LM-60

Classification of Mesons in a Compound Model(*)

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Summary.--Mesons are considered as bound states of antibaryon-baryon pairs. The bound states of lowest quantum numbers correspond to the observed mesons, to the extent that the quantum numbers of the latter are known. In the model it is possible to have mesons of zero strangeness which are not eigenstates of G parity.

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1.—Introduction

Many authors have considered models in which mesons are treated as bound states of nucleons and antinucleons[1-5]. This idea is especially attractive for two reasons. First, a number of heavy mesons, the ρ, ω, η, K and K*, have been discovered whose existence can be qualitatively understood in terms of a bound state model. Second, evidence from the nucleon-nucleon interaction is consistent with the point of view of such a model. We also mention that a compound model of the mesons fits into a picture suggested by Chew and Frautschi[6] in which there are no elementary particles.

It is convenient to consider first how the nucleon-nucleon interaction bears on the question of the composite nature of the mesons. It has been assumed by several authors[3,7,8] that the gross features of the nucleon-nucleon scattering amplitude (at least in the higher partial waves) can be accounted for by the exchange of single mesons of various kinds. With this assumption, to get qualitative agreement with experiment, the coupling constant g_ω between the ω-meson and the nucleon must be made quite large, for example, larger than the coupling between the ρ-meson and the nucleon. The value of g_ω depends in detail on the assumptions, and different authors obtained different numbers. We merely quote one such value, $g_\omega^2/4\pi \approx 30$, obtained by Wong[8].

Now a vector meson like the ω contributes to a short range repulsion in nucleon-nucleon (NN) states, but it leads to an attraction in states of an antinucleon-nucleon ($\bar{N}N$) pair. Since g_ω is large, it is reasonable that this attractive $\bar{N}N$ interaction can lead to bound states of the $\bar{N}N$ system[9]. One of these bound states might be the ω itself. Thus we can have a bootstrap mechanism to account for the ω, in which an $\bar{N}N$ pair is bound by the exchange of bound $\bar{N}N$ pairs[10].

We can test the hypothesis that the ω is solely responsible for bound $\bar{N}N$ states. Since the ω has isospin $I = 0$, the binding energy of an $\bar{N}N$ state should be independent of whether the pair is in an $I = 0$ or $I = 1$ state. Thus, if we observe an $I = 0$ ($I = 1$) meson, we should expect also to see an $I = 1$ ($I = 0$) meson of the same mass and with the same spin and parity. However, the G parity, which is related to the isospin, must be opposite for the $I = 0$ and $I = 1$ mesons.

Some experimental information on the mesons is listed in Table I. From Table I, we see that the existence of the ω and ρ with similar masses supports the viewpoint that an $I = 0$ meson is responsible for the binding. On the other hand, the $I = 0$ counterpart of the pion has not yet been seen unless it is the η, a particle with a much larger mass than the π. Thus, the rule breaks down. This is not surprising, since the exchange of $I = 1$ mesons (the π and ρ for example) break the symmetry. Nevertheless, if the ω coupling is dominant, for every $I = 0$ meson, there should exist an $I = 1$ counterpart (of different mass) with the same spin and parity and opposite G parity. The experimental information in Table I is too meager to test this prediction.

2.—Antinucleon-nucleon states

If the attractive $\bar{N}N$ interaction arises primarily from ω exchange, it must be of short range. Therefore, it should be most effective in leading to bound $\bar{N}N$ states with orbital angular momentum $L = 0$ because of the absence of a centrifugal barrier in these states. Although the orbital angular momentum is not always a good quantum number in an $\bar{N}N$ state, we shall treat it as such because of the centrifugal barrier. There are four possible $\bar{N}N$ states with $L = 0$. The parity P and G parity of these states are specified, since for an $\bar{N}N$ state we have

$$P = (-1)^{L+1}, \quad G = (-1)^{L+I+S} \tag{1}$$

where S is the spin of the $\bar{N}N$ pair. The quantum numbers of the $L = 0$ states are listed in Table II. Three of the states correspond to well-known particles, the π, ρ and ω. It remains to be seen whether the fourth is in fact the η.

We next consider the quantum numbers of the $L = 1$ states of $\bar{N}N$; these are listed in Table III. There is not much experimental evidence as to whether mesons with these quantum numbers exist. If any do exist, however, we can guess the ordering of their masses by looking at the spin-orbit interaction. The spin-orbit interaction arising from the exchange of a single ω is intrinsically repulsive in an $\bar{N}N$ state; i.e., the sign of the quantity which multiplies $\vec{L} \cdot \vec{S}$ is positive. Then the magnitude of the expectation value $< \vec{L} \cdot \vec{S} >$ in a particular $\bar{N}N$ state is roughly proportional to the strength of the spin-orbit interaction[11], and its sign tells whether the interaction is repulsive (positive sign) or attractive (negative sign). The expectation values $< \vec{L} \cdot \vec{S} >$ for $\bar{N}N$ states with $L = 1$ are given in Table III.

We see from Table III that the most negative (attractive) interaction with $L = 1$ is in states of total angular momentum $J = 0$. Thus, if any of the states of Table III correspond to mesons, the $J = 0$ states are most likely. Possible candidates are the ABC particle[12] (see Table I) with quantum numbers 00^{++} (IJ^{PG}) and the ζ with $I = 1$, other quantum numbers unknown. From Table III we predict that the ζ (if it is more than a statistical fluctuation) has quantum numbers 10^{+-}.

Another particle for which there is some experimental evidence[13] has mass 625 Mev, $I \geqslant 1$, and decays into three pions (and perhaps other things). There is plenty of room for this particle in Table III. If we take the spin-orbit interaction argument seriously and say this particle is the lowest energy state (which can decay into three pions) not yet occupied by a meson, its quantum numbers are 11^{+-}. Alternatively, Bég and DeCelles[14] have suggested that this particle has quantum numbers 10^{--}. If so, it would be interpreted in our model in terms of a second bound state with these quantum numbers (the pion being the first). If second bound states of any of the other particles exist, presumably they would be much higher in energy.

3.—Other antibaryon-baryon states

Sakurai[3] has suggested that the ω is the quantum of a conserved vector current associated with the conservation of baryon number. If so, the coupling of the ω to all baryons should be strong and lead to additional bound antibaryon-baryon ($\bar{B}B$) states if bound $\bar{N}N$ states exist. However, not all possible $\bar{B}B$ states should lead to new mesons. This is because it is a simplification to speak of a meson as a bound $\bar{N}N$ pair. The numbers of nucleons and antinucleons are not separately conserved in such a system. A meson which is primarily a bound $\bar{N}N$ pair should have components in its wave function which correspond to $\bar{\Lambda}\Lambda$, $\bar{\Sigma}\Sigma$, $\bar{\Xi}\Xi$, and to many-particle states of still higher energy with the same quantum numbers. Thus, in looking for additional mesons we need consider only $\bar{B}B$ states which have quantum numbers which cannot be reached in the $\bar{N}N$ system.

It is convenient to divide $\bar{B}B$ states into two categories depending on whether the strangeness $\mathcal{S}$ is zero or not. Consider strangeness zero combinations first. There are no states of $\bar{\Lambda}\Lambda$ or $\bar{\Xi}\Xi$ which lead to different quantum numbers from the possible $\bar{N}N$ states[15]. States of $\bar{\Sigma}\Sigma$ exist, however, with $I = 2$. Their quantum numbers, other than isospin, are the same as the quantum numbers of the $I = 0$ states listed in Tables II and III.

Now consider possible bound states of $\bar{\Lambda}\Sigma$ and $\Lambda\bar{\Sigma}$ pairs. Such states are not eigenfunctions of G, and therefore it is possible in the model to produce mesons of zero strangeness which are mixtures of states of opposite G. In general, states which are not eigenfunctions will be characterized by non-unique masses and lifetimes. For this reason it is convenient to construct linear combinations of such mesons which are eigenfunctions of G with even and odd G parity respectively. To the extent that G is conserved in the decays of such mesons, these eigenfunctions will have unique masses and lifetimes. If the mass splitting between a state of even and odd G is caused only by the decay interaction, the mass difference should be of the same order of magnitude as the decay width of the meson with the shorter lifetime.

We shall classify $\Lambda\bar{\Sigma}$ and $\bar{\Lambda}\Sigma$ states in terms of the eigenfunctions of G. The classification also depends on the parity of the Σ[16]. In the case of even Σ parity, half of these states merely duplicate possible $\bar{N}N$ states; the other half lead to the states of Tables II and III ($I = 1$ states only), but with opposite G from the assignments in the tables. In the odd Σ parity case, the states do not duplicate the possible $\bar{N}N$ states; but lead to the $I = 1$ states of Tables II and III with opposite parity and both positive and negative G. There is no definite experimental evidence for any of these states.

There are still other possible $\bar{B}B$ states of zero strangeness. To cite only one example we mention states of $\overline{N^*}N^*$, where N^* is the (33) resonance at a total energy of 1240 Mev. Since the N^* has isospin and angular momentum equal to 3/2, there are possibilities for additional bound states with $I \leqslant 3$ and $J \leqslant 3$. It is easy to construct a table of the quantum numbers of such states, but we shall not do so here.

Now consider states with strangeness not equal to zero. There are two such states of $\bar{\Lambda}N$ corresponding to $L = 0$, and they are listed in Table IV.

We have tentatively identified these particles with the K and K*, although the parity of the K and the quantum numbers of the K* have not been definitely determined. States with L = 1 are also listed, although there is not much evidence for mesons with any of these quantum numbers.

Turning to states of $\bar{\Sigma}N$, we again have two situations depending on whether the Σ parity is even or odd. For even Σ parity, the $I = 1/2$ states of $\bar{\Sigma}N$ have the same quantum numbers as the $\bar{\Lambda}N$ states listed in Table IV. Thus, the lowest energy $I = 1/2$ states will in general be linear combinations of the $\bar{\Lambda}N$ and $\bar{\Sigma}N$. On the other hand, if the Σ parity is odd, we have additional $\bar{\Sigma}N$ $I = 1/2$ states with the same quantum numbers as those in Table IV except for opposite parity.

The $I = 3/2$ states of $\bar{\Sigma}N$ do not duplicate $\bar{\Lambda}N$ states. For even Σ parity these states have the same quantum numbers (other than isospin) as in Table IV. (For odd Σ parity, the parity assignments in Table IV should be reversed.)

The possible bound states of $\bar{\Xi}N$ are likely to be more massive than the $\bar{\Sigma}N$ states which have not been seen. For a Ξ with even parity, these states have the same quantum numbers as those listed in Tables II and III except that $\mathcal{S} = 2$ and G is not a good quantum number. The effect of odd Ξ parity is merely to reverse the parity assignment in the tables.

4.—Discussion

It is apparent that a compound model contains within it the possibility for a rich supply of mesons. The fact that only a few of these possibilities have been seen suggests that most of them are either very massive or not bound at all.

We have already suggested that the centrifugal barrier in states with $L \neq 0$ will limit the number of mesons. It is also reasonable that the larger the sum of the rest masses of a $\bar{B}B$ pair the more massive will be the bound states. This rule is in qualitative agreement with experiment. For example, the pion (the lightest meson) has the same quantum numbers as a possible state of $\bar{N}N$, the lightest $\bar{B}B$ pair. As another example, the lowest mass state of $\bar{\Sigma}N$ having different quantum numbers from a $\bar{\Lambda}N$ state has $I = 3/2$ in the case of even Σ parity. Such a meson, We have tentatively identified these particles with the K and K*, although the parity of the K and the quantum numbers of the K* have not been definitely determined. States with $L = 1$ are also listed, although there is not much evidence for mesons with any of these quantum numbers.

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Since all but a few of these bound states should have large masses, we have a natural way to limit the number that can be easily observed. The larger a meson's mass, the less frequently it should be produced relative to background events, since the number of possible reactions is a rapidly increasing function of energy. Furthermore, with increasing mass, the width of a state should increase as the number of possible decay modes increases. As there should be a large number of high-mass states, it may be that their widths are larger than the average "level spacing." In these circumstances, these states will be extremely difficult to detect.

Confining ourselves to the lowest mass states, bound states of $\bar{N}N$ and $\bar{\Lambda}N$, we have just the Sakata model(2). If we add the further restriction $L = 0$, we obtain an octet (counting each charge state) of pseudoscalar mesons and an octet of vector mesons. These are the same mesons as those predicted in the eightfold way version of the unitary symmetry model(4). Also, the mesons for which there is the best experimental evidence may be just these $L = 0$ bound states as remarked by Heisenberg(17). The quantum numbers of the π, ρ and ω, and probably K are in agreement with the predictions of the model, while the quantum numbers of the η and K* are not yet known.

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(FOOTNOTES AND REFERENCES)

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(11)The proportionality is not exact because of other velocity dependence of the interaction.

(12)The ABC may not be a resonant state but a large non-resonant pion-pion interaction.

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(14)M. Bég and P. DeCelles, Phys. Rev. Lett. 9, 30 (1962).

(15)We are assuming that the equation $P = (-1)^{L+1}$ holds for any system of a given baryon and its antiparticle.

(16)We assume the Λ parity is positive.

(17)W. Heisenberg, Intl. Conf. on High Energy Phys. (CERN, 1962). See also the comment (last paragraph) of A. Rosenfeld, D. Carmony, and R. Van de Walle, Phys. Rev. Lett. 8, 293 (1962).

TABLE I.-Some mesons reported in the literature. The quantum numbers specifying a meson are its isospin I, spin J, parity P, G-parity, and strangeness $\mathcal{S}$. A meson with $\mathcal{S} \neq 0$ is not an eigenfunction of G. A question mark after the name of a meson indicates that it has not been definitely established; i.e., the experimental evidence may result from interference phenomena, a nonresonant interaction or simply a statistical fluctuation.

Meson	Mass (Mev)	I	J	P	G	$\mathcal{S}$
π	139	1	0	-	-	0
η(a)	550	0	0 ?	- ?	+ ?	0
ρ(b)	750	1	1	-	+	0
ω(c)	780	0	1	-	-	0
ABC(d)?	310	0	0	+	+	0
ζ(e)?	570	1	?	?	?	0
PRS(g)?	625	$\geqslant 1$	?	?	?	0
ρ'(f)?	720	?	?	?	?	0
K	495	$\frac{1}{2}$	0	?		1
K*(h)	885	$\frac{1}{2}$	1 ?	- ?		1
K'(i)?	730	$\frac{1}{2}$?	?	?		1
$K\bar{K}$(j)?	1000	0	0	+	+	0
(k)						

(Footnotes to Table I)

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TABLE II.-Quantum number of mesons predicted by bound state model of $\overline{N}N$ pair with orbital angular momentum $L = 0$.

Meson	I	J	P	G
π	1	0	-	-
η(*)	0	0	-	+
ρ	1	1	-	+
ω	0	1	-	-

(*)The quantum numbers of the η have not yet been experimentally determined.

TABLE III.-Quantum numbers of $\overline{N}N$ system with orbital angular momentum $L = 1$. Here S denotes the spin of the $\overline{N}N$ pair and $< \vec{L} \cdot \vec{S} >$ denotes the expectation value of the spin-orbit operator.

Meson(*)	I	J	P	G	S	$< \vec{L} \cdot \vec{S} >$
ζ	1	0	+	-	1	-2
ABC	0	0	+	+	1	-2
PRS	1	1	+	-	1	-1
	0	1	+	+	1	-1
	1	1	+	+	0	0
	0	1	+	-	0	0
	1	2	+	-	1	1
	0	2	+	+	1	1

(*)These assignments have not been verified.

TABLE IV.-Quantum numbers of possible states of the $\bar{\Lambda}N$ system with $L = 0$ and $L = 1$. For all of these states, we have $\vec{I} = 1/2$ and strangeness $\mathcal{S} = 1$. The $\bar{\Sigma}N$ states depend on the Σ parity and are discussed in the text.

Meson	L	J	P	S	$\langle \vec{L} \cdot \vec{S} \rangle$
K	0	0	-	0	0
K* ?	0	1	-	1	0
	1	0	+	1	-2
	1	1	+	1	-1
	1	1	+	0	0
	1	2	+	1	1