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A High-Resolution 20-BeV Spectrometer Resolving Momentum and Production Angle Independently

by

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The extension of magnetic spectrometer techniques to high energies involves several new problems. The principal factors, aside from minimizing cost for a given momentum acceptance $(\Delta p/p)_{max}$, solid angle $(\Delta\Omega)$, and momentum resolution $(\Delta p/p)_{min}$ are:

(1) At high energies, cross sections are rapidly varying functions of production angle θ, and therefore, information on θ within the solid angle accepted must be preserved.

(2) Liquid or gas targets are in general horizontal line sources of small vertical height.

(3) The spectrometer must permit reliable particle identification consistent with large ratios of rejected to accepted particle fluxes.

At these high energies physical dimensions of beam transport equipment are large and therefore it is highly preferable to confine the plane of bend of momentum-dispersive elements to the horizontal plane. In this paper we will compare the properties of a magnetic analyzing system of novel design meeting these requirements with those of a more conventional design in which θ information is displayed in the horizontal plane while the momentum dispersion plane is vertical.

A proposed spectrometer which illustrates the principle operates as follows (Fig. 1): In the horizontal plane a quadrupole Q_1 focuses a horizontal line source to a point, thus giving dispersion in the focal plane corresponding to the production angle θ. A defocusing quadrupole Q_2 is placed at this intermediate focus but has no effect on the final image in the horizontal plane. The remaining part of the spectrometer, two bending magnets M_1 and M_2 and a focusing quadrupole Q_3, images the intermediate focal point at the final focal plane. This final image (in the horizontal plane) is displaced from the central trajectory by a distance which is linearly proportional to both production angle and momentum. In the vertical plane, the point-to-point focal properties are determined by the quadrupoles, Q_1, Q_2, and Q_3 respectively.

Now if the second quadrupole, Q_2, is rotated about the beam axis through an angle α, the production angle θ in the <u>horizontal</u> plane will be dispersed in the <u>vertical</u> plane. Since Q_2 is at a horizontal focus, the rotation will have no effect on the first-order horizontal optics. Thus the production angle θ is determined by the vertical position of the image and the particle momentum is found from the horizontal position.

It should be emphasized that the rotated quadrupole is the only novel concept in the example just presented. Any other combination of magnetic elements preceding or following the rotated quadrupole will function in a similar manner provided that

(a) an intermediate focus of the line source exists at the principal plane of the rotated quadrupole

(b) point-to-point focusing exists from this intermediate focus to the final image point in the horizontal plane

(c) point-to-point focusing exists in the vertical plane from the target to the final image point

(d) all momentum dispersion occurs in the horizontal plane.

Briefly, the principal of operating the proposed system is as follows: Let x and x_f be the horizontal coordinate at the target and the final focal plane respectively, y and y_f the vertical coordinates and θ, θ_f and φ, φ_f be the corresponding angular variables. Let $\delta = \Delta p/p$ be the deviation from the central momentum. To first-order in these variables it follows from the above discussion that

$$x_f = (x_f|\theta)\theta + (x_f|\delta)\delta \qquad (1)$$

$$y_f = (y_f|\theta)\theta + (y_f|y)y \qquad (2)$$

are the only non-vanishing coefficients determining the focal coordinates x_f and y_f. We thus find that constant values of δ and constant θ are independently focused in the focal plane, but the lines of constant θ and constant δ in general are not orthogonal in the focal plane. From the above equation:

$$\delta = \frac{x_f}{(x_f|\delta)} - \frac{(x_f|\theta)}{(x_f|\delta)} \left[\frac{y_f}{(y_f|\theta)} - \frac{(y_f|y)}{(y_f|\theta)} y \right] \tag{3}$$

$$\theta = \frac{(y_f|y)y}{(y_f|\theta)} \tag{4}$$

Hence, neglecting second-order effects, the attainable momentum resolution is limited by the source height y through the last term of Eq. (3):

$$\delta = \frac{(x_f|\theta)}{(x_f|\delta)} \frac{(y_f|y)}{(y_f|\theta)} y \tag{5}$$

The matrix $(y_f|\theta)$ is proportional to $\sin 2\alpha$, where α is the rotation angle of Q_2.

If the vertical beam height $y = \pm 0.3$ cm, then for the spectrometer arrangement shown in Fig. 1 (with parameters listed below) the minimum momentum resolution is limited to about 0.1%.

Second-order coefficients (aberrations) for the system have been evaluated. In general terms the dominant aberrations are of two categories: (1) coefficients which are equivalent to tilting the normal to the focal plane from the beam direction, and (2) chromatic aberrations in the production angle, i.e., coefficients which relate x_f and y_f to the products of $\theta\delta$ and $\varphi\delta$.

Depending on the arrangement of counters, a tilt of the focal plane may be acceptable; if not the tilt can be controlled by the introduction of appropriate sextupole correction lenses ahead of and following the dispersive magnets. The "chromatic-angular" aberrations are basic properties of strong focusing (quadrupole) lens-systems; they can be reduced but not eliminated by replacing the single input quadrupole Q_1 by a doublet or triplet.

Since production angle and momentum are focused independently, a two-dimensional array of counters can gate any subsequent detector over a predetermined range of kinematic production variables. Such a detector can then carry out a particle identification function with a minimum background from accidental coincidences arising from particles falling outside the range of kinematic interest.

It is of further advantage to replace the quadrupoles ahead of the rotated quadrupole by a system of alternating gradient bending magnets. This substitution does not change the principle of operation of the system, but permits rejection of low energy-charged particle fluxes at a point far from the detector and also makes it possible to operate the instrument at or near a production angle of zero degrees.

If dispersion in the vertical plane is not mechanically objectionable, then a system such as the one shown in Fig. 2 produces not only independent but also orthogonal separation in the variables θ and δ. In this case the first-order resolution is given by the expression

$$\delta = \frac{(y_f|y)}{(y_f|\delta)}\, y \tag{6}$$

where y is the vertical source height. The second-order aberrations of this system are of a similar nature to the previous case, but they are in general somewhat smaller than the comparable "flat" system.

Table I gives the computed values for the first-order coefficients in the expansion of x_f and y_f as a function of the source variables x, y, δ, θ, and φ. These are tabulated for the system shown in Fig. 1 with the following parameters:

$$
\begin{aligned}
p &= 20 \text{ GeV/c} \\
\delta_{max} &= \pm 2\% \\
\Delta\Omega &= 0.1 \text{ millisteradian} \\
\text{Bend angle} &= 2 \times 7.5^{\circ} = 15^{\circ} \\
\alpha &= 10^{\circ}
\end{aligned}
\qquad (7)
$$

Table II gives the physical parameters of this system.

The assistance of Sam Howry in making the computations is gratefully acknowledged.

TABLE I

Transfer Matrix From Target Coordinates x, θ, y, φ and Deviation in Momentum (dp/p) in percent to Focal Plane Coordinates x_f, θ_f, y_f and φ_f

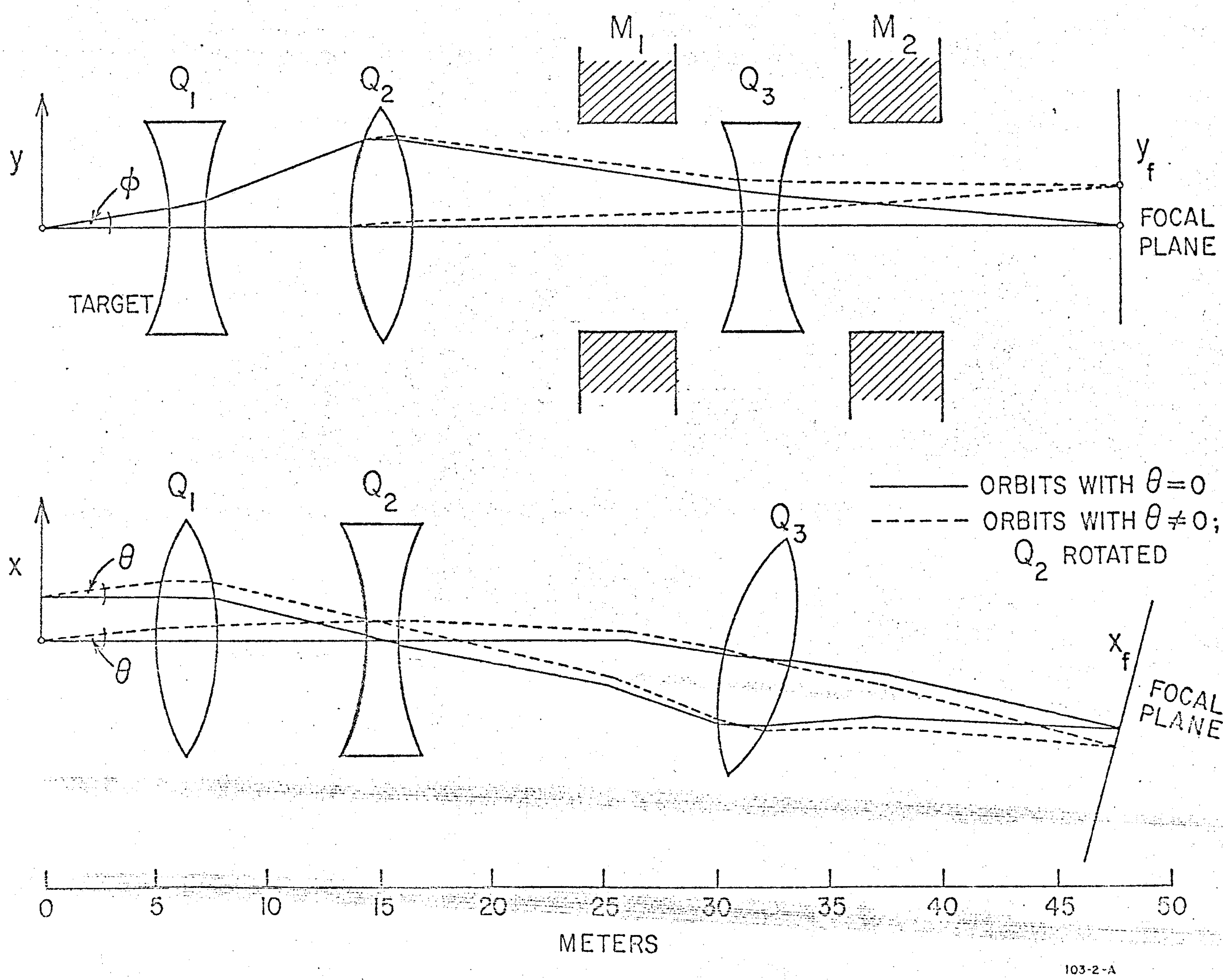
Dimensions are in cm and angles in milliradians

	x	θ	y	φ	$\frac{dp}{p}$
x_f	0	-0.85	0	0	3.17
θ_f	1.18	-2.76	-1.06	-1.09	1.97
y_f	0	2.92	-3.16	0	0
φ_f	0	1.37	-1.79	-0.317	0

TABLE II

Physical Parameters of 20 BeV Spectrometer with a 100 microsteradian Solid Angle and ±2% Momentum Band

	Field Gradient (kg/cm)	Field (g)	Length (meters)	Aperture (approximate)
Q_1	0.385	—	2	±5 cm
Q_2	0.466	—	2	±12 cm
M_1	—	14,600	6	14 cm gap; 40 cm width
Q_3	0.410	—	2	±20 cm
M_2	—	14,600	6	18 cm gap; 40 cm width



M1
M2
Q1
Q2
Q3
y
φ
TARGET
yf
FOCAL PLANE
ORBITS WITH θ=0
ORBITS WITH θ≠0; Q2 ROTATED
x
θ
θ
xf
FOCAL PLANE
0
5
10
15
20
25
30
35
40
45
50
METERS
103-2-A

FIGURE 1

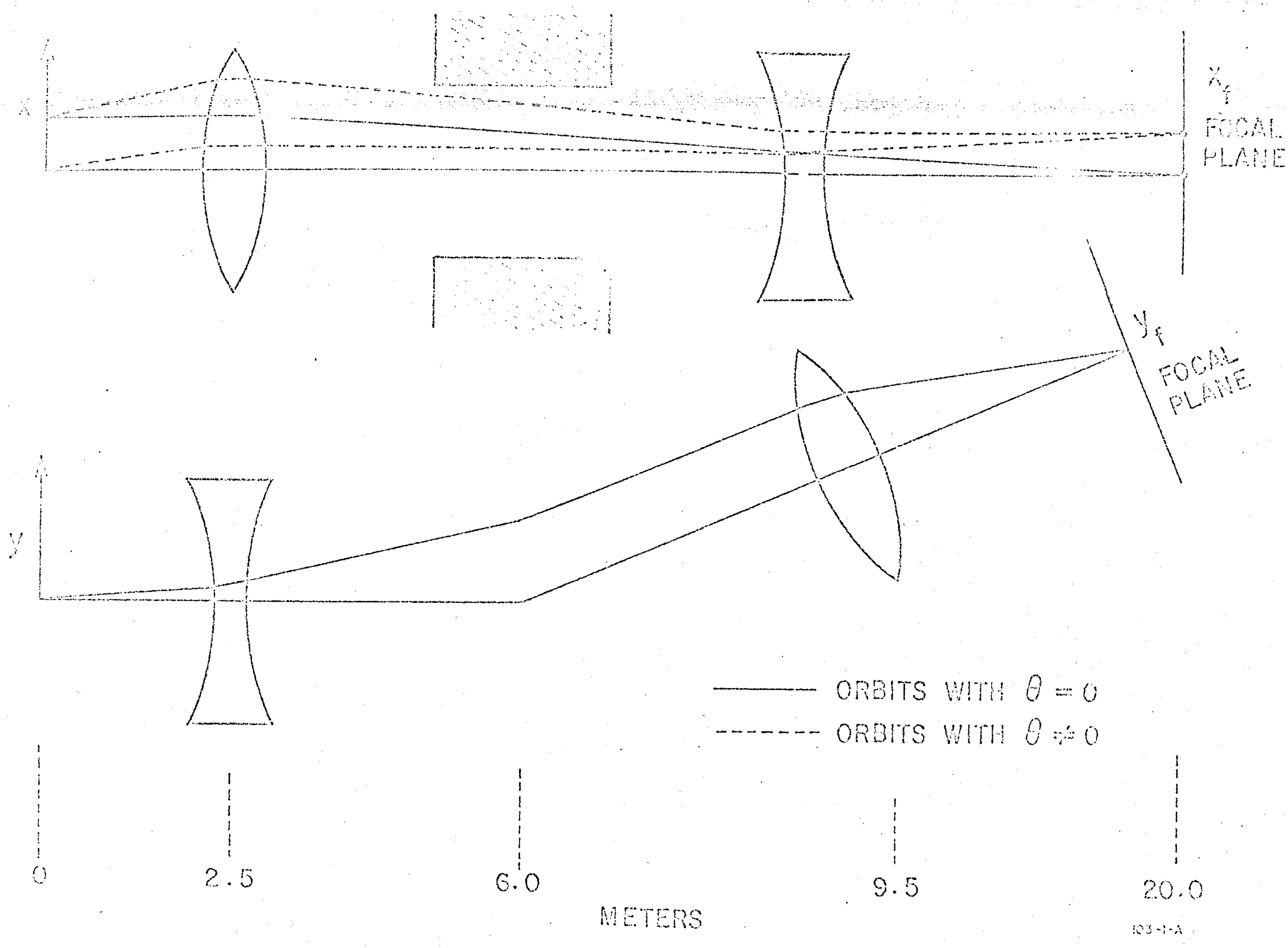

x
x_f
FOCAL
PLANE
y
y_f
FOCAL
PLANE
ORBITS WITH $\theta = 0$
ORBITS WITH $\theta \neq 0$
0
2.5
6.0
9.5
20.0
METERS
103-1-A

FIGURE 2