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DETERMINATION OF THE PROTON-PROTON $^{1}S_{0}$ SHAPE PARAMETER*

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Modification of the effective range expansion for the $^1S_0$ nucleon-nucleon state to include the effect of the one pion exchange contribution (OPEC) by means of the partial-wave dispersion relation[1,2] or the fixed-angle dispersion relation,[3] leads to the prediction that the shape-parameter, P, in the expansion $q \operatorname{ctn} \delta_0 = - 1/a + \frac{1}{2} r_e q^2 - P r_e^3 q^4 + \ldots$ is positive. This is also predicted by potential models which include the long-range one pion exchange potential (OPEP) and either an intermediate range attraction plus repulsive core[4] or energy-independent boundary condition at intermediate range[5] whose parameters are adjusted to fit the effective range, r_e, and scattering length, a. A more quantitative prediction is provided by including the electrostatic repulsion in the partial wave dispersion relation[2] and using two additional parameters to fit observed phase shifts at 95 and 310 MeV as well as a and r_e; this calculation[6] gives $P = + 0.024$. This prediction is of opposite sign to that made by an energy-independent boundary condition at intermediate range,[7,8] an energy-dependent boundary condition[9] which fits the high energy (i.e., up to 310 MeV) $^1S_0$ phase shifts,[10,11] or hard-core potentials with intermediate range attractive tails,[12] which do not include the OPE effect. Since the OPE predictions have been quantitatively confirmed in higher angular momentum states,[13] and since the qualitative features of the phase shifts empirically determined in the 100-300 MeV range are in good agreement with models based on the exchange of known bosons and strongly-interacting boson systems ("resonances") between the two nucleons (for a brief discussion of these qualitative features and references cf. Ref. 11), it is important to test the consistency of these descriptions with the interaction in the S states as rigorously as possible. This is particularly true since the models in best agreement with the high energy scattering experiments predict only 2 of the observed 8 MeV binding for the three-nucleon systems,[14] and the latter calculation is more sensitive to the details of the S state interactions than the high energy scattering. One of the few tests available is the prediction of the shape parameter. Since the effective range expansion fails to converge above 10 MeV,[1] this test can only be made by means of very low energy nucleon-nucleon experiments. Existing n-p data are not of sufficient precision to yield definite conclusions.[11] In this letter we show that the recently reported experiment on p-p scattering near the interference minimum at 0.3825 MeV[15] and the p-p differential cross sections measured at 1.397, 1.855, 2.425, and 3.037 MeV[16] can be analysed to yield a precise value of the shape parameter. This analysis is only possible because the latter experiments also yield a precise value for the J-weighted average of the ^3P phase shifts, and because we claim to have a sufficiently quantitative understanding of the multirange character of the nucleon-nucleon interaction to use this value to predict the individual $^3P_{0,1,2}$ phase shifts.

The energy at which the minimum in the p-p 90° cross section occurs is claimed by Brolley, Seagrave, and Beery[15] to have been determined to better than ±200 ev, and the energy at which the minimum occurs is given by Gursky and Heller[17] as 0.3825 MeV. This value is preliminary, but even if the final result should differ by 200 or 300 ev, none of the conclusions drawn below would be affected. In the absence of vacuum polarization effects, this would imply a $^1S_0$ phase shift of 0.25408 ±0.00020 rad. at precisely that energy; the uncertainty is assigned by assuming that the minimum actually was at 0.3823 (or 0.3827) MeV and then computing the phase shift to be expected at 0.3825 MeV. The value of the phase shift and the uncertainty are independent of the value of the scattering length, but require a knowledge of the effective range. However, within the extreme limits on r_e assigned below (BC and CFS), both the value and the error are unaffected by the value of r_e to the quoted accuracy. Heller[18] has included the vacuum polarization amplitude in the calculation of the phase shift from the energy of the minimum, and for the phase shift referred to the electric (i.e., Coulomb plus vacuum polarization) amplitude gives[18] $\delta_0^E = 0.2550$ rad. at 0.3825 MeV. We have confirmed this value by an independent calculation, and the uncorrected value quoted above also agrees (cf. Ref. 18, Figure 7). The corresponding phase shift referred to the Coulomb amplitude is, in the notation of Reference 18, $K_0 = \delta_0^E + \tau_0 = 0.25317$ rad $= 14.5055^\circ \pm 0.0125^\circ$. We have also checked that this value is independent of nuclear scattering in higher angular momentum states to the quoted accuracy, using the P waves computed below.

The vacuum polarization correction for the energies and angles of the measurements made by Dahl, Knecht, and Messelt[16] has been computed by Durand[19] and more precisely by Heller,[20] which latter calculation shows that the published[19] values are accurate enough for the current purpose. I have further refined these calculations by using the S and P phases determined below, and the $\ell > 1$ scattering predicted by OPE, but obtain negligible corrections to the published values. We also find that the $\ell > 1$ scattering predicted by OPE is somewhat smaller than the statistical uncertainty in the data at the highest energy, so although included, it has little effect on the analysis. It is anticipated from the work of Breit and Hull[21] that the OPE prediction for P-waves will not be quantitatively reliable even at these low energies, and we find in fact that no value of K_0 allows a reasonable fit to the data if the P waves are taken from OPE. The difficulty is that even though the squares of the phase shifts are almost negligible, the negative Coulomb interference term proportional to $z_2 = \delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2}$[22] predicted by OPE is an order of magnitude too large. If we allow z_2 to be a free parameter in the least-squares fit and fix $\delta_{1,0}$ and $\delta_{1,1}$ at values in a range that departs no more than 50% from the OPE predictions, we find that z_2 is quite precisely determined, but that K_0 varies by several times the statistical uncertainty due to the experimental errors. Details of these results, and the accuracy of triple scattering experiments needed to give an empirical determination of K_0 at 3 MeV will be discussed elsewhere.[23]

Lacking the spin-dependent scattering experiments needed for a direct determination of K_0, we rely on the following theoretical argument. Although centrifugal shielding in the P states is not complete, we still expect OPE to be more important than the shorter range contributions to the interaction. Since the OPE prediction gives phase shifts less than 1°, we expect such small phase shifts to be calculable from the Born approximation, in which case the various contributions are additive; further, we are at low enough energy to consider only central, tensor, and $(L \cdot S)$ spin-orbit interactions. Since the $^3P_{0,1,2}$ phase shifts have the OPE tensor signature (+ - +) below 210 MeV rather than the spin-orbit signature (--+), and the $^3F_{2,3,4}$ have the OPE tensor signature at all energies where they are measured, we are confident that the spin-orbit force is short range. As the ^3P phases are considerably closer to OPE at 51 MeV[24] than at 147 MeV, we feel justified in neglecting the $L \cdot S$ interaction at 3 MeV, and keeping only the OPE tensor term so far as the J-dependent part of the interaction goes. In the additive approximation, z_2 depends only on the central part of the interaction, but z_2 we have already noted can be directly determined from experiment. Hence we assert that a good quantitative approximation for the P-waves below 3 MeV is given by[25]

$$\begin{aligned}
\delta_{1,0} &= z_2/9 + 5\,\mathcal{C}^2(1+\eta^2)(2\delta_{1,0}{}^{\pi} - 3\delta_{1,1}{}^{\pi} + \delta_{1,2}{}^{\pi})/18 \\
\delta_{1,1} &= z_2/9 - 5\,\mathcal{C}^2(1+\eta^2)(2\delta_{1,0}{}^{\pi} - 3\delta_{1,1}{}^{\pi} + \delta_{1,2}{}^{\pi})/36 \qquad (1) \\
\delta_{1,2} &= z_2/9 + \mathcal{C}^2(1+\eta^2)(2\delta_{1,0}{}^{\pi} - 3\delta_{1,1}{}^{\pi} + \delta_{1,2}{}^{\pi})/36
\end{aligned}$$

where $\delta_{\ell,J}^{\pi}$ are the usual OPE prediction and $C^2(1 + \eta^2)$ the usual P-wave Coulomb penetration factor. Quantitative justification of this model in terms of the multi-boson exchange interpretation will be given below. The values of K_0 and z_2 determined from the data of Dahl, et al.[16] under this assumption for the P phases, and the values of the P phases themselves, are given in Table 1.

Since the values of K_0 just determined still contain the physical effects of vacuum polarization, one final correction is needed before we can compute the effective range parameters. Since we wish to compare with calculations made ignoring vacuum polarization, we use the correction to $C^2 q \operatorname{ctn} K_0 + Q$ $[C^2 = 2\pi\eta/(e^{2\pi\eta} - 1),\ \eta = e^2/\hbar v_{lab},\ Q = 2q\eta\ (\sum_{p=1}^{\infty} 1/p(p^2 + \eta^2) - 0.57721.. - \ln \eta),\ q^2 = M_p T_{lab}/m_{\pi^0}{}^2 c^2]$ computed by Foldy and Eriksen[26] rather than the model-independent expansion of $\delta_0^E = K_0 - \tau_0$ given by Heller.[18] Since only the part of the Foldy correction from outside the range of nuclear forces ($\Delta_2 K$) has appreciable energy dependence between 0.35 and 3 MeV, this introduces a model-dependent correction in a which is certainly less than the total inner Foldy correction (+ 0.018 F), and does not affect the value of r_e or P to the quoted uncertainty. The results of the least-squares fit to the phase shifts of Table I so obtained are compared with the shape-dependent effective range expansion (SD) and the prediction computed from the Coulomb-corrected partial wave dispersion relation (PWDR) mentioned in the first paragraph[6] in Table II and Figure 1. For comparison with the n-p case we also give the shape-independent approximation (SI), boundary condition model (BC), and Cini-Fubini-Stanghellini approximation to the fixed-angle dispersion relation ignoring Coulomb effects (CFS); (for explicit formulae and notation cf. Ref. 11). We see that the prediction is precisely confirmed to high accuracy, and falls between the shape-independent approximation and the calculation (CFS) which includes the OPE effect but ignores the inner Coulomb correction and the short-range repulsion, as anticipated; the pure boundary-condition model is cleanly excluded, and this would still be true if the error in the 0.3825 MeV point were 50% larger than we have assigned. We therefore have achieved a quantitative confirmation of the OPE effect in the $^1S_0$ state for the first time, and have made the discrepancy in the 3-body calculation[14] more puzzling than ever.

The large departure of the empirically determined values of z_2 from the OPE prediction (cf. Fig. 2), and the failure of z_2 to exhibit the q^3 dependence usually expected for a P wave at low energy, raise a question as to the adequacy of the approximation used in Eq. (1), which must be resolved. Since the ^3P phase shifts are dominated by tensor and spin-orbit contributions in the energy region (50-300 MeV) where they are individually determined, we have little direct information about the central ^3P interaction which determines z_2 and consider first the (central) interaction in the singlet states. This is dominated by a short range repulsion (evidenced by the change in sign of $^1S_0$ at 310 MeV), an intermediate range attraction (evidenced by the failure of OPE to give enough attraction to fit the $^1S_0$ effective range if the scattering length is fitted,[1] the rapidly increasing departure of $^1D_2$ from OPE with increasing energy, and the less rapidly increasing departure of $^1G_4$ from OPE with increasing energy). The repulsion is readily understood as due to the ω (neutral vector) meson, and the intermediate range attraction as due to a π-π S wave resonance or strong correlation of some sort. The latter, corresponding to the exchange of a zero spin particle, will persist unchanged in the ^3P states, and although the interaction due to the ω is spin-dependent, it will remain repulsive in these states, while the weak OPE attraction in the singlet states will change to a still weaker repulsion only 1/3 as strong. We thus predict that the central interaction in the ^3P states will be predominantly a short range repulsion, an intermediate range strong attraction, and a long range but very weak repulsion. Due to centrifugal shielding, we can expect an approximate cancellation between the weak long range repulsion and the intermediate range attraction at very low energy, but a predominantly attractive interaction at somewhat higher energy. Since we have just seen that z_2 is in fact close to zero and negative below 3 MeV, and since it is large and positive at 50 MeV[24], this qualitative prediction of the multi-boson exchange model is brilliantly confirmed. To remove any last doubts about the peculiar behavior of z_2, I have computed it at the four energies in question using a short range repulsion, intermediate range attraction, and the (known) OPE long range repulsion. Fitting only a single strength parameter to the four values of z_2, and choosing a wide range (2-4 m_π) of values for the effective mass of the system responsible for the attraction, and ratios of interaction strength between the intermediate and short range interactions differing by a factor of 10, gives the five barely distinguishable predictions shown in Fig. 2. Since the range of values used more than covers those used in a similar model by Ramsay[27] at much higher energy, we feel that the, at first sight peculiar, behavior of z_2 has been completely explained. To justify the neglect of the spin-orbit term below 3 MeV, we have extracted the tensor and spin-orbit parts from phase shifts at the energies of interest kindly computed for us by P. Signell[28] using the Yale[29] and Hamada-Johnston[30] potentials. In both cases we find (a) that the tensor term departs from the OPE tensor contribution given in Eq. 1 by less than 10%, and (b) that the $L\cdot S$ contribution is less than 10% of the tensor contribution. These small departures from Eq. 1 have no significant effect on the values of K_0 or z_2 given in Table I. We conclude that our treatment of the low energy P-waves is quantitatively reliable for the purposes of the current analysis, and that the values of z_2 so obtained give still another check of the self-consistency of the multi-boson exchange description of the two-nucleon interaction.

We gratefully acknowledge permission from P. Dahl to present this analysis of the still unpublished Wisconsin data[16] and from J. Brolley to make use of the preliminary results of the Los Alamos measurement.[15] Development of the computer code used in the preliminary stages of this analysis would not have been possible without the active collaboration of L. Heller, and we are most grateful to him for careful checks on earlier numerical results. Computational assistance was provided by E. DeGraw of LRL, Livermore and C. Moore of SLAC.

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TABLE I

Value of the $^1S_0$ phase shift K_0 assuming that the interference minimum is at 0.3825 ± 0.0002 MeV[15,17], and values of K_0 and the J-weighted ^3P phase shift $z_2^E = \delta_{1,0}^E + 3\delta_{1,1}^E + 5\delta_{1,2}^E$ determined by a least-squares fit to the data of Dahl, Knecht, and Messelt,[16] using the vacuum polarization correction computed by Durand,[19] ^3P phases given by Eq.(1), and $\ell > 1$ scattering predicted by OPE.

Lab. Energy	K_0	z_2^E	$\delta_{1,0}^E$	$\delta_{1,1}^E$	$\delta_{1,2}^E$
0.3825 MeV	14.5055°±0.0125°				
1.397	39.232°±0.015°	-0.105°±0.055°	0.2414°	-0.1457°	0.0258°
1.855	44.266°±0.021°	-0.045°±0.085°	0.4017°	-0.2083°	0.0367°
2.425	48.287°±0.014°	-0.076°±0.060°	0.6073°	-0.3163°	0.0531°
3.037	50.943°±0.020°	-0.018°±0.077°	0.8604°	-0.4332°	0.0842°

TABLE II.

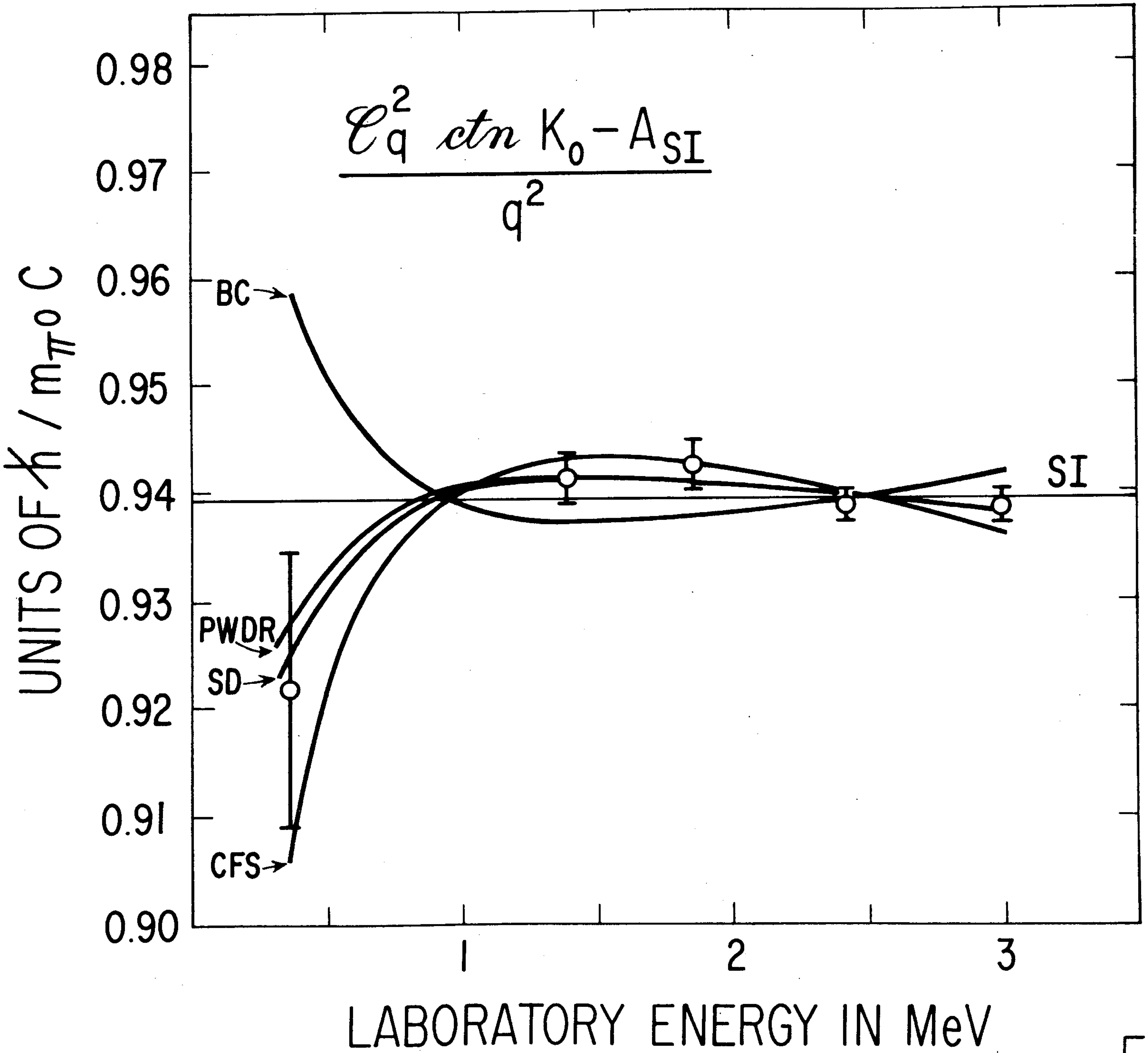
Least-squares fit to $\mathcal{C}^2\, q \,\mathrm{ctn}\, K_0 + Q = H$ using the values given in Table I and the vacuum polarization correction computed by Foldy and Eriksen;[26] this correction contributes a model-dependent term of 0.018 F to a, but does not give any model-dependence to r_e or P. The models are: BC: $H = (B + q^2 t)/(1 - Bt)$, $t = (1/q) \tan q\bar{r}$; SI: $H = A + Rq^2$; PWDR: solution of partial wave dispersion relation with Coulomb and OPE effects, fitted to phase shifts at 95 and 310 MeV, and with a and r_e adjusted to fit this data; SD: $H = A + Rq^2 + Sq^4$; CFS: $H = A + Rq^2 + Cq^4/(1 + Dq^2)$ with $D = (2 - f^2M(\frac{3}{2}\sqrt{2} + 4A - R))/(1 - f^2M(\frac{1}{4}\sqrt{2} + A))$, $C = -(1 - \frac{1}{2} D)(2\sqrt{2} - 2R + 4A)$, $f^2M = G^2 m_{\pi^0}/4M_p$. Note that SD has three degrees of freedom, but all others have only two, and have the same errors as given under SI in the table.

Model	BC	SI	PWDR	SD	CFS
Parameter values in neutral pion units	B = 0.163076	A = 0.186996	A = 0.186763	A = 0.186703	A = 0.186366
	$\bar{r}$ = 0.794875	R = 0.939514	R = 0.953191	R = 0.955884	R = 0.976131
			S = -0.16161	S = -0.18285	C = -0.45470
					D = 1.43918
a (Fermis)	-7.8009	-7.8163±0.0048	-7.8259	-7.8284±0.0030	-7.8426
r_e (Fermis)	2.687	2.746 ±0.014	2.786	2.794 ±0.026	2.853
P	-0.036	0	0.024	0.026 ±0.014	0.0612
x^2	20.64	5.46	1.74	1.71	5.67
% Probability	less than 0.1	14.9	63.3	43.6	13.5

FIGURE CAPTIONS

Figure 1. Since the various predictions and experimental errors are barely distinguishable on a conventional effective range expansion plot, we give instead the difference between $C^2 q$ ctn $K_0 + Q$ and the constant term $A_{SI} = -1/a$, divided by the c.m. momentum squared. Empirical values of K_0 are given in Table I, and the parameters of the models in Table II.

Figure 2. Values of $z_2 = \delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2}$ given in Table I are compared with the predictions of a 3-range potential model. The short range repulsion is assumed to have a range corresponding to the ω mass of 5.8 m_π, and is varied between 0.3 and 3 times the strength of the intermediate range attraction. The intermediate range attraction is assumed to have a range corresponding to 2 or 4 m_π and the strength adjusted for a best fit. The long range repulsion is computed from the central part of OPE with $G^2 = 14$ and $m_\pi = 135$ MeV. Coulomb correction is made by multiplying by the penetration factor. Values of z_2 for the Hamada-Johnston and Yale potentials computed by P. Signell are shown for comparison, as is the plane-wave OPE prediction with and without the Coulomb correction.



0.98
0.97
0.96
0.95
0.94
0.93
0.92
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$\frac{\mathcal{C}_q^2 \, ctn \, K_0 - A_{SI}}{q^2}$
BC
PWDR
SD
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SI
1
2
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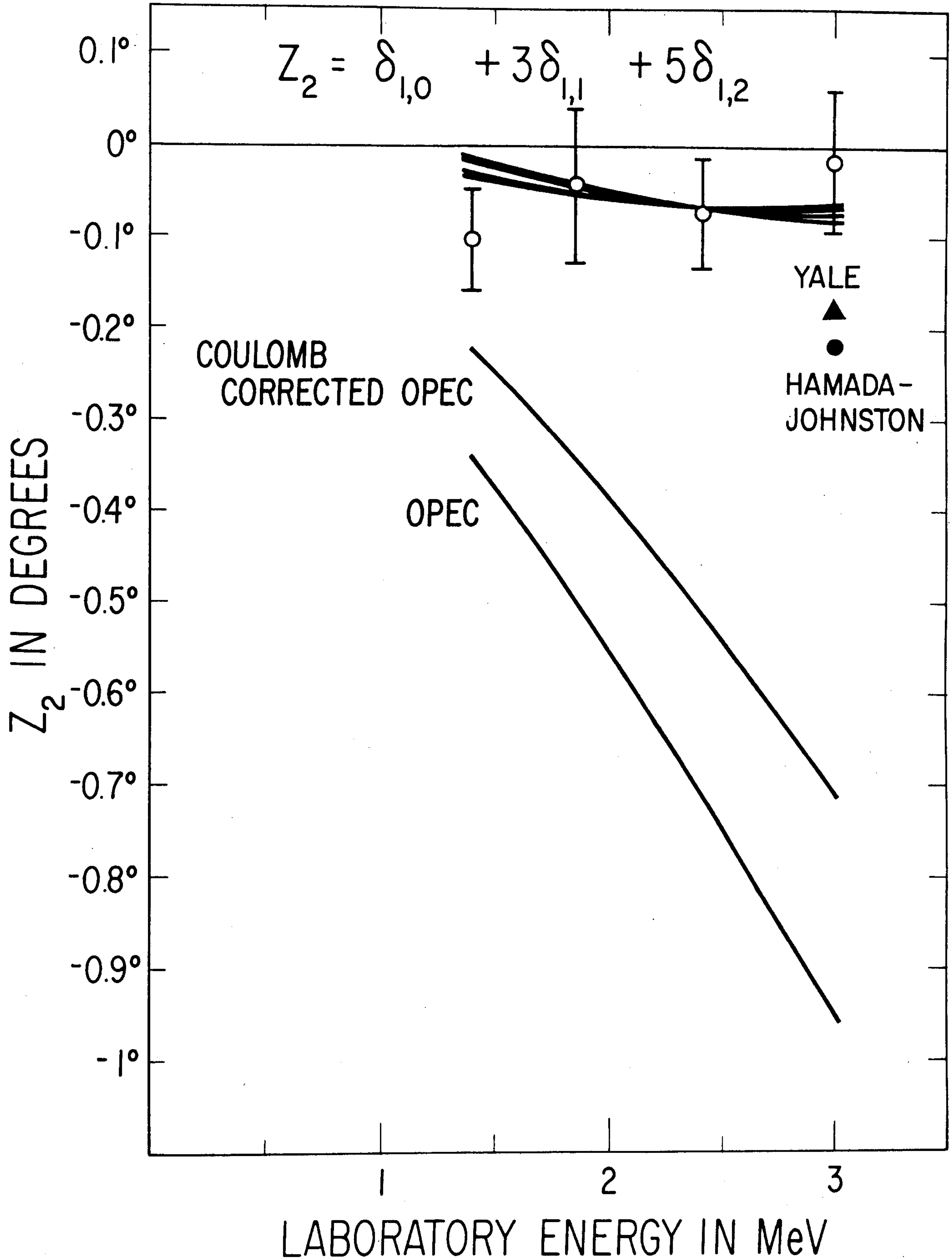
LABORATORY ENERGY IN MeV

FIG. I



$Z_2 = \delta_{1,0} + 3\delta_{1,1} + 5\delta_{1,2}$
0.1°
0°
-0.1°
-0.2°
-0.3°
-0.4°
-0.5°
-0.6°
-0.7°
-0.8°
-0.9°
-1°
Z_2 IN DEGREES
YALE
HAMADA-JOHNSTON
COULOMB CORRECTED OPEC
OPEC
1
2
3
LABORATORY ENERGY IN MeV

FIG. 2