METHODS TO INCREASE LINAC EFFICIENCY*

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1. INTRODUCTION

The average gradient of an accelerator section is proportional to the average power entering the section during its filling time. As an example the SLAC the gradient is one and two thirds the peak power. The linear collider requires a gradient of 21MV/m, hence 159 MW of peak power. We do not as yet have 159 MW klystrons, therefore we multiply, by means of pulse compression the peak power we do have, P_k , by a multiplication factor $159/P_k$. This note presents methods to increase the compression efficiency so that the 21 MV/m gradient can be reached with a minimum of rf peak power, of rf average power, and of AC line power. It discusses the design of variable group velocity sections and it shows that by changing the group velocity gradient along the section, the variation of voltage with time can be used for beam loading compensation when the beam pulse width is less than the section filling time.

2. ACCELERATOR SECTION PARAMETERS

In addition to the angular frequency ω , three parameters are necessary and sufficient to characterize a lightly loaded cell: the elastance per unit length, s , the unloaded quality factor, Q , and the group velocity, v_g . They are defined as

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$$s = V^2/UL_c \quad (1)$$

$$Q = \omega U/P_d \quad (2)$$

$$v_g = PL_c/U \quad (3)$$

where V is the voltage gained by a charged particle traversing the cell, U is the energy in the cell, L_c is the cell length, P_d is the power dissipated in the cell, and P is the power transmitted thru the cell.

Dividing (1) by (3) and using $V^2 = E^2 L^2$ we obtain the average gradient along the cell

$$E = \sqrt{\frac{s}{v_g} P} \quad (4)$$

An accelerator section consists of many cells. If we know P and v_g at any cell along the section and substitute them in (4) we obtain E as a function of z , the distance along the section. The voltage gained by a particle traversing a section of length L , the section voltage, is

$$V_o = \int_0^L E dz = \int_0^L \left(\frac{sP}{v_g}\right)^{1/2} dz \quad (5)$$

The average electric field in the section, the gradient, is the section voltage divided by the section length,

$$E_a = V_o/L \quad (6)$$

For a lossless, uniform group velocity section

$$V_o^2 = \frac{s P L^2}{v_g} = s P T_f L = s U_s L \quad (7)$$

where $T_f = L/v_g$ is the section filling time and $U_s = P T_f$, the energy entering the section.

After the bunch exits the section the energy in the section is discarded, and to accelerate the next bunch it is refilled with energy. Thus for a given voltage, s is a measure of the required energy per unit length to accelerate one bunch, while $f_r s$ (where f_r is the repetition rate) is a measure of the average rf power. In a lossy section, the energy not at the beginning of the section had to pass thru cells where a fraction of the energy was given up to the walls before it had a chance to contribute to the gradient. The amount of lost energy can be accounted for in the section efficiency, which is defined

$$\eta_s = \frac{V_o^2}{s U_s L} \quad (8)$$

and

$$V_o^2 = \eta_s s U_s L \quad (9)$$

The expression for V_o and η_s for a section with uniform group velocity, constant impedance (CZ), is derived as follows.

The transmitted power as a function of distance along the section, z is¹

$$P = P_0 e^{-2\alpha z} \quad (10)$$

where $\alpha = \omega/2Qv_g$. Substituting in (5) and performing the integration we obtain

$$V_o^2 = P_0 T_f s L \frac{(1 - e^{-\tau})^2}{\tau^2} \quad (11)$$

$$\tau = \alpha L = \omega L / 2Qv_g$$

τ is the section attenuation in nepers. Substituting (11) into (8) and using $U_s = PT_f$, we obtain

$$\eta_s = \frac{(1 - e^{-\tau})^2}{\tau^2} \quad (12)$$

If we substitute $s = \omega r / Q$ and $T_f = 2Q\tau / \omega$ in (11) we obtain the traditional expression for V_o^1

$$V_o^2 = (2/\tau)(1 - e^{-\tau})^2 P_0 r L \quad (13)$$

where r is the shunt resistance per unit length. Conversely, Eq. (11) can be obtained from equation (13), which is identical to Ref. 1, Eq. (5.16), by substituting $sT_f/2\tau$ for r . Similarly, if we make the above substitutions for the constant gradient (CG) expression for V_o given by Eq. (5.18) of Ref. 1, we obtain

$$V_o^2 = sT_f P_0 L(1 - e^{-2\tau})/2\tau \quad (14)$$

We can obtain s from r and Q using $s = r\omega/Q$. Define $T_a = Q/\omega$, then $s = r/T_a$.

3. VARIABLE GROUP VELOCITY SECTION

Consider a linearly variable group velocity section with a group velocity gradient

$$dv_g/dz = v'_g = m v_{go} \quad (15)$$

where v_{go} is the input end group velocity. The group velocity as a function of z is

$$v_g = v_{go} + v'_g z = v_{go}(1 + mz) \quad (16)$$

The time it takes for the input wave front to travel a distance z into the section is

$$T_z = \int_0^z \frac{dz}{v_g} = \int_0^z \frac{dz}{v_{go}(1 + mz)} = \left(\frac{z}{v_{go}}\right) \left[\frac{\ln(1 + mz)}{m} \right] \quad (17)$$

Substitute L for z and define

$$g = mL = \frac{v'_g}{v_{go}} L \quad (18)$$

to obtain the section filling time

$$T_f = \left(\frac{L}{v_{go}}\right) \left[\frac{\ln(1 + g)}{g} \right] = \frac{\ln(1 + v'_g L/v_{go})}{v'_g} = \frac{L}{v_{go} R_v} \quad (19)$$

$$v_{ga} = L/T_f \quad , \quad R_v = v_{ga}/v_{go} = g/\ell n(1+g)$$

v_{ga} is the average group velocity, R_v is the average to input velocity ratio. If L , v_g' , and T_s are given then

$$v_{go} = \frac{v_g' L}{e^{v_g' T_f} - 1} \quad (20)$$

The expression for V_o for a section with variable group velocity is derived in the Appendix. It is

$$V_o^2 = \eta_s s P_0 T_f L \quad (21)$$

$$\eta_s = R_v F_0^2$$

$$F_0 = [(1+g)^{x_{11}} - 1]/gx_{11} \quad , \quad x_{11} = 0.5 - \tau/\ell n(1+g) \quad , \quad \tau = \omega T_f/2Q = T_f/2T_a$$

The above expression for F_0 is indeterminate for three special cases. 1. Constant Impedance, CZ. No variation of group velocity along the section. 2. Constant Gradient, CG. The group velocity varies to compensate for attenuation so as to maintain a constant field along the section. 3. Reversed Constant Gradient, RCG. A CG section with input and output interchanged. For these special cases the expression for F_0 must be calculated separately (see Appendix) and the result is:

$$\text{CZ : } v_g' = 0, \quad R_v = 1, \quad F_0 = (1 - e^\tau)/\tau$$

$$\text{CG : } v_g' = -T_a^{-1}, \quad R_v = (1 - e^{-2\tau})/2\tau, \quad F_0 = 1, \quad \eta_s = R_v$$

$$\text{RCG : } v_g' = T_a^{-1}, \quad R_v = (1 - e^{-2\tau})/2\tau e^{-2\tau}, \quad F_0 = R_v^{-2}, \quad \eta_s = R_v^{-1}$$

$$\text{For } \tau = 0 : \eta_s = 4R_v[(1+g)^{1/2} - 1]^2/g^2 = 4[(1+g)^{1/2} - 1]/g\ell n(1+g)$$

4. CHOICE OF SECTION PARAMETERS

We have at our disposal three of the four section parameters: L , T_f , v_g' and v_{go} . Any three parameters determine the fourth. To find the value of a parameter that maximizes the gradient, we plot the normalized average gradient, E_{an} , vs one of the above parameters, keeping the others constant. We define E_{an} as the average gradient divided by the SLAC section average gradient. The SLAC section is constant gradient. Its parameters are: $Q=12900$, $L=3m$, $v_g' = -1.39\mu s^{-1}$, $v_{go} = 6.133m/\mu s$, $v_{ga} = 3.66m/\mu s$. Consequently $T_f = 0.82\mu s$, $T_a = .7192\mu s$, $\tau = 0.57$, $g = -0.681$, $\eta_s = 0.597$. If the input power $P_o = 11.14$ (50 MW klystrons) then the average gradient $E_a = 11.81MV/m$.

If we are concerned only with peak power, we maximize V_s with respect to τ keeping the peak power constant and obtain $\tau = 1.26$ for a *CZ* section; for a *CG* section V_s increases asymptotically as τ . For SLC we care about pulse energy and if we keep it constant V_s is a maximum when τ is zero. Plots of E_{an} vs. T_f , with $L = 3m$, $v_g' = -1.39$, one with constant peak power and one with constant energy inputs are shown in Fig. 1. At constant peak power E_a increases as we increase T_f , but at the expense of increased input energy. If we keep the input energy, $P_k T_k$, constant then as T_f decreases E_a increases and reaches a maximum. Decreasing T_f further will decrease E_a because the decrease of elastance with increasing group velocity has a larger effect than the decrease of wall losses. For the disk loaded SLAC structure, the elastance is a weak function of hole size hence of group velocity. It can be approximated by

$$s = 88.82 - 3.07v_g \quad (22)$$

Fig. 2 shows the variation of E_{an} as a function of L with $v_{ga} = 3.66m/\mu s$ for constant peak power and for constant pulse energy. Plots of normalized average gradient vs group velocity gradient are shown in Fig. 3.

The group velocity gradient, has little effect on section efficiency. It does, however, effect the variation of voltage with time as the section fills. The distance the pulse front reaches during T_z is

$$z = (e^{-v_g' T_z} - 1)/(v_g'/v_{go}) \quad (23)$$

We vary T_z from zero to T_f and obtain the normalized voltage vs time which is plotted in Fig. 4, for a CG, CZ, and RCG section. Observe that as we approach the filling time the slope depends on the group velocity gradient. This ability to change slope without significantly reducing the maximum efficiency can be used to reduce the spectrum of multiple bunches.

5. SHORT PULSE BEAM LOADING

By a short pulse we mean a train of at least two bunches up to a bunch each rf cycle with the time between the first and last bunch less than the filling time.

Wide pulse beam loading compensation is achieved by delaying the rf input to several sections, so that the vanguard electrons receive less voltage than the electrons following them. With a short pulse the section is only partially filled when the first bunch passes thru it. The zero momentum spread of two bunches, each with a charge $q/2$ and separated by Δt is obtained as follows. Define the ratio of the difference between the voltage at time T_f and the voltage at time Δt before T_f to Δt as

$$V' = [V_o(T_f) - V_o(T_f - \Delta t)]/\Delta t. \quad (24)$$

The first bunch enters the section at $T_f - \Delta t$ and the second at T_f , as illustrated in Fig. 5. They experience identical voltages of

$$V_s = V_o - V_b - \frac{V_b}{2} = V_o - \frac{3}{2}V_b \quad (25)$$

if the voltage due to each bunch¹ $V_b = (s/2)Lq/2 = sqL/4$ also equals $V'\Delta t$. Thus the charge that equalizes the voltage of the two bunches is

$$q = \frac{2V'\Delta t}{sL}. \quad (26)$$

The number of 10^{10} electrons in the two bunches is $N = 725q$. V_b is in MV, V' in MV/microsecond, and q in microcoulombs.

With an RCG section $V' = dV/dt = \sqrt{P_o v_{go} s} = \text{constant}$ and the section voltage increases linearly with time. Therefore we can have zero momentum

spread with uniformly distributed bunches, or even with one bunch each rf cycle, that is, with a constant amplitude current of $I = 2V'/sL$ microamps, for beam pulse widths that are sufficiently short to have a linear beam induced voltage.

The loaded beam voltage is $V_s = V_o - (3/8)sLq = V_o - kq$. Here $k = (3/8)sL$. The beam energy is $U_b = (V_o - kq)q$. The rf to beam energy conversion efficiency is $\eta = U_b/P_oT_f$. The value of q that will result in maximum beam energy is $q_m = V_o/2k$. The maximum beam energy is

$$U_{bm} = V_o^2/4k = (2/3)V_o^2/sL. \quad (27)$$

The maximum efficiency is

$$\eta = \frac{U_{bm}}{P_oT_f} = \frac{\eta_s s P_o T_f L}{P_o T_f} = \frac{2\eta_s}{3}. \quad (28)$$

In terms of section parameters and power input V' and therefore N can be approximated by

$$V' = E\Delta L/\Delta t = Ev_{gL} = \sqrt{P_o e^{-2\tau} s v_{gL}} \quad (29)$$

$$N = 1250\Delta t \sqrt{P_o e^{-2\tau} v_{gL}/s} / L \quad (30)$$

. Here v_{gL} is the group velocity along ΔL .

For the SLAC section, $V' = 25\text{MV}/\mu\text{s}$, $N = 8.18$. For a constant impedance section with the same length and filling time as the SLAC section: $\eta_s = 0.581$, and $E_{an} = .997$, $V' = 32.4\text{MV}/\mu\text{s}$, $N = 10.6$. For the SLAC section reversed: $v'_g = 1.39\mu\text{s}^{-1}$, $g = 2.13$, $v_{go} = 1.963\text{m}/\mu\text{s}$, resulting in $\eta_s = 0.536$, and $E_{an} = 0.948$, $V' = 38.2$, and $N = 12.5$.

We saw that E_{an} increases as the filling time decreases (either due to decrease in length or increased group velocity) provided the peak power is increased so as to maintain $P_o T_k$ constant. One way to increase peak power is by pulse compression.

6. PULSE COMPRESSION (SLED)

After an interval T_f from the start of the rf pulse, additional energy input into the section does not increase the section voltage. Thus, the rf efficiency

$$\eta_{rf} = V_o^2 / s P_k T_k L \quad (31)$$

decreases as the klystron pulsewidth increases. To reduce this decrease in efficiency, we resort to pulse compression. At SLAC we already have a wide pulse. But even if this were not the case, the reasons for compressing wide pulses instead of generating narrow pulses directly are:

1. High peak power sources unavailable.
2. Avoid window breakdown due to peak power.
3. Both, long pulse low power and short pulse high power operation are possible.
4. The incremental cost to make the pulse wider is small, hence lower capital costs.
5. Higher ac to rf conversion efficiency.

In 1954, S. Darlington received a patent for a pulse compression system that linearly frequency modulates an rf pulse and transmits it through a network whose time delay is proportional to frequency. This method was not used because of the residual frequency modulation of the compressed pulse.

A pulse compression system (SLED) has been installed at SLAC.^{2,3,4} It uses energy storage cavities and rapid phase reversal to convert a wide klystron rectangular pulse into a narrow pulse consisting of constant and an exponential term. In Ref.s 3 V_o as a function of time for a constant gradient section with a SLEDed input is given. A FORTRAN code, RAPP⁵ (RADio frequency Pulse Power Multiplier) was written that solves for the SLEDed V_o as a function of time. The input rf pulse is sub-divided into several intervals, each interval having its own amplitude and phase. The storage cavity Q_o , Q_e can also be different during each interval. The cavities need not be tuned. The accelerator section can have either a linearly increasing or linearly decreasing group velocity.

A plot of the SLEDed pulse is shown in Fig. 6. The resulting variation with time of the voltage for a CZ, CG, and RCG section, normalized to the non-SLEDed CG section voltage, are shown in Fig. 7. Here we give only the expression for a special case of the peak section voltage which occurs at the end of the klystron pulse. The expression for this voltage, derived in the Appendix, is

$$V_o^2 = \eta_{rf} P_0 T_k s L \quad (32)$$

$$\eta_{rf} = R_v \frac{[a_0 F_0 + a_1 F_1]^2}{T_k / T_f} \quad (33)$$

$$F_1 = e^{-T_n} [(1+g)^{x_{22}} - 1] / g x_{22} \quad , \quad x_{22} = 0.5 - (\tau - T_n) / \ln(1+g)$$

$$a_0 = -(\alpha - 1) \quad , \quad \alpha = 2 / (1 + Q_e / Q_0)$$

$$a_1 = \alpha(2 - e^{-t_1/T_c}) \quad , \quad T_c = \alpha Q_e / \omega \quad , \quad T_n = T_f / T_c$$

Q_0, Q_e = internal and external cavity quality factors. F_0 was defined in eq. (21).

For a CZ and CG section respectively:

$$F_1 = e^{-T_n} \frac{1 - e^{-(\tau - T_n)}}{\tau - T_n} \quad , \quad F_1 = e^{-T_n} \frac{2\tau(1 - e^{-(2\tau - T_n)})}{(1 - e^{-2\tau})(2\tau - T_n)} \quad (34)$$

We define the power multiplication factor, M , as the ratio of V_o^2 with SLED to V_o^2 without SLED, and the SLED gain, G is the square root of M . They are given by

$$M = \frac{(a_0 F_0 + a_1 F_1)^2}{F_0^2} \quad (35)$$

$$G = a_0 + \frac{a_1 F_1}{F_0} \quad (36)$$

Equation (32) and (33) are valid for any section with a linearly increasing or decreasing group velocity, which includes CZ and CG. The compression efficiency,

the power multiplication factor divided by the ideal multiplication factor, is

$$\eta_{pc} = \frac{M}{C_f} \quad (37)$$

where $C_f = T_k/T_f$ is the compression ratio. For SLAC $Q_o = 100000$, $Q_e = 20000$, $M = 3.17$, $T_k/T_f = 6.1$, and $\eta_{pc} = 0.52$. There is room for improvement!

7. CHOICE OF SLED PARAMETERS

The efficiency is a function of Q_o , Q_e , T_k , T_f , Q_a and g . It is easy to get lost in the intricacies of how to maximize η_{rf} with respect to a given parameter. Nevertheless I shall attempt it. The root causes of reduction in efficiency and the parameters they depend on, are:

1. energy dissipated in the cavity. (Q_o)
2. energy reflected during charging. ($Q_e, T_k/T_f$)
3. energy left in the cavity after the section has been filled. ($Q_e, T_k/T_f$)
4. energy dissipated in the section during filling. (Q, T_f)

The object is to choose parameters that minimize the above losses.

Choice of Q_o , Q_e , and T_k/T_f

Define T_{on} as the ratio of the unloaded cavity time constant $Q_o/\pi f$ to charging time, t_1 , $T_{on} = Q_o/\pi f t_1$. The energy dissipated in the cavity decreases as T_{on} increases, hence for a given t_1 , Q_o should be as large as necessary to make the dissipated energy in the cavity inconsequential. The right choice of Q_e minimizes the reflected energy during charging. The efficiency of energy transfer into the cavity is⁵

$$\eta_c = \frac{\alpha(1 - e^{-T_n})^2}{T_n} \quad (38)$$

where $\alpha = 2/(1 + Q_e/Q_o)$, $T_n = t_1/T_c$, $Q_e = \omega T_c/\alpha$. Fig. 8 shows plot of η_c , maximized with respect to Q_e , vs T_{on} . We see that η_c increases monotonically

with Q_0 and inversely with t_1 , and reaches a nearly constant value at about $Q_0 = 10\pi ft_1$. There is no need to make Q_0 greater than that. The first pair plotted points are for $t_1 = 1.68$ ($T_k = 2.5 \mu s$) and the second pair are for $t_1 = 4.18$ ($T_k = 5 \mu s$), for $Q_0 = 100000$ and 250000 respectively. If $T_{on} \gg 1$ then η_c is a maximum when $T_n = 1.257$, and Q_e is $Q_e = \pi ft_1/1.257$. Fig. 8 also shows a plot of η_c (dashed line) when Q_e remains at $\pi ft_1/1.257$. Fig. 8 also shows a plot of Q_e that maximizes η_c . Unless we use amplitude modulation or Q_e switching,⁶ the losses due to reflection limit η_c to a maximum of 0.814.

The condition that causes the leftover energy in the cavity at the end of the pulse to be zero, is $T_f/T_{c2} = \ln(2 - e^{-t_1/T_{c1}})$. T_{c1} , T_{c2} are the cavity time constants during charging and discharging respectively. We substitute 1.257 for t_1/T_{c1} and obtain $T_f/T_{c2} = 0.540$. Thus $t_1/T_f = 2.328$, and $C_f = t_1/T_f + 1 = 3.328$. We obtain Q_e during discharge as follows: $T_{c1} = T_f/0.540$, $Q_L = \pi f T_{c1}$, $Q_e = Q_L/(1 - Q_L/Q_0)$. At SLAC $T_f = 0.82 \mu s$, hence $T_{c1} = 1.518$. If $Q_0 = 10^5$ then $Q_e = 15800$, and if $Q_0 = 2.5 \times 10^5$ then $Q_e = 14400$. Thus for a fixed T_f , we can obtain the values of T_k , Q_e during charging and Q_e during discharging that simultaneously maximize η_c and cause the left over energy at the end of the pulse to be zero. Since we do not Q_e switch the value of Q_e is a compromise between the value that minimizes reflection and leftover energy. The compromise Q_e is 20000.

Fig. 9 shows the variation with T_k of E_{an} , the average gradient normalized to the bench mark gradient of the SLAC section with $T_k = 5\mu s$ input pulse. With $P_o = 11.14MW$ (50MW klystrons), $E_a=21MV/m$. The pulse energy $P_k T_k$ was maintained constant. We see that E_{an} has a maximum of 1.11 at $T_k = 2.5$, $C_f = 3.05$, when $Q_0 = 10^5$, and a maximum of 1.145 at $T_k = 2.8$, $C_f = 3.41$ when $Q_0 = 250000$.

Choice of Filling Time for Fixed Section Length

The fraction of energy lost before it contributes to the accelerating electric field increases as the ratio of filling time to unloaded cell time constant increases. We can make the unloaded cell time constant large by making the section super-

conducting. Then, we do not need pulse compression because we can make the filling time arbitrarily long.⁷ For a copper section, however, in order to reduce the energy it dissipates, we make its filling time short. Therefore we need high peak power, and hence pulse compression.

There are three factors that influence the section voltage as we increase T_f : 1) the increased wall loss decreases V_o ; 2) the increase in s increases V_o ; and 3) the compression efficiency which first increases and then decreases V_o . For a given T_f the compression efficiency depends on T_k . Plots of normalized average gradient vs T_f with $T_k = 5$ and $2.5\mu s$ and a 3m fixed length are shown in Fig. 10. We have a broad maximum at $T_f = 0.6\mu s$, $v_{ga} = 5m/\mu s$. The dashed curves assume a constant s . For a fixed length, having the same gradient for a shorter T_f as for a longer T_f is an advantage because a shorter T_f implies a higher group velocity hence a larger hole size which in turn results in a lower longitudinal and transverse wake potentials.

Choice of Section Length for Fixed Average Group Velocity

For a fixed average group velocity we can reduce T_f and hence energy loss. We can not make the section arbitrarily small because then we need a large number of high peak power sources, as the gradient depends on the peak power input. An alternative to high power sources is pulse compression. If we could compress with 100 % efficiency we would choose a length that makes the section dissipation negligible. As both the compression and section efficiencies depend on length we choose a length that maximizes their product. The solid curves of Fig. 11 are plots of normalized average gradient as a function of length, for 5 and $2.5\mu s$ klystron pulse widths. The non-solid curves are for $5\mu s$ klystron pulse width and a CZ section with an average group velocity of 3.66, 2.5, and $2\mu s/m$.

Choice of Group Velocity Gradient

Because with SLED the end of the section contains rf when the first bunch goes thru, V' is a more complicated function of P and $v_g L$. With a SLEDed pulse, the difference in V' between normal and reversed section is greater than

when the input is not SLEDed. Table 1 lists the section efficiency, η_s , the SLED gain, G , their product, $\eta_s G$, V' and N for the SLAC section, for a CZ SLAC section and for the SLAC section reversed. The klystron pulse width is $5\mu s$. V' was obtained using RAPPIM.⁴

Table 1

	η_s	G	$\eta_s G$	E_{an}	V'	$N \times 10^{-10}$
<i>CG</i>	.597	1.783	1.378	1	2.1	.693
<i>CZ</i>	.581	1.834	1.398	1.015	18.6	6.135
<i>RCG</i>	.536	1.886	1.381	1.002	37.1	12.2

We see that, even more than for a rectangular pulse, we can have large changes in V' without changing E_a , and, as for the rectangular pulse case the ability to change slope without reducing gain can be used to ascertain that multiple bunches have equal energy. We can reverse a fraction of sections to obtain the required average v_g . Also we can change V' by modulating the power input just prior to the compressed pulse, and by varying the timing of the rapid phase change.

Pulse Shape – Energy Distribution Along the Section

The section voltage with the SLED pulse is less than it would be with a rectangular pulse containing the same energy. To explain this reduction we examine the influence of pulse shape on section voltage in general. To maximize the average gradient of a fixed length lossless section we want to distribute the energy uniformly along the section and therefore we would need a rectangular pulse shape. If the section is very lossy we want to concentrate most of the energy at the section input end and therefore at the lagging end of the rf pulse. To obtain a pulse shape that maximizes the average gradient and to see the reduction in average gradient due to a misshaped pulse, we proceed as follows. Divide the pulse into two equal time intervals. Let R be the ratio of the power amplitude during the first interval to the power amplitude during the second interval. It can be shown that the ratio of the section voltage with modulation to the section

voltage with no modulation, $R = 1$, but with identical pulse energies, is

$$G_r = \sqrt{\frac{2}{1+R}} \times \frac{1 + \sqrt{k_L A R}}{1 + \sqrt{k_L A}} \quad (39)$$

Here k_L is the ratio of the length of the first part of the section that contains the second half of the pulse to the second part of the section which contains the first half of the pulse. A is the attenuation of the first part of the section. For a CZ section $k_L = 1$ and $A = e^{-\tau}$. For a CG section $k_L = A = e^{-\tau}$. G_r is maximum when $R = R_m = k_L A$. A plot of G_r vs R is shown in Fig. 12. For SLAC $R_m = 3$. Thus we can increase the gradient 4 % by making the power during the second half of the pulse 3 times the power during the first half. Unfortunately, the compressed SLED pulse power is higher during the first half than during the second half of the pulse. This mal-distribution of compressed pulse energy decreases the gradient by about 10 %.

We can improve pulse shape and reduce the reflected energy by means of intra-pulse modulation.

8. INCREASING COMPRESSION EFFICIENCY USING PULSE AMPLITUDE AND PHASE MODULATION

Reducing the energy reflected during charging by amplitude modulating the incident power upon the cavity was suggested by Sundelin.⁸ With SLED piece-wise intra-pulse modulation during charging can be used to reduce reflected energy. It can be used during discharging to reshape the accelerating pulse so as to minimize the energy lost to the section walls. Both these effects are illustrated in Figs 13 and 14. Fig. 13 shows the intra pulse modulated klystron output field and Fig. 14 shows the resulting SLED waveform. An intra-pulse 4 to 1 klystron power ratio increases the compression efficiency by 27 %, from 52% to 66%.

The power can be modulated using a mod anode or by varying the anode voltage by charging different portions of the pulse forming network to different voltages. In either case the klystron efficiency should not vary with power level.

Phase modulation can also be used to increase the efficiency but only during

discharge. Unfortunately, if it is used then the section phase velocity has to vary along the section length to compensate for phase variation with time. The best improvement due to phase modulation that I could obtain, was 4%. The increase in efficiency due to phase modulation is too small and implementation too difficult to warrant installation.

9. AC EFFICIENCY

We must also consider the ac energy required to generate a given section voltage. This in turn depends on the ac energy required to generate an rf pulse. From a memo by G. Konrad⁷ we infer that the ac to rf efficiency is

$$\eta_{ar} = \frac{0.398T_k}{(T_k + 1)} \cdot T_k \text{ in } \mu\text{s} \quad (40)$$

We assume that the klystron efficiency is a constant 50% over the applied voltage range. The ac efficiency is $\eta_{ac} = \eta_{ar}\eta_{rf}$. The required ac energy per rf pulse is

$$U_{ac} = V_o^2 / \eta_{ac} s T_f L \quad (41)$$

Plots of ac and rf efficiencies vs pulse width are shown in Fig. 15. Because η_{rf} and η_{ar} depend differently on T_k , their maxima do not coincide, as can be seen in Fig. 14 and Table 2. Table 2 shows the efficiencies and klystron energy, klystron power, ac energy per period, and total ac line power (assuming 240 klystrons), P_{act} , needed to obtain 21 MV/m. It also lists the required improvement in η_{rf} and in η_{ac} , I_{rf} and I_{ac} , needed to attain 21 MV/m if the present rf and ac energies per pulse, 180 and 633 joules respectively, do not increase. The last column lists the improvement in efficiency and in P_{act} if Q_0 is increased from 10^5 to 2.5×10^5 .

Table 2

Efficiencies, klystron energy, klystron power, ac energy and ac power
as a function of T_k . $Q_0 = 10^5$, $E_a = 21$ MV/m.

T_k μs	η_{rf}	η_{ar}	η_{ac}	I_{rf}	I_{ac}	U_k joule	P_k MW	U_{ac} joule	P_{act} MW	I_{rf} I_{ac}	P_{act} MW
.82	.532	.179	.0952	.714	1.14	130	157	726	31.4	$Q_0 = 250000$	
2.5	.340	.284	.0965	1.21	1.12	202	80.9	712	30.8	1.07	28.8
3.0	.335	.299	.100	1.11	1.08	205	68.5	686	29.6	1.09	27.2
3.5	.324	.310	.100	1.18	1.08	212	60.6	684	29.5	1.10	26.8
4.0	.310	.318	.0986	1.23	1.10	212	55.5	697	30.0	1.11	27.0
4.5	.293	.326	.0956	1.32	1.13	234	52.0	719	31.1	1.13	27.5
5.0	.277	.332	.0919	1.38	1.18	248	49.6	748	32.3	1.13	28.6

The klystron peak power is important because it is not equally easy to produce a given $U_k = P_k T_k$ regardless of the relative values of P_k and T_k . However the value of the capacitance of the pulse forming network is proportional to the product $P_k T_k$ and not to their individual values. An additional 1.44 MW should be added to P_{act} to account for 6 kW for the focusing coil of each klystron.

10. CONCLUSION

We expressed the section noload voltage in terms of rf pulse energy for both a rectangular and SLEDED pulses, for a section having a linearly varying group velocity. We showed for both a rectangular and SLEDED pulse that for a given T_f , and length there is an optimum pulse width, for a given pulse width and length there is an optimum T_f and for a given T_k and T_f there is an optimum L. We also showed that the average gradient does not depend on the group velocity gradient, but which can be used for beam loading compensation. The group velocity at the end of the section, can be chosen to determine the charge per bunch that

will result in zero loaded wake potential. Lastly, we showed how to improve the compression efficiency by intra-pulse modulation and how to optimize the ac efficiency.

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APPENDIX

The transmitted power at z is obtained from the fundamental equations

$$w = \frac{P}{v_g} = \frac{Q}{\omega} \frac{dP}{dz} = T_a \frac{dP}{dz}$$

We defined $T_a = Q/\omega$. w is the energy stored per unit length.

Using the expression for v_g we have

$$\frac{dP}{P} = \frac{dz}{T_a v_{go}(1+mz)}$$

whose solution is

$$P(z) = P_o e^{-\frac{\ln(1+mz)}{T_a v_{go} m}} = P_o e^{-2\tau_z}$$

where

$$\tau_z = \frac{\ln(1+mz)}{2T_a v_{go} m} = \frac{T_z}{2T_a}$$

As the above is true for any z , the ratio of filling time T_f to section attenuation, τ , is

$$\frac{T_f}{\tau} = 2T_a.$$

Define $n = 1/2T_a v_{go} m$. Then

$$P(z) = P_o e^{-n \ln(1+mz)}$$

The accelerating field as a function of z is

$$E(z) = \sqrt{\frac{sP}{v_g}} = \sqrt{\frac{sP_o}{v_{go}}} \frac{e^{-n \ln(1+mz)}}{(1+mz)^{.5}}$$

$$E(z) = E_o(1+mz)^{-.5-n}; \quad E_o = \sqrt{\frac{sP_o}{v_{go}}}$$

The voltage as a function of z is

$$V(z) = \int_0^z E dz = E_o z \frac{(1 + mz)^{.5-n} - 1}{(.5 - n)mz}$$

Let $x_{11} = .5 - n$, $z = L$ and define $g = mL$. Then

$$V_o = E_o L \frac{(1 + g)^{x_{11}} - 1}{x_{11}g}$$

$$V_o = \sqrt{sP_o T_s R_v} \frac{(1 + g)^{x_{11}} - 1}{x_{11}g} = \sqrt{sP_o T_s R_v} F_o.$$

We can write n as

$$n = \frac{1}{2T_a v_{go} m} = \frac{\tau}{T_f} \frac{L}{v_{go} g} = \frac{\tau}{\ell n(1 + g)} = \frac{1}{2 T_a v'_g}$$

Special Cases

We will now consider 3 special cases for which the above expression for V_o is indeterminate. They are characterized by their value of v'_g . 1) The constant impedance CZ section $v'_g = 0$, is treated in the text.

$$2) \text{ CG: } v'_g = -T_a, \quad n = -.5, \quad m = \frac{v'_g}{v_{go}} = -\frac{1}{v_{go} T_a}$$

$$P = P_o e^{\ell n(1 - \frac{z}{v_{go} T_a})} P_o \left(1 - \frac{z}{v_{go} T_a}\right)$$

$$E = E_o, \quad V_o = E_o L = \sqrt{sP_o T_f R_v} L$$

$$\eta_s = R_v, \quad F_0 = 1$$

3. RCG: $v_g' = T_a^{-1}, \quad n = .5$

$$P = P_o e^{-\ell n(1+mz)} = P_o \frac{1}{(1+mz)}$$

$$E = E_o \frac{1}{(1+mz)}$$

$$V(z) = E_o z \frac{\ell n(1+mz)}{mz}$$

$$V_o = \sqrt{s P_o T_f R_v L} \frac{\ell n(1+g)}{g}$$

$$\eta_s = R_v \left[\frac{\ell n(1+g)}{g} \right]^2 = \frac{1}{R_v}$$

$$F_o = R_v^{-2}$$

Exponential Input

If the square root of the section input power is exponential of the form $a_1 e^{-t/T_c}$, as the output of a cavity, than at the time T_f ,

$$\sqrt{P}(z, T_f) = a_1 e^{-(T_f - T_z)/T_c} = a_1 e^{-T_f/T_c} e^{T_z/T_c}$$

$$\begin{aligned} E(z, T_f) &= a_1 \sqrt{\frac{s}{v_{go}}} \frac{e^{-T_f/T_c} e^{T_z/T_c} e^{-n \ell n(1+mz)}}{(1+mz)^{.5}} \\ &= a_1 \sqrt{\frac{s}{v_{go}}} e^{-T_f/T_c} (1+mz)^{-.5-n(1-2T_a/T_c)} \end{aligned}$$

Observe that the expression for E for an exponential input is that of a rectangular input multiplied by e^{-T_f/T_c} and that n is multiplied by $1 - 2T_a/T_c$. In the expression for V_o we replace x_{11} by x_{22}

$$\begin{aligned} x_{22} &= .5 - n \left(1 - \frac{2T_a}{T_c} \right) \\ &= \frac{.5 - (\tau - T_f/T_c)}{\ell n(1+g)} \end{aligned}$$

$$V_o = \sqrt{a_1 s T_s L R_v} e^{-T_f/T_c} \frac{(1+g)^{x_{22}} - 1}{g x_{22}}.$$

The SLED pulse is the sum of a constant and exponential term and the section voltage is the superposition of the voltages due to each term.

Rationale for Using Elastance per Unit Length

An astute observer would notice that the shunt resistance per unit length r was not used in the above derivation. The answer to 'why not?' is similar to the answer Laplace gave to Napoleon when asked why he omitted mention of the Deity from his "Mecanique Celeste", to wit "I had no need for it". There is no need to use two parameters, r and τ , both of which depend on losses and therefore on the section material when one, τ , is sufficient. Not only is r unnecessary but as the losses approach zero, r approaches infinity, and we know that the section voltage remains finite. For no loss, the standard expression for V_o becomes indeterminate. But the expression for V_o we derived reduces to the expression we would have obtained had we started without losses in the first place.

The shunt resistance is derived from cavity analogy to the lumped parameter parallel RLC circuit. It is the R of the RLC circuit. To carry the analogy to its logical conclusion, we should call R/Q the shunt reactance. It is the reactance of either the shunt capacitor or of the shunt inductor: $X = \frac{1}{\omega C} = \omega L$. The elastance can also be derived from the same circuit. It is C^{-1} . The shunt resistance is useful when considering average power and the elastance when considering the average power required to periodically charge the capacitor. A travelling wave section is more analogous to the latter. It is fortuitous that S follows R in the alphabet.

Presently, the elastance and shunt reactance are given in terms of R and Q . There is no basic objection for calling the elastance, $\omega R/Q$ and the shunt reactance R/Q . But why call parameters that do not depend on loss, in terms of parameters that do.

The elastance S is an International Standards Organization (ISO) symbol.

It can be derived from fundamental relations. From the energy stored per unit volume

$$\frac{1}{2}\epsilon E^2 = \frac{U}{A_e L}$$

we obtain

$$s = \frac{V^2}{UL} = \frac{E^2}{U/L} = \frac{2}{\epsilon A_e}$$

Here A_e is the effective crosssectional area of the cell.

In Ref. 1, the shunt resistance is not used in the expression for the charge induced section voltage. The elastance is used but the form of $\omega r/Q$. The expressions for the no-load voltage and for the charge induced voltage can be converted to expressions that do not become indeterminate as the resistivity of the section material approaches zero, if we replace αr by $s/2v_g$ and $r\tau$ by $sT_f/2$.

FIGURE CAPTIONS

- Fig. 1 Average gradient vs filling time. Constant length.
- Fig. 2 Average gradient vs length. Constant average group velocity.
- Fig. 3 Average gradient vs group velocity gradient.
- Fig. 4 Normalized voltage vs time.
- Fig. 5 Beam Loading Diagram
- Fig. 6 Reverse Field Amplitude and Phase.
- Fig. 7 Normalized SLED voltage vs Time.
- Fig. 8 Storage Cavity Filling Efficiency.
- Fig. 9 Normalized SLED average gradient vs pulse width.
- Fig. 10 Normalized SLED average gradient vs filling time. Constant Length.
- Fig. 11 Normalized SLED average gradient vs Length. Constant average velocity.
- Fig. 12 Intra-Pulse modulation gain G vs Modulation amplitude R
- Fig. 13 Incident field amplitude vs Time.
- Fig. 14 Reverse field amplitude vs Time. attenuation.
- Fig. 15 RF and AC to Accelerating Energy Conversion Efficiencies.

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Fig. 1

Fig. 2

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Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

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Fig. 10

Fig. 11

Fig. 12

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Fig. 13

Fig. 14

Fig. 15