SLAC/AP–67
February 1988
(AP)

On the Luminosity of Heteroenergetic Colliding-Beam Storage Rings*

JOHN REES

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

In the electron-positron colliding-beam storage-ring systems built so far, the two colliding beams have had the same energies. Consequently, the center-of-mass of the colliding system is at rest in the laboratory. Having the center-of-mass stationary in the laboratory has virtues that have been well exploited in the design and operation of detectors for these machines; however, it has also some disadvantages.

An important example is the case of the decay of a pair of B–mesons which were produced by the decay of the 4S state of the Υ meson. (The Υ was produced at rest by the colliding beams.) The B's have very little momentum in the c-m system. Most of the B–factories presently being contemplated are designed to collide beams of equal energy so that the center-of-mass is stationary. Although the time that elapses between the decay of one B and the other is, on the average, 10^{-12} seconds, the B's move so little that one cannot distinguish the two decay vertices. As a result one cannot make a direct measurement of the lifetime and furthermore he is hard put even to sort out and separate the products of the two decays.

If we collide two beams of unequal energy, the center-of-mass will move and so will the decaying B's. We can see the vertices separately and reconstruct the events with greater certainty. For these reasons we have begun studying the design of heteroenergetic colliding-beam systems. In particular, we have

*Work supported by Department of Energy contract DE–AC03–76SF50015.

concentrated our studies on a pair of storage rings, one operating at 12 GeV and the other at 2 GeV, with their beams being brought into collision in a shared straight section. We envisage PEP as the 12 GeV ring, and we suppose that we shall build a new high-current 2 GeV ring. This choice of energies sets the center-of-mass in motion with a velocity, $\beta = 0.7$, and the decay paths of the mesons can be seen in the laboratory.

In the following paragraphs, we shall estimate how the luminosity of a heteroenergetic colliding-beam storage-ring system is likely to be restrained by the incoherent beam-beam limit — or, in other words, by the tune-shift limit.

To establish our notation, let us number the two storage rings #1 and #2, and take #1 to be the higher-energy ring. When these numbers appear in subscripts, they will identify the ring to which the subscripted quantity applies. Thus, E_1 is the beam energy of ring #1 and β_{x2} is a horizontal beta function in ring #2, for example. We shall use f_B to denote the frequency at which bunches pass the interaction region and I to denote average current. The symbols e and r_e represent the charge and classical radius of the electron, and γ always signifies the energy in units of rest mass.

We begin by stating certain assumptions we shall make *ab initio.* We assume that whenever a bunch from one ring passes through the Interaction Region (IR), a bunch from the other ring does too. For the big ring, this just means that we are not storing any bunches that do not contribute to the luminosity, certainly a reasonable assumption. This first assumption implies that

$$f_{B1} = f_{B2} = f_B \quad . \tag{1}$$

Our second assumption is that the two beams are each flat as they collide. Their heights are much less than their widths at the interaction region:

$$\sigma_{y1} \ll \sigma_{x1}; \qquad \sigma_{y2} \ll \sigma_{x2} \quad . \tag{2}$$

And our third assumption is that the bunches collide head-on, not at an angle.

The vertical tune-shift parameters for the two rings of the heteroenergetic system are

$$\xi_1 = \frac{r_e \beta_{y1} I_2}{2\pi e f_B \gamma_1 \sigma_{x2} \sigma_{y2}} \quad , \tag{3}$$

$$\xi_2 = \frac{r_e \beta_{y2} I_1}{2\pi e f_B \gamma_2 \sigma_{x1} \sigma_{y1}} \quad . \tag{4}$$

For the case of two beams in a single storage ring (e^+ and e^-), the values of most of the variables in these expressions are the same for both beams. Only I_1 and I_2 are different and in practice they are not very different. The rule for single storage rings is that neither ξ_1 nor ξ_2 may grow larger than some value between 0.02 and 0.07, depending on the storage ring. This rule characterizes the limiting performance of the machine. It is called the tune-shift limit.

For the more general case we are dealing with (the case of unequal beam energies, beta functions, beam dimensions, damping times, etc.), we do not know *a priori* what values of ξ_1 and ξ_2 may be permitted; we have no experimental evidence.* Nevertheless, we can express the luminosity in terms of these parameters, and we may anticipate that they will serve as useful performance-limiting parameters as they have before.

The luminosity of the system is given by the formula

$$\mathcal{L} = \frac{I_1 I_2}{4\pi e^2 f_B \sigma_{xrms} \sigma_{yrms}} \quad , \tag{5}$$

where

$$\sigma_{xrms} = \sqrt{\frac{\sigma_{x1}^2 + \sigma_{x2}^2}{2}} \quad \text{and} \quad \sigma_{yrms} = \sqrt{\frac{\sigma_{y1}^2 + \sigma_{y2}^2}{2}} \quad . \tag{6}$$

---

* Double-ring systems have been built and have operated: the Princeton–Stanford $e^- e^-$ rings at Stanford, VEP–1 at Novosibirsk, Doris at DESY and DCI at Orsay. Their performance has sometimes been disappointing, but tune-shift parameters up to 0.03 have been attained. On the other hand, the energies, beta functions, damping times, etc., of the two beams in these cases were very nearly the same.

We may eliminate I_1 and I_2 in favor of ξ_1 and ξ_2:

$$\mathcal{L} = \frac{\pi\gamma_1\gamma_2 f_B \xi_1 \xi_2 \sigma_{x1}\sigma_{y1}\sigma_{x2}\sigma_{y2}}{r_e^2 \beta_{y1}\beta_{y2}\sigma_{xrms}\sigma_{yrms}} \quad . \tag{7}$$

To elucidate the dependence of the luminosity on the sizes and shapes of the beams at the IR, we introduce as new variables the ratios of the heights and widths of the two beams:

$$p = \sigma_{x2}/\sigma_{x1} \quad \text{and} \quad q = \sigma_{y2}/\sigma_{y1} \quad . \tag{8}$$

In terms of these variables

$$\mathcal{L} = \frac{2\pi\gamma_1\gamma_2 f_B \xi_1 \xi_2 \sigma_{x1}\sigma_{y1}}{r_e^2 \beta_{y1}\beta_{y2}} \frac{p}{\sqrt{1+p^2}} \frac{q}{\sqrt{1+q^2}} \quad . \tag{9}$$

From this expression we learn that we should make the area of beam #1 as large as we can and we should make both vertical beta values as small as we can. Having done that, we consider p and q. The bigger the better, but very large values of p and q do not help much. Clearly we want the beam in ring #2 to be at least as large as the beam in ring #1, *i.e.*, $p = q = 1$. But once it is that large, making it still larger does not do much more good. Only a factor of two remains to be gained. (We are implicitly assuming that ξ_1 and ξ_2 are independent of p and q. More on that point later.)

Another instructive way of looking at the question of how to choose p and q is to consider the current I_2. Ring #2 has the lower energy beam and beam instabilities are likely to trouble us in that ring. It may be sensible for us to strive to attain the highest luminosity for the lowest current in that ring. For that purpose we should consider the quantity

$$\frac{\mathcal{L}}{I_2} = \frac{\gamma_2 \xi_2 \sigma_{x1}\sigma_{y1}}{e r_e \beta_{y2}} \frac{1}{\sqrt{1+p^2}} \frac{1}{\sqrt{1+q^2}} \quad . \tag{10}$$

That function clearly favors small values of p and q.

On the whole then, for purposes of deriving an approximate scaling law for luminosity, we shall not go too far wrong if we choose

$$p = q = 1 \quad . \tag{11}$$

This is a good compromise between the dictates of Eqs. (9) and (10). What it describes is matched beam shapes — matched in the sense that the two beams just overlap each other laterally. With that choice, we can drop some subscripts:

$$\sigma_{x1} = \sigma_{x2} = \sigma_x \quad \text{and} \quad \sigma_{y1} = \sigma_{y2} = \sigma_y \quad . \tag{12}$$

Then, noting that

$$\gamma_{cm}^2 = 4\gamma_1\gamma_2 \quad , \tag{13}$$

we can write the luminosity

$$\mathcal{L} = \frac{\pi\gamma_{cm}^2 f_B \xi_1 \xi_2 \sigma_x \sigma_y}{4r_e^2 \beta_{y1} \beta_{y2}} \quad , \tag{14}$$

and this is our estimate of the luminosity of the heteroenergetic system.

From this formula we conclude that the attainable luminosity does not depend on the energies of the two separate beams; it depends only on the center-of-mass energy.

Some discussion of the reasoning is called for however. The terms in which we have chosen to couch the luminosity anticipate that the tune shifts will prove to be more convenient variables than the beam currents. This expectation is encouraged by our experience with the beam-beam limit in existing storage rings, which has taught us that the attainable tune shift is about the same in different machines having greatly different characteristics and parameters. On both theoretical and practical grounds, we expect the tune shift to be the appropriate variable to describe the approach to the luminosity limit.

Thus, in Eq. (9) we are tempted to regard the ξ's as quantities which may be increased to their allowed maxima and no further. Having so maximized them, we vary p and q. But how do we know that the maximum allowed values of the ξ's do not depend upon p and q? The answer is: we do not. Pending some study of the beam-beam interaction between beams of different energies, cross sections, damping times, etc., we can only speculate that the maximum values of the ξ's ought not to be very sensitive to the shape parameters, p and q. If that speculation is warranted, then our reasoning in choosing matched beam shapes is supportable.