

OFF-SHELL CORRECTION IN PION PHOTOPRODUCTION

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In this note we present some preliminary results of analyzing pion photoproduction data^{1,2} by taking into account the virtuality of the exchanged pion in the Drell model.^{3,4}

Ferrari and Selleri⁵ derived an approximate expression for the off-shell pion-nucleon ρ, ρ scattering amplitude containing an unknown pionic form factor $K(\Delta^2)$ depending only on Δ^2 , the square of the four-momentum of the virtual pion. They applied their result to an analysis,⁶ on the basis of the one-pion exchange (OPE) mechanism, of single pion production data from nucleon-nucleon collisions. It is shown that in the calculation of the amplitude for this process there occurs the function

$$\Phi(\Delta^2) = K^2(\Delta^2)K'(\Delta^2)\psi(\Delta^2) \quad (1)$$

where $K(\Delta^2)$ appears twice since it is associated with each pion-nucleon vertex, $K'(\Delta^2)$ contains all the higher order corrections to the pion propagator, and $\psi(\Delta^2)$ is a known function depending on the parameters of the ρ, ρ resonance. The moderate success they met in substantiating the OPE by fitting this data with an empirical function $\Phi(\Delta^2)$ suggests the importance of a similar calculation in the case of photoproduction.

Drell's expression³ for photoproduction of negative pions from a heavy target nucleus A is

$$\frac{d^2\sigma}{dpd\Omega} = \frac{\alpha}{8\pi^2} \left(\frac{\sin \theta}{1 - \beta \cos \theta} \right)^2 \frac{p(k - \omega)}{k^3} \sigma_A(T) \quad (2)$$

where the photon has energy k , the pion of mass μ is observed in solid angle $d\Omega$ about θ and in momentum interval dp about p . The corresponding energy is ω , the velocity is β , and total π^-A cross section

at kinetic energy $T = k - \omega - \mu$ is $\sigma_A(T)$. In Drell's terminology, the photon produces a π^+, π^- pair in the Coulomb field of the target A , the π^- is observed, while the virtual π^+ interacts strongly with the target, and initiates any undetected final states.

In case A is a proton, we assume that the π^+, p interaction occurs predominantly in the $3,3$ state so that the Ferrari-Selleri result may be applied unambiguously. If A is a complex nucleus, the situation is somewhat ambiguous, but in what follows we assume that the strong interaction within A is purely quasi-elastic (i.e., a single nucleon, treated as an unbound particle, participates in the interaction) and also occurs in the $3,3$ state. In this way, both cases are treated in a unified manner, although admittedly we may be drastically oversimplifying the treatment of complex nuclei.

The finite mass of the nucleon and, following Ferrari and Selleri,^{5,7} the off-shell nature of the exchanged pion can thus be incorporated in Eq. (2), in case A is a proton, by replacing $\sigma_A(T)$ by $\sigma_{33}(T, \Delta^2)$, where

$$\sigma_{33}(T, \Delta^2) = \sigma_{33}(T, \mu^2) c(T, \Delta^2) \quad (3)$$

$\sigma_{33}(T, \mu^2)$ is the on-shell $3,3$ pion-nucleon cross section, and

$$c(T, \Delta^2) \equiv \frac{q_L}{k - \omega} \left(\frac{p}{\omega} \right)^3 \left[1 + \frac{\mu^2 - \Delta^2}{4m^2} \right] \left[\frac{1 + 3\alpha_r}{(1 + \alpha_r)^3} \right]^2 \frac{q_i}{q_f} K^2(\Delta^2) \quad (4)$$

In Eq. (4), m = nucleon mass, $\alpha_r = (\mu^2 - \Delta^2)/2m(E_r - m)$, E_r is the $3,3$ resonance energy (1.238 BeV), q_L is the virtual pion momentum in the laboratory system, and q_i and q_f are, respectively, the initial and final momenta in the barycentric system of the virtual pion and target nucleon.

Also T must be redefined in (2), (3), and (4) to be the kinetic energy of the real pion emerging from the strong interaction in the rest frame of the recoil nucleon.

Since higher order corrections to the pion propagator and the $\gamma \rightarrow \pi^+, \pi^-$ vertex precisely cancel⁸ by Ward's identity, only $K^2(\Delta^2)$ appears in (4). It is interesting to observe that knowledge of $K^2(\Delta^2)$ from analysis of photoproduction data could, upon comparison with the empirical function $\Phi(\Delta^2)$ in (1), lead to determination of the higher order corrections $K'(\Delta^2)$ to the pion propagator. Such a determination seems unwarranted at this preliminary stage of our analysis since it would be undesirable to depend heavily on photoproduction data from a complex nucleus as we are doing in this note.

For complex nuclei, a plausible (but speculative) way to make the extensions corresponding to Eq. (3) is to replace $\sigma_A(T)$ by

$$\sigma_A(T, \Delta^2) = \sigma_A(T, \mu^2) C(T, \Delta^2) \quad (5)$$

where $\sigma_A(T, \mu^2)$ is that part of the pion-nucleus cross section attributed to quasi-elastic scattering in the 3,3 state.⁹

Equations (3) and (5) were inserted in (2) in our analysis for values of T below the 3,3 resonance. Above the resonance the cross sections on the right hand side of (3) and (5) were replaced by total cross sections since the off-shell corrections for the other states now contributing substantially to the cross sections are hopefully not considerably different from Eq. (4). This procedure is sensible for experiments where T is

restricted to the region of the $3,3$ resonance. We should point out, however, that in the experiments considered here the range of T is $\approx 0.0 - 1.0$ BeV where other states are indeed important.

Since $q_i > q_F$, the factor (q_i/q_F) in Eq. (4) when considered alone causes a possibly significant enhancement of the cross section. If we fold in a bremsstrahlung spectrum $\phi(k)dk$, the region of integration may include $q_F = 0$, where $q_i \neq 0$. A singularity is prevented in (3), as pointed out by Ferrari and Selleri,⁶ since $\sigma_{33}(T, \mu^2)$ vanishes as q_F^4 .

It is possible to unfold approximately the integration over $\phi(k)$. We define

$$N(p, \theta) \equiv \int dk \phi(k) C_1(T, \Delta^2) d^2\sigma/dpd\Omega \quad (6)$$

where the subscript 1 means that we have set $K^2(\Delta^2) = 1$ in Eq. (4), and we define $E(p, \theta)$ to be the corresponding experimental value. Then $K^2(\Delta_0^2) = E(p, \theta)/N(p, \theta)$ where Δ_0^2 , the average value of Δ^2 in the region of integration, is given by

$$\Delta_0^2 = [1/N(p, \theta)] \int dk \phi(k) C_1(T, \Delta^2) \Delta^2 d^2\sigma/dpd\Omega \quad (7)$$

This procedure correctly establishes a value of $K^2(\Delta^2)$ if $K^2(\Delta^2)$ is linear in Δ^2 over the region of integration, a justifiable assumption since the range of values of Δ^2 in the integral is small for the experiments considered here.

Figure 1 shows values of $K^2(\Delta^2)$ determined in this manner using data from pion production in hydrogen and beryllium. The curves in Fig. 1 are

crude fits to the data of the form

$$K^2(\Delta^2) = 1/[1 + (\mu^2 - \Delta^2)/a^2]^2 \quad (8)$$

with $a = 6\mu$, 8μ , and 12μ .

Figure 2 illustrates the importance of including the term $C(T, \Delta^2)$ in the calculation. The Drell angular distribution is plotted for photoproduction in beryllium from a photon spectrum with maximum $k = 5.82$ BeV and $p = 5.0$ BeV/c. Four cases are considered:

- (a) $C(T, \Delta^2)$ given by Eq. (4) is used in (5) with $K^2(\Delta^2)$ given by (8) and $a = 8\mu$,
- (b) Same as (a) but with $K^2(\Delta^2) = 1$,
- (c) $C(T, \Delta^2)$ is replaced by 1,
- (d) Same as (c) but with $m = \infty$ (static approximation).

We list some conclusions, evident from the figures:

- (i) Although there is quite a scattering of values of $K^2(\Delta^2)$ over the range of Δ^2 , there is a general trend for $K^2(\Delta^2)$ to decrease with $|\Delta^2|$ as in the corresponding analysis of Ref. 2. The main difference is that our analysis has pulled down $K^2(\Delta^2)$ by a factor of almost 2 for the larger momentum transfers.
- (ii) There is no clear separation in Fig. 1 between the data for production in hydrogen (reference 1) and that for production in beryllium (reference 2). This suggests that the quasi-elastic treatment of complex nuclei is perhaps justified.
- (iii) The data are not grossly incompatible with Eq. (8) for $|\Delta^2| > 3\mu^2$.

- (iv) The OPE represents only a small part of the physics going on for $|\Delta^2| < 3\mu^2$.
- (v) The maximum possible effect of the off-shell correction is quite significant since curve (b) of Fig. 2 is about 2.5 times larger than the uncorrected curve (c) over a substantial part of the phase space of the observed pion. Also note that the static curve (d) deviates sharply from the other curves at sufficiently large angles.

It should be emphasized that these conclusions are for the most part tentative, since it is possible that they depend crucially on the assumptions stated in this note. Further investigations of these assumptions and analysis of more recent pion photoproduction data are in preparation and will be contained in a forthcoming paper.

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FIGURE CAPTIONS

1. Pionic form factor squared vs mass squared of virtual pion. Circles indicate experimental points from reference 2, triangles from reference 1. A few data from reference 1 are omitted for clarity. The curves are plots of Eq. (8). Values of a are indicated above the curves.
2. Plot of photoproduction cross section per equivalent quanta for observing a $5.0 \text{ BeV}/c \pi^-$ produced in Be by bremsstrahlung of peak energy 5.82 BeV . See text for explanation of the curves. The experimental points are from reference 2.

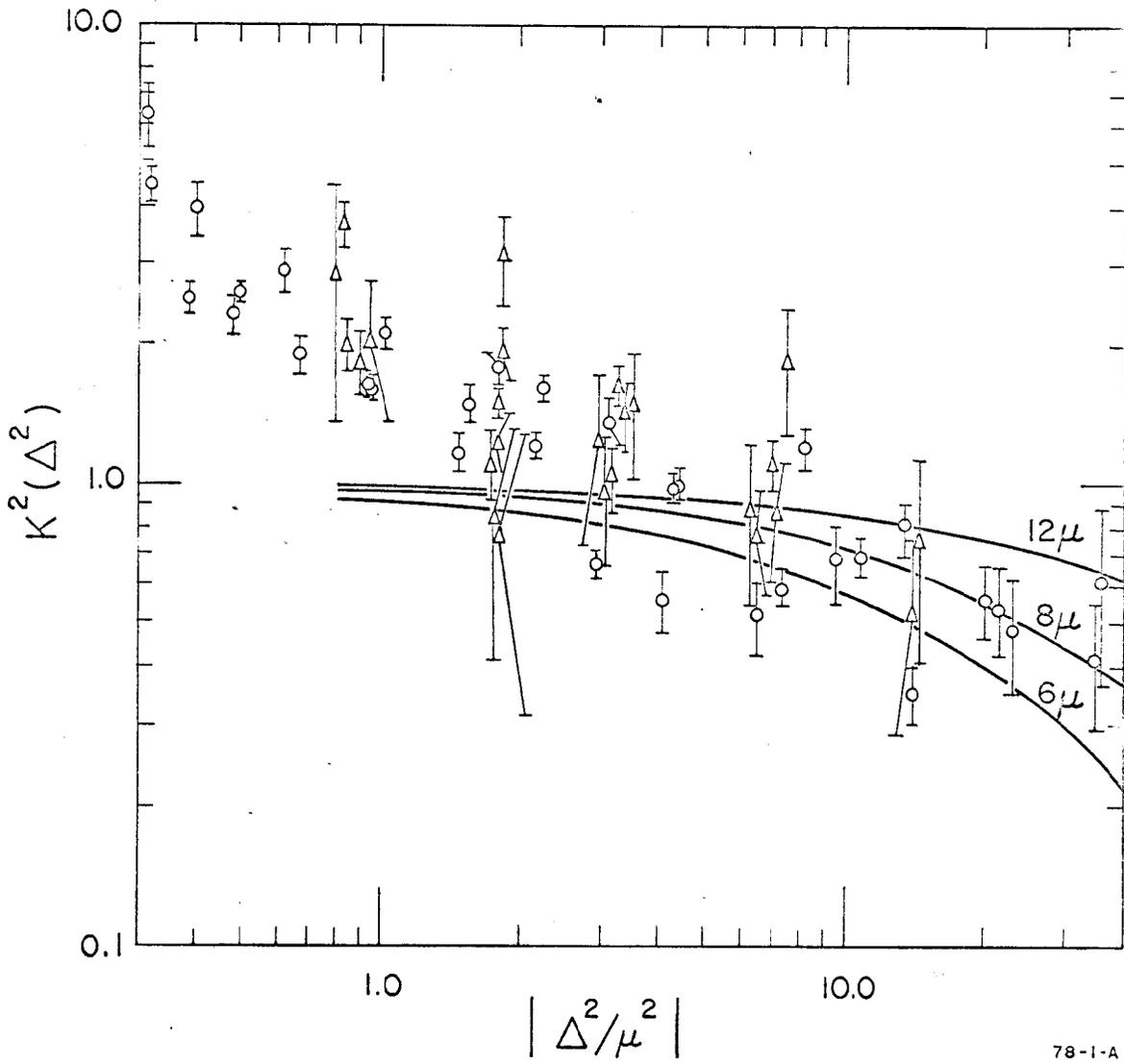


Fig 1

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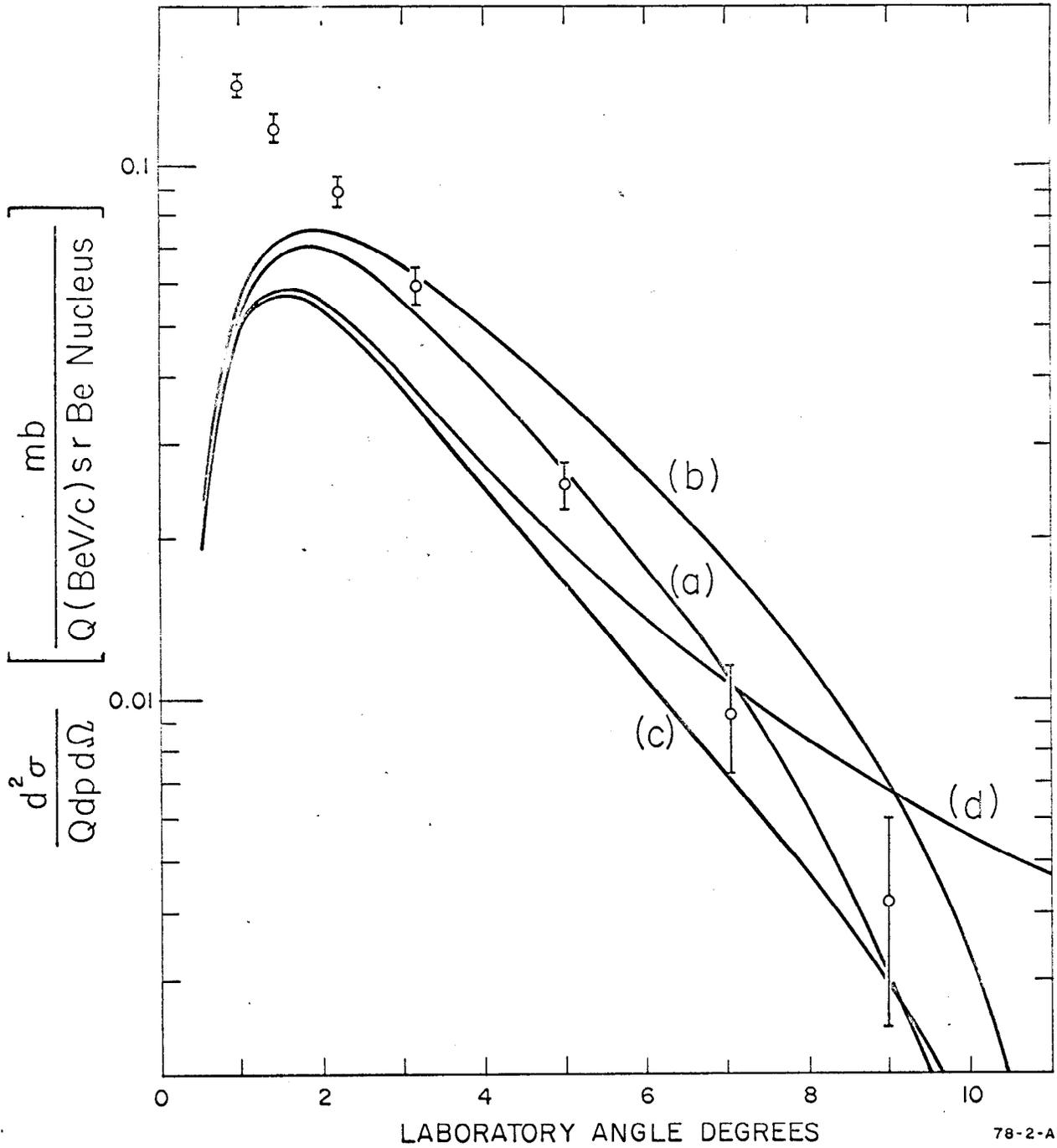


Fig 2