IN-LINE CALORIMETER FOR
MICROWAVE POWER MEASUREMENTS

by

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# TABLE OF CONTENTS

<p>| Acknowledgment | iii |
| List of figures | v |
| I. Introduction | 1 |
| II. Comparison of measurement schemes | 4 |
| A. Calorimetry | 4 |
| B. Direct electrical indication | 6 |
| C. Balanced-bridge techniques | 7 |
| D. Miscellaneous physical effects | 8 |
| III. The in-line calorimeter | 10 |
| A. The static waveguide calorimeter equations | 10 |
| B. Effect of a non-zero reflection coefficient on calorimeter indication | 19 |
| C. Calorimeter characteristics | 23 |
| D. Temperature indication | 26 |
| IV. Experimental calorimeters | 29 |
| A. Characteristics | 29 |
| B. Short calorimeter with flanges | 31 |
| C. A clamp-on calorimeter one waveguide-wavelength long | 33 |
| D. Conclusions | 39 |
| Appendix A | 40 |
| References | 41 |</p>
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Static in-line calorimeter</td>
<td>11</td>
</tr>
<tr>
<td>3.2</td>
<td>Standing wave pattern of power in a waveguide</td>
<td>21</td>
</tr>
<tr>
<td>3.3</td>
<td>Maximum error $</td>
<td>\varepsilon</td>
</tr>
<tr>
<td>3.4</td>
<td>The resistance bridge</td>
<td>27</td>
</tr>
<tr>
<td>4.1</td>
<td>Short calorimeter with flanges. Insulation removed</td>
<td>32</td>
</tr>
<tr>
<td>4.2</td>
<td>Test setup for measuring sensitivity and time constant</td>
<td>34</td>
</tr>
<tr>
<td>4.3</td>
<td>Minimum resolution test setup</td>
<td>35</td>
</tr>
<tr>
<td>4.4</td>
<td>Minimum resolution test recording</td>
<td>36</td>
</tr>
<tr>
<td>4.5</td>
<td>$\lambda$ long calorimeter with clamp-on heat sinks. Insulation partially removed at right end to show cooling clamp</td>
<td>37</td>
</tr>
</tbody>
</table>
ABSTRACT

The static in-line calorimeter is a device which measures the temperature rise in the walls of a waveguide caused by the attenuation of microwave power flowing through the waveguide. It is simple and inexpensive. It can be constructed so that it will fit on waveguide already existing in a microwave system. The device should be reliable because it uses no active circuitry. In addition, few mechanical problems are encountered in its use because the existing waveguide need not be altered.

If a section of waveguide is thermally insulated and provided with heat sinks at both ends, then the center temperature rise is directly proportional to the average waveguide power level. The temperature level attainable for a given length of waveguide is a function of the physical properties of the waveguide material and the square of its length. The rise time of such an in-line calorimeter is also dependent on the square of the length and can be fairly well described by a single exponential. If an in-line calorimeter is made an integral number of waveguide wavelengths long, it is insensitive to standing-wave patterns in the waveguide and will simply read the sum of the incident and reflected powers.

Two experimental S-band calorimeters using stainless steel waveguide and resistance-wire bridge temperature indicators are described. The measured sensitivity and time constant for both units falls within the experimental error of confirming the theoretically predicted figures.
I. INTRODUCTION

The development in recent years of microwave devices capable of generating and utilizing average powers in the range of tens of kilowatts has created many new measurement problems. Less than ten years ago it was possible to refer to average microwave power levels greater than one watt as "high power."¹ A more recent text makes a division at ten watts.² The work described here is specifically concerned with continuous and simultaneous measurements of power from a number of microwave sources where average power levels of a few hundred to tens of kilowatts and peak pulse power levels of tens of megawatts may exist.

Unlike systems operating at lower power levels, in a high-pulsed-power microwave system the possibility of electrical breakdown is an important consideration. Such systems must therefore contain few discontinuities, and often a system must be evacuated. These properties present an additional complication to any measurement to be made in a waveguide system, for not only must a proposed instrumentation scheme be capable of handling a portion of the high-pulsed power and its corresponding high average power level, but often it must also conform to the mechanical requirements dictated by the vacuum system. The mechanical requirements imposed by the problem of electrical breakdown indicate that a high-power measurement scheme which introduces no auxiliary devices in the measured section of waveguide is to be preferred. Every device inserted into a high-power waveguide system imposes the dual problems of electrical breakdown caused by configuration alone and breakdown caused by any vacuum leaks which may occur where the device was inserted. If practical, then, a high-power measurement scheme should have a simple waveguide configuration. If a power measurement is to be made on a continuous basis, as might be done in a surveillance radar or in a linear electron accelerator, the necessary instrumentation should be as simple and reliable as practically possible. If a large number of power measurements from different sources of a complex system

¹All references will be found at the end of the text.
are to be made simultaneously, then of course each measurement device should be inexpensive and easily serviced.

The microwave power meter described in this report was designed with a view toward achieving a satisfactory synthesis of the criteria noted above. Since it is designed to operate at high power levels, many of the problems normally associated with the detection and utilization of small signals were not encountered. In most low-level measurement schemes, the background noise (ambient temperature fluctuations in the case of the bolometer power meter) often determines the ultimate resolution of the device. In a high-power measurement system, on the other hand, the power level present most often completely masks the low-power background noise. It is, therefore, undesirable to sample a high-power signal through a directional coupler and attenuate the sampled signal until it can be read with conventional low-power meters. Not only is such a system expensive and tedious to calibrate, but it is also unnecessarily devious in the sense that an inherently simple measurement problem is made more difficult by reducing the signal to a level where a complex measuring device is required.

A good high-power measurement scheme applicable to a system where a large number of signals must be measured simultaneously should then be cheap, reliable, simple, and be so constructed that it meets the mechanical requirements imposed. The measurement system used should be one that responds to the high-power signal directly, thus eliminating the need for reduced-signal sampling methods. The device described in this paper meets all these stipulations. It consists of a static in-line dry calorimeter, a device that measures the temperature rise in the walls of a waveguide caused by the attenuation of power flowing through the waveguide. It is simple and inexpensive. It can be constructed so that it will fit on waveguide already existing in the system. The device should be reliable because it uses no active circuitry. In addition, few mechanical problems are encountered in its use because the existing waveguide need not be altered.
In this report a first-order theory of the in-line calorimeter is developed. The predictions of the theory are in reasonably close agreement with the experimental results that were obtained from two S-band models of a prototype calorimeter. As expected from a calorimetric device, the in-line calorimeter has a relatively long time constant, varying from less than half a minute to slightly over ten minutes for the practical calorimeters considered. The power sensitivity for an experimental S-band model 7.6 cm long was 12.5 millivolts output per volt into bridge circuit per kilowatt of microwave power flowing through the measured waveguide section, while for a model 14.98 cm long the corresponding figure was 47 millivolts. The error in these experimental figures, which is 10%, includes the theoretically predicted figures for sensitivity. The minimum resolution of the experimental calorimeters, which were designed to measure average power levels of a half to several kilowatts, was found to be less than half a watt.
II. COMPARISON OF MEASUREMENT SCHEMES

Although there are a number of problems peculiar to the measurement of high-power microwave signals, there is no basic physical principle that favors one power-measurement scheme over another. The advantages and limitations of the various power-measurement schemes developed to date are mostly of a practical nature; it is on these grounds that the systems will be compared. Although other measurement classification systems exist, for the purposes of this report four divisions will be made: (1) calorimetry, (2) direct electrical indication through rectification or change of some electrical parameter, (3) balanced-bridge techniques where a substitution of low-frequency or dc power is made for the microwave power, and (4) miscellaneous physical effects.

A. Calorimetry

The principle of conservation of energy based on the first law of thermodynamics forms the theoretical foundation for calorimetry; this is the most direct method of measuring power, in that energy and time are measured directly. Calorimetry is simply based upon the measurement of energy in the form of heat emitted by a system which is continuously absorbing electromagnetic energy. Once the energy-transfer mechanism is known, then the measurement of electromagnetic energy is reduced to the measurement of heat energy; it is this simple aspect of calorimetry that makes it attractive as a measurement standard. Aside from the operational classifications of flow or static calorimetry, there are two basic classifications of calorimeters: adiabatic, in which heat losses are effectively eliminated; and nonadiabatic, in which heat losses are compensated for, usually by the addition of a known amount of dc power in some substitution scheme. The former classification leads to the direct form of calorimetry, while the latter leads to adapted or balanced methods of calorimetry.\(^1\)

As noted in the previous chapter, in high pulsed-power systems it is often necessary for a waveguide to be sealed and evacuated, and it is desirable to have as few discontinuities as is practically
possible. An in-line calorimetric power meter that can provide an accurate indication of the average power present in a waveguide would be practical for applications in which it is desirable to monitor high pulsed powers. Methods of sampling a portion of the power to be measured and using it to excite an absorption type of calorimeter have been described comprehensively in the literature. The basic interest in the present work is in calorimetric systems which allow a measurement of microwave power to be made without sampling from the waveguide which carries the power.

This latter kind of calorimetry is possible with both flow and dry calorimeters. If two walls of a waveguide absorb power and then pass it on to a fluid in intimate contact, or if the fluid itself absorbs the microwave power, then ordinary flow-calorimetric means can be used to measure the heat rise in the liquid and thus provide an indication of the power present in the waveguide. Jaeger and Schneider have described a power meter in which the fluid itself is the absorbing medium. Despite the possible accuracy of a carefully made system of this type, the amount of associated instrumentation may become excessive if the power meter is to be used as a simple monitor.

The static, adiabatic, dry calorimeter has the inherent advantage that it can be built with very little associated instrumentation. If, for example, a short section of relatively lossy waveguide is arranged with a heat sink at either end, then the temperature distribution between its ends is a function of the heat dissipated in the section and thus of the total waveguide average power. A temperature-sensing device, such as a resistance-wire bridge or a thermopile, can then be used to give a direct electrical indication of the waveguide average power. The primary disadvantage of the in-line calorimeter is that, like all calorimeters, it may have a long response time. A static, adiabatic, in-line dry calorimeter as described above can be built without disturbing the existing waveguide of a system; for this reason and for reasons of simplicity, it has been selected as the most logical device for high-power monitoring.
B. Direct Electrical Indication

Direct electrical indication of microwave power is usually accomplished by rectification in either a semiconductor or vacuum diode, or by measuring the change of resistance of a bolometer either directly or in an unbalanced bridge circuit.

The use of a semiconductor diode to indicate microwave power is probably the most common method of microwave signal detection; nearly all radar systems use a point-contact semiconductor diode as a detector. In addition to being difficult to calibrate properly and having different characteristics from unit to unit, point-contact diodes used as rectifiers also have another distinct disadvantage when used as power monitors: if they are operated at a high enough signal level in order to escape their characteristically poor signal-to-noise ratio at low frequencies, then they have very poor overload characteristics; conversely, if they are operated at a low enough signal level to provide a large dynamic range and thus good overload characteristics, then they are operating in a region where the output signal-to-noise ratio may be poor. The lifetime characteristics of the microwave point-contact semiconductor diode are not well understood at the present time; all that can be said concerning their lifetime is that in radar applications they usually last until the T-R device fails. A major effort during and since World War II to improve radar reliability has been to extend the life of T-R devices and their associated components; thus very little has been done to extend the life of point-contact diodes. In favor of the point-contact diode, however, is its simplicity and ease of use. This, coupled with its short response time, makes it ideal for an instantaneous type of indicator which is fully capable of following a pulse envelope. Although far faster than a calorimeter, a point-contact diode cannot be expected to maintain its calibration for any appreciable length of time and thus is a poor choice for a calibrated power monitor.

Prior to the advent of the point-contact semiconductor diode, numerous types of planar vacuum diodes were used in conventional detector arrangements as rf power monitors. This sort of detector has nearly all of the disadvantages of the semiconductor diode except the
the low-frequency output noise, and in addition it is difficult to use
at most microwave frequencies because of its physical size.

Recently the development by Elliott in England of the rf monitor
diode has rekindled interest in vacuum diodes as microwave detectors.\textsuperscript{4} Basically, it has been found that if a coaxial diode structure is evacu-
ated and the center conductor treated with a cathode oxide material,
then (after processing) a potential difference will develop between the
inner and outer coaxial conductors when microwave power is propagated
through the structure. This potential difference exists even though
the transit time for an electron from inner to outer electrodes may
well be several periods of the microwave signal in length. Although
such diodes can be made to have a response time as short as 0.01 micro-
second, they are, at the present time, not clearly reliable and certain-
ly expensive.

The wire barretter is often used by itself as a power detector in
the probes of standing-wave indicators and other microwave instruments.
Like the point-contact semiconductor microwave diode, it has rather
poor overload characteristics and must be continuously re-calibrated,
even if used in an unbalanced-bridge type of circuit.

C. Balanced-Bridge Techniques

Bridge circuits in which known amounts of dc or low-frequency power
are substituted for an unknown amount of microwave power have been long
accepted, along with calorimeters, as power-measuring devices and have
even been built accurately enough to be used as rf power standards.\textsuperscript{5} Such techniques can be divided into two distinct categories: manually
balanced bridges and automatically balanced bridges. In either case,
the temperature dependence of bolometric devices demands that some sort
of compensation scheme, either manual or automatic, be employed to lend
reasonable accuracy to a bridge technique.

Both manual and conventional automatic bridges have been thoroughly
investigated and presented in texts on the subject,\textsuperscript{2} and will not be
discussed here. Of interest are the basic characteristics of thermistors,
wire barretters, and deposited-film bolometers which are, in themselves,
fundamentally simple devices that conceivably could be adaptable to a simple power-measuring scheme. Their relatively low burn-out power indicates that they must be used in a sampled-power-measurement type of system; a typical system might consist of a directional coupler and attenuator followed by a bolometer mount with its associated bolometer element and auxiliary circuitry.

Experimentation has shown that the minimum circuitry associated with a film bolometer power meter must embody some sort of feedback for continuous correction of changes in bolometer characteristics. The present state of the art in thermistors and wire barretters indicates that this result can be generalized to cover all bolometers available at the present time. Recently two commercial firms have made available power meters with double-bridge circuits in which two bolometers are used to provide a system that is essentially independent of ambient temperature variation and has a very low zero-drift. Such bridges, however, are complex in themselves, and with their associated rf sampling devices are far more complex than the simple calorimeter. Although their accuracy is excellent, it is limited by the accuracy to which their associated sampling devices can be calibrated.

D. Miscellaneous Physical Effects

The Hall effect, the pressure transducer, and the electron-beam power meter represent three additional possibilities for measuring microwave power.

The Hall effect is basically a mechanism whereby the mobile charges in a semiconductor crystal body carrying a current at right angles to an applied magnetic field suffer a displacement such that a potential difference is developed across the body at right angles to the plane containing the direction of current flow and the magnetic field. This developed potential difference has been found to be proportional to the applied magnetic field, thus making a Hall-effect crystal adaptable to monitoring the strength of a magnetic field. Although Hall-effect gaussmeters are by now in common usage, the Hall-effect rf power monitor is definitely still a laboratory device and is not presently adaptable
to the practical measurement of power.

Pressure-transducer power meters are in some sense the radio-
frequency analog of direct-current standards in that they achieve their
indication through the measurement of basic physical quantities. With
every electric, magnetic, or electromagnetic field in a region there is
associated an energy density which is numerically equal to a mechanical
pressure at the same point. It is through such a simple equality that
the familiar electrostatic computation of the force between parallel
capacitor plates can be made; the rf case is, of course, not as simple
but nonetheless embodies the same principle. Pressure transducers can
be accurately made and are available commercially but, like the Hall-
effect crystal, are still laboratory devices and are not well suited to
routine power measurements.

The electron-beam power meter is usually arranged so that an elec-
tron beam is so accelerated and geometrically arranged with respect to
a rectangular waveguide propagating power in its dominant mode that
maximum interaction is achieved and the beam gains energy from the
fields in the waveguide. A dc retarding potential is then used to
stop the beam and thus measure the energy that it gained in its transit
of the waveguide. The power in the waveguide is then computed from a
knowledge of the field pattern in the waveguide and the resultant
Poynting vector. This technique appears to have great value in
measuring high powers without disturbing the power being measured and,
in theory at least, is self-calibrating. It is more complex than any
of the systems discussed and is thus ruled out as a simple device for
monitoring high powers.
III. THE IN-LINE CALORIMETER

If a section of waveguide of known attenuation is provided with heat sinks at either end and is well insulated throughout its length, then absorption of power in the walls of the waveguide will cause the center to be elevated in temperature above the ends. As will be shown in this chapter, the temperature difference between the center and the ends of the section is directly proportional to the power absorbed and thus to the power transmitted through the section. Such a waveguide section, along with its auxiliary temperature measuring apparatus, comprises an in-line dry calorimeter capable of indicating the average power level in a waveguide. Once the characteristics of such a calorimeter are known, it could well serve as a transfer power standard, for its indication is derived entirely from directly measurable quantities.

In 1942 Johnson proposed a similar type of calorimeter in which portions of the wall of a section of waveguide had been replaced with thin constantin sheets.12 The device did not use heat sinks and was thus sensitive to temperature gradients along the waveguide. Further practical difficulties in calibration limited the usefulness of the device, and it was apparently discarded.

A. The Static Waveguide Calorimeter Equations

For ease of computation, the calorimeter will be assumed to be adequately represented by a rod of length and volume equal to the actual length and metal volume of the waveguide in the actual calorimeter, as shown in Fig. 3.1. The dissipation of microwave power in the walls of the waveguide will be treated as a uniformly distributed source of power. The dissipation of power normally occurs only within the skin depth of the waveguide, and the remainder of the metal present acts as a heat sink. If the waveguide is constructed of a metal having a thermal conductivity far higher than that of its surroundings and is insulated to minimize convection and radiation heat losses, then the rod model should be a quite accurate representation of the calorimeter.
A. In-line calorimeter

B. Rod approximation to calorimeter

C. Single volume element

FIG. 3.1--Static in-line calorimeter.
1. The basic equation

Let the rod model be divided into a number of infinitesimal volume elements dV, and let the opposite faces of a single elementary cube be isothermal surfaces at temperatures \( \theta_1 \) and \( \theta_2 \). The rate at which heat \( Q \) flows through the cube normal to the isothermal surfaces is equal to the product of the thermal conductivity \( K \) of the material and the gradient of the temperature distribution \( \theta^* \):

\[
\dot{Q} = -K \nabla \theta \cdot \hat{A}
\]  

(3.1)

where the dot denotes the first time derivative, and \( \hat{A} \) is the end-area normal vector. The quantity \( \dot{Q} \) is sometimes called heat current in analogy with electric current. Now by Gauss' theorem the integral of a vector over a closed surface is equal to the integral of its divergence through the volume enclosed by that surface. Thus the rate of heat energy decrease in the volume dV is

\[
\dot{U}_1 = -KV^2\theta
\]  

(3.2)

The energy per unit volume \( U \), in joules per cubic centimeter, at any point is equal to the temperature at that point times the product of the thermal capacity \( s \), in joules per degree Celsius per gram, and the density \( \rho \), in grams per cubic centimeter: \( U = s\rho\theta \) such that the rate of energy increase in the volume is

\[
\dot{U}_2 = s\rho\theta
\]  

(3.3)

Now if an external source adds an amount of energy per unit volume along the rod at a rate of \( F \), then because the volume contains neither sources

---

or sinks of heat the sum of all heat flows entering and leaving the volume must be equal to the total energy increase:

\[ \dot{U}_2 = \overline{P} - \dot{U}_1 \]

or

\[ KV^2 \theta - s \phi \dot{\theta} + \overline{P} = 0 \]  
(3.4)

which is the heat-flow equation for the distributed energy source case. For the one-dimensional case, Eq. (3.4) may be written

\[ \frac{\partial^2 \theta}{\partial x^2} - \frac{s \phi}{K} \frac{\partial \theta}{\partial t} = -\frac{\overline{P}}{K} \]

which will reduce to a steady-state and a transient equation:

\[ \frac{\partial^2 \theta}{\partial x^2} = -\frac{\overline{P}}{K} \]  
(steady-state)  
(3.5a)

\[ \frac{\partial^2 \theta}{\partial x^2} - \frac{s \phi}{K} \frac{\partial \theta}{\partial t} = 0 \]  
(transient)  
(3.5b)

2. The steady-state solution

The steady-state solution can be obtained from Eq. (3.5a) by applying the following boundary conditions:

when \( x = 0 \), \( \frac{\partial \theta}{\partial x} = 0 \)

when \( x = \pm l \), \( \theta = \theta_0 \)
which results in a parabolic solution of the form

\[ \theta = \frac{P}{2K} \left[ x^2 - x^2 \right] + \theta_0 \]  

(3.6)

3. The transient solution

The transient solution can be obtained by applying the appropriate boundary conditions to Eq. (3.5b). It is, however, simpler to demand that the transient solution be equal in magnitude and opposite in sign to the steady-state solution for zero time, and zero for infinite time. The net result is then that the entire rod is at temperature \( \theta_0 \) at \( t = 0 \) and assumes the steady-state solution of Eq. (3.6) for \( t \) infinite. The method of attack will be to study the problem of the temperature distribution \( \theta(x,t) \) in a rod when the initial temperature distribution \( \theta(x,0) \) is a parabola of the form of Eq. (3.6). For convenience, the symbol \( \theta \) will now be taken to mean the temperature above the end temperature \( \theta_0 \). Equation (3.5b) can be rewritten as

\[ \frac{\partial \theta}{\partial t} = \frac{K}{s\rho} \frac{\partial^2 \theta}{\partial x^2} \]  

(3.7)

Using Table 3.1, the quantity \( K/s\rho \) is found to be usually less than unity (0.112 cm\(^2\)/\(^\circ\)C for copper). The initial temperature distribution \( \theta(x,0) \) of Eq. (3.6) can be expanded in a Fourier series containing cosine terms only because the \( x \) coordinates chosen for the parabola make it an even function of \( x \). The Fourier series solution for \( \theta(x,t) \) can be written as

\[ \theta(x,t) = \sum_{n=1}^{\infty} A_n \exp \left[ -\left( \frac{n\pi}{2L} \right)^2 \frac{K}{s\rho} t \right] \cos \left( \frac{n\pi x}{2L} \right) \]  

(3.8)
TABLE 3.1
CHARACTERISTICS OF VARIOUS METALS

<table>
<thead>
<tr>
<th>Material</th>
<th>s</th>
<th>ρ</th>
<th>K</th>
<th>α/α_{Cu} *</th>
<th>η</th>
<th>θ(0,∞)/θ(0,∞)_{Cu}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>joules</td>
<td>gm/cm³</td>
<td>watts</td>
<td>°C/cm</td>
<td>°C·cm³</td>
<td>joules</td>
</tr>
<tr>
<td>Aluminum</td>
<td>0.896</td>
<td>2.69</td>
<td>2.135</td>
<td>1.23</td>
<td>0.511</td>
<td>2.31</td>
</tr>
<tr>
<td>Brass (yellow)</td>
<td>0.364</td>
<td>8.56</td>
<td>1.03</td>
<td>2.0</td>
<td>0.397</td>
<td>7.07</td>
</tr>
<tr>
<td>Constantin</td>
<td>0.418</td>
<td>8.88</td>
<td>0.226</td>
<td>4.97</td>
<td>0.322</td>
<td>35.6</td>
</tr>
<tr>
<td>Copper</td>
<td>0.38</td>
<td>8.89</td>
<td>3.82</td>
<td>1.00</td>
<td>0.361</td>
<td>1.00</td>
</tr>
<tr>
<td>Gold (drawn)</td>
<td>0.129</td>
<td>19.26</td>
<td>2.93</td>
<td>1.19</td>
<td>0.495</td>
<td>1.56</td>
</tr>
<tr>
<td>Nickel</td>
<td>0.440</td>
<td>8.70</td>
<td>0.595</td>
<td>1.83</td>
<td>0.323</td>
<td>12.12</td>
</tr>
<tr>
<td>Platinum</td>
<td>0.134</td>
<td>21.37</td>
<td>0.70</td>
<td>2.41</td>
<td>0.432</td>
<td>13.31</td>
</tr>
<tr>
<td>Silver</td>
<td>0.234</td>
<td>10.60</td>
<td>4.22</td>
<td>0.97</td>
<td>0.498</td>
<td>0.83</td>
</tr>
<tr>
<td>Steel, carbon</td>
<td>0.482</td>
<td>7.87</td>
<td>0.594</td>
<td>2.76</td>
<td>0.325</td>
<td>17.92</td>
</tr>
<tr>
<td>Steel, 304 stainless</td>
<td>0.502</td>
<td>7.93</td>
<td>0.151</td>
<td>7.0*</td>
<td>0.311</td>
<td>179.5</td>
</tr>
</tbody>
</table>

*computed using square roots of ratios of resistivities.

* measured value at 2.856 Gc is 7.83.14
Taking the partial derivatives of $\theta(x,t)$,

$$
\frac{\partial \theta}{\partial t} = - \sum_{n=1}^{\infty} \left( \frac{nx}{2l} \right)^2 \frac{K}{sp} A_n \exp \left[ - \left( \frac{nx}{2l} \right)^2 \frac{K}{sp} t \right] \cos \left( \frac{nx}{2l} \right)
$$

$$
\frac{K}{sp} \frac{\partial^2 \theta}{\partial t^2} = - \sum_{n=1}^{\infty} \left( \frac{nx}{2l} \right)^2 \frac{K}{sp} A_n \exp \left[ - \left( \frac{nx}{2l} \right)^2 \frac{K}{sp} t \right] \cos \left( \frac{nx}{2l} \right)
$$

shows that Eq. (3.7) is satisfied; thus Eq. (3.6) is a valid form for $\theta(x,t)$. Now for $t = 0$, $\theta(x,0)$ must be equal to the negative of the steady-state distribution,

$$
\frac{F}{2K} (l^2 - x^2) = - \sum_{n=1}^{\infty} A_n \cos \left( \frac{nx}{2l} \right)
$$

where the coefficients $A_n$ are evaluated as

$$
A_n = \frac{1}{l} \int_{-l}^{l} \theta(x,0) \cos \left( \frac{nx}{2l} \right) dx
$$

By straightforward evaluation the $A_n$'s are

$$
A_n = \begin{cases} 
\frac{16F l^2}{Kn^3 \pi^3} & \text{For } n = 1, 5, 9, 13 \ldots \\
- \frac{16F l^2}{Kn^3 \pi^3} & \text{For } n = 3, 7, 11, 15 \ldots \\
0 & \text{For } n = 0, 2, 4, 6, 8 \ldots
\end{cases}
$$
which reduces to

\[ A_n = \frac{16P\ell^2(-1)^n}{Kn^3\pi^3} \quad \text{for } n \geq 0 \]

Thus Eq. (3.8) for the transient solution becomes

\[ \theta(x,t) = \frac{16P\ell^2}{Kn^3} \sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)^3} \exp \left[ -\frac{(2n+1)^2}{2\ell} \right] \exp \left[ -\frac{(2n+1)\pi^2}{2l^2} \right] \cos \left( \frac{\pi nx}{2l} \right) \]

and the complete solution for the temperature distribution in the rod as a function of both distance along the rod and time is

\[ \theta(x,t) = \frac{P\ell^2}{2K} \left\{ \ell^2 - x^2 - \frac{32\ell^2}{\pi^3} \sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)^3} \exp \left[ -\frac{(2n+1)^2}{2\ell} \right] \exp \left[ -\frac{(2n+1)\pi^2}{2l^2} \right] \cos \left( \frac{\pi nx}{2l} \right) \right\} \quad (3.10)\]

The solution of Eq. (3.10) for \( t = 0 \) should reduce to zero. As a check, \( \theta(0,0) \) is

\[ \theta(0,0) = \frac{P\ell^2}{2K} \left\{ 1 - \frac{32}{\pi^3} \sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)^3} \right\} \]

This will reduce to zero if the summation is equal to the quantity \( \pi^3/32 \). The summation is similar to an Euler polynomial which has the form

\[ E_{2r} = \frac{(-1)^r 2r!(2)^{2r+2}}{(\pi)^{2r+1}} \sum_{n=0}^\infty \frac{(-1)^n}{(2n+1)^{2r+1}} \quad (3.11) \]
where the first four Euler numbers are: \( E_0 = 1, E_2 = -1, E_4 = 5, E_6 = -61 \). Thus for \( r = 1 \), Eq. (3.11) becomes
\[
E_2 = -1 = -\frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3}
\]
Thus the summation in question is indeed equal to \( \pi^3/32 \), and \( \theta(0,0) \) reduces to zero. Equation (3.10) may then be regarded as the complete solution for the temperature distribution in the calorimeter. The actual calorimeter measures the temperature rise \( \theta(0,t) \) of the center of the waveguide, which is given by
\[
\theta(0,t) = \frac{\bar{P}l^2}{2K} \left\{ 1 - \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \exp \left[ -\frac{(2n+1)^3t}{\tau} \right] \right\} = \frac{\bar{P}l^2}{2K} \left\{ 1 - 1.032 \frac{e^{-t/\tau}}{\tau} + 0.0381 \frac{e^{-2\tau t/\tau}}{\tau} - 0.0082 \frac{e^{-12\tau t/\tau}}{\tau} + \ldots \right\}
\]
where \( \bar{P} \) is the power per unit waveguide metal volume absorbed by the calorimeter, \( l \) is the half length of the waveguide between heat sinks, and \( \tau = 4l^2 s/p/\pi^2 K \) (as shown in Appendix A) has the dimensions of time and thus is a kind of time constant. This time constant may, for S-band waveguides of common metals 6 cm in length, fall in the range of tens of seconds to several minutes. As can be seen from Eq. (3.12), the first term of the summation for \( n = 0 \) will dominate, for the coefficients of the terms of the summation are the coefficients of the cosine terms of the series representing the steady-state parabolic distribution given by Eq. (3.6), and a parabola of small amplitude is fairly well approximated by the first term of its representative Fourier cosine series.
Both the time constant and the total temperature rise at the center of the calorimeter are seen to depend on the square of the half-length $c$. The design of a given calorimeter must then be a compromise between a long body for sensitivity and a short body for a fast rise time. The eventual temperature rise at the center of the calorimeter as $t$ approaches infinity is given by

$$\theta(0, \infty) = \frac{P L^2}{2K}$$  \hspace{1cm} (3.13)

It is interesting to note that the ratio of the calorimeter steady-state temperature rise per unit of input energy from Eq. (3.13) to the time constant of Eq. (3.12) is dependent only on the product of the thermal capacity $s$ and density $\rho$:

$$\frac{\theta(0, \infty)}{\tau} = \frac{\pi^2}{8s \rho} = \eta$$  \hspace{1cm} (3.14)

where $\eta$, as shown in Appendix A, has the dimensions of temperature divided by energy per unit volume.

Because the product of the thermal capacity $s$ and the density $\rho$ is fairly constant for metals, $\eta$ will be roughly the same for any metal, as illustrated by Table 3.1 ($\eta$ for copper is 0.361). This, then, indicates that for a given absorbed power per unit volume, the ratio of expected temperature rise to time-constant is approximately constant.

B. Effect of a Non-Zero Reflection Coefficient on Calorimeter Indication

Even if the calorimeter itself is perfectly matched, a standing wave of the $H$ field in the waveguide will result in a standing wave of waveguide wall current. Since the power loss to the waveguide walls is proportional to the square of the wall current, a standing wave of power dissipated in the waveguide walls will exist. The calorimeter might then be expected to respond to this pattern in some way other than simply reading the sum of the incident and reflected power passing through the
waveguide section.

Equation (3.13), which expressed the temperature rise at the center of the calorimeter caused by the absorption of power in the walls of the waveguide, was based on the assumption that the \( E \) field, and thus the power absorbed in the waveguide walls, was uniform along the axis of the waveguide. This, of course, implies that the load reflection coefficient is zero. If, however, the load reflection coefficient is not zero, and there exists a standing-wave pattern of electric and magnetic fields within the waveguide, then the power dissipated in the walls of the waveguide will not be uniform along the axis of the calorimeter. In what follows, the absorbed-power standing-wave pattern will be assumed to be directly proportional to the square of the wall current standing-wave pattern and thus to the square of the \( E \) field standing-wave pattern as shown in Fig. 3.2.

Assuming that the waveguide is lossless, then the ideal transmission-line equations apply. The power absorbed in the waveguide walls as a function of the reflection coefficient \( \Gamma \) can then be written

\[
P = P_i + P_r + 2|P_i| \cos \frac{4\pi(x - a)}{\lambda_g}
\]

(3.15)

where \( P_i \) is the incident power, \( P_r \) is the reflected power, and \( a \) is the phase of the pattern and is a random variable assumed to have a uniform distribution between 0 and \( 2\pi \). If the sum of \( P_i \) and \( P_r \) is set equal to a nominal power, \( P_0 = P_i + P_r \), then

\[
P = P_0 \left[ 1 + \frac{2\Gamma}{1 + \Gamma^2} \cos \frac{4\pi(x - a)}{\lambda_g} \right]
\]

(3.16).

Equation (3.16) expresses the relationship between absorbed waveguide power \( P \) and axial displacement \( x \). Thus the steady-state
FIG. 3.2--Standing-wave pattern of power in a waveguide.
Eq. (3.5a) is modified as follows:

$$\frac{\partial^2 \theta}{\partial x^2} = - \frac{P_o}{K} \left[ 1 + \frac{2\Gamma}{1 + \Gamma^2} \cos \frac{4\pi(x - a)}{\lambda_g} \right] \tag{3.17}$$

Integrating Eq. (3.17) twice with respect to $x$ results in an equation with two unknown constants:

$$\theta(x,t) = - \frac{P_o}{K} \left[ \frac{x^2}{2} - \frac{2\Gamma}{1 + \Gamma^2} \left( \frac{\lambda_g}{4\pi} \right)^2 \cos \frac{4\pi(x - a)}{\lambda_g} \right] + C_1x + C_2 \tag{3.18}$$

Using the boundary conditions of $\theta = 0$ at $x = +l$ and $x = -l$ separately, two equations involving the unknown constants $C_1$ and $C_2$ can be obtained from Eq. (3.18). These two equations can then be solved algebraically for $C_1$ and $C_2$. With these values for the constants, the temperature rise in the center of the calorimeter is

$$\theta(x,t) = \frac{P_o l^2}{2K} \left\{ 1 + \frac{\lambda_g^2}{8\pi^2 l^2 (1 + \Gamma^2)} \left( \cos \frac{4\pi a}{\lambda_g} \right) \left( 1 - \cos \frac{4\pi l}{\lambda_g} \right) \right\} \tag{3.19}$$

Now defining the nominal center temperature rise according to Eq. (3.13) as $\theta_0 = \frac{P_o l^2}{2K}$, normalizing the length $\ell$ and phase $\alpha$ such that $L = 2\pi\ell/\lambda_g$ and $A = 2\pi\alpha/\lambda_g$, and defining the per-unit error as

$$\epsilon = \frac{[\theta(0,\infty) - \theta_0(0,\infty)]}{\theta_0(0,\infty)}$$

results in a simple error expression:

$$\epsilon = \frac{2}{L^2} \frac{\Gamma}{1 + \Gamma^2} \cos 2A \sin^2 L$$

- 22 -
The largest error would be for \( A = n\pi \) (\( n \) an integer), so the absolute value of the error is bounded:

\[
|\epsilon| \leq \left( \frac{\sin L}{L} \right) \frac{2|\Gamma|}{1 + |\Gamma|^2}
\]

Equation (3.20) may easily be plotted, since \( \sin L/L \) is well tabulated and since the error \( |\epsilon| \) as a function of length \( L \) with reflection coefficient \( |\Gamma| \) as a parameter is of interest. This is shown in Fig. 3.3.

It should be noted that the error \( |\epsilon| \) expresses the extent to which the calorimeter does not read the sum of the incident and reflected powers passing through its waveguide section. If the error is zero, the calorimeter simply reads the sum of \( P_i \) and \( P_r \); the reflection coefficient must then be known in order to extract either \( P_i \) or \( P_r \) from the calorimeter reading. In order for the error \( |\epsilon| \) to vanish, the normalized length \( L \) must be a multiple of \( \pi \), or the half-length \( \ell \) must be an integral number of half-waveguide wavelengths, \( \ell = n\lambda_g/2 \), as illustrated by Fig. 3.3. In this case Eq. (3.19) reduces to Eq. (3.13), and the center temperature rise and thus the output indication is dependent only on the sum of the powers passing through the calorimeter.

C. Calorimeter Characteristics

Provided that the length of the calorimeter is chosen to be an integral number of waveguide wavelengths, in accordance with Fig. 3.3, the steady-state temperature rise at the center of the calorimeter is

\[
\theta(0,\infty) = \frac{\bar{P}\ell^2}{2K}
\]

But \( \bar{P} \) is the power per unit volume dissipated in the waveguide material; the length of the calorimeter is \( 2\ell \); and if the cross-sectional metal area is \( a \), the temperature rise due to the total absorbed power
FIG. 3.3--Maximum error $|\epsilon|$ vs normalized length $L$. 

Legend:
- $|\Gamma| = 1.0$
- $|\Gamma| = 0.5$
- $|\Gamma| = 0.1$
If the waveguide has an attenuation of $\alpha$ db per unit length, and if its length is short, so that the average power leaving ($P_{out}$) is essentially the same as the average power entering ($P_{in}$), then $P$ may be written

$$P = P_{in} \left(1 - 10^{-0.2l\alpha}\right)$$

which gives

$$\theta(0,\infty) = \frac{lP_{in}}{4K_{a}} \left[1 - 10^{-0.2l\alpha}\right]$$

If the total attenuation through the waveguide section is very small, that is, if $0.2l\alpha \ll 1$, then $\theta(0,\infty)$ may be approximated by expanding $(1 - 10^{-0.2l\alpha})$ in a series and taking the first term:

$$\theta(0,\infty) \approx \frac{0.115l^{2}P_{in}\alpha}{aK}$$

Equation (3.22) gives the temperature rise to be expected in the middle of a calorimeter constructed of a given metal of given dimensions. If, for example, a calorimeter were constructed of 1-by-3 inch copper waveguide with a total length of 6 cm, Eq. (3.22) would give a center temperature rise of $5.5 \times 10^{-5} \, ^{\circ}C$ per watt of input power.

The selection of a material for the waveguide section of a specific calorimeter is based on a compromise among several factors. The major factor of concern is, of course, the temperature rise that can be expected per unit of microwave power flowing through the waveguide.
section; Eq. (3.22) can be used to predict $\theta(0,\infty)$ for any given calorimeter configuration. It is convenient to compare various materials using one as a reference. If copper is chosen as a reference, then $\theta(0,\infty)$ for various metals may be tabulated as a function of $\theta(0,\infty)$ for copper. From Eq. (3.22)

$$\frac{\theta(0,\infty)}{\theta(0,\infty)_\text{Cu}} = \frac{K \alpha}{K \alpha_{\text{Cu}}}$$

(3.23)

The various parameters for several metals are tabulated in Table 3.1, along with the results of Eq. (3.23) for each case. The center temperature ratios vary from 0.88 for silver to 179.5 for stainless steel.

D. Temperature Indication

Given a specific temperature difference between the center and the ends of the calorimeter, some method must be used to give an indication of this difference so that the waveguide power may be measured. Two methods immediately suggest themselves: a resistance-wire bridge, and a thermopile. The resistance-wire bridge has the advantage that as large a signal as desired can be obtained; it was, therefore, selected as the method of indication.

If four identical temperature-sensitive resistances are placed in contact with the waveguide and connected in a bridge circuit, then the output of the bridge becomes an indication of the temperature difference between the middle and the ends of the waveguide (see Fig. 3.4). Suppose that the ends of the waveguide along with resistances $R_1$ and $R_4$ are at some reference temperature, and the center of the waveguide along with resistances $R_2$ and $R_3$ is elevated in temperature by $\theta$ degrees Celsius. The ratio of the detector voltage $V_d$ due to the unbalanced bridge to the source voltage $E$ is

$$\frac{V_d}{E} = \frac{k\theta}{2 + k\theta}$$

(3.24)
A. Location of resistance wire windings on calorimeter.

B. Resistance bridge circuit.

FIG. 3.4--The resistance bridge.
where $\theta$ is the temperature difference between the center and the ends of the calorimeter, and $k$ is the temperature coefficient of resistance of the wire used. If both $k$ and $\theta$ are relatively small, so that their product is much less than 2, then Eq. (3.23) may be approximated as

\[
\frac{V_d}{E} \approx k\theta \frac{1}{2}
\]

Combining this with Eq. (3.22) and rewriting gives an expression for the voltage output of the bridge per unit power flowing through the waveguide:

\[
\frac{V_d}{P_{in}} = \frac{0.0575 k E}{\alpha k E} = \frac{0.0575}{\alpha} = \frac{0.0575}{0.0043} \approx 13.3
\]

Equation (3.25) may then be used to predict the amount of output signal expected from a given calorimeter. For example, a calorimeter 6 cm long using standard S-band copper waveguide at a frequency of 3.0 GHz and using a resistance wire with $k = 0.0045$ will give an output signal of 45 microvolts per volt into the bridge per kilowatt of power flowing through the waveguide. For the same waveguide dimensions in stainless steel, the output would be about 8 millivolts per volt into the bridge per kilowatt of power flowing through the waveguide.
IV. EXPERIMENTAL CALORIMETERS

A. Characteristics

If a resistance-wire bridge is used to indicate the temperature rise in the center of a calorimeter, then Eqs. (3.12) and (3.25) can be combined to give an expression for the voltage output of the bridge circuit as a function of bridge voltage input, waveguide power input, and time:

\[
V_d = \frac{0.0575 \zeta^2 k a E P_{in}}{a K} \left\{ \begin{array}{l}
1 - \frac{32}{\pi^3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^3} \exp\left[ -\frac{(2n+1)^2 t}{\tau} \right] \\
1 - 1.032 e^{-t/\tau} + 0.0381 e^{27t/\tau} \\
- 0.0082 e^{125t/\tau} + \ldots \end{array} \right. 
\]

where \( \tau = \frac{4 \zeta^2 s_p}{a K} \) as in Eq. (3.12). Equation (4.1) along with Table 3.1 can then be used to predict the behavior of a calorimeter constructed using a given waveguide material. There is a slight disadvantage to building a calorimeter using a waveguide of a uniform material in that it is desirable to have a high waveguide attenuation in order to have a reasonable output as well as desirable to have a high thermal conductivity in order to have a short time constant. To examine this, Eqs. (3.12), (3.22), and (3.14) can be combined and rewritten

\[
\theta(0,\infty) = \frac{0.115 \zeta^2 P_{in} \alpha}{a K} \quad (4.2a)
\]

\[
\tau = \frac{\zeta^2}{2K\eta} \quad (4.2b)
\]
Thus, to have a reasonable center temperature rise and a short time constant, it is desirable to select a material with large thermal conductivity and large attenuation. These are of course almost mutually exclusive qualities, so the obvious solution is to coat the inside of a high-thermal-conductivity waveguide with a lossy material to give the waveguide a high attenuation. This arrangement is a bit impractical from the experimental standpoint, although for a production device it might be an excellent way of achieving both reasonable sensitivity and a short time constant.

According to Table 3.1, a calorimeter constructed using stainless steel would be the most sensitive, but would also have the longest time constant. Of the materials tabulated in Table 3.1, copper and stainless steel would be the best combination for a coated waveguide type of construction. For the present experimental purposes, however, the stainless steel is the most satisfactory. The results obtained using stainless steel can be extrapolated to cover other waveguide materials.

The resistances of the arms of the calorimeter bridge circuit are chosen to minimize the error due to heating the calorimeter from power losses in the bridge circuit itself. The power dissipated in a calorimeter of length $2\ell$ and of attenuation per unit length $\alpha$ is $0.46\ell P_{in}$, while that dissipated in the bridge is $\ell^2/R$. If the power dissipated in the bridge is to be limited to 1-2% of the absorbed waveguide power for the smallest expected waveguide input power, then for a given bridge input voltage the minimum bridge arm resistance is determined.

Two S-band calorimeters using stainless steel waveguide have been designed and tested: a short model with water-cooled flanges, and a longer model designed to be built on the existing waveguide of a system. The theoretical and experimental values of sensitivity and time constant have been compared for both models, and the minimum resolution has been measured for the short model. The results are described in the following pages.
B. Short Calorimeter with Flanges

In order to determine experimentally the feasibility of an in-line calorimeter, a model 3-3/4 inches long was constructed using 3.00 x 1.50 inch x 0.065 inch wall stainless steel waveguide and copper water-circulating vacuum flanges (see Fig. 4.1). An alloy wire of 70 percent nickel and 30 percent iron ("Balco," a trademark of the Wilbur B. Driver Co.) was selected for the resistance bridge windings as the best compromise between a high temperature coefficient of resistance and high tensile strength. The resistances of the bridge windings were selected to be approximately 200 ohms, so that the total bridge generated power is less than 2 percent of the power absorbed by the calorimeter when the input power is 100 watts and the bridge input is 1 volt. The resistance windings are 200 ohms each at 26°C; a small 1-ohm trimming potentiometer is included to balance the bridge. The entire unit is insulated with foam plastic to reduce convection and radiation heat losses.

The pertinent constants for this calorimeter are as follows:

\[ l = 3.80 \text{ cm} \quad K = 1.506 \times 10^{-1} \text{ watts/°C-cm} \]
\[ a = 3.66 \text{ cm}^2 \quad s = 0.502 \text{ joules/°C-gm} \]
\[ k = 4.5 \times 10^{-3}/\text{°C} \quad \rho = 7.93 \text{ gm/cm}^3 \]
\[ E = 1.0 \text{ volt assumed} \quad \alpha = 0.04698 \text{ db/ft (see ref. 14)} \]

Substituting these values in Eq. (4.1) gives, for the first three terms of the summation,

\[ V_d = 13.25 \text{ P}_{\text{in}} \left(1 - 1.032 e^{-t/156.5} + 0.0384 e^{-t/17.4} - 0.008 e^{-t/1.25}\right) \]

where time \( t \) is in seconds, power input \( P_{\text{in}} \) is in kilowatts, and detector voltage to open circuit \( V_d \) is in millivolts. The sensitivity of this calorimeter should then be 13.25 millivolts out per volt input to the bridge per kilowatt flowing through the calorimeter, which is larger than that of the example of the previous chapter because the half-length \( l \) is longer and the area \( a \) is less.
FIG. 4.1--Short calorimeter with flanges. Insulation removed.
The first term of the summation is dominant, so the rise time can be assumed to be controlled by a single time constant, in which case the computed time constant is \( \tau = 2 \text{ minutes, } 36.5 \text{ seconds} \).

The sensitivity and time constant of the actual calorimeter were measured in the ring-waveguide test setup shown in Fig. 4.2. Average power levels up to 1 kilowatt were used to determine the sensitivity, while the time constant was determined by measuring the decay of the bridge output voltage with time after the exciting power in the ring circuit was turned off. Under the assumption that the rise and fall times of the calorimeter are dominated by a single exponential, the time constant was taken as being the time necessary for the bridge output to decrease to \( 1/e \) of its steady-state excited value. The calibration of the ring power monitor is accurate to ten percent, so any experimental figure for sensitivity may be inaccurate by ten percent. The experimental results are as follows:

- **Sensitivity:** \( 12.5 \pm 1.25 \text{ millivolts out per volt into bridge per kilowatt flowing through calorimeter} \)
- **Time constant:** 2 minutes 30 seconds

This model of the calorimeter was also checked for minimum resolution using the setup of Fig. 4.3. The ambient air temperature was maintained at \( 25^\circ \pm 2^\circ \text{C} \), while the flange cooling water was maintained at a temperature of \( 18.5^\circ \pm 0.6^\circ \text{C} \). The microwave power through the calorimeter was cycled: 0.9 watts average for one-half hour, zero power for one-half hour. The recording of the bridge output is shown in Fig. 4.4. As can be seen from the recording, the instabilities in the calorimeter output are equivalent to no more than about 0.5 watt of power transmitted through the device. It might then be concluded that the ultimate resolution of the calorimeter is 0.5 watts.

C. **A Clamp-On Calorimeter One Waveguide-Wavelength Long**

To illustrate the simplicity of the in-line calorimeter, a model with clamp-on heat sinks was constructed (see Fig. 4.5). This is
FIG. 4.2--Test setup for measuring sensitivity and time constant.

A. Waveguide ring setup

B. Bridge circuit
FIG. 4.3--Minimum resolution test setup.
FIG. 4.4—Minimum resolution test recording.
FIG. 4.5-λ long calorimeter with clamp-on heat sinks. Insulation partially removed at right end to show cooling clamp.
distinctly different from the previous model in that it uses an existing waveguide and does not require two flange connections.

In order to minimize the error in reading the true sum of the powers passing through the calorimeter, the half-length \( l \) should be a multiple of half waveguide wavelengths. The total length of the shortest calorimeter possible is then one waveguide wavelength. The waveguide wavelength at 2.856 Gc in the stainless steel waveguide was measured by sliding a short through the waveguide positioned in the output arm of a detuned three-arm reflectometer, and was found to be 14.98 cm. Water cooling blocks were clamped 14.98 cm apart on a length of stainless steel waveguide to form the calorimeter. The bridge resistances are 380 ohms, so that the maximum bridge power for an input waveguide power of 100 watts and a bridge input of 1 volt will be only 1.5 percent of the power absorbed by the waveguide. Other than the length and the arm resistances, the constants for this model of the calorimeter are identical to those of the short calorimeter. The expression for the output voltage of the bridge of this calorimeter is then

\[ V_d = 51.5 \, P_{in} \left( 1 - 1.032 \, e^{-t/600} + 0.0384 \, e^{-t/66.7} - 0.00826 \, e^{-t/4.8} \right) \]

to the first three terms of the summation. The sensitivity of this model is then 51.5 millivolts output per volt input per kilowatt flowing through the calorimeter. The first term of the summation is again dominant, so the time constant can be taken as \( \tau = 10 \) minutes.

The sensitivity and time constant were measured in the test setup shown in Fig. 4.2. Average power levels up to 2450 watts were used to determine the sensitivity, while the time constant was determined in the same fashion as was that for the short model. The experimental results are as follows:

Sensitivity: \( 47 \pm 4.7 \) millivolts output per volt input to bridge per kilowatt flowing through calorimeter

Time constant: 11 minutes
D. Conclusions

The close agreement between the theoretical and experimental values of sensitivity and time constant for the two experimental calorimeters illustrates the utility of the in-line calorimeter in that it is not only a simple device but also has predictable characteristics. These results seem to indicate that if a calorimeter were made with extreme care, insulated as well as possible, and supplied with a constant-temperature, constant-flow-rate coolant to its heat sinks, it could easily serve as a secondary high-power standard. The chief disadvantage of the in-line calorimeter is its inherently long time constant: ten minutes for the longer calorimeter may well be far too long for most applications. If, however, the longer calorimeter were constructed of copper waveguide with a coating of stainless steel on the inside, Eqs. (4.2a) and (4.2b) indicate that the time constant would be 21 seconds, while the sensitivity would be 2.02 millivolts output per volt input per kilowatt flowing through the calorimeter.

It may then be concluded that the inherently simple in-line calorimeter is potentially an accurate device which can be made to have a reasonable time constant while maintaining an adequate sensitivity.
APPENDIX A
DIMENSIONAL CHECKS

The various equations derived herein are shown to be dimensionally correct below. The fundamental quantities involved, and their respective dimensions are, in terms of mass $M$, length $L$, time $T$, and temperature $\theta$, as follows:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calorie or joule (units of energy)</td>
<td>$ML^2/T^2$</td>
</tr>
<tr>
<td>Power $P$</td>
<td>$ML^2/T^3$</td>
</tr>
<tr>
<td>Thermal conductivity $K^*$</td>
<td>$ML/T^3\theta$</td>
</tr>
<tr>
<td>Attenuation $\alpha$</td>
<td>$1/L$</td>
</tr>
<tr>
<td>Area $A$</td>
<td>$L^2$</td>
</tr>
<tr>
<td>Density $\rho^*$</td>
<td>$M/L^3$</td>
</tr>
<tr>
<td>Thermal capacity $s^*$</td>
<td>$L^2/T^2\theta$</td>
</tr>
</tbody>
</table>

Equation (3.3):

$$ U = s\rho\theta: \quad \frac{ML^2}{L^3T^3} = \frac{ML^2}{L^3} = \frac{\text{energy}}{\text{volume}} $$

Equation (3.12):

$$ \tau = \frac{4L^2s\rho}{\pi^2K}: \quad \frac{L^2ML^3\theta L^2}{L^3ML^2\theta} = T $$

Equation (3.14):

$$ \frac{\theta(0,\infty)/P}{\tau} = \frac{\pi^2}{\delta s\rho} = \eta: \quad \frac{\theta L^3T^3}{ML^2T} = \frac{L^3T^2\theta}{ML^2} $$

Equation (3.22):

$$ \theta(0,\infty) = \frac{0.115L^2\rho P_{in}}{\sigma K}: \quad \frac{L^2ML^2T^3\theta}{T^3ML^3L} = \theta $$

*Typical values may be found in Table 3.1.
REFERENCES


7. G. F. Engen, private communication.


