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ASYMMETRIC μ-PAIR PHOTOPRODUCTION AT SMALL ANGLES*

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Photoproduction of μ-pairs has been measured recently[1] with the aim of further probing the theory of quantum electrodynamics as applied to muons. A symmetric experimental arrangement as used in these measurements offers two advantages:[2]

1) In both Bethe-Heitler diagrams (1a) and (1b) the virtual muon is equally removed from its mass shell by a space-like amount $(k - p_{\pm})^2 - m_\mu^2 \approx -kE\theta^2$ corresponding to $\approx -(450 \text{ Mev})^2$ for the extreme energies and angles of the recently reported observations. These diagrams can be computed to order Ze^3 and expressed in terms of measurable structure factors for scattering, elastic or total inelastic, from nucleons or nuclei.[3]

2) Interference of the Bethe-Heitler with the virtual Compton diagrams (1c) vanishes identically for the symmetric arrangement and the squares of the Compton amplitudes are estimated to be negligibly small for this condition in which momentum transfer to the nucleus is very low.

In this letter an extremely assymmetric kinematic condition for photoproducing μ-pairs is discussed for probing electrodynamics, as well as, possibly for studying the photoproduction of vector resonances. This arrangement is designed to be of maximum advantage to electron linacs which provide beams of electrons and photons with very high currents but in short pulses that hamper coincidence relative to singles counting experiments.

We consider photoproduction of μ-pairs with all of the energy concentrated on one member of the pair which is detected while the other one emerges almost at rest - i.e. with nonrelativistic energy. We have in mind specifically the photoproduction from hydrogen, detecting the μ^- with an energy E_- within ≈ 40 Mev of the maximum possible energy. Since photoproduction of only a single π^- from hydrogen is excluded by charge conservation this region is not kinematically available to a decay μ^- from a π^- which at lower energies swamps the events of interest.

In the limit of low momentum p_+ for the undetected μ^+, — i.e., to leading order in $p_+/m_\mu \to 0$ — and in the high energy approximation, $\frac{m_\mu}{k} \to 0$, the Bethe-Heitler formula becomes

$$\frac{d\sigma}{dE_- d\Omega_-} = \frac{\alpha^3}{2\pi} \frac{p_+ k}{m_\mu^5} \left\{ \frac{\theta_-^2 k^2/m_\mu^2}{\left[\frac{1}{2} + \frac{1}{2}(\theta_-^2 k^2/m_\mu^2)\right]^3} \right\} \tag{1}$$

This cross section is approximately equal to $\sigma \approx \frac{10^{-34}}{k(\text{BeV})}$ cm^2 for μ^-'s emerging with an energy $0 < k - m - E_- < 40$ MeV into a solid angle of $\sim\frac{15}{k^2(\text{BeV})}$ millisterad about the peak angle of the angular distribution at $\theta_- = .7 \ (m_\mu/k) = \frac{1}{14k_{\text{BeV}}}$.
The corresponding counting rate presents a possible if difficult experiment for the Stanford Linacs at 1 BeV (Mark III) and at 10 BeV (SLAC).

We comment on several interesting features of (1) and corrections to it:

a) The two Bethe-Heitler diagrams (1a) and (1b) must be added together to give a gauge invariant amplitude and any statement about the amplitude corresponding to an individual graph depends on choice of the gauge. However, it is useful in assessing the value of an experiment such as proposed here to probe electrodynamics to determine what momentum transfers, or virtual muon propagator masses, are contributing. In the laboratory system of reference and in transverse gauge for the incident photon both diagrams contribute comparably. In (1a) the intermediate μ propagator is removed from the mass shell by $(k - p_+)^2 - m_\mu^2 \approx -2km_\mu$ while in (1b) it is much closer to the mass shell with

$$(k - p_-)^2 - m_\mu^2 = -m_\mu^2 \left(1 + \theta_-^2 \frac{k^2}{m_\mu^2}\right)$$

However the virtual Coulomb scattering from the proton in (1a) occurs at an energy of $\sim k$ whereas in (1b) it is at the reduced energy of $\sim m_\mu$. Since the momentum transfer to the proton is the same in both diagrams and, for a fixed momentum transfer, Coulomb scattering increases as k, the two factors of k in the numerator and denominator compensate each other leading thereby to comparable amplitudes.[4] We conclude from this that measurement of (1) is sensitive to possible electrodynamics modifications for muons removed from the mass shell by $\sim -2km_\mu$. For $k = 1$ Bev this corresponds to a mass of $-(450\ \text{Mev})^2$ as achieved in recent coincidence experiments[1] at 5 Bev, and at $k = 10$ Bev, a probing to masses of $-2km_\mu = -(1.4\ \text{Bev})^2$ will be possible.[5]

b) The virtual Compton terms interfere with the Bethe-Heitler amplitude but are estimated to be negligibly small for the above conditions. Firstly only the real part of the Compton terms are in phase with and will interfere with the Bethe-Heitler ones. For Compton scattering of real photons the forward amplitude, as is relevant here, is given by the Thomson limit at threshold plus a dispersion integral over the photon absorption cross section

$$\text{Re } f_c(\omega,0^\circ) = -e^2/M + \frac{\omega^2}{2\pi^2}\, P \int_0^\infty \frac{\sigma_{abs}(\omega')\, d\omega'}{\omega'^2 - \omega^2} \qquad (2)$$

The principal value integral contributes only as a result of the energy variation of $\sigma_{abs}(\omega')$ from an observed high energy approximately constant value of 50 μb. Inserting its threshold variation and observed resonances we estimate it to add to the Thomson amplitude an increment of less than 30%.

Using then simply the Thomson limit, $-e^2/M$, we compute the interference terms and obtain a correction factor to (1) of

$$\left\{1 - \frac{m_\mu^2}{kM}\left[1 + k^2\,\theta_-^2/m_\mu^2\right]\right\} \tag{3}$$

which comes to 1.5% at 1 BeV and 0.15% at 10 BeV for $\theta_- \sim .7\, m_\mu/k$ as considered above.

Whereas it is important for (3) to be a small correction factor to (1) in testing quantum electrodynamics, virtual Compton scattering and photoproduction of vector "heavy photon" resonances is in its own right a topic of great interest and importance. Proposals for directly measuring this amplitude as well as experimentally limiting its correction to (1) are discussed in the following. In concluding this discussion we note that three additional corrections to (1) must be included before applying it as a quantitative formula. These are the kinematic corrections due to proton recoil and due to finite velocity of the slow μ^+, Coulomb distortion of the final slow μ^+ wave function, and especially the radiative correction appropriate for the experimental resolutions.

Turning next to the virtual Compton terms we note from (1) and (3) that the Bethe-Heitler amplitude vanishes at $\theta_- = 0$ since there is no transverse current in the $p_+ \to 0$ limit. The cross section to produce the high energy μ^- as $\theta_- \to 0$ $(< m_\mu/k)$ will come from the virtual Compton process as well as from corrections to (1) due to finite momentum of the slow μ^+. These latter can be computed directly from the Bethe-Heitler formula and to leading order in p_+ correct (1) by the factor

$$\left\{1 - \frac{1}{6}\,p_+^2/m_\mu^2 - \frac{1}{3}\,\frac{p_+^2/m_\mu^2}{\left[\frac{1}{2} + \frac{1}{2}\,k^2\,\theta_-^2/m_\mu^2\right]} - \frac{p_+^2/m_\mu^2}{\left[\frac{1}{2} + \frac{1}{2}\,k^2\,\theta_-^2/m_\mu^2\right]^2} + \frac{2}{3}\,\frac{p_+^2}{k^2\,\theta_-^2}\right\} \tag{4}$$

Contributions from the virtual Compton terms (1c) can be estimated from the recent theoretical discussions[6] and measurements.[7] Although reliable quantitative predictions are not possible it is suggested that at incident photon energies appropriate to vector resonance photoproduction these contributions will be enhanced and observable. The kinematics for such events are as follows: We detect a μ^- at $\theta \to 0^\circ$ with essentially all of the incident photon energy - i.e. $k - E_- < \mu_\pi$. This means that the virtual intermediate photon of energy = k has "mass" $\approx \sqrt{2km_\mu}$ which is to equal that of a vector resonance, m_V, such as ρ^0, ω, or φ for example; i.e. $m_V^2 = 2km_\mu$. At this energy the resonance, moving in the forward direction, decays to a μ_+ which is at rest if emitted in the backward direction, and all the energy appears on the μ_- as detected.

Two particular mechanisms as computed earlier[6] give an idea of the magnitudes anticipated for the virtual Compton terms to produce μ pairs via vector resonance channels. The one pion exchange production of an ω-meson which decays subsequently to a μ pair as in Fig. 2a with the μ^- emerging in the forward direction and the μ^+ emerging with non relativistic energies $p_+ < m_\mu$ is

$$\left(\frac{d\sigma}{dE_- d\Omega_-}\right)_{\substack{\theta_- \to 0 \\ p_- \to k \\ p_+/m_\mu \to 0}} = \frac{3}{4k} \left(\frac{\Gamma_{\omega\to\pi\gamma}}{\Gamma_\omega}\right) \left(\frac{g^2_{\pi N}}{4\pi}\right) \frac{\alpha^2}{\gamma_\omega^{\ 2}} \left(\frac{m_\omega^{\ 2}}{m_\mu^{\ 2}\, M^2}\right) \left(\frac{(1 - \mu_\pi^{\ 2}/m_\omega^{\ 2})^3}{(1 + \mu_\pi^{\ 2}/m_\omega^{\ 2})^2}\right) \cdot \tan^{-1} \left\{ \frac{4k \sqrt{(k-E_-)^2 - m_\mu^{\ 2}}/\Gamma_\omega\, m_\omega}{1 + \left[(2m_\mu k - m_\omega^{\ 2})^2/\Gamma_\omega^{\ 2} m_\omega^{\ 2}\right]} \right\} \tag{5}$$

where m_μ, μ_π, m_ω, and M are the μ, π, ω, and nucleon masses respectively, $g^2_{\pi N}/4\pi = 14$ is the π-nucleon coupling constant, $\alpha = 1/137$, γ_ω characterizes the reciprocal strength of the electromagnetic interaction of a neutral meson as introduced by Gell-Mann, Sharp, and Wagner[8] and used in Ref. 6, $\Gamma_\omega \sim 10$ Mev is the total decay width of the ω meson and $\Gamma_{\omega \to \pi\gamma} \sim 1$ Mev is its approximate radiative decay partial width.[9] At the resonance condition for the ω-meson on its mass shell $k = m_\omega^2/2m_\mu$ and (5) reduces to

$$\left(\frac{d\sigma}{dE_- d\Omega_-}\right)_{\substack{\theta_- \to 0 \\ \text{Resonance}}} \approx \frac{3\pi}{4} \left(\frac{\Gamma_{\omega\to\pi\gamma}}{\Gamma_\omega}\right) \left(\frac{g_{\pi N}^2}{4\pi}\right) \frac{\alpha^2}{\gamma_\omega^2} \frac{1}{m_\mu M^2} \frac{1}{(1 + \mu_\pi^2/m_\omega^2)^2} \quad (6)$$

To determine under what conditions it may be possible to detect this process we compare the cross section (6) integrated over an energy interval $m_\mu < E_- - k < m_\mu(1+\epsilon)$ about the resonance, and over a solid angle $d\Omega_- = 4\pi\left(m_\mu^4/m_\omega^4\right)f$ about the forward direction, where ϵ, $f < 1$, with the corresponding result from (1) for the Bethe-Heitler process. Their ratio is given by

$$R = \frac{\text{Resonance}}{\text{Bethe-Heitler}} = \left(\frac{\Gamma_{\omega\pi\gamma}}{\Gamma_\omega}\right) \frac{0.4}{\gamma_\omega^2} \frac{(1 + f)^2}{f\sqrt{\epsilon}} \left(\frac{m_\omega^2}{M^2}\right) \quad (7)$$

Whether or not R can be actually detected depends on precisely how far one can push the limits of energy and angular resolution in present spectrometer designs for SLAC and on the actual machine intensities that are achieved. The situation for the ω, with $\Gamma_{\omega\pi\gamma}/\Gamma_\omega \sim 0.1$, and $\gamma_\omega \sim 1 - 2$, is at best difficult. The same is true if the ω is replaced by a ρ^0. Recent CEA measurements of the ρ^0 photoproduction cross section[7] at ~2 - 3 Bev show a predominance of low momentum transfer events and a cross section $\sigma_{\gamma\to\rho} \sim 10$ μb. If interpreted in terms of a one pion exchange amplitude, this is consistent with $\Gamma_{\rho\to\pi\gamma} \sim \frac{1}{2} - 1$ Mev $\sim \Gamma_{\omega\to\pi\gamma}$. However, $\Gamma_{\rho\to\pi\pi} \sim 100$ Mev and the smaller ratio $\Gamma_{\rho\to\pi\gamma}/\Gamma_{\rho\to\pi\pi} \sim 1\%$ decreases the ratio (7) for comparable $\gamma_\omega \sim \gamma_\rho$.

There are, in addition to one pion exchange, the diffraction production mechanism (Fig. 2b) as well as pion exchange with production of nucleon isobars. These may be expected[6] to play an important role in the 10 BeV energy region and above. Beyond the ω photoproduction, photoproduction of additional (new) vector mesons of narrow decay widths will contribute and may be studied by running through a range of photon energies k and detecting μ^- mesons with energy $E_- > k - m_\mu - 40$ MeV in the forward direction. The angular interval must be restricted to $\theta_- < m_\mu/k$ in order to avoid comparable or larger Bethe-Heitler contributions although the angular dependence of the virtual Compton process is flat throughout the forward cone of half width $\theta_- \sim m_\mu/k$. Under these kinematic conditions the "large" value of the muon rest mass keeps the "intermediate photon" removed from the mass shell by $\sim 2km_\mu$ which corresponds to known vector resonances for $k \sim 3$ BeV. What other resonances as well as radiative decay widths it may be possible to probe in this way will be interesting to learn.

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4. In the transverse gauge the virtual Coulomb scattering by the muon behaves here in a very similar way to real Coulomb scattering and these results obtain.

5. For comparison with the most accurate g-2 measurements of the muon, we comment that one can regulate the muon propagator according to

$$S_F(k) \rightarrow \frac{1}{\not{k}-m} \left\{ 1 - \left(\frac{k^2 - m^2}{\Lambda^2} \right) \right\}$$

   Ward's identity is then preserved by a minimal vertex replacement

$$\bar{u}(p)\, \gamma_\mu \rightarrow \bar{u}(p)\, \gamma_\mu \left\{ \frac{\Lambda^2}{\Lambda^2 - (p^2 - m^2)} \right\}$$

   Choice of $\Lambda \sim 3$ BeV decreases the Bethe-Heitler cross section by 5% and likewise changes the calculated g-2 value of the muon by ~5 parts per million from the Schwinger value α/π, which is the limit of present experimental accuracy.

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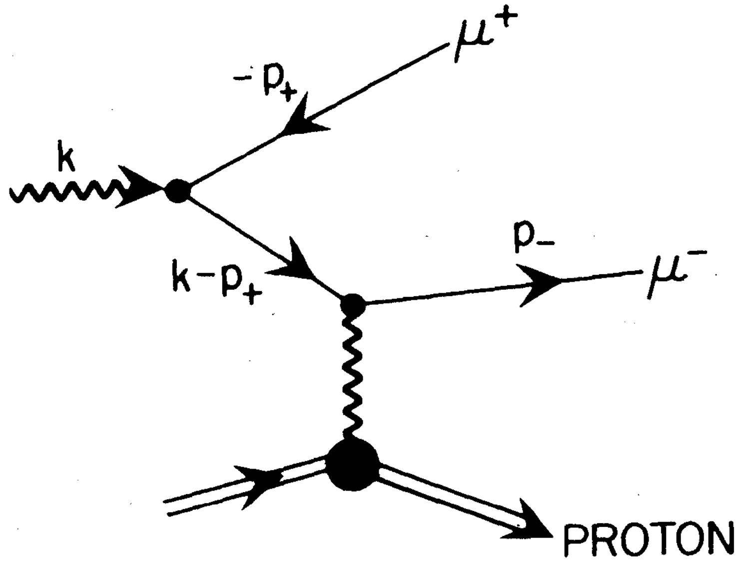
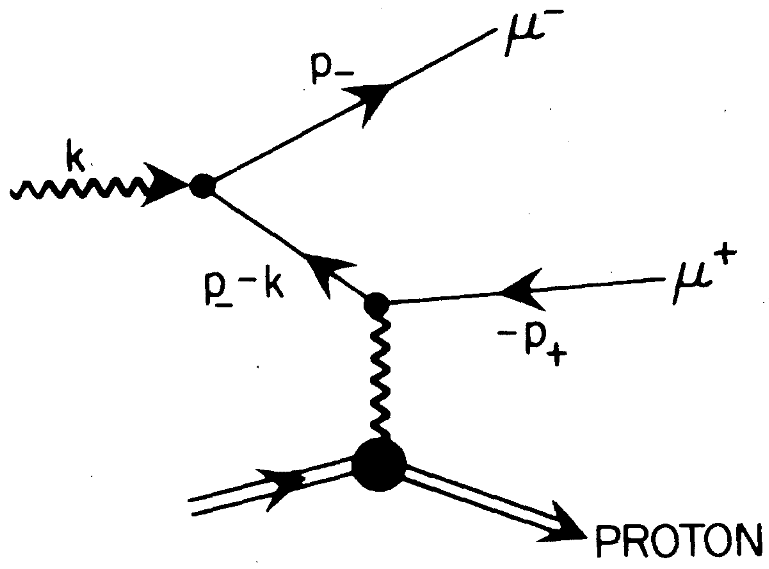
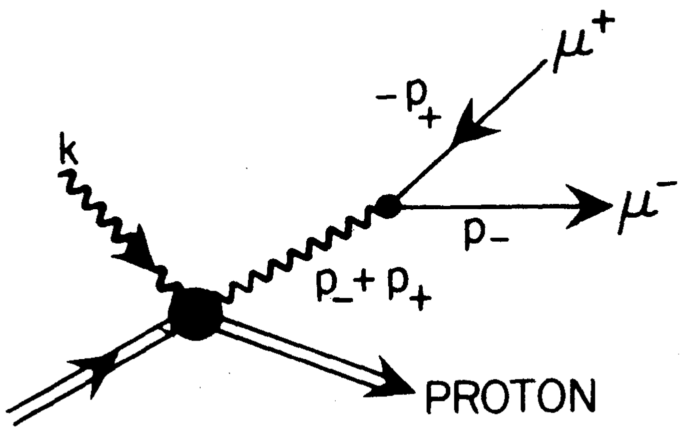


(a)



(b)



(c)

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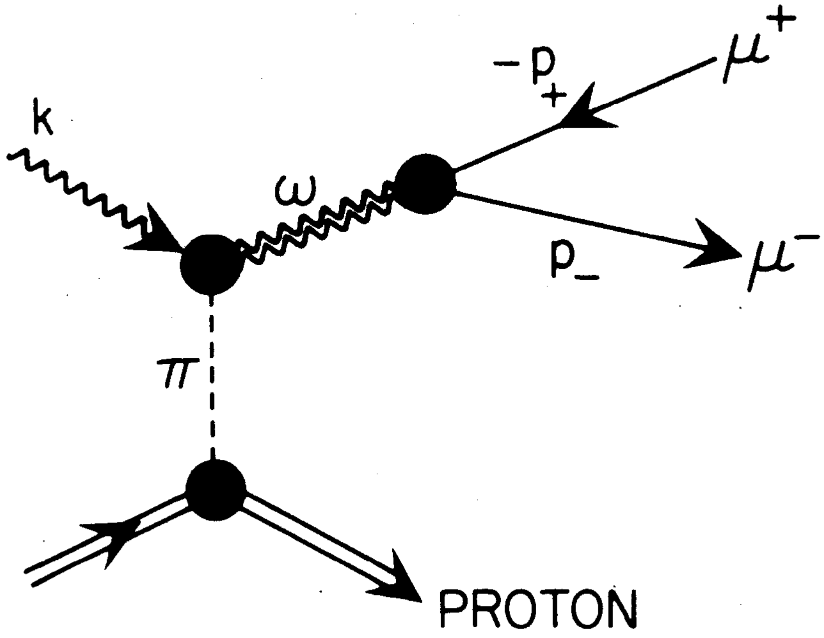
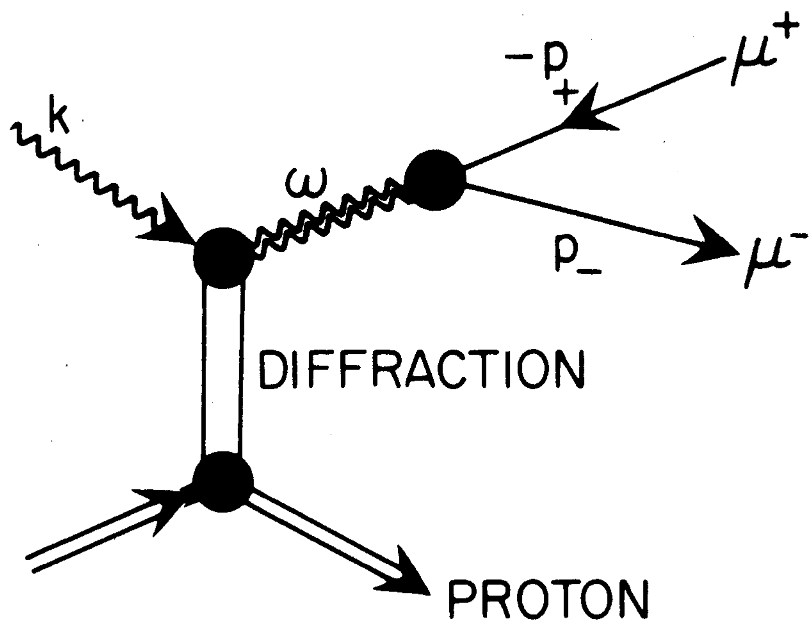
Fig 1



(a)



(b)

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Fig 2