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EQUIVALENT CIRCUIT ANALYSIS OF SLED*

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ABSTRACT

A direct application of equivalent circuit concepts leads to: (1) confirmation of Perry Wilson's SLED equation; (2) an equation that applies to a SLED device with input and output waveguides of different characteristic impedances; and (3) an equation that results if we demand that no power be lost by reflection from SLED. If the incident voltage is tailored as prescribed by this equation, the cavity voltage tracks the incident voltage and the reflected voltage is zero.

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1. Introduction

In this note, we show how the SLED equivalent circuit may be used to analyze SLED's dynamical behavior. In Chapter 1, we confirm the validity of Perry Wilson's equation;[1] in Chapter 2, we generalize the equation slightly, by assuming input and output waveguides of differering characteristic impedances; in the last Chapter, we attempt to find the conditions that the incident wave must satisfy if the reflected wave is to be zero.

The use of equivalent circuits to analyze the behavior of microwave devices has two great advantages: first, the statement of the problem is transparent and unambiguous; and, second, the physical processes at work are clearly represented. Of course, the equivalent circuit does not solve the field problem; rather, it is the other way around: after the solution of the field problem has been found, the result can be cast in the form of an equivalent circuit representation. This does not mean that the field problem must be solved completely before the equivalent circuit can be determined. In most cases, the *form* of the equivalent circuit can be fixed from general considerations; however, the specific numerical values belonging to the elements in the equivalent circuit must be obtained from experiment, or from a detailed solution of the field problem.

In the present case, we shall not discuss how the equivalent circuit is derived, but shall assume that this has already been done, with the result shown in Figure 1. Here, G_0 is the characteristic admittance of the line, G_c is the admittance of the cavity, C and L are the cavity capacitance and inductance, while I_c and V_c are the current and the voltage applied to the cavity. A generator, of voltage V_g, is connected to the left side of the line. Assuming it to be matched to the line, its internal admittance is the same as the characteristic admittance of the line, G_0.

The cavity parameters, G_c, C, and L, are determined in the following way. One chooses a plane at some point along the line, where the higher modes excited by the cavity discontinuity are negligible and only the fundamental mode is present. At this *reference plane*, various amplitudes of the incident voltage (and various frequencies) are assumed and the response of the cavity to each is measured (or calculated). This data permits the cavity parameters to be determined. These parameters vary as the reference plane is moved along the line, although any two points spaced by a half-wavelength will give the same cavity parameters. In what follows, we assume that a reference plane has been chosen and we assign $z = 0$ to the position of this reference plane.

2. Perry Wilson's Equation[1]

In this Chapter, we derive Perry Wilson's equation by a direct analysis of the equivalent circuit.

The incident and reflected voltage waves, evaluated at $z = 0$, are assumed to have the form $V_i exp(j\omega t)$, and $V_r exp(j\omega t)$, where V_i and V_r are slowly varying; that is, their change in a period $2\pi/\omega$ is negligible. Similarly, the cavity voltage and current are assumed to have the form $V_c exp(j\omega t)$, and $I_c exp(j\omega t)$, where V_c and I_c also are slowly varying.

The reference plane is merely a mathematical construct; there is no physical difference between the two sides of $z = 0$. Therefore, the current and voltage on either side of $z = 0$ are continuous:

$$V_i + V_r = V_c$$

$$G_0(V_i - V_r) = I_c$$

Here, the current due to V_i is G_0V_i; that due to V_r is $-G_0V_r$, since the reflected wave propagates in the negative z direction.

The circuit equation for the cavity is

$$I_c exp(j\omega t) = C\frac{d}{dt}(V_c exp(j\omega t)) + G_cV_c exp(j\omega t) + \frac{1}{L}\int_0^t dt'V_c' exp(j\omega t')$$

$$= G_0(2V_i - V_c)exp(j\omega t)$$

The last equation follows by eliminating V_r from the first two equations and solving for I_c. The V_c' in the integral is primed to emphasize that it is evaluated at t'.

The usual definitions of the unloaded Q, the loaded Q, and the resonant (angular) frequency, are

$$Q_0 = \frac{\omega_0 C}{G_c}$$

$$Q_L = \frac{\omega_0 C}{G_0 + G_c}$$

$$\omega_0^2 = \frac{1}{LC}$$

With these definitions, the cavity circuit equation becomes

$$\frac{dV_c}{dt} + j\omega V_c + \frac{\omega_0}{Q_L}V_c + \omega_0^2 exp(-j\omega t)\int_0^t dt'V_c' exp(j\omega t') = \frac{2\omega_0}{Q_0}V_i$$

Suppose the integral has n full periods, each $2\pi/\omega$, plus a fraction of a period that goes from $t' = n2\pi/\omega$ to $t' = t$. In each of the full periods, V_c' is approximately constant; only the exponential varies, and this, integrated over a complete period, yields zero. Hence, the integration over the n full periods is zero; the remaining integration, over the partial period from $t' = n2\pi/\omega$ to $t' = t$, integrates to

$$\frac{\omega_0^2}{j\omega}V_c(1 - exp(-j\omega t)).$$

When this is put into the circuit equation, we find:

$$\frac{dV_c}{dt} + \left(j\omega - j\frac{\omega_0^2}{\omega} + \frac{\omega_0}{Q_L}\right)V_c + j\frac{\omega_0}{\omega}exp(-j\omega t) = 2\frac{\omega_0}{Q_0}V_i$$

The rapidly-varying term, $exp(-j\omega t)$, is eliminated by averaging over a period $2\pi/\omega$; we then put $\omega = \omega_0$, $\omega_0/Q_L = 1/T_L$, and $\alpha = 2Q_L/Q_0$.

The result is Perry Wilson's equation:[1]

$$T_L\frac{dV_c}{dt} + V_c = \alpha V_i.$$

3. Different Input and Output Admittances

The case where the input and output waveguides have different characteristic admittances amounts to a simple modification of what has gone before. The equation expressing the continuity of current changes to

$$\begin{aligned} I_c &= G_0V_i - G_rV_r \\ &= (G_0 + G_r)V_i - G_rV_c \end{aligned}$$

where G_0 and G_r refer to the input and output waveguides, respectively.

Repeating our previous procedures, with the new definitions

$$Q'_L = \frac{\omega_0 C}{G_c + G_r}$$

$$T'_L = \frac{\omega_0}{Q'_L}$$

$$\alpha' = \frac{G_r + G_0}{G_r + G_c}$$

our new result is

$$T'_L \frac{dV_c}{dt} + V_c = \alpha' V_i.$$

One sees that the cavity decay time, T'_L, depends only on G_r and G_c; however, the coupling term, α', depends on these plus G_0. Making the input and output characteristic admittances different gives us another knob to turn in optimizing SLED.

4. No Reflected Wave

Is it possible to arrange things so that there is no reflected wave? To investigate this, let $G_r = G_0$ and set $V_r = 0$. Then, $V_c = V_i$, $I_c = G_0 V_i$, and the circuit equation, which can be written as a condition on either V_c or V_i, since they are equal, is

$$\frac{dV_i}{dt} + \left(j\omega - j\frac{\omega_0^2}{\omega} + \frac{G_c - G_0}{C} \right) V_i = 0.$$

Setting $\omega = \omega_0$, this becomes

$$\frac{dV_i}{dt} + \left(\frac{G_c - G_0}{C} \right) V_i = 0.$$

If, as is the usual case, the quantity in parentheses is negative, this equation requires that V_i grows exponentially. If V_i can be tailored to grow at the exponential rate given by this equation, V_c tracks V_i and V_r is zero. This does not affect the switching action of SLED; for, when V_i is reversed, to $-V_c$, the equation above is not valid and V_r no longer is zero.

One can play games with these equations. For example, if $V_i = -mV_c$, then $V_r = (m+1)V_i$, and the equation for V_i is ($\omega = \omega_0$):

$$\frac{dV_i}{dt} + \left(\frac{G_0 + (2m+1)G_c}{C}\right) V_i = 0.$$

An equation of this type, describing an exponential decay of V_i (and V_c), may be useful in the second part of the SLED pulse, when the voltage V_i is reversed to obtain a high voltage on the load.

The practical application of these results requires r.f. sources that can be modulated as prescribed here. Presently available sources probably lack this capability, but perhaps such sources can be developed.

REFERENCES

1. Z. D. Farkas, H. A. Hogg, G. A. Loew, and P. B. Wilson, SLAC-PUB-1453, June, 1974.

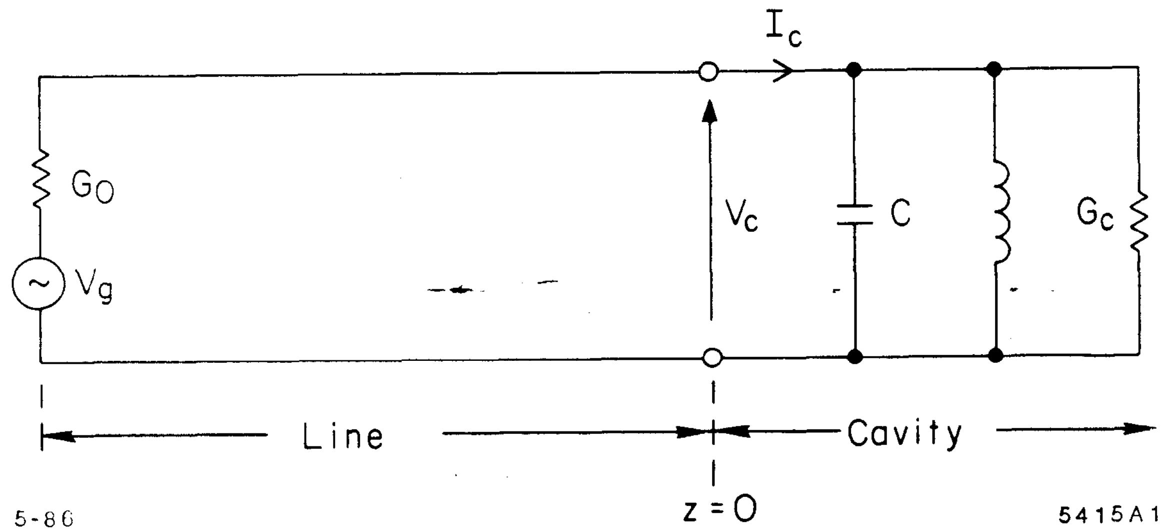

Figure 1. SLED equivalent circuit