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INTERMEDIATE BOSON PAIR PRODUCTION AS A MEANS FOR DETERMINING ITS MAGNETIC MOMENT*

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It has been proposed that the weak interactions are mediated by a vector particle (w).[1] In addition to its weak interactions the charged w can interact with the electromagnetic field through its charge, magnetic moment and quadrupole moment. Lee and Yang[2] and Lee[3] have examined the electrodynamics of a vector boson which has no strong interaction and have shown that a kind of renormalizable theory can be constructed in which the boson has arbitrary magnetic moment and, to lowest order in the fine structure constant, no anomalous quadrupole moment.[4] In this note we show that, under the assumption of zero anomalous quadrupole moment, the reaction

$$\gamma + P \longrightarrow w^+ + w^- + P \tag{1}$$

has a cross section which is very sensitive to the magnetic moment and, therefore, an investigation of this process may serve as a means for its determination.

The coherent process

$$\gamma + z \longrightarrow w^- + w^+ + z \tag{2}$$

is also calculated. However, in the range of photon energies and w masses considered here,

$$E_\gamma < 20 \text{ BeV},\ 0.8 < m_w < 2.0 \text{ BeV},$$

this coherent process is generally much smaller than the incoherent process (1). This is because even the minimum momentum transfer q_m, which for an infinitely heavy target has the value $q_m = 2m_w^2/E_\gamma$, is large compared to the inverse nuclear radius.

To lowest order in the fine structure constant α the Feynman diagrams for pair production are shown in Fig. 1. The form of coupling between photons and vector mesons with zero anomalous quadrupole moment is chosen to be that of reference 3. The matrix element for process (1) can be readily determined by quite standard and well-known techniques. After using the relations[5] $qJ = ke = w^+\varphi^+ = w^-\varphi^- = 0$ and some rearrangings, the matrix element can be conveniently written as

$$m = m_I + m_{II} + m_{III}$$

where

$$\begin{aligned}
m_I = \frac{1}{-2kw^-} \Big[& -4(w^-e)(w^+J)(\varphi^+\varphi^-) + 2(1+\kappa)(w^-e)\left\{(q\varphi^-)(J\varphi^+) - (q\varphi^+)(J\varphi^-)\right\} \\
& + 2(1+\kappa)(w^+J)\left\{(k\varphi^+)(e\varphi^-) - (k\varphi^-)(e\varphi^+)\right\} + (1+\kappa)^2(k\varphi^-)\left\{(qe)(J\varphi^+) - (q\varphi^+)(Je)\right\} \\
& + (1+\kappa)^2(e\varphi^-)\left\{(kJ)(q\varphi^+) - (J\varphi^+)(qk)\right\} + \left\{(k\varphi^-)(w^-e) - (kw^-)(e\varphi^-)\right\}\left\{(q\varphi^+)C_+ + (J\varphi^+)B_+\right\} \Big]
\end{aligned}$$

$$m_{II} = m_I(w^+ \longleftrightarrow w^-;\ \varphi^+ \longleftrightarrow \varphi^-); \qquad m_{III} = -2(\varphi^+\varphi^-)(eJ)$$

$$C_+ = \frac{-(1-\kappa)^2}{m^2}(w^+J), \qquad B_+ = \frac{(1-\kappa)}{m^2}\left\{(1-\kappa)(qw^+) + \kappa q^2\right\} \tag{3}$$

$$J_\mu = \bar{u}(p_2)\left[\gamma_\mu G_m + \frac{(p_1+p_2)_\mu}{2M}\left(\frac{G_e - G_m}{1 - \frac{q^2}{4M^2}}\right)\right] u(p_1)$$

The expression for the total cross section can be written as

$$\sigma = \frac{\alpha^3}{16E^2}\int_{t_{min}}^{t_{max}} \frac{dt}{t^2}\int_{u_{min}}^{u_{max}} \frac{du}{u-t}\int_{x_{min}}^{x_{max}} dx \int_0^{2\pi} \frac{d\varphi}{2\pi}\sum_{spins}\left|m_1 + m_2 + m_3\right|^2 \tag{4}$$

where φ is the azimuthal angle of w^+ meson with respect to the plane defined by the incident photon and proton momenta in the rest frame of the combined $w^{\pm}$ pair;

$$x = (kw^-),\ x_{\substack{max\\min}} = \frac{u - t}{2}\left[1 \pm \left(1 - \frac{2m^2}{u}\right)^{\frac{1}{2}}\right];$$

$$u = \frac{(w^+ + w^-)^2}{2},\ u_{max} = \left(1 + \frac{E}{M}\right)t + \frac{E}{M}\left[-t(2m^2 - t)\right]^{\frac{1}{2}},\ u_{min} = 2m^2;$$

$$t = \frac{q^2}{2},\ t_{\substack{max\\min}} = \frac{-ME^2 + 2m^2(E + M)}{M + 2E} \pm \frac{E}{M + 2E}\left[M^2E^2 - 4m^2M(M + E) + 4m^4\right]^{\frac{1}{2}}$$

where E is the laboratory photon energy, M and m the proton and boson mass respectively. The expression for the square of the matrix elements after summation over the final boson polarizations and averaging over the initial photon polarizations was calculated independently by both authors and complete agreement on the entire differential cross section has been obtained. The expression has several hundred terms and is too long to be given here. In Eq. (4) the azimuthal angle of the recoil proton has been already integrated. The subsequent φ integration is trivial. The remaining three integrations were performed with the assistance of an IBM 7090. For the determination of the cross section, we have used the proton and neutron form factors of Hand, Miller, and Wilson.[5] The range of momentum transfer in the numerical cases considered here is such that values of q^2 are needed beyond the presently measured region. However because of the presence of q^{-4} term in the differential cross section, the value of the total cross section for the process (1) will not depend too sensitively on the presently unknown high momentum transfer region of the form factors for the ranges of parameters chosen here.

The result for the total cross section for process (1) is given in Table I. Each value in the table uses on the average 3 minutes of machine time and the accuracy of the integration is 1%. The very great sensitivity of the cross section to the value of the magnetic moment is readily seen for the cases considered here.

It is of further interest to investigate the dependence of the w pair cross section on an assumed non-zero value for the anomalous quadrupole moment. Since the cross section is quite sensitive to the magnetic moment it may also show great sensitivity to an anomalous quadrupole moment. Furthermore if the anomalous moment is not zero (to first order in α) then the problem of separating the magnetic moment contributions from the anomalous quadrupole contribution may require detailed studies of the energy and angle dependences of the cross section. These problems are now under investigation.[12]

Because of the large background of electron and μ pairs it is our opinion that the most feasible method for detecting the process is by an e, μ, p triple coincidence; the e and μ each coming from one of the w's respectively. The detection of the recoil proton is necessary in establishing the identifying kinematics as well as establishing that the final state was unaccompanied by pion production. In Fig. 2 we show some typical differential cross sections as a function of q^2 (all other variables having been integrated out). Note that q^2 is directly proportional to the laboratory kinetic energy of the recoil proton $q^2 = -2M(E_2 - M)$. We observe that the energy distribution of the recoil proton has quite a sharp peak. The position of this peak moves closer to the origin as one increases the incident photon energy and decreases the boson mass. From Fig. 2 we see that in the cases considered here the main contributions to the cross section come from values of momentum transfer where the proton form factors are quite well known.

If a nucleus is used as a target instead of hydrogen, then in principle one can calculate the pair production cross section exactly within the one photon exchange approximation provided the elastic and inelastic form factors from electron nucleus scattering are given. However these form factors are in general unknown except for the elastic charge form factor at $|\vec{q}|$ less than 400 MeV which corresponds approximately to the 2nd minimum in the elastic form factor. For coherent process (2), we have considered as an example the element iron with $Z = 26$. To calculate the cross section we used Eq. (4) with $M = 52$ BeV, $G_m = 0$ $G_e = Z$ multiplying the Fourier transform of fermi charge distribution as given by Hofstadter et al.[7] The form factor has minima at $|\vec{q}| = .21, .36, \ldots$ BeV/c. Experimentally the form factor is known to within approximately 10% up to about the 2nd minimum. For high boson masses and low incident photon energy, the contribution to the cross section comes entirely from the unmeasured region of the form factor. However in such cases the contribution from the coherent process is completely negligible compared with incoherent processes. Table II shows the result for the coherent cross section.

Since the inelastic form factors of nuclei are completely unknown at present, one can only guess the magnitude of the total cross section from nuclei. In general the inelastic nuclear form factors contain excitation of nuclear levels, quasi-elastic region and meson production region. The excitation of individual nuclear levels is completely negligible compared with the latter two regions at the momentum transfer considered here. The contribution from the quasi-elastic region can be approximated by a factor Z multiplying the free proton cross section plus a factor $(A-Z)$ multiplying the free neutron cross section with an appropriate fermi suppression factor. We have computed the free neutron cross section at $E = 10$ BeV, $m_W = 0.8$ BeV and $\kappa = 0$, and 2. In both cases the numerical values are approximately 30% of the corresponding free proton cross section. The suppression of low momentum transfer events in the quasi-elastic region due to Pauli exclusion principle can be estimated by the Fermi gas model. This effect can decrease the incoherent cross section as much as 30% in our numerical examples for the cases where the incident photon energy E is large and the boson mass small.

In view of the uncertainties in form factors and nuclear physics we believe that the best estimate for the cross section in complex nuclei is a factor Z times the free proton cross section except in regions where the coherent process dominates the proton cross section.

The results here may be compared with those of Wu et al[8] who have made a similar calculation. These authors do not use the most recent values of the proton and neutron form factors for process (1) but rather use the older values of Hofstadter[9] et al up to value of $q^2 = (1\ \mathrm{BeV})^2$ and for $q^2 > (1\ \mathrm{BeV})^2$ use the arbitrary extrapolation that $F = 0.4$ and $F_2 = 0$. For the form factors used in this work we find cross sections which are a factor 1.5 to 10 times smaller than those of Wu et al. If we use the same form factors as used by Wu et al then our results agree with these authors within 0.5%. The different results which come from the different choice of form factors indicate some sensitivity of the cross section to the largest values of the momentum transfer. If one uses the even older values of the proton form factors of Hofstadter[10] et al where $F_1 = F_2$ then we find the results given here are changed only by approximately 10%. Thus we see that it is the core term in F_1 used by Wu et al which is responsible for the main difference between our results. The most recent form factors[6] indicate that there is no core term as large as that used by Wu et al and therefore we conclude that those authors have given a cross section which is somewhat on the high side.

Similar differences between the results quoted here and those of Wu et al occur in the coherent cross section and these are again due to the special form factor chosen by these authors rather than the measured iron form factor.[11]

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   R. P. Feynman, Phys. Rev. 109, 193 (1958).

   T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960).

2. T. D. Lee and C. N. Yang, Phys. Rev. 128, 885 (1962).

3. T. D. Lee, Phys. Rev. 128, 899 (1962).

4. By no anomalous quadrupole moment we mean that the coupling between photons and vector bosons is given by equations (3) and (5) if reference 3 and that the Lagrangian contains no terms of the form

$$Q\left[w^{+}_{\mu\nu}\, w_{\lambda} - w^{+}_{\lambda}\, w_{\mu\nu}\right]\partial_{\lambda}F_{\mu\nu}$$

   where $F_{\mu\nu}$ is the electromagnetic tensor and $w_{\mu\nu}$ is the analogous tensor formed from the boson field w_{μ}.

5. The metric $(ab) = a_o b_o - \vec{a}\cdot\vec{b}$ is employed. The four momenta and polarization of the photon, positively (negatively) charged boson are respectively k, e ; w^{+}, φ^{+}; w^{-} φ^{-}. The momentum transfer $q = p_1 - p_2$ where p_1 and p_2 are the initial and final momenta of the proton. The total magnetic moment is $1 + \kappa$, Bohr magnetons $\hbar = c = 1$ and $\alpha \approx \frac{1}{137}$.

6. L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. 35, 335 (1963).

7. We used the expression of the Fourier transform of the Fermi distribution given by R. Blankenbecler, Am. J. Phys. 25, 279 (1957) Eq. (7) with parameters given by R. Hofstadter, Ann. Rev. Nucl. Sci. 7, 231 (1957) Eq. (192).

8. A.T.C. Wu, T. T. Wu and K. Fuchel; to be published in Phys. Rev. Letters.

9. R. Hofstadter, F. A. Bumiller and M. G. Croissiaux, Phys. Rev. Letters 5, 263 (1960).

10. R. Hofstadter, F. A. Bumiller and M. R. Yearian, Rev. Mod. Phys. 30, 482 (1958).

11. We can compare the coherent cross section here with the results obtained by Bludman and Young (S. Bludman and T. Young, Phys. Rev. 126, 303 (1962) who used the Weizsacker-Williams approximation which supposedly applies in the asymtotic region $m_W/E_\gamma \gg 1$. For the values $m_W = .8$ BeV and $E_\gamma = 20$ BeV, our results are approximately an order of magnitude smaller than those of Bludman and Young.

12. Private communication from J. Mathews, California Institute of Technology.

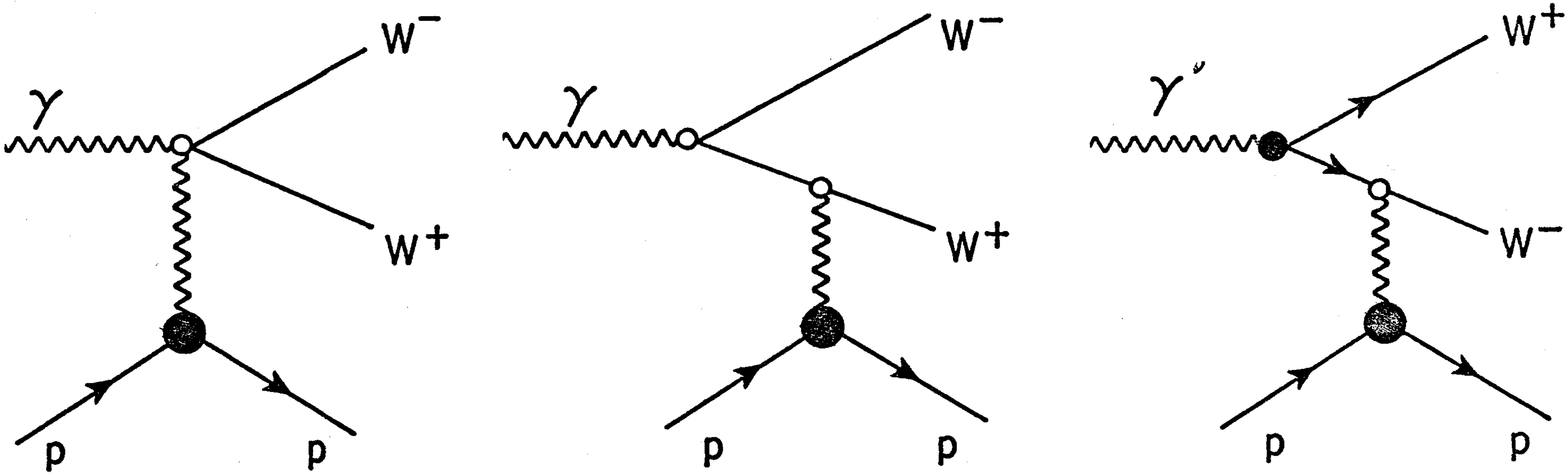
FIGURE CAPTIONS

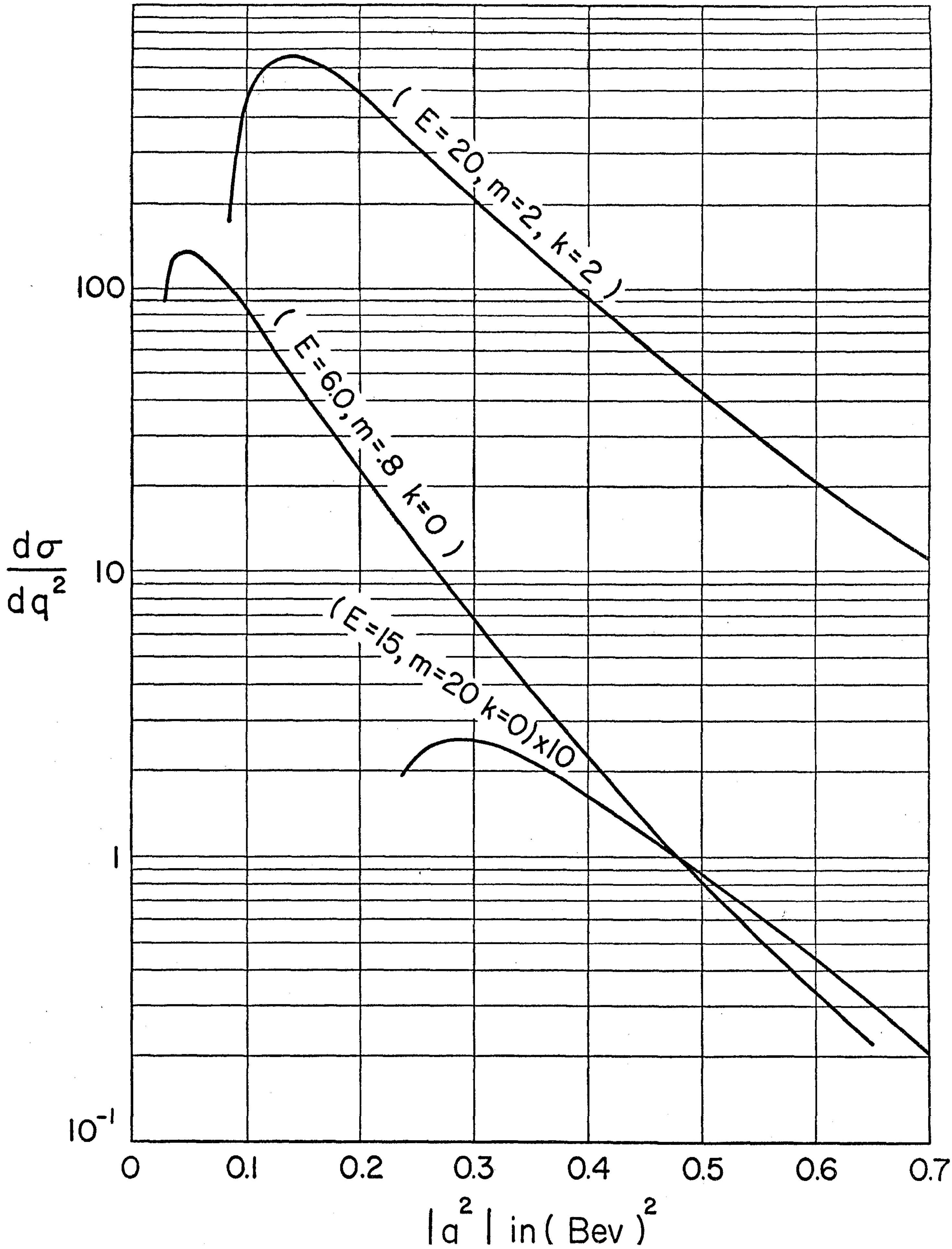
Table 1 Numerical values of the cross section for the process $\gamma + p \rightarrow w^- + w^+ + p$ using the proton form factors of reference 6. The values in parenthesis are obtained using the old values given in reference 9.

Table 2 Numerical values of the cross section per proton ($\sigma_{coherent}/26$) for the coherent process $\gamma + F_e \rightarrow w^- + w^+ + F_e$.

Fig. 1 Feynman diagrams for w-pair production.

Fig. 2 The differential cross section $(d\sigma/dq^2)$ as a function of q^2 for three sets of parameters.


γ
W⁻
W⁺
p
p
γ
W⁻
W⁺
p
p
γ
W⁺
W⁻
p
p



(E = 20, m = 2, k = 2)
(E = 6.0, m = .8 k = 0)
(E = 15, m = 20 k = 0) x 10
100
10
1
10^-1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
dσ/dq^2
|q^2| in (Bev)^2

Table I

κ	Photon Energy (BeV)	m(BeV)			
		0.8	1.0	1.4	2.0
2	6.0	$17.8 \times 10^{-34} cm^2$	$17.5 \times 10^{-35} cm^2$		
	10.0	68.0	191.0	$43.4 \times 10^{-36} cm^2$	
	15.0				$13.5 \times 10^{-37} cm^2$
	20.0	222.0	704	860	241
1	6.0	2.27	2.27		
	10.0	8.54	19.5	5.77	
	15.0				1.81
	20.0	27.0	88.5	113.0	32.9
0	6.0	.324 (.357)	.284		
	10.0	1.56	2.99	.728	
	15.0				.225
	20.0	6.90	18.8	18.2	4.27
-1	6.0	1.14	.910		
	10.0	5.74	9.92	2.07	
	15.0				.582
	20.0	27.1	67.4	56.7	112.0
-2	6.0	10.3 (11.6)	8.32		
	10.0	48.3	86.1	18.3	
	15.0				5.01
	20.0	210.0	539	478	96.6

TABLE III

κ	Photon Energy (BeV)	m(BeV) 0.8	1.0	1.4	2.0
2	6.0	$0.342 \times 10^{-35} cm^2$	$0.0319 \times 10^{-36} cm^2$		
	10.0	8.65	3.85	$.260 \times 10^{3}\ cm^2$	
	15.0				$0.257 \times 10^{-40} cm^2$
	20.0	735.0	366.0	235	11.2
1	6.0	.0488	0.00462		
	10.0	1.20	.542	.0382	0.00381
	15.0				
	20.0	101.0	51.2	33.0	1.62
0	6.0	0.00681	0.000669		
	10.0	0.171	.0742	0.00554	
	15.0				0.00558
	20.0	14.5	7.12	4.50	0.225
-1	6.0	.0160	0.00149		
	10.0	.435	.178	0.0118	
	15.0				.0114
	20.0	38.1	17.4	10.7	.488
-2	6.0	.130	0.0120		
	10.0	3.60	1.47	.0935	
	15.0				.0877
	20.0	316.0	143.0	88.5	3.92