350 KILOVOLT PULSE VOLTAGE DIVIDER

By
M. Michael Brady

Technical Report
A.E.C. Contract AT(04-3)-21
(Project Agreement No. 1)
M.L. Report No. 788
M Report No. 247
January 1961

W. W. Hansen Laboratories of Physics
Stanford University
Stanford, California
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# TABLE OF CONTENTS

I. Introduction ................................................. 1

II. A review of high-voltage measurement schemes ........ 3
   A. Voltage transformers ..................................... 3
   B. Calibrated spark gaps ................................... 3
   C. Electric-field instruments ................................. 4
   D. Rotary voltmeters ........................................ 4
   E. Dividers .................................................. 6

III. The three-terminal capacitive divider ................... 9
   A. Electrode characteristics ................................. 9
   B. Geometrical design ....................................... 10
   C. Errors in computed capacitance due to geometrical
      anomalies ................................................. 15
   D. Effect of temperature variation on division ratio ... 21
   E. Divider circuitry ......................................... 23
   F. Divider design ............................................ 27

IV. Pulse comparator circuit .................................... 30

V. Calibration and test of the divider ...................... 34
   A. Division ratio $K$ of the divider alone ................. 34
   B. Division ratio of the entire divider circuit .......... 36
   C. Measurement and test of the divider .................... 38

References .................................................................... 40

Acknowledgment .................................................... 41
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1a</td>
<td>Typical voltage transformer</td>
<td>5</td>
</tr>
<tr>
<td>2.1b</td>
<td>Calibrated spark gap</td>
<td>5</td>
</tr>
<tr>
<td>2.1c</td>
<td>A type of electrostatic voltmeter</td>
<td>5</td>
</tr>
<tr>
<td>2.2a</td>
<td>Typical rotary voltmeter</td>
<td>8</td>
</tr>
<tr>
<td>2.2b</td>
<td>Typical resistive and capacitive voltage dividers</td>
<td>8</td>
</tr>
<tr>
<td>3.1a</td>
<td>Arbitrary-shaped electrodes in a dielectric</td>
<td>11</td>
</tr>
<tr>
<td>3.1b</td>
<td>The three-terminal capacitor</td>
<td>11</td>
</tr>
<tr>
<td>3.2</td>
<td>Minimum outer radius b vs. radius ratio σ</td>
<td>13</td>
</tr>
<tr>
<td>3.3</td>
<td>Side view of coaxial capacitor using guard rings</td>
<td>14</td>
</tr>
<tr>
<td>3.4a</td>
<td>Conformal transformation</td>
<td>17</td>
</tr>
<tr>
<td>3.4b</td>
<td>Deformed cylinder</td>
<td>17</td>
</tr>
<tr>
<td>3.5</td>
<td>Anaxial alignment of cylinders</td>
<td>20</td>
</tr>
<tr>
<td>3.6a</td>
<td>Low-frequency portion of division ratio frequency characteristic</td>
<td>26</td>
</tr>
<tr>
<td>3.6b</td>
<td>Complete circuit and reduced circuit of divider circuit</td>
<td>26</td>
</tr>
<tr>
<td>3.6c</td>
<td>Circuit for evaluating effect of divider input impedance</td>
<td>26</td>
</tr>
<tr>
<td>3.7</td>
<td>Side view of divider structure</td>
<td>28</td>
</tr>
<tr>
<td>4.1a</td>
<td>Basic chopper pulse comparitor circuit</td>
<td>31</td>
</tr>
<tr>
<td>4.1b</td>
<td>Typical output presented by an oscilloscope</td>
<td>31</td>
</tr>
<tr>
<td>4.2</td>
<td>Complete diagram of pulse comparitor</td>
<td>32</td>
</tr>
<tr>
<td>5.1a</td>
<td>Bridge circuit for measuring division ratio K</td>
<td>37</td>
</tr>
<tr>
<td>5.1b</td>
<td>Bridge circuit for measuring divider ratio A</td>
<td>37</td>
</tr>
</tbody>
</table>
I. INTRODUCTION

The accurate measurement of high voltages of different characters is of considerable importance in many fields. Historically, the problem of high-voltage measurement first appeared in the field of ac electric power transmission while the later development of the X-ray tube introduced the problem of accurate measurement of high dc potentials. The problem of measuring high pulse voltages, other than power-system transients, did not occur until high-power, high-voltage pulsed radiofrequency tubes came into use. With the advent of multi-megawatt pulsed microwave amplifiers, a need arose to accurately determine the magnitude and character of pulsed voltages of magnitudes approaching several hundred kilovolts and durations of one to a few microseconds. In the testing and operation of high-power microwave tubes it is, of course, necessary to know the peak values of voltage and current to the electron gun of a tube; it is, however, also useful to know the exact character of a voltage pulse applied to a tube cathode.

The power developed in a microwave amplifier is dependent on the total energy delivered during a pulse. Any change in the amplitude of a voltage pulse applied to a high-power microwave amplifier can change the power output or cause phase modulation of the rf signal being amplified. In the design of large systems using several microwave tubes (for example, a linear electron accelerator), the efficiency of the microwave amplifiers, as well as that of the modulators that drive them, depends on the shape of the voltage pulses delivered to the cathodes of the tubes. The utilization of energy in the modulator pulse-forming networks and in the microwave tubes is of great interest in a large system. Any deviation from a zero rise-and-fall time of a pulse represents wasted energy in the pulse-forming networks, while a non-flat-topped pulse represents a variable rf power output or undesirable phase modulation of the rf signal in a microwave amplifier. The ratio of the power in the rise and decay times of a pulse to the power in the rectangular flat-topped portion is:

1. All References are at the end of this report.
\[
\frac{\text{Power in rise and decay times}}{\text{Power in flat top}} = \frac{2 \frac{T_R}{T_F} + 1}{\frac{7}{T_{FT}}} \quad (1.1)
\]

where \(T_R\) is the rise time, \(T_F\) is the decay time, and \(T_{FT}\) is the duration of the flat-topped portion of a voltage pulse delivered to an electron gun which follows the \(3/2\) power law.

The character as well as the maximum amplitude of a voltage pulse is thus seen to be of importance. An ideal device for measuring pulsed high voltages would then present the pulse shape along with a complete time and voltage calibration.

In any measurement of high voltages it is first necessary to reduce, divide, or otherwise process information about the voltage so that it may be presented using conventional low-voltage instruments. In the special case of interest concerning microwave tubes it is desirable to visually monitor high-voltage pulses; a voltage divider must then be employed to reduce the high-voltage pulses to a level that can be presented on an oscilloscope. Such a divider must have an accurately known voltage division ratio and must not distort its pulse input or give a distorted pulse output.

Any high-voltage measurement scheme then consists of a voltage ratio device and an information processing device. These two problems can be treated separately, but in any detailed analysis they must, of course, be considered together. Since high-voltage measurements have been of interest many measurement schemes have been developed and used. In Section II a review of the most-used high-voltage measurement schemes is presented. In Section III the capacitive voltage divider is analyzed in detail, and a measurement system using such a divider is developed. Section IV is devoted to pulse information processing devices, while Section V gives experimental results.
II. A REVIEW OF HIGH-VOLTAGE MEASUREMENT SCHEMES

A. VOLTAGE TRANSFORMERS

The voltage instrument transformer (see Fig. 2.1a) is quite similar in behavior and design to the power transformer. It is, in effect, a power transformer with a large turns ratio in which the secondary is virtually under no load, for the load is always small, corresponding to the power required by a voltmeter or similar instrument. For accuracy in division ratio it is important that a voltage transformer have a good regulation while maintaining a constant relationship between primary and secondary phase. The accuracy demanded of a voltage transformer for measuring voltages of 100 kv and above usually dictates a rather complex design. Less than ten years ago Harris reported that a 110-kv voltage transformer built by a leading manufacturer had an overall height of 14 feet, required nearly 800 gallons of oil, and weighed almost 5 tons. Such a transformer, needless to say, would be totally impractical for the measurement of pulsed voltages. In general, instrument transformers are of little use in modulator-driven pulsed-voltage systems, for they not only load the system in an undesirable way and thus provide an additional load to the modulator, but also are far too large to be economical.

B. CALIBRATED SPARK GAPS

Under specified conditions of temperature, pressure, and moisture content, the breakdown strengths of air and other gases are fairly well known. If an electrical geometry allows the precise calculation of electric field, and thus potential gradient at any point, then the breakdown voltage between two parts of the system can be calculated. A common configuration has been the double-sphere gap (see Fig. 2.1b); such gaps have been long used as standards for the calibration of high-voltage instruments. There are many obvious limitations to the accuracy of such gaps: the accuracy to which air conditions are known, the accuracy to which the effective gap is known, and a host of other second-order effects. Nonetheless, such gaps served as standards as stipulated by the AIEE. In the 1930's a number of workers began to doubt the
validity of the spark-gap standard and some suggested a revision of the standards\textsuperscript{4}.

The spark gap is still a valid standard for direct or periodic voltages of known waveform\textsuperscript{5}. It is also used as a shop instrument in the television industry. In a pulsed system where waveform and character are subject to change, the spark gap is of little use. At best, a spark gap could only provide an indication of the maximum height of a pulse, be it the flat top or a leading-edge spike.

C. ELECTRIC-FIELD INSTRUMENTS

If two bodies carry differing electrical charges, then there exists a force between them. This principle has been used since the time of Lord Kelvin as a basis for instruments to measure high voltages. Instruments based on this principle fall into two general classes: electrostatic instruments which give a direct indication proportional to a force due to the applied voltage, and oscillating electrode instruments in which the force due to the applied voltage determines the period of some oscillating system. Both classes are said to indicate dc and rms ac voltages, although the dc case and ac case are not always exactly equivalent\textsuperscript{6}.

Electric-field instruments can be made to operate directly on a high-voltage circuit or can use a capacitive divider\textsuperscript{7}. Electrostatic voltmeters (see Fig. 2.1c), when used directly, form absolute reference standards in that their indication can be related directly to the fundamental definition of the volt. Units have been constructed which give an accuracy of a tenth of one percent in the 30 to 350 kilovolt range\textsuperscript{8}.

Like the calibrated-spark-gap type of instrument, electric-field instruments can only be used on direct or periodic voltages of known waveform. However, if an aperiodic or pulse-type signal is rectified to give an average value, then an electric-field type instrument may be used as an indicator.

D. ROTARY VOLTMETERS

In the 1930's new techniques were developed which allowed X-ray tubes to be built using dc accelerating voltages in the hundreds of
Fig. 2.1a--Typical voltage transformer

Fig. 2.1b--Calibrated spark gap

Fig. 2.1c--A type of electrostatic voltmeter
kilovolts. The sources used to supply such tubes were of relatively high impedance and thus could not supply a resistive divider. The rotary voltmeter was devised to provide a system which could measure high voltages at high impedance levels.

Fundamentally, a rotary voltmeter comprises a conducting cylinder split longitudinally in half and so arranged that it can be rotated in an electric field established by two larger electrodes (see Fig. 2.2a). A commutator is used to extract a direct-current signal from the cylinder when it is rotated in an electric field. If $C$ is the capacitance from the split cylinder to either larger electrode and $V$ is the voltage applied between these electrodes, then for a rotary speed of $n$ revolutions per second a direct current of $I = 2CVn$ amperes flows through the external circuit. In 1935 rotary voltmeters were developed which could measure direct voltages up to 830 kV. Like the electric-field instruments, the rotary voltmeter is limited to direct or periodic voltages.

E. DIVIDERS

From the standpoint of adaptability, the voltage divider is the most logical device to use in high-voltage measurements. A properly-constructed divider can drive a variety of instruments and work on either periodic or aperiodic waveshapes with essentially the same results. The dividers useful for the presently considered pulse application fall into two rough classes: resistance dividers, and capacitance dividers. There are, of course, dividers to achieve given mathematical relations (logarithmic dividers, square dividers, etc.) and combinations of resistance-capacitance dividers to achieve certain desired frequency characteristics.

The principle of the resistance divider has long been used in constructing multi-range voltmeters, although it has only fairly recently been applied to high-voltage measurements. Although resistance dividers do have the disadvantage of loading in a high-voltage application, they can be made to have wideband frequency characteristics. It is possible with present techniques to construct resistance dividers having an input impedance of one megohm and a division ratio accurate to one part in a million. To achieve such accuracy it is usually necessary to immerse
the divider in a circulating constant-temperature bath. Resistance dividers are limited in voltage by the maximum tolerable voltage gradient along any one of their component resistances. A simple solution is to use a large number of resistances. This obviously increases the size of the divider to what may be unreasonable for a laboratory application. Nonetheless, resistance dividers can be made accurately and are stable over long periods of time. The present military standard on voltage measurement for crossed-field tubes, MIL-E-11D, specifies a resistance divider circuit.

The capacitive divider (see Fig. 2.2b) is the most logical choice for measuring high pulsed voltages, for it provides a negligible load on the measured circuit and can be accurately made to give a known division ratio. As will be shown in the next section, the capacitive divider can be made to operate over a wide range of temperatures without affecting its division ratio. If used with electrostatic instruments, a capacitive divider can measure direct voltages, or with ac instruments it can be used for a variety of input waveshapes. The upper voltage limit of a capacitive divider is set by geometry and dielectric properties.
Fig. 2.2a--Typical rotary voltmeter.

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_2}{R_1 + R_2}
\]

Fig. 2.2b--Typical resistive and capacitive voltage dividers.

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{C_1}{C_1 + C_2}
\]
III. THE THREE-TERMINAL CAPACITIVE DIVIDER

Three-terminal capacitors constructed using guard-ring principles have long been used as standards of capacitance, although multi-rod capacitors are capable of greater precision. A capacitive divider is a three-terminal capacitor, yet varies from a standard capacitor in two respects: one of the capacitors is one or two orders of magnitude larger than the others, and the divider must be constructed to work at relatively high voltages.

A. ELECTRODE CHARACTERISTICS

Any capacitor must, of course, be formed by two electrodes separated by or imbedded in some dielectric. The capacity of the system is dependent on the dielectric constant of the dielectric and the geometrical configuration of the electrodes, but the product of the resistance between and capacity of any two electrodes is dependent only on the characteristics of the dielectric. This can be illustrated using two electrodes of arbitrary shape and position imbedded in a dielectric having a dielectric constant \( \varepsilon \) and a conductivity \( \gamma \) (see Fig. 3.1a).

By Gauss' Law:

\[
Q = \int \vec{D} \cdot d\vec{a}
\]

around either electrode. The definition of capacity is \( C = Q/V \), so

\[
C = \frac{\int \vec{E} \cdot d\vec{a}}{V}
\]

by Ohm's law \( R = V/I \); thus

\[
I = \gamma \int \vec{E} \cdot d\vec{a}
\]

The resistance \( R \) between the two electrodes then becomes \( R = \varepsilon/\gamma C \), and the product of the resistance between and capacity of the electrodes is

\[
RC = \frac{\varepsilon}{\gamma}
\]

(3.1)
The division ratio $K$ of the three-terminal capacitor (see Fig. 3.1b) is:

$$K = \frac{V_0}{V_i} = \frac{R_2(l + sC_1R_1)}{R_2(l + sC_1R_1) + R_1(l + sC_2R_2)}$$

now $R_1C_1 = R_2C_2 = e/\gamma$, so

$$K = \frac{R_2}{R_1 + R_2} = \frac{C_1}{C_1 + C_2} \quad (3.2)$$

Thus if neither the dielectric constant or the conductivity of the dielectric change with frequency, the division ratio $K$ is frequency independent. This property of electrodes in a dielectric medium has been used to make potentiometers working over a wide frequency range.\(^{14}\)

B. GEOMETRICAL DESIGN

Small capacities (in the neighborhood of a few picofarads) are difficult to measure accurately, while larger capacities can be measured to high accuracy using bridge techniques. For this reason, the small high voltage to signal ring capacitor $C_1$ should be readily computable from the geometry of the divider; $C_2$ may more easily be measured for it is in the neighborhood of 1000 times $C_1$ (a division ratio of 1000). The maximum value of $C_1$ determines the values of the capacitances of the actual divider. The geometry of the divider should be so chosen as to minimize machining time and facilitate the assembly of the structure. The dimensions of the divider are determined by the magnitude of $C_1$ and by the electric strength of the dielectric oil used. In view of considerations such as shielding, ease of assembly, ease of computing the various capacities, and probable ease of mechanical reproducability, the coaxial geometry seems best suited for a high-voltage divider. The coaxial capacitive divider is not a new development, for free-air coaxial capacitive dividers were used on 150-kv electric power circuits in the early 1920's.\(^{3}\)

Taking the inner conductor radius as $a$ and the outer conductor radius as $b$, the capacity of a coaxial capacitor is

$$C = \frac{2\pi e}{\ln(b/a)} \quad (3.3)$$
Fig. 3.1a--Arbitrary-shaped electrodes in a dielectric.

Fig. 3.1b--The three-terminal capacitor.
where the dielectric constant \( e \) is equal to the product of the relative dielectric constant \( k \) and the dielectric constant of free space \( \varepsilon_0 \):

\[ e = k \varepsilon_0. \]

The electric field in the radial direction, \( E_r \), is

\[ E_r = \frac{V_{in}}{r[\ln(b/a)]} \]  \hspace{1cm} (3.4)

The maximum electric field occurs at the surface of the inner conductor for \( r = a \). Letting the radius ratio \( b/a = \sigma \), Eq. (3.4) becomes:

\[ E_r = \frac{V_{in}}{b[\sigma/\ln(\sigma)]} \]

now differentiating \( E_r \) with respect to \( \sigma \):

\[
\frac{dE_r}{d\sigma} = \left[ \frac{d}{d\sigma} \frac{\sigma}{\ln(\sigma)} \right] \left( \frac{V_{in}}{b} \right)
\]

The expression

\[
\frac{d}{d\sigma} \frac{\sigma}{\ln(\sigma)} = 0
\]

is zero for \( \sigma = e \) (the base of natural logarithms). Thus to minimize the over-all dimensions for a given permissible electric-field strength, the radius ratio \( b/a \) should be equal to \( e \).

The maximum electric field permissible in the divider is determined by the electric strength of the dielectric oil used. For a given maximum expected input voltage to the divider, there is a minimum value of inner radius corresponding to the permissible value of electric field. For each value of inner radius there corresponds an outer radius determined by the radius ratio. The relationship between the minimum outer radius and the radius ratio for two permissible values of maximum electric field (using the electric strength of Dow-Corning 200 Silicon Fluid, Electrical Grade: 350 volts/mil) is plotted in Fig. 3.2. The curves of Fig. 3.3 show, as did Eq. (3.5), that the minimum over-all diameter of a coaxial divider is achieved for a radius ratio of \( e \).

In order that the capacitance \( C_1 \) be computed as accurately as possible, the divider should be constructed using guard-ring techniques.
Fig. 3.2--Minimum outer radius $b$ vs. radius ratio $\sigma$. 

Maximum electric field $E_r = 280 \text{ v/mil}$

Maximum electric field $E_r = 350 \text{ v/mil}$
Fig. 3.3--Side view of coaxial capacitor using guard rings
(see Fig. 3.3 for a side view of a coaxial divider). The small gaps between the signal electrode and the guard electrodes have no effect on the capacity $C_1$ provided that their width $\Delta$ is much less than the interelectrode space: $\Delta \ll (b - a)$.

In the region of the gaps, electric-field lines from the center electrode terminate either on the guard electrode or the signal electrode. If $\Delta$ is very small, the number of lines emanating from the center electrode is unaltered by the presence of the gaps; the lines curve on entering the gap so that half of them terminate on the signal ring and the other half on the guard electrode. The effective length for computation of $C_1$ is then $L + \Delta$.

C. ERRORS IN COMPUTED CAPACITANCE DUE TO GEOMETRICAL ANOMALIES

In general, there are a large number of geometrical anomalies which may give rise to deviations in the capacitance of a coaxial system from that computed assuming a perfectly cylindrical coaxial geometry. Either electrode may be out-of-round, the axes of the electrodes may be skewed, or the length of the outer electrode may vary. Assuming that the inner electrode is machined from solid stock while the outer electrode is machined from tubing, then the out-of-round problem may be confined to an out-of-round of the outer electrode. The general case of cylinders on skewed axes is difficult to solve analytically; the anaxial problem can, however, give an indication of the seriousness of skewed axes. The variation of the length of the outer electrode is a simple problem.

1. Out-of-round Outer Electrode

The case of the out-of-round outer electrode is best approached through considering the capacity of a coaxial system comprising confocal elliptical cylinders. The capacity of a confocal elliptical system can be found through considering a coordinate transformation such that the ellipses are transformed into coaxial cylinders in another plane. Consider the two planes:

$$w = u + jv = re^{j\phi}$$
$$z = x + jy$$

(3.6)
Now considering the mapping function,

\[
z = \frac{1}{2} \left[ w + \frac{1}{w} \right]
\]

or

\[
x + jy = \frac{1}{2} \left[ \rho e^{j\phi} + \frac{1}{\rho} e^{-j\phi} \right]
\]

\[
x = \frac{1}{2} \left[ \rho + \frac{1}{\rho} \right] \cos(\phi)
\]

\[
y = \frac{1}{2} \left[ \rho - \frac{1}{\rho} \right] \sin(\phi)
\]

Equations (3.8) transform concentric circles and radial lines in the \( z \) plane into confocal ellipses and hyperbolas in the \( w \) plane. By elimination of \( \rho \) or \( \phi \):

\[
\frac{x^2}{\left( \rho + \frac{1}{\rho} \right)^2} + \frac{y^2}{\left( \rho - \frac{1}{\rho} \right)^2} = \frac{1}{4}
\]

\[
\frac{x^2}{\cos^2 \phi} + \frac{y^2}{\sin^2 \phi} = 1
\]

The family of curves \( \rho \) = constant in the \( w \) plane corresponds to ellipses in the \( z \) plane, while the family of radial lines for \( \phi \) = constant corresponds to hyperbolas in the \( z \) plane. The unit circle \( \rho = 1 \) in the \( w \) plane is a two-sided infinitesimally thin slit between \( x = 1 \) and \( x = -1 \) in the \( z \) plane. Thus the entire \( z \) plane maps into the exterior of the unit circle in the \( w \) plane. The coordinate transformation is illustrated in Fig. 3.4a.

For a point in Fig. 3.4a:

\[
x = b = \frac{1}{2} \left[ b_2 + 1/b_2 \right], \ y = 0
\]

or

\[
b = b_2 + b_1
\]
Fig. 3.4a--Conformal transformation

Fig. 3.4b--Deformed cylinder
In the same fashion the transformation applying to the small ellipse is
\[ z = a_1 + a_2. \]
Now the capacity of a cylindrical coaxial system is
\[ C = \frac{2\pi k e_0}{\ln(b/a)} \]

So the capacity of the confocal ellipsoidal system is
\[ C = \frac{2\pi k e_0}{\ln\left(\frac{b_1 + b_2}{a_1 + a_2}\right)} \] \hspace{1cm} (3.11)

If the semimajor and semiminor axes of the inner ellipse are equal, then it is a circle. If the deformation of the outer conductor of a coaxial system is considered to be towards an ellipsoidal shape, then Eq. (3.11) can be used to indicate the deviation from the capacitance computed for the coaxial system. If \( b \) is the radius of the perfectly round outer conductor, then in the deformed ellipsoidal shape the semimajor and semiminor axes can be expressed as:
\[ b_1 = b\left[1 + \delta_1\right] \] \hspace{1cm} (3.12)
\[ b_2 = b\left[1 + \delta_2\right] \]

Substituting the relations of Eq. (3.12) into Eq. (3.11) along with \( a = a_1 = a_2 \):
\[ C' = C + \Delta C = \frac{2\pi k e_0}{\ln\left(\frac{b}{a}\right) + \ln\left(\frac{1 + \delta_2 - \delta_1}{2}\right)} \] \hspace{1cm} (3.13)

where \( \Delta C \) is the deviation in capacity from the true value \( C \) computed for the coaxial system. The per-unit change in capacity is then
\[ \frac{\Delta C}{C} = \frac{\ln\left[1 + \frac{\delta_2 - \delta_1}{2}\right]}{\ln\left[\frac{b}{a}\right] + \ln\left[\frac{1 + \delta_2 - \delta_1}{2}\right]} \] \hspace{1cm} (3.14)
Now expanding the function \(\ln \left[ 1 + \frac{\delta_2 - \delta_1}{2} \right]\) in a logarithmic series and taking the first term, Eq. (3.14) becomes

\[
\frac{\Delta C}{C} \sim \left[ \frac{\delta_2 - \delta_1}{\sqrt{2} + (\delta_2 - \delta_1)} \right]
\] (3.15)

To a good approximation a metal cylinder will distort into an ellipse such that the per-unit decrease along one axis is almost equal to the per-unit increase along another. If this is true, then Eq. (3.15) shows that small deviations in the roundness of the outer electrode are of little consequence in the accuracy of the determination of the capacity of the system.

2. Anaxial Alignment of Cylinders

The capacity of a system of two anaxial cylinders (see Fig. 3.5) is

\[
C = \frac{2\pi ke_0}{\ln \left( \frac{(a + b)^2 - c^2 + (m)^2}{(a + b)^2 - c^2 - (m)^2} \right)}
\] (3.16)

where

\[
m = c^2 - 2c(b^2 + a^2) + (b^2 - a^2)^2
\]

If the deviation from coaxial alignment \(c\) is zero, Eq. (3.16) reduces to the equation for the capacity of two coaxial cylinders. Letting the radius ratio \(b/a = \sigma\) as before, and the per-unit deviation from coaxial alignment \(c/a = \delta\), the per-unit deviation from the computed coaxial system capacitance \(C\) is:

\[
\frac{\Delta C}{C} = -\frac{\ln \left( \frac{2(b + a) - \delta^2 b/\sigma^2}{2(b + a) - \delta^2 a} \right)}{\ln(\sigma) + \ln \left( \frac{2(b + a) - \delta^2 b/\sigma^2}{2(b + a) - \delta^2 a} \right)}
\] (3.17)

As for the out-of-round cylinder case, the logarithm expressing the small deviations is expanded in a logarithmic series and the first term
Fig. 3.5--An axial alignment of cylinders
is taken to give an approximate expression for the per-unit deviation in capacitance. For the case of a radius ratio of $\sigma = e$, $\ln(\sigma) = 1$, Eq. (3.17) can be approximated as

$$\frac{\Delta C}{C} \sim \frac{\delta^2}{11.75 - 0.586^2}$$ (3.18)

Obviously, then, deviations in the alignment of the cylinders are of little consequence. For example, a deviation of ten percent, $\delta = 0.1$, in the alignment of the center conductors on axis will result in a per-unit error in the computed capacity of only 0.0008.

3. Undetermined Length of Outer Cylinder

The error in the capacity of the coaxial system will be directly proportional to the per-unit error in the known length of the outer electrode. The length of the cylinder can, using normal machine-shop methods, be measured to a precision of a thousandth of an inch over a length of two inches. An error of a thousandth of an inch in two inches would result in a per-unit error in the capacity of 0.0005.

D. EFFECT OF TEMPERATURE VARIATION ON DIVISION RATIO

The effects of temperature variations on the division ratio of the divider may be divided into two classes: effects due to changes in the mechanical dimensions of the divider and effects due to changes in the characteristics of the dielectric oil used in the divider.

1. Changes in Geometry with Temperature

If the inner and outer electrodes are made of the same material, then the radius ratio $\sigma$ will remain unchanged with temperature variation, and the capacitance per unit length of the coaxial capacitor will be invariant with temperature. The length of the outer electrode, however, varies directly with temperature so that the capacity of the coaxial system is

$$C'_1 = C_1(1 + K_t)$$ (3.19)
where $C_1$ is the capacity computed at some reference temperature, $K$ is the coefficient of expansion of the outer cylinder, and $t$ is the temperature variation from the reference temperature. If the large signal electrode capacitor comprises several annular discs and a uniform material is assumed for the discs and their spacers, its area expansion is

$$A' = A(1 + Kt)^2$$

and its spacing expansion is obviously $d' = d(1 + Kt)$. Thus its capacitance variation with temperature is

$$C_2' = C_2(1 + Kt)$$

Thus the division ratio $K = C_1 / (C_1 + C_2)$ remains unchanged with changes in mechanical dimensions due to temperature variation.

The dielectric constant of the oil used in the divider changes with temperature; however, since all capacitances are directly proportional to dielectric constant, the division ratio remains unchanged with changes in dielectric constant due to temperature variation.

2. Effect of Variation of Loss Tangent of Dielectric

Although the relative dielectric constant of silicone oils (as used in the divider under consideration) is essentially constant up to $3 \times 10^9$ cps, the loss tangent is a function of frequency. As pointed out previously, the division ratio $K$ of the divider itself is independent of frequency. The over-all system (divider plus output cable and termination) division ratio is, however, a function of the loss tangent of the dielectric in the divider proper. The actual transfer function as developed in the following section has one zero and two poles. Due to a finite resistance in parallel with every capacitor in the divider, the zero of the transfer function is not at zero frequency, nor is the magnitude of the transfer function zero for zero frequency. The approximate transfer function for the divider circuit (see Fig. 3.6b) at low frequencies is

$$A = \frac{V_0}{V_1} = \frac{KR_1}{R_L + KR_1 + sR_L(CR_1 + KC_1 \omega_1)}$$

(3.20)
which will give a frequency characteristic as shown in Fig. 3.6a. Both $C_1$ and $R_1$ are in the divider proper, so the relation between the product $R_1 C_1$ and the loss tangent, $\tan \delta$, is:

$$\tan \delta = \frac{1}{\omega R_1 C_1} = \gamma / \omega \epsilon$$

or

$$\frac{1}{R_1 C_1} = \omega \tan \delta$$  \hspace{1cm} (3.21)

The lower break-point in Fig. 3.6a is then seen to occur at a frequency where $\tan \delta = 1$. No exact data is available on the variation of the loss tangent with frequency, save that it is $0.8 \times 10^{-4}$ at 100 cps and $0.4 \times 10^{-4}$ at 1000 cps\textsuperscript{19}. Extrapolating these values on a logarithmic scale gives a frequency of less than 1 cps for the lower break-point.

The second break-point is relatively unaffected by variations in the $R_1 C_1$ product with frequency. In summary, then, the loss tangent can be said to control the low-frequency response of the divider circuit, while values of the divider components determine the magnitude of the division ratio.

E. DIVIDER CIRCUITRY

In actual use the divider may be considered to be driven by a voltage source in parallel with a resistance (the conduction resistance of a klystron gun), and to drive a signal circuit comprising a properly terminated length of coaxial cable and a load resistance. There is a certain self-inductance associated with the single lead wire from the cathode of the klystron to the divider. The entire circuit is shown in Fig. 3.6b. The conduction resistance of the klystron gun is represented by $R_5$; $L$ is the lead inductance of the input lead; $C_1$, $C_2$ and $C_3$, and $R_1$, $R_2$, and $R_3$ are the capacitances and resistances of the divider proper; $R_m$ is the termination for the input end of the coaxial output cable; $C_L$ is the capacity of the cable; and $R_L$ is the load resistance.

The self-inductance of a wire 10 inches long is normally less than one microhenry. As will be pointed out later, the input impedance of the divider is almost equal to $1/s[3C_1]$; if $C_1$ is of the order of 8 pf,
then a break-point due to \( L \) and \( C_1 \) would be at a radian frequency of \( 7 \times 10^9 \). To a first-order approximation, then, the inductance \( L \) may be neglected in this analysis.

The remainder of the circuit may be represented in a topological fashion by lettering the various branches to represent their admittances:

\[
\begin{align*}
a &= \frac{1}{R_1} + sC_1 \\
c &= \frac{1}{R_s} + \frac{1}{R_3} + sC_3 \\
e &= \frac{1}{R_L} + sC_L \\
b &= \frac{1}{R_m} \\
d &= \frac{1}{R_2} + sC_2
\end{align*}
\]

By inspection the driving point impedance and transfer ratio \( A = \frac{V_o}{V_in} \) are found to be

\[
Z_{in} = \frac{(a + d)(b + e) + be}{ac(b + e) + d(a + c)(b + e) + be(a + c)}
\]

\[
A = V_o/V_in = \frac{ab}{(a + d)(b + e) + be}
\]

(3.22)

In the divider under consideration, \( C_3 \) is almost double \( C_1 \), and \( C_2 \) is about two orders of magnitude larger than \( C_1 \). With these assumptions, the input impedance of the divider circuit becomes

\[
Z_{in} = \frac{1}{s|3C_1|}
\]

(3.23)

The exact expression for the division ratio is

\[
A = \frac{\frac{1}{R_1} \frac{1}{R_m} \left[ 1 + sC_1 R_1 \right]}{\frac{1}{R_1} \left[ 1 + sC_1 R_1 \right] \left[ \frac{1}{R_m} + \frac{1}{R_L} \left[ 1 + sC_1 R_L \right] \right] + \frac{1}{R_m} \left[ 1 + sC_2 R_L \right]}
\]

(3.24)

At mid- and high-frequencies the reactances of the divider capacitors are far less than the resistances between the capacitor electrodes. The divider may then be considered to consist of capacitors only \( (R_1, R_2 \text{ and } R_3 \text{ infinite}) \) without affecting the over-all character of the division ratio. The low-frequency portion of the division ratio has been discussed in the previous section, and was seen to be controlled by the loss tangent of the divider dielectric. The approximate expression for the over-all division ratio is:

- 24 -
The steady-state frequency characteristic corresponding to the transfer function of Eq. (3.25) is not easily determined in simple terms. However, once the parameters of the divider are quantitatively determined, the character of the over-all division ratio is set. The values of the various circuit components are determined by the limits set on the divider input impedance and by the over-all magnitude of the division ratio desired.

Equation (3.23) states that the input impedance of the divider is practically equal to one-third of the reactance of \( C_1 \) at most frequencies. The eventual impedance criterion determined will depend on the magnitude of the resistance \( R_s \). A 75-megawatt pulse to a klystron having a gun perveance of \( 2 \times 10^{-6} \) gives an equivalent \( R_s \) of 925 ohms. Basically, the criterion on the magnitude of \( C_1 \) is based on the extent to which the shunt input capacity of the divider will degrade a pulse at the cathode of the klystron.

The combination of the input impedance of the divider and the conduction resistance of the klystron can be looked upon as forming a circuit which will determine the criterion for the value of \( C_1 \). The output voltage from the simple circuit of Fig. 3.6c for a unit step input voltage \( V_{in}(t) = u(t) \); \( V_{in}(s) = 1/s \) is

\[
V_{out}(s) = \frac{V_{in}/3R_s C_1}{s(s + 1/3R_s C_1)}
\]

This corresponds to a time-domain function of

\[
V_{out}(t) = \frac{1}{3R_s C_1} \left[ 1 - e^{-t/3R_s C_1} \right]
\]

Adopting the standard ten-to-ninety percent definition of rise time, the rise time of the output voltage for a unit step input voltage is:

\[
T_{rise} = 6.6 R_s C_1
\]
Fig. 3.6a--Low-frequency portion of division ratio frequency characteristic

\[
\frac{1}{C_1 R_1} \quad \frac{R_L + K R_1}{R_L (C_1 R_1 + K C_L R_L)}
\]

Fig. 3.6b--Complete circuit and reduced circuit of divider circuit

Fig. 3.6c--Circuit for evaluating effect of divider input impedance
This, then, determines the maximum permissible value of $C_1$ and thus the values of the other capacitances in the divider proper.

F. DIVIDER DESIGN

If some maximum rise time corresponding to Eq. (3.26) is selected, then the maximum value of $C_1$ is determined. If a maximum rise time of 0.1 microsecond is selected, then for a source resistance of 1000 ohms the maximum value of $C_1$ is found to be 15.2 picofarads.

The exact dimensions of the divider are somewhat dependent on the availability of tubing sizes. The outer conductor and divider shields should be chosen so that they may be made of readily available tubing. Using a radius ratio of $\sigma = e$ and 6.750-inch i.d. tubing for the outer electrode, the diameter of the inner electrode is 2.482 in. If $C_1$ is about half the maximum value determined above ($C_1 = 8$ pf), then the length of the signal electrode plus the gap spacing must be $8/3.81 = 2.1$ in.

Let the division ratio $A$ be 1000. If ten feet of RG-55A/U cable is used as the output coaxial cable, then $C_L$ is $10(28.5$ pf/ft) = 285 pf. Because $A = C_1/[C_1 + C_2 + C_L]$ at mid-band frequencies, $C_2$ must be 7707 pf. The inner diameter of the divider shield is chosen as 9.750 inches to agree with commercially available stainless-steel tubing. The capacitor $C_2$ can then be formed from annular discs fitting between the outer conductor of the coaxial system and the divider shield. In order to provide sufficient clearances, the inner and outer radii of the discs are 3/16 in. greater and less than the outer radius of the coaxial system and the inner radius of the shield, respectively. The inner and outer radii of the annular discs are then 3.6875 in. and 4.6875 in., giving a plate area of 26.3 square inches. With a 50-mil spacing between plates, the capacity per gap of a capacitor made of such annular rings is about 330 pf/gap, making the number of gaps required 24 to achieve the required 7707 pf for $C_2$.

A side view drawing of the divider structure is shown in Fig. 3.7. The divider shield, along with a bottom plate and upper mount ring, serves as a tank for the dielectric oil as well as a support for the electrode structures. The outer electrode rings of the signal and guard capacitors ($C_1$ and $C_3$) are assembled as a unit along with the annular ring output capacitor ($C_2$); the entire structure is suspended by eight rods from a
FIG. 3.7--Side view of divider structure.
ring at the top of the tank. The center electrode is suspended from a polystyrene disk which also serves as the top of the tank; sealing is accomplished through the use of O-rings. A top bushing serves as an oil expansion tank and is mounted above the center electrode. The entire tank is filled under vacuum and sealed.
IV. PULSE COMPARITOR CIRCUIT

Once the unknown high voltage has been reduced by a known ratio by the voltage divider, it is necessary to process it in some way to obtain information as to its magnitude and character. If the shape of a pulse is accurately known and only its magnitude is desired, then it is possible to use peak-reading or slide-back type voltmeters to give a metered indication of the pulse height. In the operation of high-power microwave tubes it is, however, also desirable to continuously monitor the pulse shape. This, then, indicates that some sort of oscilloscope presentation is necessary.

A commonly used oscilloscope comparison scheme comprises a variable dc power supply, a simple switching circuit to compare the dc against the unknown divided pulse height, and a voltmeter or potentiometer comparison circuit to read the dc potential. Such systems may use the vertical deflection plates of a cathode-ray tube directly or may use a stabilized dc oscilloscope amplifier.

One disadvantage of such comparitors is that a position reference on the face of a cathode-ray tube must be retained visually. This difficulty can be overcome by comparing the unknown divided pulse against a known dc potential with a contact-modulator type switch (chopper). In this fashion, the output of the comparator is an ac signal and thus may be fed directly to the vertical deflection amplifiers of any oscilloscope; the vertical gain of the amplifiers can be made very large so that the top of the pulse can be accurately aligned with the top of the rectangular wave produced by chopping the reference dc. A simplified diagram of the comparitor is shown in Fig. 4.1a. The oscilloscope is triggered at the rate of the unknown high-voltage pulses; thus a pulse will always occur at a fixed place on the time axis of the oscilloscope presentation. The chopper is not synchronized with the pulses, so the variable dc potential will be presented on some sweeps while the pulses are presented on others. The oscilloscope presentation of a compared pulse is shown in Fig. 4.1b.

The complete schematic diagram of the pulse comparitor is shown in Fig. 4.2. Power transistor Q1 is connected in a series regulator...
Fig. 4.1a--Basic chopper pulse comparator circuit

Fig. 4.1b--Typical output presented by an oscilloscope
Fig. 4.2--Complete diagram of pulse comparator
configuration to provide a constant 100 volts across the output load: a 10-turn, 5,000-ohm precision potentiometer. A separate power supply is used to provide 22 volts to run the regulating amplifier Q2 - Q3 and to supply power to the 1N755 Zener reference diode operating at 7.5 volts. Any changes in the output voltage appearing across the precision potentiometer appear as changes in the base-to-emitter potential of Q3, and are amplified and applied to the base of Q1, thus controlling the load voltage. Because the base-to-emitter potential of Q3 is always very small and the voltage across the Zener diode is a constant 7.5 volts, the current flowing through resistor R13 is practically constant at 2 milliamperes. The base current of Q3 is negligible compared to 2 milliamperes, so the current through resistors R14 and R15 is constant at about 2 milliamperes. Thus the output voltage level can be varied over a 4-volt range by varying resistor R15. Meter M is calibrated to give a 7-volt deflection on a 10-volt scale when the output voltage is exactly 100 volts. Either positive or negative output can be selected and fed to either a manual push button or a 60-cycle chopper.

The linearity of the precision potentiometer used (Helipot type A) is ± 0.5 percent; thus the basic limitation to the accuracy of the comparator is ± 0.5 percent. It is felt that this combination of a well-regulated power supply and a precision potentiometer is easier to use and equally as accurate as a variable power supply and a precision voltmeter.

The power supply was found to provide an essentially constant 100 volts output while the input line voltage was varied from 95 to 125 volts ac.
V. CALIBRATION AND TEST OF THE DIVIDER

A. DIVISION RATIO K OF THE DIVIDER ALONE

There are available many different methods of determining the ratio of a divider, three of which have been selected for use here: bridge voltage-ratio measurement, computation of ratio using measured and computed capacities, and voltage-ratio measurement by comparison with known ratios.

The circuit for measuring the division ratio \( K \) is shown in Fig. 5.1a. Capacitors \( C_1 \) and \( C_2 \) and resistors \( R_1 \) and \( R_2 \) are the circuit elements of the divider itself as shown in Fig. 3.1b. Resistors \( R_a \) and \( R_b \) are precision decade resistors with shunt capacitances \( C_a \) and \( C_b \). The detector \( D \) has an input impedance consisting of \( R_d \) shunted by \( C_d \). The ratio of the output voltage of the divider \( V_2 \) to the input voltage \( V_1 \) is

\[
\frac{V_2}{V_1} = \frac{1}{R_1} \left( 1 + \frac{1}{sC_1R_1} \right) + \frac{1}{R_2} \left( 1 + \frac{1}{sC_2R_2} \right) + \frac{1}{R_d} \left( 1 + \frac{1}{sC_dR_d} \right)
\]  

By Eq. (3.1), \( R_1C_1 = R_2C_2 = e/\gamma \), and by Eq. (3.2) \( K = \frac{R_2}{R_1 + R_2} = \frac{C_1}{C_1 + C_2} \), so Eq. (5.1) may be rewritten as

\[
\frac{1}{K} = \frac{V_1}{V_2} = \frac{1}{C_1R_d} \left[ 1 + \frac{1}{sC_dR_d} \right]
\]  

The ratio of the output voltage of the resistive divider \( V_3 \) to the input voltage \( V_1 \) is

\[
\frac{V_3}{V_1} = \frac{1}{R_a} \left( 1 + \frac{1}{sC_aR_a} \right) + \frac{1}{R_b} \left( 1 + \frac{1}{sC_bR_b} \right) + \frac{1}{R_d} \left( 1 + \frac{1}{sC_dR_d} \right)
\]  

At bridge balance there is no voltage across the detector \( D \), thus \( V_2 = V_3 \), and Eqs. (5.2) and (5.3) may be combined to give
\[
\frac{1}{K} \left[ 1 + sC_a R_a \right] = 1 + \frac{R_a}{R_b} + \frac{R_a}{R_d} + sR_a \left[ C_a + C_b + C_d \right] - \frac{\left( 1 + sC_a R_a \right) \left( 1 + sC_d R_d \right)}{C_1 R_d \left( s + \gamma/e \right)}
\]  

(5.4)

At all but the very lowest frequencies, \(|s| \gg \gamma/e\), so the numerator of the last term reduces to \(sC_1 R_d\). If only the steady state is considered where \(s = j\omega\), the real and imaginary parts of Eq. (5.4) can be equated to give

Real part: \[
\frac{1}{K} = 1 + \frac{R_a}{R_b} + \frac{R_a}{R_b} \left( 1 - \frac{C_a}{C_1} \right) - \frac{C_d}{C_1}
\]  

(5.5a)

Imaginary part: \[
\frac{1}{K} = 1 + \frac{C_b}{C_a} + \frac{C_1}{C_a} - \frac{C_d}{C_1}
\]  

(5.5b)

Thus the division ratio \(K\) can be readily determined by the parameters of the balanced bridge circuit.

If the capacities \(C_1\) and \(C_2\) are known, \(K\) may be computed. The relatively large capacitor \(C_2\) may easily be measured using a laboratory impedance bridge, but \(C_1\) is not so easily measured and is more conveniently computed. The dimensions for the computation of \(C_1\) are:

- \(2a = 2.482\) in., \(2b = 6.7445\) in., \(L = 2.050\) in., and \(\delta = 0.050\) in.

Now the capacity \(C_1\) is:

\[
C_1 = \frac{2\pi k \varepsilon_0 (L + \delta) / \ln(b/a)}{\ln(6.7445/2.482)}
\]

The capacity \(C_1\) is then:

\[
C_1 = 2.97k pf
\]

The relative dielectric constant \(k\) is a function of temperature, which to a first-degree approximation is \(k = k_0 - k_1 T\), where \(k\) is the relative dielectric constant at centigrade temperature \(T\), \(k_0\) is the relative dielectric constant at \(0^\circ\)C, and \(k_1\) is the temperature coefficient of the relative dielectric constant. For Dow-Corning 50 centistokes 200 silicone fluid electrical grade, \(k\) is 2.73 at \(25^\circ\)C and 2.36 at \(150^\circ\)C. Using these values, \(k_0\) and \(k_1\) can be determined:

\[
k = 2.797 - 2.91 \times 10^{-3} T
\]

The capacity \(C_1\) is then
The division ratio computed from the capacities is, of course, 
\[ K = \frac{C_1}{C_1 + C_2} \].

B. DIVISION RATIO OF THE ENTIRE DIVIDER CIRCUIT

The addition of an output cable and termination to the divider circuit results in a division ratio \( A \) which is different from the pure divider ratio \( K \) (see Eq. 3.24). In addition to causing a difference in the magnitude of the division ratio, the addition of the extra components will contribute to a phase difference between \( V_1 \) and \( V_2 \). With reference to the complete bridge diagram of Fig. 5.1b, if \( V_1 \) is selected as a reference, \( V_3 \) can be written as

\[ V_3 = AV_1 \left[ \cos \theta + j \sin \theta \right] \]  

where \( \theta \) is the phase-angle difference between \( V_1 \) and \( V_3 \). At balance there is no potential difference across detector \( D \), so \( V_2 = V_3 \). This means that the magnitude and phase angle of the division ratio must be the same on both sides of the bridge, thus permitting both \( 1/A \) and \( \theta \) to be computed from the constants on the right side of the bridge circuit. Equation (5.3) can be rewritten to include the phase angle \( \theta \):

\[ \frac{1}{A} \left[ \cos \theta - j \sin \theta \right] \left[ 1 + j \omega C a \frac{R_a}{R_a} \right] = 1 + \frac{R_a}{R_b} + j \omega R_a \left[ C_a + C_b' \right] \]  

where \( C_b' \) is the parallel combination of \( C_b \) and \( C_d' \) and \( R_d \) has been neglected in parallel with \( R_b \). At most frequencies, the small matching resistance \( R_m \) can be neglected. Equating real and imaginary parts of Eq. (5.8) and rewriting gives

\[ \frac{A}{\cos \theta} = \frac{1 + \omega C a R a \tan \theta}{1 + R_a/R_b} \]  
\[ \tan \theta = \omega R_a \left[ C_a - \frac{A}{\cos \theta} (C_a + C_b') \right] \]  
\[ -36 - \]
Fig. 5.1a--Bridge circuit for measuring division ratio K

Fig. 5.1b--Bridge circuit for measuring divider ratio A
Combining these two equations and solving for $1/A$ and $\tan \phi$ gives the final expressions for the division ratio and phase error of the voltage divider:

$$\frac{1}{A} = \left[ 1 + \frac{R_a}{R_b} \right] \left[ \frac{\sqrt{1 + \tan^2 \phi}}{1 + \alpha C_a R_a \tan \phi} \right]$$

(5.10)

$$\tan \phi = \frac{\frac{\alpha R_a}{R_a + R_b} [C_a R_a - R_b C_b']}{1 + \frac{\alpha R_a}{R_a + R_b} \left[ \alpha R_a C_a \left( C_a + C_b' \right) \right]}$$

(5.11)

If Eq. (5.1) is rewritten to include the phase angle $\phi$ and the cable capacity $C_L$ and load resistance $R_L$, equating its imaginary parts results in

$$\frac{1}{A} = \left[ \frac{1}{K} + \frac{C_d + C_L}{C_1} \right] \left[ \frac{\sqrt{1 + \tan^2 \phi}}{1 - \tan \phi / \alpha C_1 R_1} \right]$$

(5.12)

Equations (5.10) and (5.12) then provide two measurements of the overall divider ratio $1/A$.

C. MEASUREMENT AND TEST OF THE DIVIDER

1. Divider Ratio $1/K$.

All bridge measurements were made at 1000 cycles using General Radio decade resistance boxes and a standard capacitor. The precise value of each decade resistance box setting was determined by bridge measurement at 1000 cycles with an ESI bridge. All capacitances were measured at 1000 cycles using a General Radio 716-C capacitance bridge. The capacitance $C_2$ was measured at 28.6°C as 7951.4 pf. Capacitance $C_1$ was computed using Eq. (5.6) as 8.060 pf. These two values give a $1/K$ of

$$\frac{1}{K} = \frac{7951.4 + 8.060}{8.060} = 987.53$$

Bridge measurements using the circuit of Fig. 5.1 and Eq. (5.5a) yield

$$\frac{1}{K} = 987.30 \pm 2.74$$
where the value of 987.30 is the arithmetic mean of a number of measurements, and 2.74 is the standard deviation. The average of these two determinations of $1/K$ is 987.41.

2. Overall 1000-cycle Division Ratio $1/A$.

Twenty-one measurements of the division ratio $1/A$ were made at 1000 cycles with the bridge circuit of Fig. 5.2 using Eqs. (5.10) and (5.11) to compute $1/A$. The arithmetic mean of the measurements was 1054.1 with a standard deviation of ±2.52.

Capacities $C_L$ and $C_d$ were measured as totaling 471.8 pf. The average value of $1/K$ and $\tan \phi$ substituted in Eq. (5.12) give a $1/A$ of 1061.3 with an error of ±5.31 due to an assumed error in the accuracy of the capacitance measurements. If the errors in the bridge determinations of resistance values are included, the two values of $1/A$ are $1054.1 \pm 3.57$ and $1061.3 \pm 5.31$. A value of 1056.0 for $1/A$ at 1000 cycles then seems reasonable.

When the divider is used to measure pulses of the order of a few microseconds pulse width, the frequencies of interest are of the order of hundreds of kilocycles. The $\tan \phi$ correction factor in Eq. (5.12) for the 1000 cycle bridge measurements then approaches zero. The overall division ratio for pulses of the system comprising the divider, the comparator and an oscilloscope indicator, is $1045.6 \pm 6.0$. This division ratio is expected to change 0.01% per degree centigrade.

3. High-voltage Test.

The divider and its associated voltage comparator were used to calibrate another divider immersed in a pulse-transformer tank. At a test voltage of 87 kv, the divider and associated circuitry performed as expected. The divider itself has been tested at pulsed voltages up to 150 kv.
15. K. Dedrick, Measurement of High Voltage Pulses with the Coaxial Voltage Divider, Microwave Laboratory Internal Memorandum, ML 556, Stanford University (1956).
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