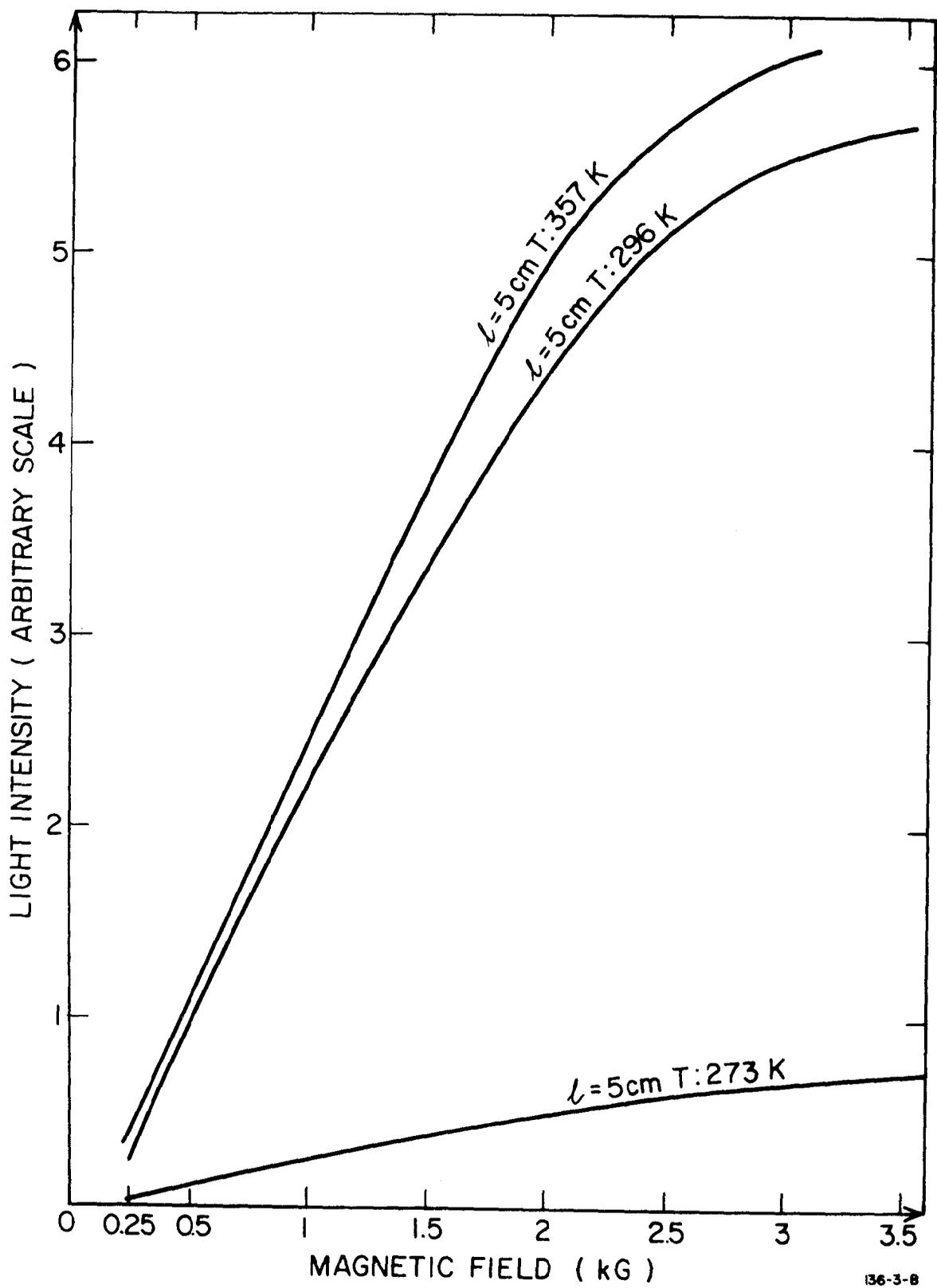


FIGURE 6

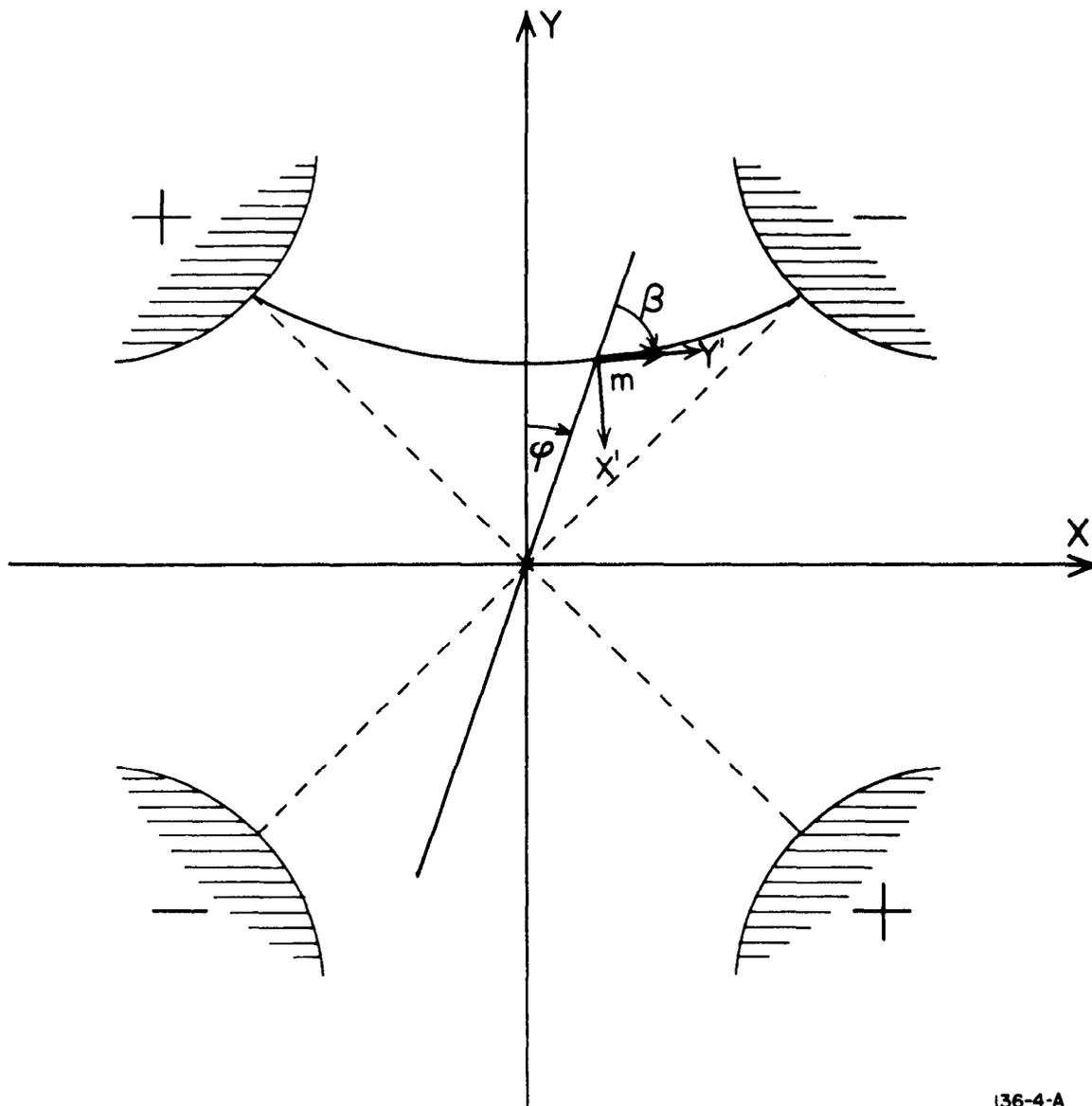
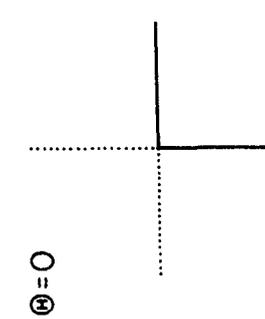
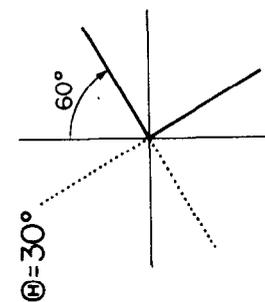
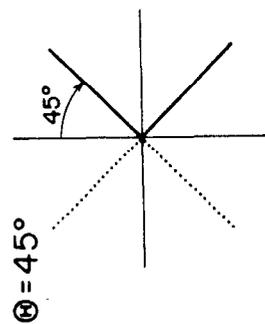
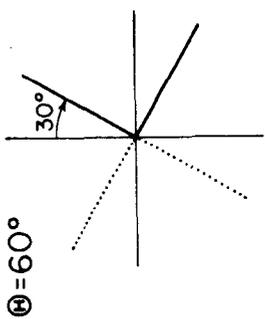


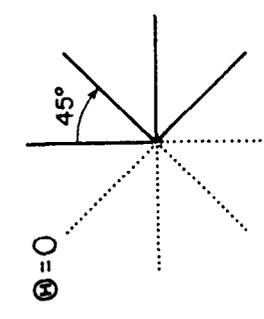
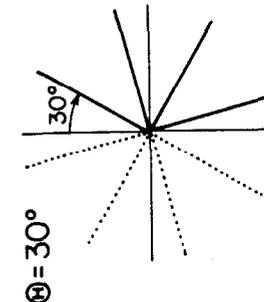
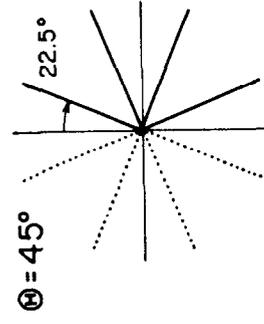
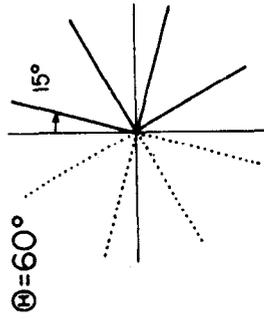
FIGURE 7

136-4-A

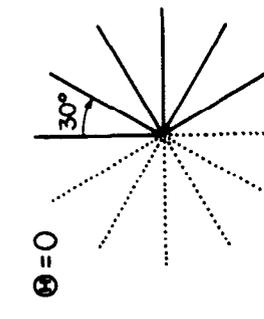
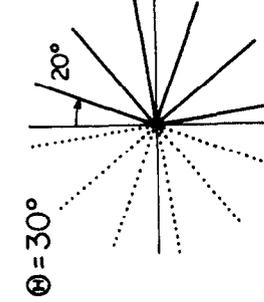
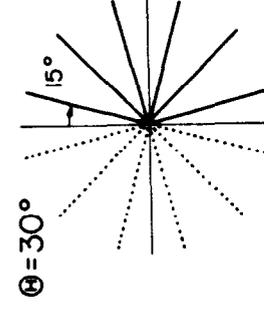
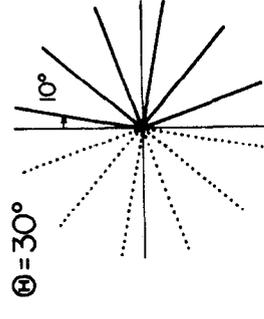
QUADRUPOLE



SEXTUPOLE



OCTUPOLE



136-5-B

FIGURE 8

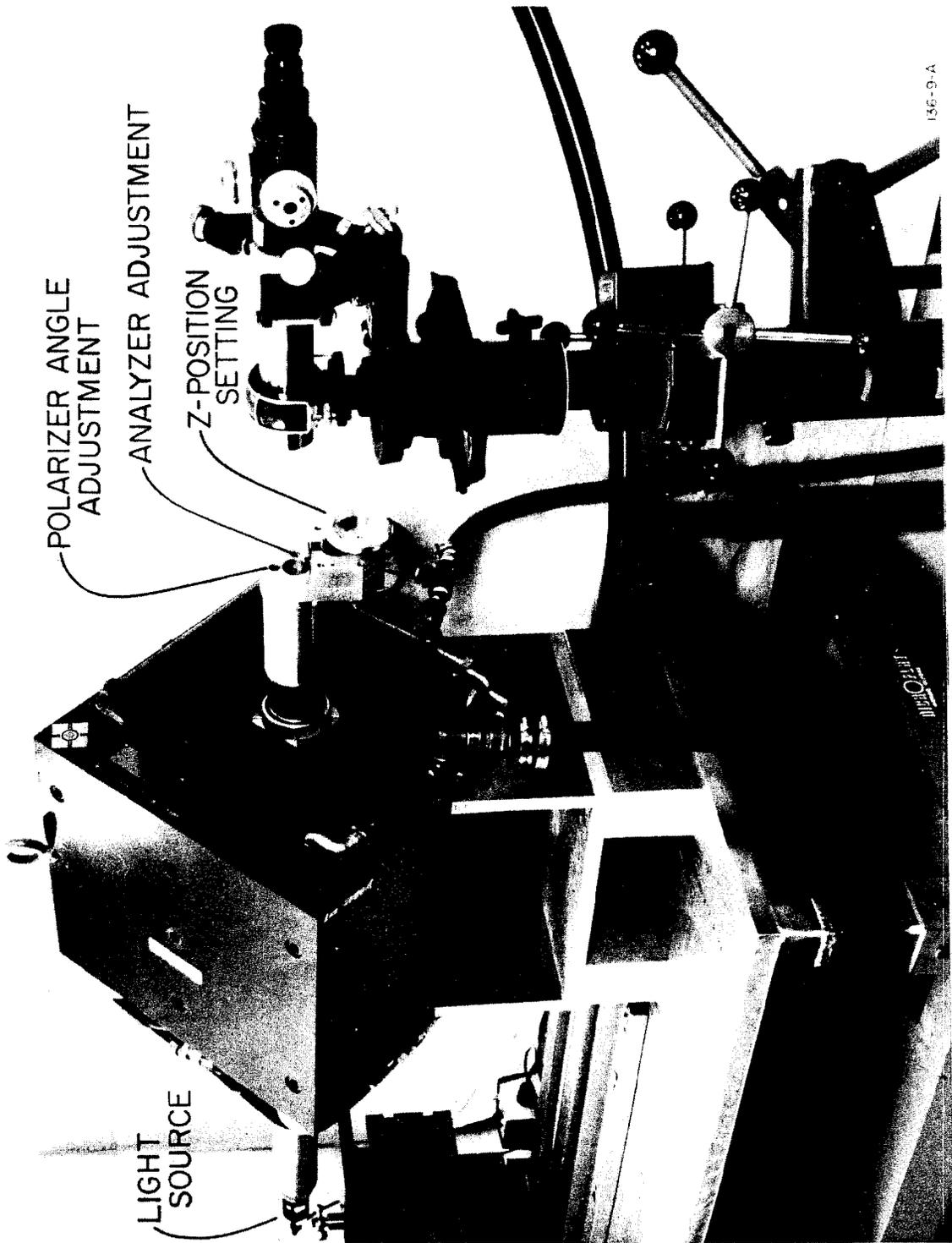


Figure 9