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THE INTERPRETATION OF LOW ENERGY NUCLEON-NUCLEON SCATTERING

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It is shown that the observed 10-20 per cent difference between the n-p and p-p effective ranges in the $^1S_0$ state implies a corresponding difference in the effective mass of the boson system exchanged between the two nucleons which cannot be accounted for by known charge-dependent effects arising from the one-pion-exchange. Since current models for the intermediate and short range interactions in this state ascribe them primarily to the exchange of $I = 0$ boson systems, we conclude that a substantial $I \neq 0$ component of the crossed channels may have been omitted from these models.

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The value of 2.786 ± 0.014F for the ${}^1S_0$ effective range recently obtained from p-p scattering [1] disagrees with the value of 2.51 ± 0.11F determined from n-p scattering [2], indicating a departure from charge independence which is hard to understand in terms of current models. For a zero-energy wave function which is close to zero at short distances (as required by the change in sign of the phase shift near 255 MeV) and which approaches the asymptotic value rapidly at long range (as required by the relative weakness of OPE), the observed effect, naively interpreted, requires the n-p intermediate range attraction to be of shorter range and/or weaker than the corresponding p-p interaction, which is opposite to the expected coulomb effect. We present here a dispersion-theoretic model which confirms this qualitative conclusion.

The model adopted for the ${}^1S_0$ amplitude, $\exp(i\delta_0)\sin\delta_0/\mathcal{C}^2 q = 1/(S(q^2) + (-q^2)^{\frac{1}{2}}f)$ is to take $S(q^2) = A + Rq^2 + Cq^4(1-r_0q^2)/(1+Dq^2)(1-r_1q^2)$. One-pion-exchange can be approximately represented by requiring this amplitude to have a pole at $q^2 = 2m^2_{\pi^0}c^2/M_pE_{lab} = -\frac{1}{2} = -K^2$ of residue $\Gamma h = G^2m^2_{\pi^0}h/8M_p$. This is accomplished by calculating D and C from the equations

$$D = 2 - 2J/(1 - J) \quad ; \quad 1/J = 1/F^2 - K^2(r_0 - r_1)/(1 + r_0K^2)(1 + r_1K^2) \quad ;$$

$$F^2 = (A + Kf - R)/K^2(1/\Gamma h + g/2K - R) \quad ;$$

$$C = -(1 - DK^2)(A + Kf - R)(1 + r_1K^2)/K^4(1 + r_0K^2) \; .$$

Here f, g, h, and $\mathcal{C}^2$, which take account of coulomb effects, are given by Wong and Noyes [3], and are equal to unity in the n-p case. To take account of mass differences in the n-p case, we let $M_p \rightarrow 2M_nM_p/(M_n + M_p)$ and $K \rightarrow (m_{\pi^0} + 2m_{\pi^+})/3m_{\pi^0}2^{\frac{1}{2}}$; a possible charge dependence of the pion-nucleon coupling constant can be allowed for by letting G^2 differ in the two cases. If we let $r_0 = 0 = r_1$, this simple model is in good quantitative agreement with the integral equation derived in [3]; further, the shape parameter so predicted is in excellent agreement with the experimental value given in [1], in contrast with the CFS model obtained by letting $f = g = h = 1$ in the above equations. This two-pole formula, since it contains only attractive forces, predicts a phase shift which is everywhere positive and which lies somewhat above the observed values in the high energy region. To give a change of sign in the phase shift between 259 and 250 MeV in agreement with experiment, we need simply take $0.15 < r_1 < 0.155$. At first sight we might also fit an empirical phase shift at lower energy by an appropriate choice of r_0, but this turns out to require $-Cr_0/Dr_1 < R$, and hence a model with a CDD pole that causes the phase shift to pass through 90^0 at very high energy. We therefore require $r_0 = (DRr_1/-C) + \epsilon$, $\epsilon > 0$. This insures that if two additional poles occur for $q^2 < 0$, the intermediate range force will be attractive and the short range force will be repulsive, in agreement with our current understanding of the ${}^1S_0$ interaction. It does not guarantee that these two poles will occur on the physical sheet, the alternative being that they both occur on the second sheet, corresponding to a different type of CDD situation. Fortunately, for $r_1 = 0.15$, ϵ small, and $G^2 < 15$, solutions can be found. These three pole models predict a phase shift lying everywhere below the empirical values for $E < 250$ MeV, so the two and three pole models bracket the empirical situation. Further, although the predicted shape parameter is somewhat smaller for the three-pole model, it still lies within the experimental error assigned in [1].

Having determined the parameters of the models from p-p experiments, we locate the positions of the one or two remaining poles and evaluate their residues. The corresponding n-p scattering can then be predicted by solving the usual N/D equations. We find that a slight modification of the strength of the intermediate range attraction gives agreement with the observed n-p scattering length, but that the predicted effective range is longer than the p-p value for all two and three pole models. By a correlated change in the strength and range of this attraction, we can achieve agreement with the observed n-p effective range as well as the scattering length, but only if the effective mass of the system exchanged is 10-20 per cent heavier than in the p-p case. In spite of the extreme sensitivity of the interaction parameters due to the closeness to the CDD situation, this remains true when the charge-dependent effects in the one-pion-exchange are included, if due care is exercised in the numerical work.

The substantial charge-dependent effect found is hard to understand in terms of current models. These explain the intermediate range attraction by the exchange of the ABC or some other $I = 0$ π-π S wave effect and the $I = 0$ η, while they attribute the short range repulsion primarily to the $I = 0$ ω and φ, none of which can explain the observed effect. We conclude that there must be a substantial intermediate range contribution from the ρ, which in itself would be surprising, or from some as yet not suspected $I \neq 0$ component of the π-π system. Hence, until this effect is understood, models of the $^1S_0$ interaction which are dominated by the exchange of $I = 0$ boson systems should be viewed with suspicion.

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1. H. P. Noyes, Phys. Rev. Letters 12, 171 (1964).

2. H. P. Noyes, Phys. Rev. 130, 2025 (1963).

3. D. Y. Wong and H. P. Noyes, Phys. Rev. 126, 1866 (1962).

Addendum to

"The Interpretation of Low Energy Nucleon-Nucleon Scattering" (SLAC-PUB-29)

H. Pierre Noyes

December 4, 1964

Since the above mentioned paper was written, Heller, Signell, and Yoder (Phys.Rev.Letters 13, 557 (1964)), have shown that the pole approximation used therein is unreliable for the scattering length in that if a potential model is fitted to p-p scattering the n-n scattering length predicted is about -17 F, in agreement with experiment, rather than the -28 F predicted by the prescription of Wong and Noyes (Phys.Rev. 126, 1866 (1962)). We have therefore reinvestigated the dependence of the intermediate range attraction (f_σ^2 and m_σ) on the n-p effective range using a potential model consisting of one, two, or three yukawa potentials. The results are tabulated below, and compared with the results of the pole approximation in the accompanying figure.

A more detailed discussion of the question of charge independence, using the Signell-Yoder-Heller potential, is inculded in Section 2. of SLAC-PUB-59 "The interaction effect in n-p capture", which should also be consulted.

Explicitly, the yukawa potential models are given by

$$V(r) = -G^2 (m_\pi / 2M_{np})^2 \frac{e^{-m_\pi r}}{r} - f_\sigma^2 \frac{e^{-m_\sigma r}}{r} + f_\omega^2 \frac{e^{-m_\omega r}}{r}$$

$m_\pi = 138.07$ Mev/c^2 $M_{np} = 938.9025$ Mev/c^2 $m_\omega = 782.8$ Mev/c^2

Remaining parameters are given in the accompanying tables. The (ALGOL) computer code is also given.

One Yukawa Model

G^2	m_π	f_ω^2	m_ω
0.0000000000@+00	1.3607000000@+02	0,0800000000@+00	7.8280000000@+02

f_σ^2	m_σ	a_s^{np}	r_s^{np}
2.8451610361@-01	1.6973844225@+02	-2.3679988316@+01	2.6800089588@+00
2.8808176357@-01	1.7173844225@+02	-2.3679966168@+01	2.6469866743@+00
2.9164796788@-01	1.7373844225@+02	-2.3680056001@+01	2.6147147452@+00
2.9521457790@-01	1.7573844225@+02	-2.3680055521@+01	2.5831783325@+00
2.9878152174@-01	1.7773844225@+02	-2.3680000198@+01	2.5523516909@+00
3.0234896871@-01	1.7973844225@+02	-2.3680044781@+01	2.5222160314@+00
3.0591671398@-01	1.8173844225@+02	-2.3680053391@+01	2.4927485581@+00
3.0946483762@-01	1.8373844225@+02	-2.3680008246@+01	2.4639297002@+00
3.0794618388@-01	1.8287718975@+02	-2.3679967518@+01	2.4762643323@+00
3.0973233915@-01	1.8387718975@+02	-2.3680001002@+01	2.4619543858@+00
3.1151653830@-01	1.8487718975@+02	-2.3680017352@+01	2.4478039619@+00
3.1330080081@-01	1.8587718975@+02	-2.3679992435@+01	2.4338095373@+00

Two Yukawa Model

G^2	m_π	f_ω^2	m_ω
1.4000000000@+01	1.3807000000@+02	0.0000000000@+00	7.8280000000@+02
f_σ^2	m_σ	a_s^{np}	r_s^{np}
2.2038817185@-01	1.9382044668@+02	-2.3679995486@+01	2.6792084691@+00
2.2663943531@-01	1.9882044668@+02	-2.3679991558@+01	2.6404080377@+00
2.3291494596@-01	2.0382044668@+02	-2.3680011612@+01	2.6037513458@+00
2.3921444908@-01	2.0882044668@+02	-2.3680001273@+01	2.5690789798@+00
2.4553755160@-01	2.1382044668@+02	-2.3679992731@+01	2.5362466217@+00
2.5188392426@-01	2.1882044668@+02	-2.3680012700@+01	2.5051170066@+00
2.5825315740@-01	2.2382044668@+02	-2.3680015225@+01	2.4755714704@+00
2.6464498020@-01	2.2882044668@+02	-2.3680010018@+01	2.4474984896@+00
2.7105904823@-01	2.3382044668@+02	-2.3679982705@+01	2.4207971114@+00
2.6208110239@-01	2.2681697748@+02	-2.3680001155@+01	2.4585779083@+00
2.6335037279@-01	2.2781697748@+02	-2.3680006338@+01	2.4530210555@+00

Three Yukawa Model

G^2	m_π	f_ω^2	m_ω
1.4000000000@+01	1.3807000000@+02	3.5000000000@+00	7.8280000000@+02
f_σ^2	m_σ	a_s^{np}	r_s^{np}
1.2391727499@+00	3.2686858240@+02	-2.3680006835@+01	2.6800665451@+00
1.3024938795@+00	3.3686858240@+02	-2.3680025333@+01	2.6383923918@+00
1.3670441430@+00	3.4686858240@+02	-2.3679995440@+01	2.5988343372@+00
1.4327783570@+00	3.5686858240@+02	-2.3680022151@+01	2.5612053164@+00
1.4996492151@+00	3.6686858240@+02	-2.3679975711@+01	2.5253325913@+00
1.[illegible]376074360@+00	3.7686858240@+02	-2.3679985337@+01	2.4910666438@+00
1.5266012089@+00	3.8686858240@+02	-2.3679993570@+01	2.4582724391@+00
1.6180331742@+00	3.8419878600@+02	-2.3679996887@+01	2.4668878380@+00
1.6319489997@+00	3.8619878600@+02	-2.3680049805@+01	2.4604249846@+00
1.6458542274@+00	3.8819878600@+02	-2.3680028061@+01	2.4540140545@+00

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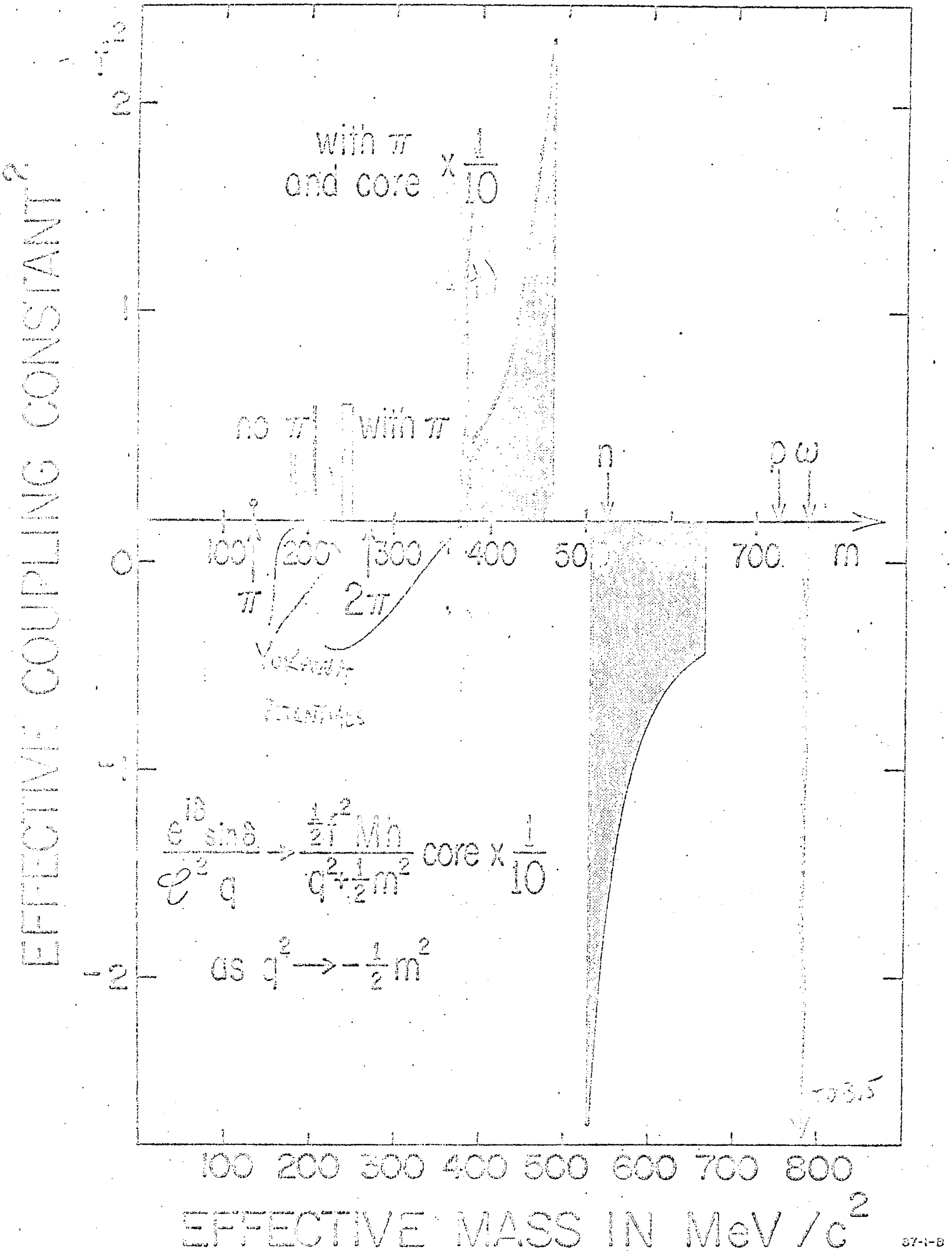
[illegible]
        FOR A RANGE OF VALUES OF MSIG AND COMPUTE THE EFFECTIVE RA

LABEL ITERP,ITERS,SEARCH,LOOP,STOP;
   REAL ARRAY A[0:1000],US [0:1000],ASS[0:1000],I[0:1000];
REAL DR,NN,NX,N,R,LAS,MUS,AS,CON,G2,MPI,M,LAP,MUP,PN,PX,DP,P1,CHI1,
          FW,MW,LAW,MUW,
 Q,       IP1,AI,SE,AS1,AS2,DP1,CHI2,NORM,RS;
PROCEDURE MILNE(A,U); REAL ARRAY A[*],U[*]; BEGIN
     REAL DR2; REAL ARRAY Y[NN:NX]; DR2 ← DR*2;
     Y[NN]←(1-DR2×A[NN]/12)×U[NN]; Y[NN+1]←(1-DR2×A[NN+1]/12)×U[NN+1];
FOR N←NN+2 STEP 1 UNTIL NX DO BEGIN Y[N]←2×Y[N-1]+DR2×A[N-1]×U[N-1]
   - Y[N-2];  U[N] ← Y[N]/(1-DR2×A[N]/12); END; END;
REAL PROCEDURE TRAP; BEGIN REAL N,SUM;
SUM ← 0.5×(I[0]+I[NX]); FOR N←1 STEP 1 UNTIL NX-1 DO SUM ←SUM  + I[N];
TRAP←DR×SUM ;END;
PROCEDURE YUK2; BEGIN R←DR; FOR N←1 STEP 1 UNTIL NX DO  BEGIN
        A[N] ← A[N] -(LAS/R)×EXP(-MUS×R);R←R+DR;END;MILNE(A,US);
        R←DR;               FOR N←1 STEP 1 UNTIL NX DO   BEGIN
        A[N] ← A[N] +(LAS/R)×EXP(-MUS×R);  R←R+DR; END;
    AS← DR×(NX×US[NX-10]-(NX-10)×US[NX])/(US[NX-10]-US[NX]);
END;

DR←.01;NN←0;NX←1000;US[0]←0;US[1]←1;A[0]←0;CON←197.322;M←938.9025;
ITERP:        READ(G2,MPI,  FW,MW)[STOP];  WRITE(G2,MPI,  FW,MW);
                 LAW ← FW×M/CON;    MUW ← MW/CON;
        LAP←G2×(.5×MPI/M)*2×M/CON; MUP←MPI/CON;     R←DR;
        FOR N←1 STEP 1 UNTIL NX DO BEGIN A[N]←-(LAP/R)×EXP(-MUP×R)
                 +(LAW/R)×EXP(-MUW×R);      R← R+DR;.END;
ITERS:      READ(P1,IP1,PN,PX,DP,AI,SE)[STOP];
     FOR Q←1 STEP 1 WHILE PN < PX DO
BEGIN MUS←P1/CON; LAS←P1×M/CON;YUK2;  AS1←AS;  CHI1←(1/AI-1/AS)*2;
SEARCH: LAS←(P1+IP1)×M/CON;YUK2;AS2←AS;DP1←IP1×(1/AI-1/AS1)/(1/AS2-1/
                                                AS1);
LOOP:       LAS←(P1+DP1)×M/CON;YUK2;CHI2←(1/AI-1/AS)*2;
       IF CHI1<CHI2 THEN BEGIN DP1←.5×DP1;GO TO LOOP;END;
       IF 10-14<CHI2 THEN BEGIN CHI1←CHI2;AS1←AS;P1←P1+DP1;
          GO TO SEARCH; END;
    NORM←(1-NX×DR/AS)/US[NX]; R←0;
  FOR N←0 STEP 1 UNTIL NX DO BEGIN US[N]←US[N]×NORM;ASS[N]←1-R/AS;
  I[N]←ASS[N]*2-US[N]*2; R←R+DR; END; RS←2×TRAP;
          WRITE(P1+DP1,PN,AS,RS); PN← PN + DP;
END; IF SE=1 THEN  GO TO ITERS; GO TO ITERP; STOP: END.
```

POLE PARAMETERS TO FIT P-P SCATTERING



CHANGE IN P-P POLE PARAMETERS TO FIT a_{np} = 23.68 F