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MESON EXCHANGE EFFECTS IN ELASTIC e-D SCATTERING*

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This letter reports a calculation of an effect of the 3 pion exchange current on the electromagnetic interaction of the deuteron.

Since the deuteron has isotopic spin $I = 0$, only the isotopic scalar part of the electromagnetic current contributes to elastic scattering. In the language of dispersion theory, this selection rule removes all even pion states, and in particular the simple pion exchange current illustrated by Fig. 1a. The least massive state is then that with 3 pions. Its contribution, via the 3 pion resonance (the ω or φ), as in Fig. 1b, has been studied extensively, most recently by Jones[1] and Gourdin.[2] In addition it gives rise to an exchange current contribution which we study here in the approximation that the 3π state may be approximated by a two particle (ρ,π) system, with the ρ and π landing on different nucleons and thus constituting an exchange current as illustrated in Fig. 1c.

In reporting this calculation we wish especially to emphasize the importance of a measurement of the $\rho\pi\gamma$ coupling strength. On the basis of a polology interpretation of the cross section for ρ photoproduction in terms of one-pion exchange this coupling strength has already been estimated, although rather crudely.[3] Using this estimate we have calculated the contribution of Fig. 1c to the deuteron magnetic moment, obtaining $\sim \pm(1\text{-}2) \times 10^{-2}$ nuclear magnetons. This result is proportional to the $\rho\pi\gamma$ and ρ-N coupling strengths[4] and is comparable in magnitude with the existing discrepancy of $+1.7 \times 10^{-2}$ nuclear magnetons between the observed moment $\mu_D = 0.857$ nm and the value calculated using a 7% D-state probability for the deuteron as indicated by other experiments[5]:

$$\left(\mu_D\right)_{th} = \left(\mu_n + \mu_p\right) - \frac{3}{2} P_D \left(\mu_p + \mu_n - \frac{1}{2}\right) = 0.840 \quad \text{with} \quad P_D = 0.07 \quad (1)$$

To reconcile Eq. (1) with experiment in the absence of an exchange current requires $P_D = 0.039$, i.e., the small difference between 0.857 nm and 0.840 nm in μ_D corresponds to a large difference, 0.039 versus 0.07, in P_D. The dominant contribution to the theoretical value for the magnetic moment comes from the cross term between the S-state and D-state amplitudes and is relatively insensitive to details of the particular deuteron model for a fixed D-wave percentage if one is reasonably discriminating in choosing the wave function.[6,7]

The suggestion made here that the $\rho\pi\gamma$ exchange current may help resolve this long standing mystery between the measured magnetic moment and the D-wave probability of the deuteron can be confirmed or defeated by a measurement of the coupling constant $g_{\rho\pi\gamma}$ in the gauge invariant phenomenological interaction

$$H_{int} = \frac{1}{2} g_{\rho\pi\gamma} \epsilon_{\alpha\beta\sigma\tau} F^{\alpha\beta} \underset{\sim}{\rho}^{\sigma} \cdot \underset{\sim}{\pi}^{\tau}: \qquad \pi^{\tau} \equiv \frac{\partial\pi}{\partial x_{\tau}} \tag{2}$$

where $F^{\alpha\beta}$ is the electromagnetic field of the photon, $\underset{\sim}{\rho}$ the ρ-meson amplitude, and $\underset{\sim}{\pi}$ the pion amplitude. For simplicity we have formed an isotopic scalar product of the ρ and π isotopic vectors in Eq. (2) though in general the neutral and charged fields need not experience the same couplings. Otherwise Eq. (2) is unique up to a momentum dependent form factor.

The above calculation of $\Delta\mu_D = \pm(1\text{-}2) \times 10^{-2}$ nm was based on a value of $g^2_{\rho\pi\gamma}/4\pi \sim 0.02$ corresponding to a $\rho \to \pi + \gamma$ decay width of 1/2 MeV as estimated in Ref. (3). Crude application of universal coupling arguments leads to a decay width and coupling constant smaller by as much as an order of magnitude, i. e., $g^2_{\rho\pi\gamma}/4\pi \sim 2 \times 10^{-3}$. This reduces the resulting contribution to the exchange moment to $\sim(3\text{-}6) \times 10^{-3}$ nm which is too small to count. Small values for the $\rho\pi\gamma$ interaction are also suggested by the Bronzan-Low[8] selection rule. If, however, the larger value used here is confirmed by direct measurement we will have a natural and simple mechanism for reconciling the deuteron magnetic moment with a 7% D-wave probability. What is urgently needed is a direct accurate measurement of $g_{\rho\pi\gamma}$ in Eq. (2) by coherent electromagnetic excitation of the π to a ρ as suggested in Ref. (3) or by a peripheral analysis of low momentum transfer events in photo-ρ production.

This interaction also gives a contribution to the deuteron electric quadrupole moment, Q; with the above parameters it changes the value of Q by no more than $\frac{1}{2}$% which is within experimental uncertainties and not at present very interesting.[7] Similarly the contribution to the electric form factor for $q^2 \neq 0$ as presently studied in elastic e-D scattering is not large enough to be interesting, but for larger q^2 both the electric and quadrupole form factor corrections may become more important. Lastly we find that our correction to the magnetic form factor provides improved agreement with the recent measurements of Goldemberg and Schaerf[9] for elastic e-D scattering at backward angles and momentum transfers of $q^2 = 0.26$ and 0.41 $(\text{fermi})^2$, and is consistent with more recent results of Drickey et al.,[10] at $q^2 = 3$, 4, and 5, $(\text{fermi})^2$. (See Fig. 2.)

The calculation is straight forward and goes as follows. The exchange current contribution of Fig. 2 is computed using perturbation theory and added to the usual impulse approximation current of Fig. 1. For the $\rho\pi\gamma$

interaction we use Eq. (2), and for the ρ-N and π-N couplings we use the standard nonrelativistic limits with Pauli spinors x.

$$e\left(x_f^\dagger \Gamma^\mu x_i\right) \rho_\mu; \quad \Gamma^\mu = \left[a,\ a\frac{p_j}{2M} - b\frac{i}{2M}\epsilon^{jkl}\ell^k\sigma^\ell\right] \tag{3}$$

$$G_{N\pi}\frac{i}{2M}x_f^\dagger(\vec{p}\cdot\vec{\sigma})\,x_i$$

Here p_j is the sum of the nucleon momenta before and after interaction with the ρ meson, and ℓ is the momentum transfer at the ρ meson vertex. This ρ-N interaction is just the analogue of the isovector γ-N vertex[4] with F_1^V replaced by "a" and G_M^V by "b". The pion-nucleon vertex is standard; p represents three-momentum transfer. The final result obtained by adding contributions from Figs. 1 and 2 is an effective deuteron current given by

$$J^\mu = x_f^\dagger\left[\left(G_{ED} + \Delta G_{ED}\right) - \left(G_{QD} + \Delta G_{QD}\right)\frac{S_{12}(\hat{q})}{8}\right., \tag{4}$$

$$\left.\frac{-i}{2M_D}\epsilon^{jmn}q^m\left(\sigma_N^n + \sigma_P^n\right)\left(G_{MD} + \Delta G_{MD}\right)\right]x_i$$

where G_{ED}, G_{QD} and G_{MD} are the electric quadrupole and magnetic form factors given by Clendenning and Kramer,[11] and the "ΔG"s are corrections due to the meson exchange current. The x here represents triplet deuteron spin functions, and S_{12} is the usual tensor operator. In particular with $e^2/4\pi = 1/137 = \alpha$

$$\Delta G_{MD}(q^2) \approx - \frac{G_{N\pi}\ a\ g_{\rho\pi\gamma}}{16\ e\ m_\rho} \int \frac{d^3_\tau \left(\tau_x\right)^2}{(2\pi)^3 \left[\left(\frac{\vec{q} + \vec{\tau}}{2}\right)^2 m_\pi^2\right]\left[\left(\frac{\vec{q} - \vec{\tau}}{2}\right)^2 + m_\rho^2\right]}$$
$$\int_0^\infty \left\{ \left(u^2 - \frac{w^2}{2}\right) j_0\left(\frac{\tau r}{2}\right) - \sqrt{2}\, w \left(u + \frac{w}{\sqrt{2}}\right) j_2\left(\frac{\tau r}{2}\right) \right\} dr \tag{5}$$

which for $q = 0$ is the correction to the magnetic moment; one obtains by a contour integration

$$\Delta\mu = \Delta G_{MD}(0) = \left[\frac{-g_{\rho\pi\gamma} G_{N\pi}\ a}{16\ m_\rho e}\right] \frac{8}{3\pi\left(m_\rho^2 - m_\pi^2\right)} \int \left\{ \left(m_\rho^2 \frac{e}{r}^{-m_\rho r} - m_\pi^2 \frac{e}{r}^{-m_\pi r}\right)\left(u^2 - \frac{w^2}{2}\right)\right.$$
$$\left. + \left(m_\rho^2\left[1 + \frac{3}{m_\rho r} + \frac{3}{m_\rho^2 r^2}\right]\frac{e}{r}^{-m_\rho r} - m_\pi^2\left[1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right]\frac{e}{r}^{-m_\pi r}\right)\sqrt{2}\, w\left(u + \frac{w}{\sqrt{2}}\right)\right\} dr \tag{6}$$

The numerical value of $\Delta\mu$ has been computed for Breit wave functions and Partovi wave functions.[12] It turns out that the 2nd term of Eq. (6) dominates by a factor of 10^2 and is relatively model independent. The weighting factor in front of u^2 leads to a near cancellation in the 1st term. The results are

$$\Delta\mu = 0.99 \times 10^{-2} \times (1\text{-}2) \quad \text{Breit}$$
$$\approx 10^{-2} \times (1\text{-}2)$$
$$\Delta\mu = 0.94 \times 10^{-2} \times (1\text{-}2) \quad \text{Partovi}$$

A similar process for the quadrupole moment, which we only mention here, gives a correction of only 10^{-3} (fermi)^2 versus a total value of 0.282 ± 0.001 (fermi)^2.

We have also applied our results to re-examining the data of Goldemberg and Schaerf and Drickey et al., Eq. (9) by assuming that ΔG_{MD} is roughly constant to about $q^2 \approx 5$ (fermi)^2 which is reasonable from Eq. (5). The result is that the theoretical curve shifts upward to where the experimental points of Goldemberg and Schaerf lie on it well within their experimental error.[9]

Lastly we remark that other resonances may contribute to an exchange current contribution in addition to the $\rho\pi\gamma$ one considered here. If the ρ' (or B) resonance[13] at 1220 MeV has the same quantum numbers as the ρ and couplings similar to Eq. (3) it will contribute to $\Delta\mu$ but serves to change its magnitude by only 20% because of its larger mass. Another possibility comes from an $\omega(ABC)\gamma$ exchange term, where (ABC) here stands for a scalar channel of $I = 0$. Such an interaction gives an approximate representation of a 5-pion intermediate state but we have found it to contribute negligibly to the electromagnetic amplitudes for the small values of q^2 considered here. In particular there is no change in $\Delta\mu$.

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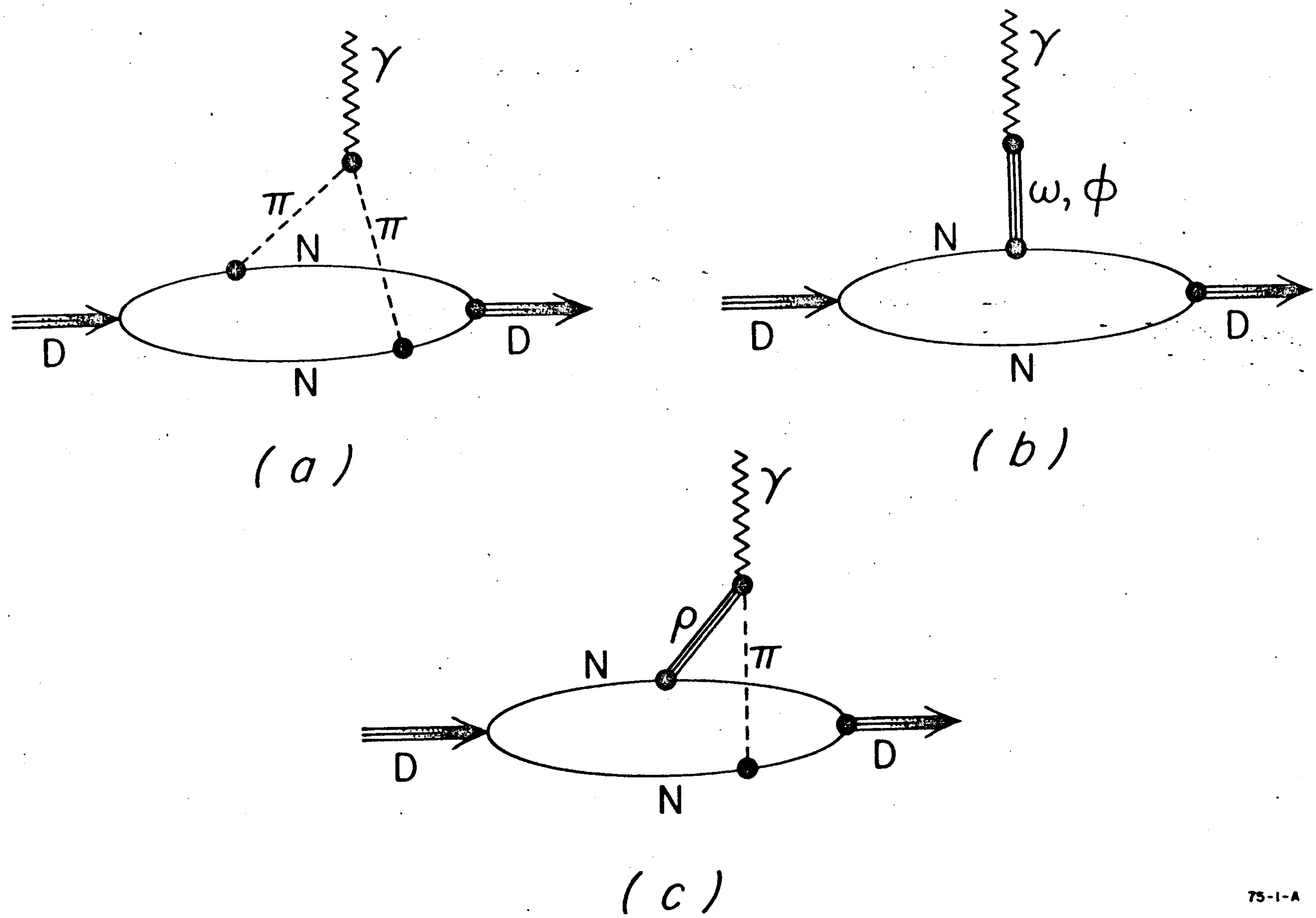
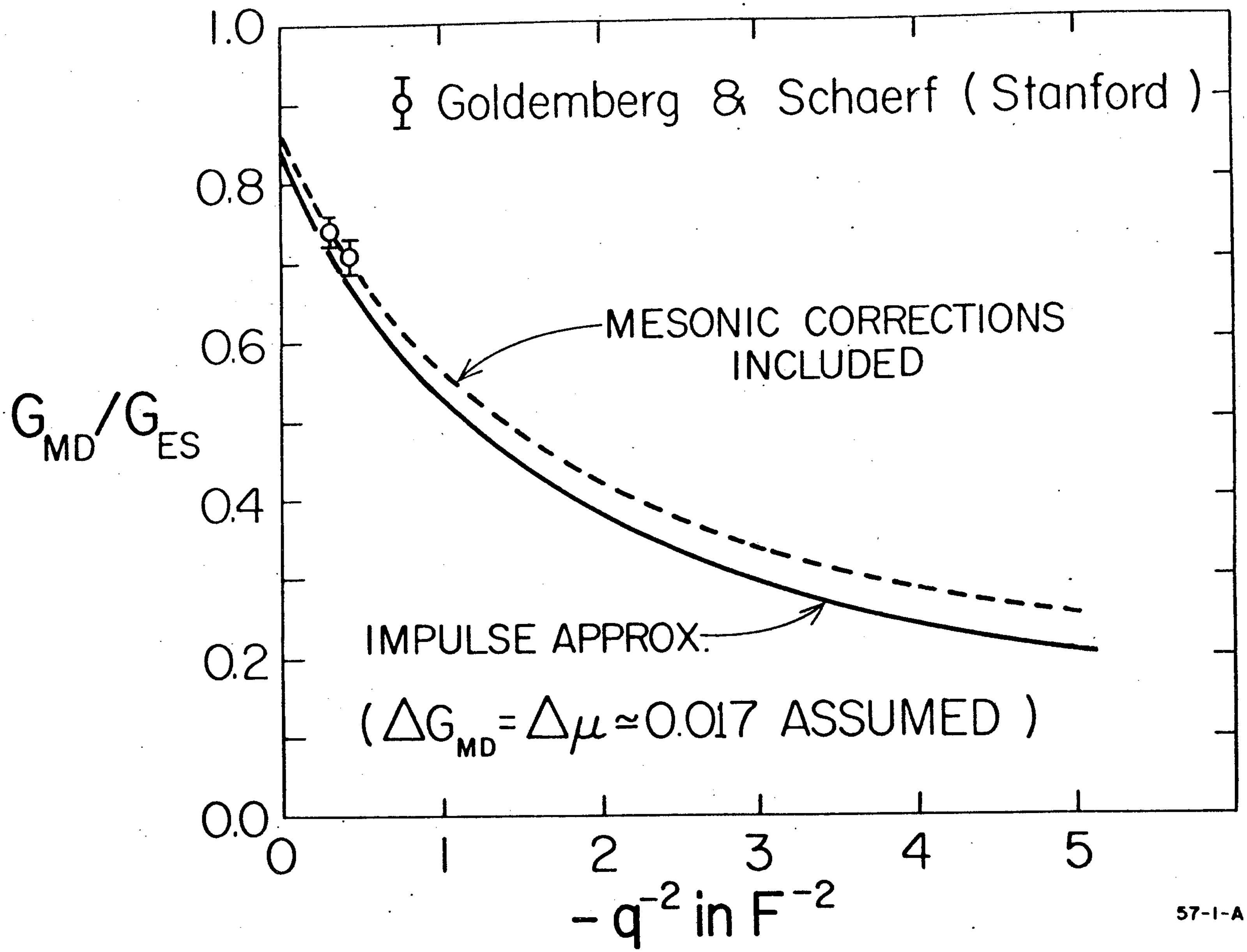


Fig 1



Goldemberg & Schaerf (Stanford)
MESONIC CORRECTIONS INCLUDED
IMPULSE APPROX.
(ΔG_MD = Δμ ≃ 0.017 ASSUMED)
G_MD/G_ES
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
-q^-2 in F^-2
57-1-A

Fig 2