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# CALCULATION OF TWO-PARTICLE REACTION KINEMATICS USING THE I.B.M. 650 COMPUTER

By

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This report describes a program for the I.B.M. 650 computer, operating in the Bell System, which calculates quantities of interest in the relativistic kinematics of reactions of the type

$$P_1 + P_2 \rightarrow P_3 + P_4. \tag{1}$$

Here $P_1$ is the bombarding particle with kinetic energy $E_{k1}$. The target particle is $P_2$ and has zero initial kinetic energy. $P_3$ and $P_4$ are the products of the reaction. The rest mass of the bombarding particle may be taken to be zero so that photon-induced reactions may be considered. For stated values of the bombarding energy $E_{k1}$ and center-of-mass angle $\theta'$, the program calculates the laboratory kinetic energies of the reaction products $P_3$ and $P_4$ and the laboratory angles $\theta$ (in degrees) at which these particles will appear. The program also yields values of $d(\cos\theta')/d(\cos\theta)$ for both $P_3$ and $P_4$. The formulae* used are as follows:

$$E_{k3} = \frac{M_1 + M_2 + E_{k1}}{\gamma_3 + \gamma_4}\left(\gamma_3 + \beta^2 \cos\theta_3'\right) - M_3 \tag{2a}$$

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*These formulae are worked out in "The Kinematics of High-Energy Collisions", K. G. Dedrick, M.L. Report No. 574, 1959.

$$E_{k4} = \frac{M_1 + M_2 + E_{k1}}{\gamma_3 + \gamma_4} \left( \gamma_4 + \beta^2 \cos \theta_4' \right) - M_4 \tag{2b}$$

$$\frac{d(\cos \theta_3')}{d(\cos \theta_3)} = \frac{\left[ (\gamma_3 + \cos \theta_3')^2 + (1 - \beta^2) \sin^2 \theta_3' \right]^{3/2}}{(1 - \beta^2) \left| 1 + \gamma_3 \cos \theta_3' \right|} \tag{3a}$$

$$\frac{d(\cos \theta_4')}{d(\cos \theta_4)} = \frac{\left[ (\gamma_4 + \cos \theta_4')^2 + (1 - \beta^2) \sin^2 \theta_4' \right]^{3/2}}{(1 - \beta^2) \left| 1 + \gamma_4 \cos \theta_4' \right|} \tag{3b}$$

$$\theta_3 = \tan^{-1} \left[ \frac{\sqrt{1 - \beta^2} \sin \theta_3'}{\gamma_3 + \cos \theta_3'} \right] \tag{4a}$$

$$\theta_4 = \tan^{-1} \left[ \frac{\sqrt{1 - \beta^2} \sin \theta_4'}{\gamma_4 + \cos \theta_4'} \right] , \tag{4b}$$

where

$$\gamma_3 = \beta \left[ 1 - \left( \frac{2 E' M_3}{E'^2 - M_4^2 + M_3^2} \right)^2 \right]^{-1/2} \tag{5a}$$

$$\gamma_4 = \beta \left[ 1 - \left( \frac{2 E' M_4}{E'^2 - M_3^2 + M_4^2} \right)^2 \right]^{-1/2} \tag{5b}$$

$$\beta = \frac{1}{c} \times (\text{velocity of center of mass}) = \frac{\sqrt{1 + (2m_1/E_{k1})}}{1 + (m_1 + m_2)/E_{k1}} \qquad (6)$$

$E'$ = total energy available in the center of mass

$$= (m_1 + m_2)\sqrt{1 + \left[ 2m_2 E_{k1}/(m_1 + m_2)^2 \right]} \qquad (7)$$

$p'c$ = (momentum of either particle in the center of mass) $\times$ c

$$= \frac{E'\beta}{\gamma_3 + \gamma_4} \qquad (8)$$

$E_{k1}$ = initial kinetic energy of bombarding particle

$E_{k3}$ = final kinetic energy of $P_3$

$E_{k4}$ = final kinetic energy of $P_4$

$M_1$ = rest energy of $P_1$

$M_2$ = rest energy of $P_2$

$M_3$ = rest energy of $P_3$

$M_4$ = rest energy of $P_4$

The energy scale is arbitrary, but clearly whatever scale is used must be adhered to. The energy scale is introduced by the values of $M_1 \ldots M_4$, and the energies $E_{k3}$ and $E_{k4}$ are then calculated in the same units. The momentum in the center of mass $p'$ has the units $(1/c) \times$ (energy units used).

The program is arranged so that it first calculates $\beta, \sqrt{(1 - \beta^2)}$,

$\gamma_3$, $\gamma_4$, and $p'c$ for all values of $E_{k1}$ inserted by the user. Next, for the first value of $E_{k1}$, the quantities $\theta_3$, $\theta_4$, $d(\cos \theta_3')/d(\cos \theta_3)$, $d(\cos \theta_4')/d(\cos \theta_4)$, $E_{k3}$, and $E_{k4}$, are calculated for $\theta' =$ zero, then for $\theta' = 10^\circ$, etc., until $\theta' = 180^\circ$ is reached. This is repeated until all values of $E_{k1}$ have been stepped through. The computer then stops showing 7000000050 + on the display lights.

The values of $E_{k1}$ used must be inserted by the user. The first value goes in cell 800, the second in cell 801, etc. If n values of $E_{k1}$ are used, the number (n-1) must be put in cell 041. The program restricts n to be no greater than 25, hence we may use only 25 values of $E_{k1}$.

Examples:

Consider the kinematics of the reaction

$$\text{photon} + \text{proton} \rightarrow \text{neutron} + \pi^+.$$

Here,

$M_1 = 0$ (put in cell 020)

$M_2 = 0.93820$ Bev (put in cell 021)

$M_3 = 0.93949$ Bev (put in cell 022)

$M_4 = 0.13963$ Bev (put in cell 023)

The kinematics are desired for photon energies $(E_{k1})$ of 0.100 Bev, 0.200 Bev ... 0.500 Bev. There are five values of $E_{k1}$; hence put the number 4 in cell 041. We require the following cards:

| Column → | 7 8 9 | 10 | 11 | 12 | 13 14 15 | 16 17 18 | 19 20 21 | Remarks |
|---|---|---|---|---|---|---|---|---|
| | 8 0 0 | 1 | + | 1 | 0 0 0 | 0 0 0 | 0 4 9 | $E_{k1}$ = 0.1 Bev |
| | 8 0 1 | 1 | + | 2 | 0 0 0 | 0 0 0 | 0 4 9 | 0.2 Bev |
| | 8 0 2 | 1 | + | 3 | 0 0 0 | 0 0 0 | 0 4 9 | 0.3 Bev |
| | 8 0 3 | 1 | + | 4 | 0 0 0 | 0 0 0 | 0 4 9 | 0.4 Bev |
| | 8 0 4 | 1 | + | 5 | 0 0 0 | 0 0 0 | 0 4 9 | 0.5 Bev |
| | 0 4 1 | 1 | + | 4 | 0 0 0 | 0 0 0 | 0 5 0 | (n - 1) = 4 |
| | 0 2 0 | 1 | + | 0 | 0 0 0 | 0 0 0 | 0 0 0 | $M_1$ = zero |
| | 0 2 1 | 1 | + | 9 | 3 8 2 | 0 0 0 | 0 4 9 | $M_2$ = 0.93820 Bev |
| | 0 2 2 | 1 | + | 9 | 3 9 4 | 9 0 0 | 0 4 9 | $M_3$ = 0.93949 Bev |
| | 0 2 3 | 1 | + | 1 | 3 9 6 | 3 0 0 | 0 4 9 | $M_4$ = 0.13963 Bev |

Loading Order

1. Bell System Deck
2. Memory Reset Card
3. The Kinematics Program
4. $E_{k1}$ Values (n cards)
5. One card with (n-1) for cell 041
6. Four cards with values of $M_1$, $M_2$, $M_3$, $M_4$.
7. Bell System Punch Mode Deck ("six-per-card" punch)
8. Transfer card to location 109

Operation

This program is written in The Bell Interpretive System and so the console switch settings, etc., are the usual Bell System settings.

Printed Output Form (In Bell Floating Decimal Form)

| $M_1$ | $M_2$ | $M_3$ | $M_4$ |
|---|---|---|---|

n values of $E_{k1}$

| $E_{k1}$ | $\beta$ | $\sqrt{1-\beta^2}$ | $\gamma_3$ | $\gamma_4$ | $p'c$ |
|---|---|---|---|---|---|
| | | | | | |

| First value of $E_{k1}$ |
|---|

| $\theta' = 0^\circ$ |
|---|

| | | |
|---|---|---|
| $\theta_3$ (degrees) | $d(\cos\theta_3')/d(\cos\theta_3)$ | $E_{k3}$ |
| $\theta_4$ (degrees) | $d(\cos\theta_4')/d(\cos\theta_4)$ | $E_{k4}$ |

| $\theta' = 10^\circ$ |
|---|

| | | |
|---|---|---|
| $\theta_3$ (degrees) | $d(\cos\theta_3')/d(\cos\theta_3)$ | $E_{k3}$ |
| $\theta_4$ (degrees) | $d(\cos\theta_3')/d(\cos\theta_3)$ | $E_{k4}$ |

| $\theta' = 20^\circ$ |
|---|

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etc.

Remarks

The program deck may be obtained from the author. The running time is about 100 seconds for each value of $E_{k1}$ used.