SOME ASPECTS OF TARGET AREA DESIGN
FOR THE PROPOSED
STANFORD TWO-MILE LINEAR ELECTRON ACCELERATOR

By

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PREFACE

The papers in this Report were written by members of a group assembled at Stanford University in the summer of 1960 to study problems related to the design of target facilities for the proposed two-mile electron linear accelerator.

All of the members of the group were not present at the same time, and in some cases work which was done late in the summer changed part of the possible conclusions of earlier work. Several of the reports were revised, and as a result it may appear that optimum conditions were not chosen in some of the discussions.

In particular the ideas of Ballam and Drell led to a calculation of electromagnetic production of pairs of secondary particles in which one member of the pair interacts strongly with the nucleon. This mechanism gave such a high yield of secondary particles as to qualitatively change our ideas about the relative usefulness of electron and proton machines as sources of secondary particles.

This work is considered an internal memorandum and not a part of the scientific literature. It should not be cited without the permission of the author concerned.

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NOTE ON THE FEASIBILITY OF ELECTRON-BEAM STRETCHERS

by

D. M. Ritson

This note considers systems which accept short bursts of electrons and spill them out over a long time period. All formulae used below, the derivations, and the appropriate literature can be found in R. R. Wilson, Electron Synchrotron, Handbuch Der Physik, Vol. XLIV, 170 (Berlin, Springer-Verlag, 1959). The desirable characteristics of a beam stretcher are that it have an energy acceptance of \( \sim 1\% \) and a pulse rate of \( \sim 360 \) pps.

This performance might be achieved by a single-turn pulsed inflection of a linac beam into a storage ring, followed by a slow spill-out of the beam onto a target before the next linac pulse. This slow spill could be achieved in several ways: mechanically, by programmed deflection, or by utilizing the natural beam instabilities of an electron machine.

The following limitations exist: (1) the beam must be confined to a given machine aperture. The maximum feasible aperture is a question of economics. (2) The synchronous beam oscillations must have a phase angle \( < 90^\circ \). This is an unalterable limitation. (3) The rf energy necessary to keep the beam in orbit must be economically realizable. (4) The instabilities associated with the beam must not be too large.

The relevant machine parameters are as follows: An electron beam radiates energy at a rate proportional to \( E^2 H \) sec\(^{-1}\). Characteristic times are of the order of

\[
\tau = \frac{\text{Electron energy}}{\text{Energy radiated per sec}} \approx E^{-1} H^{-2} \text{ secs.}
\]
The radiation is in the form of quanta whose average energies are 
\[ \xi_c \propto E^2. \]

The question of dumping times is fairly complex. In a strongly-focusing machine, horizontal radial betatron oscillations are induced and blow up exponentially with a lifetime of approximately \( T \). However, by coupling oscillation modes, all oscillations can in principle be damped with a period \( T \). The beam from the linac is probably sufficiently well-collimated that we can ignore betatron oscillations in this discussion.

If we consider a beam for a period of time \( T \), we will radiate a number \( N \approx \xi_c T \) quanta, and the resulting energy spread is 
\[ (\Delta E/E) \approx N^{1/2} \xi_c E^{-1} \sim EH^{3/2} T^{-1/2} \] due to quantum fluctuations. If this quantity is less than the initial energy spread of the beam from the linac, this does not provide a good method for dumping the beam.

The energy and radial acceptance is given by

\[ \frac{\Delta r}{r} = \alpha \frac{\Delta E}{E} \approx 2 \left( \frac{W_r}{2\pi \rho E} \right)^{1/2} \]

where \( \alpha \) is the momentum-compaction factor defined by the relation \( \Delta r/r = \alpha(\Delta E/E) \); \( k \) the harmonic number is the ratio of rf frequency to rotation frequency; and \( V \) is the accelerating voltage.

The parameters of the CEA machine are as follows:

- n-value: 91
- mean machine radius: 118 ft
- rf frequency: 475 MHz (Mc/sec)
- harmonic number: 360
- momentum compaction: \( \alpha = 0.042 \)
- radiation loss at 6 Bev: 4.5 Mev/turn
- maximum field: 7.8 k gauss
- \( \xi_c \), mean energy of quanta at 6 Bev: 16.3 kv
- \( \tau \): \( \sim 1 \) msec
The CEA machine has, at 6 Bev, \((V/E) \approx 5 \times 10^{-4}\). For these conditions the full synchronous oscillations only occupy 1/5 inch \((\alpha \Delta r/r \approx 2 \times 10^{-14} \text{ and } \alpha \Delta E/E \approx 4 \times 10^{-3})\). The energy input into such a system per electron would have to be about 10 Bev per 6-Bev electron.

The beam dumping times can easily be made of the order of milliseconds. (The \(\Delta E/E\) arising in a period of 1 msec is \(= 2 \times 10^{-3}\) and is of the same order as the machine acceptance.)

Thus, using the CEA magnet and rf structure, and given an 0.4% momentum spread from the linac beam, we could construct a "beam stretcher" (at a cost of \(\sim \$10\) million) for a 6-Bev machine. This "stretcher" would have a 30% duty cycle and could (by suitable modulation of the linac) have 100% acceptance.

Modifications could be made in several ways: (1) Larger magnets and hence smaller radiation losses and lower rf voltage. However, if the rf voltage is cut, the momentum acceptance also is cut (a step in the wrong direction, as we are already below a desirable figure).

Secondly, if the radius is increased, we either have to scale the rf frequency down (undesirable, as the system would become less efficient), or we again cut the momentum acceptance. (2) Smaller radius. This gives a rapid increase proportional to \(R^{-2}\) in rf power requirements. (3) A machine scaled down to 3 Bev but with the same rf frequency and voltage. Here we gain by a factor of two on momentum acceptance. (4) A weak instead of a strong-focusing machine. Much simpler construction but otherwise pure loss as the momentum compaction factor \(\alpha\) is much nearer unity. (5) Lower rf frequency. Pure gain theoretically, but in practice it is harder to obtain the necessary voltage.

Our conclusion can be most optimistically stated as follows: At 6 Bev the costs of the storage ring and of a 6-Bev linear accelerator become comparable at \(\sim \$10\) million. A beam stretcher for this energy appears to be quite feasible.

If 6 Bev is the desired energy, it would appear most reasonable to construct a separate machine. Above 6 Bev the construction difficulties increase rapidly. For lower energies it does not seem
appropriate to use the Monster as an injector. It therefore does not
appear that a beam stretcher should be part of the experimental facili-
ties of the Monster.

(Everything we have said above applies to colliding beams, but
more so. For colliding-beam experiments, not only must the beams be
stacked, but they also must remain stable for minutes, not milliseconds.)
This note is intended to point out the limitations on the Monster beams that can be expected with the use of conventional focusing systems. As a rough rule, a bubble chamber is worth running with beams down to one particle per pulse, possibly to a factor of ten less. The chamber can be run in two ways. The particles put through it can be physically separated from unwanted particles, or particles can be electronically tagged as they enter the chamber. If a beam consisting of 10% antiprotons and 90% mu's and pi's is being used to run an antiproton experiment, and if we can deliver 10 antiprotons to the chamber and physically throw away the 90 unwanted particles, physical separation is desirable. If only ten particles in all can be delivered per pulse, physical separation achieves very little. If only one particle is delivered per pulse, physical separation achieves practically nothing.

For counter experiments we have an analogous situation with a 1 μsec duration pulse. At rates of 100 particles per pulse physical separation is a necessity; at 10 per pulse it is pleasant but not essential; and at less than one per pulse it is unnecessary. It is interesting that these regions coincide for counters and bubble chambers for the Monster.

The utility of a system incorporating an rf separator can therefore be gauged by the beam it will deliver; if less than 10 particles per pulse, its use is optional; if less than one particle per pulse, its use is unnecessary and may even be detrimental, since it will put constraints on the focusing systems that can be employed.
To make the discussion concrete we will consider a 2° beam of 20-Bev antiprotons from a 50-Bev machine. This is an angle for which the efficiency of production will be 1/2 that at zero degrees, and the antiprotons will constitute 10% of the beam. The cross section for antiproton production is ~ $10^{-4}$ particles per Bev/c per steradian per electron at 2°, and for all particles is about $10^{-3}$ per Bev/c per steradian. Thus a separator with a total momentum and angular acceptance of $> 5 \times 10^{-10}$ becomes interesting; with an acceptance $< 5 \times 10^{-10}$ it is uninteresting.

Acceptance of a Practical System.

\[ \frac{L}{n} \approx \frac{8 \pm A}{n} \] (1)

The solid angle accepted is

\[ \Omega \approx \frac{\pi A^2}{n^2} \] (2)

The momentum-band acceptance is calculated on the basis that the change in the angle of deflection $A/n$ for a wrong momentum particle is
\[ \Delta \theta \approx \frac{2 \Delta p A}{\rho n} \]  

(3)

The factor 2 is a function of the actual lens and arrangement used.

The momentum acceptance is then given by

\[ \frac{12}{p} \frac{\Delta p A}{\rho n} \approx A \]  

(4)

or

\[ \frac{\Delta p}{p} \approx \frac{n}{2L} \]  

(5)

Accordingly

\[ \frac{\Delta p}{p} \approx \frac{\pi A^2}{2nL} \]  

(6)

and, using (1),

\[ \frac{\Delta p}{p} \approx \frac{\pi A^3}{2L \delta} \]  

(7)

For the system we are considering, an S-band separator for 20-Bev $\bar{p}$, $L \sim 120$ feet, $A \sim 0.7$ inches, $\delta$ is set by the condition that we are observing at a $2^\circ$ angle and would be $\sim 1/16$ inch from the slant of a 6-inch target and would be another $1/16$ inch from the beam size, giving an approximate $1/8$ inch. There would probably be a factor 2 attenuation of the beam getting out of the target and through a lead filter to remove electrons, and a factor of the order of 2 introduced by the use of collimator slits.

Thus the acceptance

\[ \approx \frac{\pi A^3}{8L^2 \delta} \approx 5 \times 10^{-7} \]  

(8)
which is much larger than the figure we arrived at in the first section for the point at which the use of separators becomes marginal.

As \( L \) increases as the square of the energy, the acceptance is proportional to \( E^{-4} \) and thus gets worse. For stable particles at 20 Bev, the yield gets better as \( M^{-4} \); however, the separation length needed goes as \( M^{-2} \). Hence the acceptance goes as \( M^{+4} \), and thus the total delivery remains approximately constant. For unstable particles, the situation is worse by the fraction that have decayed in the separation distance.

Accordingly, it appears that rf beam separators as discussed by Mozley will be of wide utility.
Only a very small fraction of the energy of an electromagnetic cascade goes into the pair production of particles heavier than electrons. This fraction for ultra high energies is of the order of \((m_e/m)^2\) where \(m\) is the mass of the produced particles. However, these pairs are produced into an opening angle \(\sim mc^2/E\), where \(E\) is the energy of the secondary, and are produced with an energy spectrum that is approximately flat up to the top energy available. This is to be contrasted with nucleon-nucleon production where a large fraction of the energy goes into particle production, the typical opening angle is \(\sim (M_pE_0/E)^{1/2}\), and the energy spectrum drops off very sharply at high energies. For photon-nucleon production, the photon is probably equivalent to \(10^{-2}\) to \(10^{-3}\) of a nucleon in its ability to produce strange particles.

To carry through quantitative calculations on electromagnetic pair production, we will require information on the generation of an electromagnetic cascade, the cross sections for pair production, and the angular distributions of the products.

**Electromagnetic Cascades**

Interest in electromagnetic processes has usually centered on the total numbers of secondary electrons produced by a high-energy primary electron or gamma ray. The approximations previously used are not valid in the region we are interested in, where the secondaries have energies comparable to the primary energy. We will calculate, using the same set of approximations used in Approximation A of Shower theory,
namely a $\frac{dk}{k}$ spectrum of secondary gammas produced by a primary electron, a $\frac{dE}{k}$ (flat) spectrum of secondary electrons and positrons produced by a primary gamma ray, and asymptotic cross-sections; one additional approximation is made, that the energy $E$ of a given electron varies with depth as $E = E_0 - t$ where $t$ is the depth measured in radiation units.

The spectrum of first generation gamma rays is then given by

$$N(k)dk = \int_{0}^{t_{\text{max}}} \frac{dk}{k} dt = t_{\text{max}} \frac{dk}{k}$$

where $k$ is the $\gamma$-ray energy divided by the primary energy and can have values ranging from 0 to 1; $t$ is the depth in radiation units; and $t_{\text{max}}$ is the depth where the primary energy has dropped below $k$, i.e., $e^{-t_{\text{max}}} = k$, or $t_{\text{max}} = -\ln k$. Therefore

$$N_1(k)dk = -\ln k \frac{dk}{k} . \quad (1)$$

The spectrum of first-generation pair products is

$$N(\xi) d\xi = \int_{\xi}^{1} -\ln k \frac{dk}{k} d\xi ,$$

or

$$N(\xi) d\xi = d\xi [1 - \frac{1}{\xi} (\ln \xi + 1)] \quad (2)$$

Finally, for reference purposes, the spectrum of second-generation gamma-rays is given by

$$N_2(k)dk = 2 \int_{k}^{1} -\ln \frac{k}{\xi} dk [1 - \frac{1}{\xi} (\ln \xi + 1)] d\xi$$

$$= 2 \frac{dk}{k} \left[ -\frac{(\ln k)^3}{6} - \frac{(\ln k)^2}{2} + k - 1 - \ln k \right] \quad (3)$$
These functions are tabulated below:

<table>
<thead>
<tr>
<th>( \xi ) or ( k )</th>
<th>.9</th>
<th>.8</th>
<th>.5</th>
<th>.2</th>
<th>.1</th>
<th>.05</th>
<th>.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k N_1(k) )</td>
<td>.10</td>
<td>.22</td>
<td>.69</td>
<td>1.6</td>
<td>2.3</td>
<td>3.0</td>
<td>4.6</td>
</tr>
<tr>
<td>( k N_2(k) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>.40</td>
<td>1.6</td>
<td>6.6</td>
<td>18.4</td>
</tr>
<tr>
<td>( N(\xi) )</td>
<td>.005</td>
<td>.02</td>
<td>.38</td>
<td>4.0</td>
<td>14.0</td>
<td>40.0</td>
<td>460</td>
</tr>
<tr>
<td>( \xi^2 N(\xi) )</td>
<td>.004</td>
<td>.013</td>
<td>.094</td>
<td>.16</td>
<td>.14</td>
<td>.1</td>
<td>.04</td>
</tr>
</tbody>
</table>

It will be noted that the second generation gamma rays only become equal to the first for \( k = 1/10 \). Thus, solely the first generation is important for our considerations, and accordingly targets of two-radiation-unit thickness will produce most of the secondaries in this range.

**Pair-Production Cross Section**

One of the first questions that arises for pair production of heavy particles is whether the production is coherent. A simple argument gives the range of validity. If we consider a gamma-ray of momentum \( P_\gamma \) that forms a pair of particles of mass \( m \) with momentum \( p_+ \) and \( p_- \) in the field of a nucleus which recoils with momentum \( p \), then the condition for coherent production is \( h/p \gg \) nucleon radius. For the case of \( p \) having a minimum value, conservation of momenta gives

\[
P = P_\gamma - p_+ - p_-
\]

and the conservation of energy gives

\[
cP_\gamma = \sqrt{p_+^2 c^2 + m^2 c^4} - \sqrt{p_-^2 c^2 + m^2 c^4} = 0
\]
Thus, for example, a 50-Bev γ ray imparts a minimum momentum transfer between 10 to 40 Mev/c in forming a proton-antiproton pair, and the process will accordingly be coherent for low-Z nuclei.

Reference to W. Heitler gives the cross-section for pair-production in the extreme relativistic range as

\[ \phi_{\text{pair}} = \frac{Z^2 e^4}{137 m c^2} \left( \frac{28}{9} \log \frac{2k}{m} - \frac{218}{27} \right) \]

for the unshiled case, and

\[ \phi_{\text{pair}} = \frac{Z^2 e^4}{137 m c^2} \left( \frac{28}{9} \log \left( \frac{1832^{1/3}}{2} \right) - \frac{2}{27} \right) \]

for the shielded case. The appropriate modification appears to be

\[ \phi_{\text{pair}} = \frac{Z^2 e^4}{137 m c^2} \left( \frac{28}{9} \log \frac{\gamma_{\text{max}}}{\gamma_{\text{min}}} \right) \]

where

\[ \gamma_{\text{max}} = \frac{h}{p_{\text{min}}} = \frac{h k}{m c} \]

and \( \gamma_{\text{min}} \) for the electron case \( \sim h/mc \), and \( \gamma_{\text{min}} \sim R \) the radius of the nucleus for the heavy-particle case. With these modifications,
Using the above formulae for 50-Bev γ-rays incident on carbon, the ratio of the cross section for μ- and π-pair to electron-pair production would be
\[ \sim 1/2 \left( \frac{m_e}{m_\mu} \right)^2, \]
and for the antiproton-proton pairs the ratio would be
\[ \sim 1/4 \left( \frac{m_e}{m_p} \right)^2. \]
In addition, there is a factor of 1/7 for the production of spin-0 particles.

**Opening Angle of Pairs**

Bethe (Proc. Cam. Soc. 30, 524, 1934) gives the formulae for the angle of pair-produced particles relative to the incident γ-ray:

\[
\sigma(\theta) d\theta = \frac{\theta d\theta}{(\theta_0^2 + \theta^2)^2} \ln \left( \frac{\theta_0^2 + \theta^2}{\theta_0^2} \right) + B
\]

where \( \theta_+ \) is the angle between the positively charged member of the pair and the incident γ-ray, and \( \theta_0 = mc^2/E_+ \). At small angles we shall use the approximation

\[
\sigma(\theta) d\theta = \frac{\theta d\theta}{(\theta_0^2 + \theta^2)^2} \cdot 2\theta_0^2
\]

or

\[
\sigma d\Omega = \frac{\theta_0^2 d\Omega}{\pi(\theta_0^2 + \theta^2)^2}
\]

where \( d\Omega \) is the solid angle. For \( \theta_+ \rightarrow 0, \)
\[ \sigma d\Omega \rightarrow d\Omega/m\theta_0^2. \]

**Yields for Pair-Production**

Combining the results of the previous three sections, the yields of muons \( Y(E) \) per 50-Bev electron per steradian per Bev/c is given by
\[ Y(E) = \left( \frac{50 \text{ Bev}}{m_\mu c^2} \right)^2 \left( \frac{m_e}{m_\mu} \right)^2 \frac{1}{\pi} \frac{1}{2} \frac{1}{2} N(\xi) \frac{1}{50} \]

\[ = 4 \times 10^{-2} \xi^2 N(\xi), \]

where \( \xi = E/50 \text{ Bev}; \) \( \xi^2 N(\xi) \) is tabulated in Table I.

**TABLE II.** Pair-production yield of \( \mu \)-mesons

Yields in units of number per Bev/c per steradian per 50-Bev electron of both positive and negative \( \mu \)-mesons.

<table>
<thead>
<tr>
<th>Secondary Energy</th>
<th>45 Bev</th>
<th>40 Bev</th>
<th>25 Bev</th>
<th>10 Bev</th>
<th>5 Bev</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muons</td>
<td>(1.6 \times 10^{-4})</td>
<td>(5.2 \times 10^{-4})</td>
<td>(3.6 \times 10^{-3})</td>
<td>(6.4 \times 10^{-3})</td>
<td>(5.6 \times 10^{-3})</td>
</tr>
</tbody>
</table>
Introduction. The possibility of getting a monochromatic photon beam by annihilation of positrons at Gev energies has been discussed before, notably by Devons and by Gunn and Moorhouse in reports prepared for the U. K. National Institute for Research in Nuclear Science. The basic physical data required for evaluating the possibility are well known (v. Heitler, 3rd Edition, S. 27), but certain technical data are also required. In particular it is necessary to know the efficiency with which electrons can be shower-converted into positrons acceptable to a linear accelerator. An experimental study of this problem has been made at Stanford by Pine and Yount, and the purpose of this note is to reconsider the situation in the light of their results. The rather different problems and possibilities of getting monochromatic 100-Mev photons will not be considered at all.

Kinematics of High-Energy Positron Annihilation. A positron of energy $E_+ (\gg \mu c^2)$ approaches an electron at rest in the laboratory system (Fig. 1a). In the C. M. system (Fig. 1b) each particle has energy $E_0 \sim \sqrt{1/2 \mu c^2 E_+} \gg \mu c^2$. In the same system the annihilation photons also have energy $E_0'$, and an examination of the differential cross section shows that they are strongly collimated along the axis at angles of the order of $\mu c^2/E_0'$. After transformation back to the
FIG. 1--Kinematics of high-energy positron annihilation.
laboratory system (Fig. 1c) the forward photon is collimated along the axis at angles of the order of $\mu c^2/E^+$. The energy of the backward photon is $1/2 \mu c^2$ for axial annihilation, and this increases only to about $\mu c^2$ for off-axis annihilation at typical angles. To a very good approximation the forward photon may be considered to have the unique energy $E^+_+$. 

**Experimental Layout.** A possible experimental layout is shown in sketch form in Fig. 2. An injector linac accelerates primary electrons which are converted into positrons by showering. These positrons are accelerated by the main linac, and pass through a magnetic switchyard and slit system for energy selection. Then they traverse a liquid hydrogen annihilation target and are finally dumped by a third magnet. The annihilation photons and bremsstrahlung background continue forward into an experimental area. The numbers indicated refer to operation at 2 Gev.

**Injector Linac.** This should give the highest possible current but must match the main accelerator in frequency, phase, and duty ratio. Energy droop during the pulse is not important, however.

Selected parameters: $E_0 = 300$ Mev, $i_{\text{peak}} = 1$ amp, duty ratio = $3 \times 10^{-4}$, $\langle i \rangle = 300$ $\mu$amp $= 1.8 \times 10^{15}$ $e^{-} \text{sec}^{-1}$.

**Converter.** Typically an optimum converter would be $\sim 3$-$1/2$ radiation lengths of tungsten, producing positrons of roughly the critical energy ($10 \pm 5$ Mev). Conversion efficiencies, including the effect of solenoidal focusing of emergent positrons, are quoted from the work of Pine and Yount. They refer to a 10-Mev band of positron energies trapped in the main linac.

<table>
<thead>
<tr>
<th>$E_0$ (Mev)</th>
<th>Positrons per electron in 10-Mev band</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>$\sim 5 \times 10^{-5}$</td>
</tr>
<tr>
<td>50</td>
<td>$\sim 5 \times 10^{-6}$</td>
</tr>
</tbody>
</table>
FIG. 2--Schematic layout
The proposed injector linac would therefore give an injected beam of $9 \times 10^{10}$ e$^+$ sec$^{-1}$ in a 10-Mev band injected into the main linac.

**Main Linac.** This accelerates the positrons under light loading conditions but is likely to introduce an additional energy spread of the order of $1/2\%$ of $E_+$, which must be compounded in square-law fashion with the source spread. The resolved beam available after energy definition at the slits may be estimated as follows:

<table>
<thead>
<tr>
<th>$E_+$ (Gev)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>e$^+$ sec$^{-1}$ in 10-Mev band</td>
<td>$8 \times 10^{10}$</td>
<td>$6 \times 10^{10}$</td>
<td>$3 \times 10^{10}$</td>
<td>$1.8 \times 10^{10}$</td>
</tr>
</tbody>
</table>

**Hydrogen Target.** The thickness of the hydrogen target is limited by the ionization loss of about 5 Mev-grm$^{-1}$ cm$^2$, which introduces an additional energy spread. A thickness of 1 grm-cm$^2$ will be adopted. This gives an annihilation peak 15-Mev wide at the base.

The total 2-quantum annihilation cross section of an electron is,

$$\phi = \frac{\pi r_0^2 \mu c^2}{E_+} \log \left( \frac{2E_+}{\mu} - 1 \right)$$

<table>
<thead>
<tr>
<th>$E_+$ (Gev)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ (µb)</td>
<td>910</td>
<td>498</td>
<td>223</td>
<td>120</td>
</tr>
</tbody>
</table>

Approximately $\phi \propto 1/E_+$. The positrons also produce bremsstrahlung in the liquid hydrogen. It is assumed that the atomic electron produces as much radiation as the proton (correct to $\sim 15\%$). At the upper end of the spectrum the differential cross section for producing a photon of energy between $k$ and $k + dk$ is then
\[ \dot{\Phi}_k = \frac{4r_0^2}{137} \left[ 2 \log \left( \frac{E_+ - k}{1/2 \mu c^2} \right) - 1 \right] \frac{dk}{k} \]

while for slightly larger \( E_+ - k \) the screened formula,

\[ d\Phi_k \approx \frac{8r_0^2}{137} \log(183) \frac{dk}{k} \]

should be used. This last formula correctly predicts a radiation length \( x_0 \) of approximately 69 g/cm\(^2\).

At 1 Gev the computed \( d\Phi_k \) are:

<table>
<thead>
<tr>
<th>Formula</th>
<th>Unscreened</th>
<th>Screened</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_+ - k ) (Mev)</td>
<td>50 20 10 5 2.1/2</td>
<td>100 50</td>
</tr>
<tr>
<td>( k ) (Mev)</td>
<td>950 980 990 995 997.5</td>
<td>900 950</td>
</tr>
<tr>
<td>( d\Phi_k ) (ub-Mev(^{-1}))</td>
<td>22.7 19.4 14.9 11.6 8.4</td>
<td>29.6 25.3</td>
</tr>
</tbody>
</table>

When \( E_+ \) is varied, \( \dot{\Phi}_k \) for a given \( E_+ - k \) is approximately proportional to \( 1/E_+ \). Therefore the ratio of annihilation to bremsstrahlung photons in a given band of energy at the top end of the spectrum is nearly independent of \( E_+ \).

At 1 Gev the annihilation peak amounts to

\[ 8 \times 10^{10} \text{ e}^+ - \text{sec}^{-1} \times 9.1 \times 10^{-28} \text{ cm}^2 \times 6 \times 10^{23} \text{ atoms/cm}^2 \]

\[ = 4.4 \times 10^7 \text{ photons/sec}^{-1}. \]

The overall situation at this energy is summarized in Fig. 3; for \( n \) Gev all ordinates in Fig. 3 should be scaled down together by the factor.
For higher energies multiply all ordinates by $\rho$.

Total in peak $4.4 \times 10^7$ photons/sec-l

FIG. 3--Annihilation peak and bremsstrahlung background at 1 Gev.
where \( \frac{1}{n} \) takes care of cross-section changes, and the factor in the brackets arises from acceleration energy spread. Computed values are:

<table>
<thead>
<tr>
<th>( E_+ ) (Gev)</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho )</td>
<td>1</td>
<td>0.39</td>
<td>0.081</td>
<td>0.022</td>
</tr>
<tr>
<td>photons-sec(^{-1}) in peak</td>
<td>(4.4 \times 10^7)</td>
<td>(1.7 \times 10^7)</td>
<td>(3.6 \times 10^6)</td>
<td>(1.0 \times 10^6)</td>
</tr>
</tbody>
</table>

Comments. No definite evaluation of the usefulness of this technique will be presented here, but some comments seem appropriate.

1. Photon difference experiments can be done by switching acceleration from shower positrons to shower electrons; the latter of course will not produce the annihilation peak. Success depends on having a high detection threshold in the counting apparatus, or otherwise the peak will produce an insignificant fraction of the counting rate.

2. A two-body strange particle reaction (e.g., \( \gamma + p \rightarrow k^+ + \Lambda \)), having a differential cross-section of about \(10^{-31} \text{ cm}^2 \text{-steradian}^{-1}\) at 1 Gev, would produce about one count per minute in a detection system accepting \(10^{-2}\) steradian from a 1 gm-cm\(^2\) liquid-hydrogen target.

3. An electron synchrotron of Cambridge-M.I.T. specification produces at 1 Gev about 15 times as many photons-Mev\(^{-1}\) in a bremsstrahlung spectrum as there are in the annihilation peak discussed here. Use of the bremsstrahlung is in any case better for doing any experiment in which the photon energy can conveniently be defined in the detection apparatus (e.g., \( \gamma + p \rightarrow k^+ + \Lambda \) or \( \gamma + p \rightarrow k^+ + \Sigma^0 \)). For photon difference experiments in 3-body reactions the synchrotron produces a thin-target type of bremsstrahlung spectrum, which allows an energy resolution of about \(\pm 15\) Mev in a photon difference type
of experiment. A reasonably high detection threshold is again required. The annihilation peak seems to have better resolution by a factor 2 but an order of magnitude smaller absolute counting rate.
ELECTRON-PROTON SCATTERING IN THE GEV RANGE

by

J. M. Cassels

1. Introduction

The general process of scattering an electron off a proton in the Gev range is illustrated in Fig. 1, which serves also to introduce various four-vectors $p, p', q, P, P_n$. The electron is extremely relativistic, so that $E = |\vec{p}|$ and $E' = |\vec{p}'|$; in the laboratory space it is easy to see that

$$ P = 0, \quad \text{im} $$

$$ q^2 = 4 E E' \sin^2 \frac{\theta}{2} $$

$$ -q \cdot P = M(E - E') $$

where $\theta$ is the laboratory angle of scattering.

The $n$ particles and momenta into which the target may disintegrate present a rich variety of physical situations, of which three are of special interest. These are

1. The final state contains only one strongly interacting particle, a proton. This is elastic scattering, for which it is easy to see (e.g., in C.M. space) that $-q \cdot P = 1/2 q^2$.

2. No notice whatever is taken of the nature of the final state of the target. In this case certain invariance arguments can be applied to the integrated inelastic cross section.
FIG. 1--Electron-proton scattering.
(3) The final state consists of \( \pi K \) or \( \Sigma K \). To extract information from this situation calls for the application of particular, not general, theory. At present \( m_{\pi}/M \) is apt to be put equal to zero, a not specially realistic assumption. No further reference will be made here to this possibility.

Before situations (1) and (2) are considered in a little more detail, attention may be directed to Fig. 2. This shows the available regions of \( q^2, -q \cdot P \) space for primary electrons of \( E \leq 10 \text{ Gev} \). The momenta are expressed in units of inverse fermis (1 fermi\(^{-1}\) = 197 Mev/c); this has become standard practice at lower energies, but a changeover to proton-mass units would have advantages.

The elastic scattering line \( -q \cdot P = l/2 q^2 \) acts as the boundary between physical and unphysical pairs of \( q^2, -q \cdot P \) values. Along the horizontal axis \( q^2 = 0 \) is reached when the electron is brought to rest in the laboratory frame \( (E' = 0) \). In this special situation the virtual photon has no mass \( (q^2 - q_4^2 = 0) \), so that there is a physical connection with the photoproduction process in which a real photon of energy \( k \) in the laboratory strikes the target \( (k = E - E' = l/M q \cdot P) \).

Six typical lines drawn on the diagram correspond to the scattering of 5 and 10 Gev electrons at \( 10^\circ, 30^\circ \) and \( 180^\circ \). For a particular value of \( E \) the lines converge to a point on the \( -q \cdot P \) axis, where \( E' = q^2 = 0 \) and the angle of scattering is irrelevant. Along each line \( E' \) varies linearly from zero to its specified maximum value, reached of course when the scattering is elastic.

The diagram illustrates an essential feature, that each point in \( q^2, -q \cdot P \) space can be reached with an infinite number of sets of \( E, \theta \) and \( E' \) values. Except when a special effort to the contrary is made, the value of \( \theta \) is apt to be small, since large areas of \( q^2, -q \cdot P \) space are swept out by keeping \( E \) constant and changing \( \theta \) from, say, \( 10^\circ \) to \( 30^\circ \). Correspondingly \( E' \) is apt to run up to a maximum value which is a large fraction of \( E \).

In practice it will be an objective to reach each point with at least two measurements, one with \( \theta \) small and another with \( \theta \sim \pi \). It is natural to specify two types of spectrometer,
FIG. 2--Accessible regions of $q^2$, $-q \cdot P$ space.

All energies are in Gev.
(i) a large-solid-angle, low-E' type for $\theta \sim \pi$ measurements, and
(ii) a small-solid-angle, high-E' type for $10^3 \leq \theta \leq 45^\circ$.

These will be discussed in a little more detail in Section 4.

2. Elastic Scattering.

The elastic scattering is traditionally described by the modified Rosenbluth formula,

$$
\frac{d\sigma}{d\Omega} = \frac{e^2}{4E} \cos^2 \frac{\theta}{2} \left( \frac{1}{1 + \left( \frac{2E}{M} \right) \sin^2 \frac{\theta}{2}} \right)
$$

$$
\times \left\{ F_1^2 + \frac{q^2}{4M^2} \left[ 2(F_1 + KF_2)^2 \tan^2 \frac{\theta}{2} + K^2 F_2^2 \right] \right\}
$$

where $F_1$ is associated with the charge and Dirac magnetic moment of the proton, and $F_2$ with its Pauli magnetic moment ($K = 1.79$). The two form factors $F_1, F_2$ are functions only of $q^2(-2 q \cdot P)$ and they can be separated by approaching the same $q^2$ by way of large and small values of $\theta$.

The formula is exact so long as one photon only is exchanged in the scattering process (Born approximation). Drell and Fubini argue that this is good to 1% for $q^2 \lesssim 25$. However the form factors are rapidly falling functions of energy, since the proton finds difficulty in hanging together as $q^2$ rises. Probably 2-photon exchange will take over at $q^2 \sim 100$, and this will cause a breakdown of the scattering formula, revealed by the inability of two form factors to collate all the results at different $E, \theta$ but the same $q^2$.

The point of onset of this condition will be qualitatively interesting, but it will certainly be a complicated and unattractive problem to learn much theoretically from scattering results at higher $q^2$.

For orientation some numbers may be considered:
The cross sections are given both for a point proton \((F_1 = F_2 = 1)\), and for form factors computed from a formula which gives fair agreement with experiment up to \(q^2 \leq 25\). As already stated, even the order of magnitude of the cross sections should be viewed skeptically whenever \(F_1^2, F_2^2 \leq 10^{-3}\); probably the form factors will be larger or 2-photon exchange will intervene.

The values of \(E'\) for \(\theta = 180^\circ\) are given by the formula

\[
E' = \frac{ME}{M + 2E}
\]

which approaches the limiting value \(M/2\) as \(E \to \infty\). It therefore makes extremely good sense to propose, as Hofstadter has done, a very large solid angle 500 Mev/c spectrometer especially for measurements in the backward direction.

The spectrometer resolution also requires some comment. The maximum \(E'\) for an inelastic event occurs when a nucleon and a pion recoil together with zero relative velocity. As compared with elastic scattering this situation produces an \(E'\) which is smaller by a factor

\[
1 - \frac{\mu^2}{2M^2E^2} \sim 1 - \frac{\mu}{E} \frac{\mu}{2ME}
\]

To resolve the elastic line completely therefore requires a resolution of \(\mu/E \sim 1.5\%\) for \(E = 10\) Gev. However a somewhat poorer
resolution would probably show an elastic peak above an extrapolated tail from the inelastic events.

3. Inelastic Electron Scattering.

The cross section for inelastic scattering integrated over all details of the final state has been considered in two unpublished memoranda by Bjorken, and independently by von Gehlen [Phys. Rev. 118, 1455 (1960)].

The differential cross section for scattering the electron into the solid angle $d\Omega$ and the energy band $dE'$ must have the form

$$
\frac{d\sigma}{d\Omega \, dE'} = \frac{\alpha^2}{4E^2 \sin^2 \frac{\theta}{2}} \left\{ G_1^2(q^2, -q \cdot P) \cos^2 \frac{\theta}{2} + G_2^2(q^2, -q \cdot P) \sin^2 \frac{\theta}{2} \right\}
$$

where $G_1, G_2$ are functions simply of $q^2$ and $-q \cdot P$, the only suitable invariants available in the problem. In this form $G_1, G_2$ have the dimensions of an inverse energy. The whole relation is based again on one-photon exchange (Born approximation), but this is expected to be a good assumption at all times since there are no constraints to make the 1-photon cross-section especially small.

The principal experimental objective is obviously to approach the same point on the $q^2, -q \cdot P$ plane with $\theta$ small and $\theta \sim \pi$, so separating $G_1, G_2$. An important project would be to determine $G_1, G_2$ also from muon scattering, so checking that the muon-nucleon interaction is purely electromagnetic, apart from Fermi interactions.

In order to help estimate the actual order of magnitude of the inelastic cross section Bjorken has applied the Weizächer-Williams method to get

$$
\frac{d\sigma}{d\Omega \, dE'} = \frac{\alpha}{4x^2} \frac{1}{E \sin^2 \frac{\theta}{2}} \frac{1}{k} \left\{ \cos^2 \frac{\theta}{2} + \frac{k^2}{2} \frac{2k}{2EE'} \frac{\sin^2 \frac{\theta}{2}}{M} + f(q^2, k) \right\}
$$

where $\sigma(q^2, k)$ is the photo-disintegration cross section for photons of laboratory energy $k = E - E' = -\frac{1}{M} q \cdot P$, transferring momentum.
corresponding to \( q^2 \). The formula is claimed to be exact, if vague, because \( \sigma(q^2,k) \) can only be measured for real photons with \( q^2 = 0 \); also the factor \( f(q^2,k) \) has not been investigated in detail, although it is known to approach a constant value as \( q^2 \to 0 \).

Bjorken recommends that \( \sigma(q^2,k) \) should have the value \( \sigma(0,k) \) multiplied by a nucleon form factor, which is presumably not specially small because there are no restrictions on the final state. In fact \( \sigma(q^2,k) \) will here be given the constant value of \( 5 \times 10^{-29} \text{ cm}^2 \).

In order to compute some numbers \( f(q^2,k) \) will be taken as zero, and it will be assumed that the spectrometer takes a bite \( \Delta E' \) equal to 5% of the elastic \( E' \) for the same \( E \) and \( \theta \),

\[
\Delta E'(E,\theta) = \frac{1}{20} E'_{\text{elastic}}(E,\theta)
\]

<table>
<thead>
<tr>
<th>( E ) (Gev)</th>
<th>( \theta )</th>
<th>( q^2 (f^{-2}) )</th>
<th>(-q \cdot P (f^{-2}))</th>
<th>( k ) (Gev)</th>
<th>( E' ) (Gev)</th>
<th>( \frac{d\sigma}{d\Omega dE'} \times \Delta E' ) (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>9</td>
<td>65</td>
<td>2.7</td>
<td>2.3</td>
<td>( 6.5 \times 10^{-32} )</td>
</tr>
<tr>
<td>180</td>
<td></td>
<td>110</td>
<td>115</td>
<td>4.8</td>
<td>0.22</td>
<td>( 1.9 \times 10^{-35} )</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>34</td>
<td>137</td>
<td>5.7</td>
<td>4.3</td>
<td>( 5.5 \times 10^{-32} )</td>
</tr>
<tr>
<td>180</td>
<td></td>
<td>230</td>
<td>237</td>
<td>9.8</td>
<td>0.23</td>
<td>( 1.0 \times 10^{-35} )</td>
</tr>
</tbody>
</table>

4. Practical Measurements

It has already been remarked that two spectrometers of quite different type are needed for forward and backward measurements.

The solenoid \( E' \leq 500 \text{ Mev}/c \) spectrometer proposed by Hofstadter has an aperture \( \Delta \Omega \) (in steradians) related to its measured resolution

\[
\frac{\Delta E'}{E'} = 10 \left( \frac{\Delta \Omega}{4\pi} \right)^2
\]
so that backward measurements with a solid angle of 1/2 steradian seem practical. The spectrometer would be about 20 ft long, and the incident beam would pass along its axis on the way to the target.

An $E' \leq 10$ Gev/c spectrometer for small-$\theta$ experiments is being considered by Penner. It seems possible to contemplate an aperture of $5 \times 10^{-3}$ steradians, with a system about 60 ft long. Some counting rates are given below for a hydrogen target 1 gram/cm$^2$ thick struck by a 10 $\mu$amp electron beam.

<table>
<thead>
<tr>
<th>$E$ (Gev)</th>
<th>$\theta$</th>
<th>$E'$ (Gev)</th>
<th>Remarks</th>
<th>Counts/minute</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>10</td>
<td>4.6</td>
<td>Elastic</td>
<td>$4.1 \times 10^6$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>2.3</td>
<td>Inelastic</td>
<td>$7.2 \times 10^5$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>2.9</td>
<td>Elastic</td>
<td>$2.2 \times 10^3$</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.43</td>
<td>Elastic</td>
<td>$3.9 \times 10^1$</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.22</td>
<td>Inelastic</td>
<td>$2.1 \times 10^4$</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>8.6</td>
<td>Elastic</td>
<td>$1.9 \times 10^5$</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>4.3</td>
<td>Inelastic</td>
<td>$6.0 \times 10^5$</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>4.1</td>
<td>Elastic</td>
<td>$1.5 \times 10^1$</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.45</td>
<td>Elastic</td>
<td>$5.5 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>180</td>
<td>0.23</td>
<td>Inelastic</td>
<td>$1.1 \times 10^4$</td>
</tr>
</tbody>
</table>

The numbers are somewhat encouraging, except for elastic scattering in the backward direction, where the estimates of cross section are especially subject to uncertainty.
NOTES ON POWER DISSIPATION FROM COLLIMATORS AND TARGETS FOR HIGH-ENERGY, HIGH-POWER ELECTRON BEAMS

by

S. Penner

Introduction

Since a 60-μamp electron beam at 45 Bev has 2.7 megawatts of power in the beam, the removal of heat from collimators and targets is a difficult problem. The purpose of this note is to point out some general principles for the design of such elements.

Power Deposited in Thick Collimators

To obtain the maximum number of shower electrons per incident 45-Bev electron, we refer to Rossi, High-Energy Particles, Fig. 5.13.2, page 258. By interpolating this graph, we find the maximum multiplicities that are listed in Table I of this memo. These multiplicities vary logarithmically with incident electron energy. Multiplying the multiplicity for each substance by the critical energy and dividing by the radiation length gives the energy deposited per gram per cm$^2$ of material per incident electron. Assuming an incident beam of 60 μamp = $3.8 \times 10^{14}$ electrons/sec, and a beam cross section of one cm$^2$, we obtain the power deposited per gram of material. At the peak of the shower, the beam cross-section may exceed one cm$^2$ due to multiple scattering, so the values listed in the last column of Table I are probably overestimates.
Table I

<table>
<thead>
<tr>
<th>Material</th>
<th>Critical Energy (MeV)</th>
<th>Radiation Length (gm/cm²)</th>
<th>Maximum Multiplicity</th>
<th>Max. Power Deposited (kw/gm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>102</td>
<td>45</td>
<td>59</td>
<td>8.0</td>
</tr>
<tr>
<td>Water</td>
<td>84</td>
<td>37</td>
<td>71</td>
<td>9.7</td>
</tr>
<tr>
<td>Aluminum</td>
<td>49</td>
<td>25</td>
<td>115</td>
<td>13.5</td>
</tr>
<tr>
<td>Iron</td>
<td>27</td>
<td>14</td>
<td>220</td>
<td>23</td>
</tr>
<tr>
<td>Copper</td>
<td>22</td>
<td>13</td>
<td>235</td>
<td>24</td>
</tr>
<tr>
<td>Lead</td>
<td>8</td>
<td>6.5</td>
<td>600</td>
<td>44</td>
</tr>
</tbody>
</table>

Temperature Rise: One Burst

The temperature rise per burst is given by

$$\Delta T = \frac{P_0}{360 \cdot J \cdot C} \quad (1)$$

for a pulse rate of 360 per second. $P_0$ is the power deposited (watts/gm) from Table I, $J$ is the conversion factor to thermal units (4.18 joules/cal), and $C$ is the specific heat in calories/cm°C.

The temperature rise per burst is listed in Table II. From this table we see that lead is ruled out as a collimator or target material. Other heavy metals (tungsten, platinum, etc.) have specific heats comparable to lead and are thus also excluded. Although these materials may not melt, the temperatures are so high that they exclude the possibility of water cooling without boiling.
<table>
<thead>
<tr>
<th>Material</th>
<th>Temp. Rise Per Burst (°C)</th>
<th>Heat Conductivity (cal/sec cm)</th>
<th>Specific Heat (cal/gm°C)</th>
<th>Density (gm/cm³)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon</td>
<td>27</td>
<td>0.01</td>
<td>0.2</td>
<td>1.0</td>
<td>Heat conductivity too low</td>
</tr>
<tr>
<td>Water</td>
<td>6</td>
<td>0.001</td>
<td>1.0</td>
<td>1.0</td>
<td>Water cools by convection</td>
</tr>
<tr>
<td>Aluminum</td>
<td>45</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Iron</td>
<td>140</td>
<td>0.1</td>
<td>0.10</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Copper</td>
<td>180</td>
<td>0.09</td>
<td>0.09</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Lead</td>
<td>980</td>
<td>0.08</td>
<td>0.05</td>
<td>11.3</td>
<td>Specific heat too low</td>
</tr>
</tbody>
</table>
Steady-State Temperature

Next we shall consider the steady-state temperature distribution in a block of material bombarded by an electron beam. We shall consider two geometries.

In the geometry of Fig. 1, an incident beam of radius \( r_0 \) deposits \( P_0 \) watts/gm along the axis of a cylinder of radius \( r_1 \). We assume the cylinder to be infinitely long and to be held at a fixed temperature \( T_0 \) at its outer surface by water cooling. The temperature distribution is given by

\[
\nabla^2 T(r) = -\frac{Q(r)}{k}
\]

(2)

where \( k \) is the thermal conductivity, and \( Q \) is the heat input per unit volume. For purely radial flow, the solution is

\[
T(r)-T_0 = \left( \frac{r_0^2 Q}{2k} \right) \left( \ln \frac{r_1}{r} \right), \quad r > r_0,
\]

\[
T(r)-T_0 = \frac{Q}{4k} (r_0^2 - r^2) + \left( \frac{r_0^2 Q}{2k} \right) \left( \ln \frac{r_1}{r_0} \right), \quad r < r_0
\]

(3)

The maximum temperature obviously occurs along the axis. Assuming \( \pi r_0^2 = 1 \text{ cm}^2 \), and using the fact that \( Q = \rho P_0/J \), where \( \rho \) is the density of the material, the temperature difference between axis and outer surface is

\[
\Delta T = \frac{\rho P_0}{4Jk} r_0^2 \left[ 1 + 2 \ln \frac{r_1}{r_0} \right],
\]

or

\[
\Delta T = \frac{\rho P_0}{52.5k} \left[ 1 + \ln \left( \pi r_1^2 \right) \right]
\]

(4)
FIGURE 1

FIGURE 2
where \( r_1 \) is expressed in centimeters, \( \rho \) is in gm/cm\(^3\), \( P_0 \) is in watts/gram, and \( k \) is in calories/°C sec cm. Even for the extreme case where \( r_1 = r_0 \), \( \Delta T = 1400^\circ C \) for aluminum, which is the least unfavorable case. Therefore, this type of collimator structure is not satisfactory.

The other geometry to be considered is shown in Fig. 2. The electron beam is normally incident on a stack of foils of thickness \( \ell \), with cooling water at temperature \( T_0 \) flowing between the foils. If we consider the case \( \ell \ll r_0 \), the heat flow is essentially along the beam direction. The solution to Eq. (2) then becomes

\[
T(x) - T_0 = \frac{\rho P_0}{2 J k} x (\ell - x)
\]

(5)

where \( x \) is the distance into the foil from one surface.

The highest temperature in the foil occurs at \( x = \ell/2 \),

\[
T(x/2) - T_0 = \frac{\rho P_0 \ell^2}{8 J k}
\]

(6)

In Table III we list the foil thicknesses for various materials corresponding to a 100°C temperature rise. The values for the three materials chosen are reasonable. We may now determine the water flow required to remove the heat. The requirement is 0.6 gpm/kw for a 20°C water temperature rise. The necessary water-flow rates per channel are given in Table 3, assuming a water channel of 1/8-inch thickness, and a power deposition per gram in the water equal to that in the foil material (since the shower is largely generated in the metal). Both volume and linear flow-rates are given. To calculate the linear flow-rate we assume the channel width to be 0.45 inches (the diameter of a 1-cm\(^2\) beam spot). While these flow rates are large, they are not impossible, and therefore, high-power collimators of this type are feasible.
Thick Collimators

All above considerations apply only to very thick collimators and targets. The maximum energy dissipation at these high energies occurs at depths greater than 6 radiation lengths. Since essentially all of the initial electrons have lost at least 10% of their energy in the first two radiation lengths, any collimator which is followed by a bending magnet need be no more than two radiation lengths thick. The shower multiplicity at this depth is < 20% of the maximum values listed in Table I. Thus for two-radiation-length collimators the temperature rise per burst will be less than 20% of the values given in Table II. The temperature rise in the foil thicknesses given in Table III will be less than 20°C, and the water-flow per channel need be only 20% of the values in Table III.

Very Thin Targets

Targets much thinner than one radiation length will often be used. In this case the temperature rises and water flows listed in Tables II and III can be scaled down by the multiplicity factors in Table I. In this limit, edge-cooled targets are also possible (geometry of Fig. 1). The temperature rise in the center of a thin (< 1 cm) aluminum target one cm² in area will be about 12°C. For a one-cm² copper target thinner than about 2 mm, the temperature rise will be about 20°C. A thin lead target will have a temperature rise of the order of 500°C, which is prohibitive, but seems to allow thin targets of other heavy metals which have higher melting points.

<table>
<thead>
<tr>
<th>Material</th>
<th>Foil Thickness for 100°C Rise (inches)</th>
<th>Water Flow per Channel (gpm)</th>
<th>Water Speed in Channel (ft/sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>.083</td>
<td>7.2</td>
<td>42</td>
</tr>
<tr>
<td>Iron</td>
<td>.016</td>
<td>8.8</td>
<td>51</td>
</tr>
<tr>
<td>Copper</td>
<td>.047</td>
<td>20.0</td>
<td>115</td>
</tr>
</tbody>
</table>
Conclusions

With careful designing, it is possible to remove the heat from materials struck by the beam. Materials of low Z, low density, large heat conductivity, and large heat capacity are favored. Aluminum is the best all-around structural metal for collimators and targets when heat dissipation is a serious problem. Beryllium, which has not been considered in detail, would also be good.
PRODUCTION OF PARTICLE BEAMS AT VERY HIGH ENERGIES*

by

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I. INTRODUCTION

We consider here the production of beams of very high energy particles. Our primary interest is to predict how effective an electron accelerator such as the Stanford Monster will be in producing high-energy beams of strongly interacting nuclear particles (such as pions, kaons, antinucleons, and other baryons). For purposes of comparison we present also some estimates of high-energy beam production by a proton accelerator. Finally, we suggest an experiment which can be performed now at Berkeley, CERN and Dubna on the proton synchrotron or at Brookhaven on the AGS to check our prediction here.

The main point we will make in what follows is this: electromagnetic pair production is as good a way as exists to produce collimated beams of very high-energy nuclear particles. By this mechanism it is possible to produce particles of mass $m$ and energy $\omega$ in a cone of half-angle $m/\omega \ll 1$, with cross sections down from geometric ($\sim \pi/\mu^2 \approx 60 \text{ mb}$ for a proton target) by roughly the fine-structure

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constant $\alpha = 1/137$. On the other hand, for nucleon-initiated processes, although one avoids the fine-structure constant, the statistical model predicts emergence of very high energy particles in only a very small fraction of the collisions.

II. ELECTROMAGNETIC PION PRODUCTION

Consider the process shown in Fig. 1. An unpolarized quantum of energy $k$ ($h = c = 1$) is incident on a nuclear target $X$. We observe a particle produced with energy $\omega_q$ at an angle $\theta_q$ with the incident direction. We do not look at what else is produced, and therefore sum over all possible final states $(n)$ which can be made along with the observed particle.

![FIGURE 1](image-url)

We consider observed particles which interact electromagnetically via either a charge or magnetic moment; to start with, we consider in particular a charged pion $\pi^\pm$ with mass $\mu$. Our basic approximation now in calculating the cross section for producing a $\pi^\pm$ with energy $\omega_q > \frac{1}{2}k$ and within the forward angular cone with $\theta_1 \sim \mu/\omega_q$ is to assume that the amplitude corresponding to Fig. 2 makes the major contribution to this process.
In Fig. 2 the photon produces a pair of charged pions, one of which emerges directly with very high energy \( \omega_q > \frac{1}{2} k \) at an angle \( \theta_q \sim \mu/\omega_q \), and the other of which plows into \( X \) initiating reactions to various final states \( (n) \). The cross section of interest is the sum of the squares of matrix elements for all possible states \( (n) \).

The conditions for the validity of this approximation are the following:

1. The propagator for the pion which plows into \( X \) in Fig. 2 is

\[
\left[ (k - q)^2 - \mu^2 \right]^{-1} = (-2q \cdot k)^{-1}
\]

where \( k = (k, \not{k}) \) is the four-momentum of the photon, and \( q = (\omega_q, \not{q}) \) is that of the emerging pion; and

\[
2q \cdot k = 2\omega_q k \left( 1 - \beta^2_q \cos \theta_q \right)
\]

with \( \beta_q = |\not{q}|/\omega_q \). For small angles and high energies we have

\[
2q \cdot k \approx 2\omega_q k \left( \frac{\mu^2}{2\omega_q^2} + \frac{\beta^2_q}{2} \right) \approx 2\mu^2 ,
\]

and we are near the pole of the pion propagator. The point is that this propagator is large \( (\mu^2 \gg \omega_q^{-2}) \), and therefore emphasizes the importance of this diagram relative to others which correspond to two
or more pions or heavier particles propagating from the point (a) at which the $\gamma$ is absorbed to the interaction (b) with $X$, as shown in Fig. 3.

For a two-pion state propagating between (a) and (b) of squared total mass $\mu^2 > (2\mu)^2$, the propagator is less in magnitude than

$$\frac{-1}{(k - q)^2 - (2\mu)^2} = \frac{1}{2q \cdot k + 3\mu^2}. \quad (2)$$

2. The observed pion in Fig. 2 is concentrated in a cone of angle $\theta_1 \sim \mu/\omega_q$ as we see from the expression in (1). (Within this angle the propagator is $\sim 1/\mu^2$ as compared with $1/\omega_q^2 \ll 1/\mu^2$ at larger angles.) For other processes in which states of higher mass emerge from the photon than drawn in Fig. 2, the observed pion will not be so sharply concentrated. For a state of total mass $\mu$ in Fig. 3, the angle of the cone for which the propagator (2) falls to half-value is $\theta_2 \sim \mu/\omega_q > 2\theta_1$, and so the flux per solid angle at $\theta \sim \mu/\omega_q$ is down by a factor of $< 1/4$. This argument also applies in considering processes of the type shown in Fig. 4, in which more than one pion emerges from (a), the electromagnetic part of the interaction. Integration over the undetected final particle group (p) spreads the cone into which the observed pion emerges to a broad angle of width.
\[ \frac{\text{energy in the particle group } (p)}{\omega_q} > \mu/\omega_q. \] Therefore, the amplitude corresponding to Fig. 2 will dominate for the flux per solid angle at \( \theta \sim \mu/\omega_q \).

3. Neglected in Fig. 2 are processes in which the high-energy pion \( \omega_q \) interacts with the nuclear target \( X \) or with the products \( (n) \) in the final state. We are justified in neglecting them for two reasons. First of all, in the multi-Bev region the major fraction of the interaction cross section of pions with nucleons is inelastic. A high-energy incident pion is degraded in energy, producing secondary pions as well as heavier particles in an interaction. Since we are computing the cross section for a very high energy pion to emerge, our error in neglecting its scattering in Fig. 2 is measured by the ratio of elastic to total interaction cross sections at high energies. The cross section corresponding to Fig. 2 will be proportional to the total interaction cross section for an incident pion of approximate energy \( \omega_q \) to produce any states \( (n) \). Processes which correspond to an interaction of the high-energy pion \( \omega_q \) will be proportional to the elastic cross section only at \( \omega_q \). The elastic cross section has dropped to \( \sim 1/6 \) of the total cross section at \( \sim 4 \) Bev, and presumably becomes even smaller at higher energies. The second and more important reason for our neglect of scatterings of the high-energy pions is that we are interested only in those in a narrow forward cone, and the scatterings will not lead to the sharp peak with \( \theta_2 \sim \mu/\omega_q \) that follows from Fig. 2.
For the three reasons given above, the process drawn in Fig. 2 should dominate in the production of very high energy pions \((\omega_q > \frac{1}{2}k \gg \mu)\) in the forward cone at angles \(\theta \sim \mu/\omega_q\). This diagram also has the great and worthy advantage of being readily calculable. The intermediate pion is almost "real" under these conditions; its total four-momentum squared is \((k - q)^2 = \mu^2 [1 - (2k/\omega_q)]\), by (1), and misses the mass shell only by \(\approx \mu \ll \omega_q\). Therefore we obtain directly an expression in terms of the amplitude to produce electromagnetically a \(\pi^\pm\) pair, with one of the pions emerging in a given energy and solid angle interval about \(\omega_q\) and \(\theta_q\)' times the total interaction cross section for the other pion of energy \((k - \omega_q)\) incident on the target X:

\[
d^2 \sigma_{\gamma\pi}(k, \omega_q, \theta_q) = \left( \frac{\alpha}{2\pi} \right) \frac{\sin^2 \theta_q}{[1 - \beta_q \cos \theta_q]^2} \frac{d\omega_q}{4\pi} \times \left[ \frac{\omega_q(k - \omega_q)k\omega_q}{k^3} \right] [\sigma_{\pi + X, \text{total}}(k - \omega_q)] . \tag{3}
\]

Integrating over angles from \(\theta_{q_{\text{min}}}\) to \(\theta_{q_{\text{max}}} \approx \text{several times } \mu/\omega_q\), we obtain

\[
d\sigma_{\gamma\pi}(k, \omega_q) = \frac{\sigma}{2\pi} \left\{ \ln \left[ \frac{(\mu^2/\omega_q^2) + \theta_{q_{\text{max}}}^2}{(\mu^2/\omega_q^2) + \theta_{q_{\text{min}}}^2} \right] \right\} \times \left[ \frac{\omega_q(k - \omega_q)k\omega_q}{k^3} \right] [\sigma_{\pi + X, \text{total}}(k - \omega_q)] . \tag{4}
\]

We ask next how this result compares with other electromagnetic and nonelectromagnetic mechanisms for producing a \(\pi^\pm\) beam. First, we may compare it with the cross section for electromagnetic production of \(\pi^\pm\) pairs in the Coulomb field of a charge \(Z\alpha\) (Fig. 5). This was
calculated by Pauli and Weisskopf.\textsuperscript{1,2} The formula below has been re-derived to express the cross-section in terms of the angle and energy distributions of one of the pions. Numerically the result disagrees with their published numbers, being a factor of two smaller than reference 2 and a factor of four smaller than reference 1. We find (neglecting \( 1 \ll \ln \omega / \mu \))

\[
\frac{d^2\sigma}{d\gamma d\Omega} (k, \theta, \omega) = \frac{\alpha}{2\pi} (2\pi)^2 \frac{d\Omega}{4\pi} \left[ \frac{(\mu/\omega)^4 + \sin^4 \theta \omega^2}{q^4} \right] 2\pi \left[ \frac{\omega (k - \omega) \omega}{q^2} \right] \ln \frac{\omega}{\mu} ;
\]

\begin{equation}
(5)
\end{equation}


These formulas are derived for Coulomb fields of point sources and are valid only for small angles where the momentum transfers are \( \approx \mu^2/k \). Finite nuclear size effects alter the cross section only at larger angles corresponding to momentum transfers of \( \approx \mu \) and are best included by Bethe's method of integration [Proc. Cambridge Phil. Soc. \textbf{30}, 524 (1934)].
intergrating over angles,

\[ \frac{d\sigma}{d\omega} = \frac{\alpha}{2\pi} \left( \ln \frac{\omega}{\mu} \right) \frac{\omega}{k^2} \frac{d\omega}{q} \left[ \frac{(Z\alpha)^2}{3\mu^2} \right] \]  

Equations (6) and (4) are comparable in magnitude if in (4) we use for \( \sigma_{\pi^+X,\text{total}} \) a geometric cross section \( \sim \pi/\mu^2 = 60 \text{ mb} \) as appropriate for \( X \) = one nucleon, say, while in (6) we choose \( Z = 82 \) corresponding to lead. However, there is one important advantage for process (4). Pair-production of \( \mu \)-mesons by the Bethe-Heitler process, Fig. 5 for muons, gives larger cross sections than (5) or (6) for Dirac particles of the same mass \( \mu \) by a factor of roughly 7, which arises from the spin degree of freedom (a factor 2 comes from summing over two final spin states without spin flip, and the rest from the spin-flip amplitude).

This means a large \( \mu \)-background coming from targets of high \( Z \), which can be avoided entirely by working with light targets and using (4). For convenient reference we include here the Bethe-Heitler formulas corresponding to (5) and (6), again in a form valid for high energies and small angles \( \theta \sim \mu/\epsilon_q \):

\[ \frac{d^2\sigma}{d\theta d\epsilon_q'} = \frac{\alpha}{2\pi} \left( \ln \frac{\epsilon_q}{\mu} \right) \frac{d\epsilon_q}{\mu} \]

\[ \times \frac{\left[ k^2/\epsilon_q (k - \epsilon_q) \right]}{\left[ (\mu/\epsilon_q)^2 + \sin^2 \theta \right]^{1/4}} \]

\[ \times \frac{2\pi \left[ \epsilon_q (k - \epsilon_q) d\epsilon_q \right]}{\epsilon_q^2 k^3} \frac{\ln \epsilon_q}{\mu} \]  

\[ \frac{d\sigma}{d\omega} = \frac{\alpha}{2\pi} \left( \ln \frac{\omega}{\mu} \right) \frac{d\omega}{k} \left[ 1 - \frac{4 \epsilon_q (k - \epsilon_q)}{k^2} \right] \left[ \frac{(Z\alpha)^2}{3\mu^2} \right] \]  

Next we compare (3) and (4) with pion production by an incident proton beam. Recent p-p experiments\(^3\) at 25 Bev at CERN indicate total

3. Private communication from J. Adams to B. Cork.
interaction cross sections that are roughly two-thirds geometric, or 40 mb. Calculations have been made by Hagedorn and Behr\textsuperscript{4} at these energies on the basis of the statistical model. They predict the fraction of interactions leading to pions in various given momentum and angular bins.

To get some idea of the numbers we consider an incident 25-Bev gamma producing a 20-Bev pion at small angles. According to (3) the cross section per unit solid angle and per Bev/c of momentum interval is

$$\frac{d^{2}\sigma(k,\omega_{q},\theta)}{d\omega_{q} d\omega_{\theta}} = \frac{\alpha \omega_{q} (k - \omega_{q})}{2\pi \ell \pi k^{3}} \frac{\frac{\theta^{2}}{\omega_{q}^{2}}}{\left(\frac{\mu^{2}}{\omega_{q}^{2}} + \theta^{2}\right)^{2}} \sigma_{\pi X, total} (k - \omega_{q})$$

$$\approx \frac{1}{20} \frac{\theta \omega_{q} / \mu}{1 + \left(\frac{\theta \omega_{q} / \mu}{\omega_{q}^{2}}\right)^{2}} \sigma_{\pi X, total} (k - \omega_{q}) \text{ster}^{-1} \text{Bev}^{-1}. \ (9)$$

Since $$\omega / \mu = 20/0.14 = 140$$, the half-width is $$\sim \frac{1}{2}^0$$, and the intensity at $$1^0$$ is $$\sim (0.008) \sigma_{\pi X, total} (k - \omega_{q}) \text{ster}^{-1} \text{Bev}^{-1}$$.

This is considerably larger than the predictions of the statistical model which leads to $$\approx 0.0007 \sigma_{\pi X, total} (E_{p}) \text{ster}^{-1} \text{Bev}^{-1}$$ according to Behr and Hagedorn\textsuperscript{4} for $$X$$, a target proton, and a 20-Bev pion emerging at $$1^0$$ from an incident proton of $$E_{p} = 25$$ Bev. Since both pion and nucleon cross sections are approximately geometric above a few Bev,\textsuperscript{3,5} the ratio of (9) to the statistical model is $$\geq 10$$. To be sure, the statistical model is least reliable in its predictions at the high-energy tip of the spectrum of produced particles, but it appears that a gamma is at least as effective as a proton in producing a high-energy pion beam.

The approximation in the foregoing discussion can be checked by experiments that can now be performed on existing high-energy proton machines (such as the Bevatron at Berkeley, or at CERN or Dubna).

\textsuperscript{4} J. V. Behr and R. Hagedorn, CERN 60-20 (1960); R. Hagedorn, Nuovo Cimento \textbf{15}, 434 (1960).

\textsuperscript{5} For an energy $$k - \omega_{q} \approx 5$$ Bev, the pion cross-section is $$\approx 30$$ mb; V. I. Veksler, report to 9th Annual High-Energy Physics Conference, Kiev (1959).
We consider the process illustrated in Fig. 6. A proton $p$ of energy $E_p \gg M$ is incident on $X$, producing a neutron (or proton) of energy $E_q \approx E_p$, and anything else (n). For $\Delta = E_p - E_q \leq (\mu/M)E_p$ the high-energy nucleon with $E_q$ will again emerge within a forward cone of half-angle $\theta_1 \sim \mu/E_p$ when one pion is exchanged between (a) and (b) in Fig. 6. The corresponding cross section is

$$d^2\sigma(E_p, E_q, \theta_q)$$

$$= \frac{1}{2\pi^2 \Gamma^2} \frac{p \cdot q - M^2}{[p \cdot q - M^2 + \mu^2/2]^2} \frac{d\Omega}{4\pi} \frac{E_q}{E_p} \frac{dE_q}{dE_{q+X, \text{total}}} \Delta \frac{dE_q}{E_q}$$

where $G^2/4\pi = (2M/\mu)^2 r^2$ with $r^2 = 0.08$ the pion-nucleon coupling constant, and $p \cdot q = E_p E_q (1 - \beta_p \beta_q \cos \theta_q)$ is the invariant product of the incident and emerging nucleon four-momenta. Simplifying (10) for forward angles $\theta_q \sim \mu/E_p$ we find
\[ d^2 \sigma(E_p, E_q, \theta_q) = \frac{\lambda^2}{\pi} \frac{\left[ \theta(E_p/\mu) \right]^2 + \left[ (\Delta/E_p)(M/\mu) \right]^2}{\left\{ \left[ \theta(E_p/\mu) \right]^2 + \left[ (\Delta/E_p)(M/\mu) \right]^2 + 1 \right\}^2} \]

\[ \frac{M^2 \Delta}{\mu^2} \frac{d\Omega}{d E_q} \frac{d \sigma}{d \theta_q} \sigma_{\pi X, \text{total}}(\Delta). \]  

(11)

For \( E_p = 6 \text{ Bev}, \ E_q = 5 \text{ Bev} (\Delta = 1 \text{ Bev}), \) and \( \theta_q = \mu/E_p = 1.4^0 \) this gives a cross section of

\[ \frac{d^2 \sigma}{d \Omega \ d E_q} \simeq 5 \left[ \sigma_{\pi X, \text{total}}(1 \text{ Bev}) \right] \text{ ster}^{-1} \text{ Bev}^{-1}. \]  

(12)

This very intense flux in the forward cone of \( \theta_q \sim 1^0 \) should be readily measurable. We conclude this section with three comments:

1. The large cross sections (11) and (12) may be contrasted with the forward elastic scattering peak due to single pion exchange, as in Fig. 7, which has been suggested by Chew as a means of determining

![Diagram](image)

FIGURE 7

\( r^2 \) for the one-pion contribution to the nucleon interaction. The factor \( \frac{p \cdot q - M^2}{[p \cdot q - M^2 + (\mu^2/2)]^2} \) in (11) comes about as follows: the numerator comes from "real" pion emission from \( a \) in Fig. 6 and vanishes in the forward direction as \( E_q + E_p \) corresponding to the zero amplitude to emit \( p \)-wave mesons of zero velocity; the denominator is just the square of the pion propagator. In the elastic process, Fig. 7, there will appear the factor as above but with squared numerator, \( \frac{[p \cdot q - M^2]^2}{[p \cdot q - M^2 + (\mu^2/2)]^2} \), since the "zero velocity" pion propagating from \( a' \) to \( b' \) must be both emitted and absorbed. Therefore the elastic amplitude vanishes at forward angles on the way to the pole at \( p \cdot q - M^2 + \mu^2/2 = 0 \), whereas the inelastic cross-section remains finite and large for \( \Delta = E_p - E_q > 0 \).

2. One cannot calculate the process of pion production by an incident proton beam by this mechanism, as shown in Fig. 8, because we never get near enough to the pole for the propagation of one nucleon from \( a \) to \( b \) to be justified in neglecting numerous pions also accompanying it. By experience we expect inclusion of these pions to greatly alter the strong coupling constant appearing at the vertex \( a \). On this point it is best to measure the forward angle high-energy pion flux at Berkeley, CERN or Dubna and check with the predictions of the statistical model.
3. Even at present electron machine energies of \( \approx 1 \) Bev, one can check the mechanism of Eq. (3) by looking at very high energy \( \pi \)'s in a forward cone of \( \approx 10^0 \) and thus validate or refute our approximation.*

III. KAYON, ANTINUCLEON, AND HYPERON PRODUCTION

The results of the preceding section may be transcribed directly to the process of \( K^+K^- \) pair production. In (3) and (4) it is only necessary to change the masses from pion (\( \pi \)) to kayon (\( K^- \)) and the total cross-section \( \sigma_{\pi+X} \) to \( \sigma_{K^+X} \). Since kayons are appreciably heavier than pions, up to several pions can accompany the kayon from (a) to (b) in Fig. 3 without appreciably broadening the cone of emission of the high-energy kayon. Therefore in this application the results are to be given only qualitative significance. It is interesting to observe in (3) and (4) that the ratio of high-energy \( K^+ \) to \( K^- \) intensities is just the ratio of total \( K^- \) to \( K^+ \) interaction cross sections. This prediction can also be checked experimentally. First one measures the total absorption experiments for \( K^- \) and \( K^+ \) on \( X \) at an energy \( (E - \omega) \); then one measures \( K^+ \) and \( K^- \) production at very high energy \( \omega \) and forward angles from a \( \gamma \)-ray beam incident on \( X \) at energy \( k = E \). The ratio of these measurements should check what, if any, is the quantitative significance of (3) and (4).

To calculate production of nucleon and hyperon particle and antiparticle beams by the mechanism of Fig. 2, it is necessary only to replace the boson electromagnetic vertex there by that for a Dirac particle of charge \( eA \) with \( |A| = 1 \) for charged and \( A = 0 \) for neutral particles, and of Pauli magnetic moment \( \lambda \) (\( \lambda = 1.79 \) for p, = -1.91 for n). Equation (3) then becomes

*Since this was written the July 15, 1960 Physical Review has arrived with the Cornell experiments on multiple photoproduction of mesons which is in qualitative accord with Eq. (3).

\[
\frac{d^2 \sigma_{\gamma p}(k, E_q, \theta_q)}{d\theta_q dE_q} = \frac{\alpha}{2\pi} \left[ \frac{E_q \left( k - E_q \right) dE_q}{k^3} \right] \left[ \frac{4\pi}{\sigma_{N,X(\text{total})}(k - E_q)} \right] \left\{ \frac{2 \left( (k/E_q) \right) \sin^2 \theta_q + \lambda^2 \sin^2 \theta_q}{(1 - \beta_q \cos \theta_q)^2} \right\}
\]

This is only to be viewed as a very crude qualitative result.

Since a heavy baryon is in transit between points (a) and (b) in Fig. 3, it can just as well be accompanied by pions and kaons. Neglect of these diagrams is not justified. Comparing with the statistical analysis we find by (13) for antinucleon production with \( k = 25 \text{ Bev} \), \( E_q = 20 \text{ Bev} \), and \( \theta_q = M/E_q = 3^\circ \),

\[
\left[ \sigma_{N,X(\text{total})}(5 \text{ Bev}) \right]^{-1} \frac{d^2 \sigma_{\gamma p}(25, 20, 3^\circ)}{d\theta_q dE_q} = \frac{1}{4} \times 10^{-3} \left[ \left( \frac{3}{4} \right) (a + \lambda)^2 + \lambda^2 \left( \frac{11}{8} \right) \right]
\]

\[
= \frac{1}{4} \times 10^{-2} \quad \text{for antiprotons}
\]

\[
= \frac{1}{5} \times 10^{-2} \quad \text{for antineutrons},
\]

whereas Behr and Hagedorn obtain \( \approx 10^{-4} \) for this ratio for the production of antinucleons of both charges on target nucleons. Qualitatively we predict an enhancement of \( \approx 50 \) for gammas relative to protons in high-energy antinucleon beam production. Similar results follow for other baryons with either or both charge and magnetic moment non-vanishing.

IV. CONCLUSIONS

These calculations have shown that incident \( \gamma \)-rays can be very effective as a source of secondary beams of high-energy, collimated nuclear particles. Our results are most reliable for pion production. They lead to a high-energy pion yield in the forward direction per
incident particle an order of magnitude stronger for incident $\gamma$-rays than the statistical model gives for incident protons. The predictions for production of the heavy baryons are least reliable. Neglect of heavier than one-particle states in these cases in transit between (a) and (b) in Fig. 3 is not justified, but we doubt this alters the qualitative results. Neglect of the interaction of the high-energy particle that is produced with the target nucleus may lead to an overestimate because we have neglected the possibility that it is absorbed before leaving the target nucleus. Even an order-of-magnitude reduction in our estimates from this cause leaves the cross-section for gamma production of, say, antinucleons comparable with the statistical theory predictions. Experiments which can be performed now have been proposed to test our predictions.

We conclude with two final comments about this type of approach to the pole in the inelastic process. The analysis in Eq. (13) of nucleon-antinucleon pairs can be supplemented by considering their production due to the Bethe-Heitler process. Even neglecting the anomalous moment of the proton entirely and using Eq. (7), one finds a high-energy antiproton yield of an intensity of the order of five times (for $Z = 82$) that predicted by Behr and Hagedorn for a beam of antiprotons emerging at 2 degrees and 20 Bev from an incident photon of 25 Bev on target protons. Also it seems quite feasible (according to D. Ritson) to measure the cross-section predicted for the production of two high-energy pions in the forward cone by an incident pion or gamma.\textsuperscript{7}

\begin{equation}
\frac{d^2 \sigma_{\text{pp}}}{d \omega_1 d \omega_2 \omega_1 \omega_2} \frac{16 \, d\Omega_1 \, d\Omega_2}{\pi^2 4\pi 4\pi} \frac{1}{k} \left[ A(S_1 S_2 S_3) \right]^2 \sigma_{\text{total}} \left( k - \omega_1 - \omega_2 \right)
\end{equation}

\begin{equation}
\left[ S_1 + S_2 + S_3 - 3 \mu^2 - \mu_0^2 \right]^2
\end{equation}

\textsuperscript{7} This is an extension to inelastic processes of the proposal of Goebel, Phys. Rev. Letters 1, 337 (1958), and of Chew and Low, Phys. Rev. 113, 1640 (1959).
where \( k, \omega_1, \) and \( \omega_2 \) are the energies of the incident pion or gamma and of the two emerging high-energy pions in \( d\sigma_1 \) and \( d\sigma_2 \), respectively;

\[
\mu^2 \rightarrow \text{pion} \\
\mu_0^2 = 0 \rightarrow \text{photon}
\]

\[ S_1 = (p_1 + p_2)^2, \quad S_2 = (k - p_1)^2, \quad S_3 = (k - p_2)^2 \]

are squares of invariant momentum transfers; \( A \) is the invariant amplitude at the vertex \( (a) \);

\[
(k - p_1 - p_2)^2 - \mu^2 = S_1 + S_2 + S_3 - 3\mu^2 - \mu_0^2.
\]

Since all invariant momentum transfers are of order of magnitude \( \mu^2 \) it is reasonable to make an effective range analysis of the \((\pi\pi\pi)\) or \((\gamma\pi\pi)\) interaction. For an incident pion, \( A \) is the Chew-Mandelstam scattering length in the appropriate isotopic state and perhaps \( \approx 1 \). (Similar remarks apply for two K-mesons emerging in Fig. 9.)

Very valuable discussions, especially with D. Ritson and J. Ballam, have contributed to these remarks.
1. General Considerations

As a result of calculations by Drell,\(^1\) it now appears that a high-intensity electron linear accelerator, in comparison with circular proton machines of the AGS type, will produce substantially larger-intensity beams of pions, kaons, neutrinos, and antinucleons having energies close to that of the primary energy and produced in the forward direction. In addition, the electron linear accelerator can produce a high-intensity beam of muons by direct electromagnetic production; such beams are considerably more intense than those originating from \(\pi\rightarrow\mu\) decay, and would be very useful for experiments on these particles.

The purpose of these notes is to present the results of computations based on the Drell calculations and on other mechanisms in order to compare the yields from a 25-Bev electron linear accelerator with those from an AGS proton machine operating at the same energy.

Beyond the uncertainties inherent in Drell's calculations, these computations are approximate in that they do not precisely compute the

secondary-particle yields integrated along the entire track length of a shower; rather, we compute only the yields from the first generation of $\gamma$-rays produced in fractional-radiation-length targets. This method will give quite reliable results for the secondary-particle fluxes of energy near that of the primary, but will somewhat underestimate the fluxes at lower secondary energy.

Behr and Hagedorn\(^2\) have calculated the number of particles per unit momentum per unit solid angle produced when a 25-Bev proton hits a hydrogen target. Preliminary measurements on the PS machine at CERN have substantiated these calculations for pions within an order of magnitude. These will therefore be used as the basis of comparison for pion production.

Drell's\(^1\) calculations are based primarily on the process described by the following diagram:

Strongly interacting particles X having charge or magnetic moment or both are pair-produced at A and emerge with a characteristic angle $\theta \sim M_X/E_X$, where $M_X$ is the mass of X and $E_X$ is its energy. One of the pair fragments interacts with a nucleon at B. The vertex A is calculable, and at B the X-nucleon total experimental cross-section is used since the incident X-particle is almost real. The

\(^2\)Behr and Hagedorn, CERN Report No. 60-20 (1960).
other member of the pair then comes off at a very small angle with an
energy restricted by the theory to be between $k$ and $k/2$, where $k$
is the momentum of the incoming $\gamma$-ray.

The secondary-particle-yields from the electron linear accelerator
are mostly in the forward angles, from roughly $6^\circ$ to $\frac{\pi}{2}^\circ$, depending
on the mass. This has the disadvantage that separation from the main
electron beam is difficult, and the advantage that almost the entire
yield is available for many experiments. Furthermore, the ratio of
usable directed secondary beam to total radiation and neutron yield is
improved.

2. Formulas Used in Computations

The results of Drell's calculations are given here as yields of
the number of secondary particles per Bev per steradian produced per
incident $\gamma$-ray. In all formulas, $\hbar = c = 1$.

A. Charged spin-zero particles

For $\pi$ production,

\[
\frac{d^2 \sigma(k, \omega, \theta)}{d\Omega d\omega} = \left( \frac{\alpha \omega(k - \omega)}{2\pi \frac{4\pi k}{3} \left( \frac{\mu}{\omega} \right)^2 + \frac{\theta^2}{2}} \right) \frac{4\theta^2}{\sigma_{\pi+p, \text{total}}(k - \omega)} ,
\]

where $k$ is the energy of the $\gamma$-ray, $\omega$ is the energy of the emerging
fast pion, $\mu$ is the pion rest mass, $\theta$ is the angle which the fast
pion makes with the $\gamma$-ray, and $\alpha = 1/137$.

Equation (1) can be immediately used for the production of $K^-$
and $K^+$ by substituting the $K$ rest mass for the pion rest mass and
$\sigma_{K+p}$ for $\sigma_{\pi+p}$.
B. Antiproton production

\[ \frac{d^2\sigma_{\mu}(k,E,\theta)}{d\Omega \, dE} = \frac{\alpha}{2\pi} \left( \frac{2\pi}{4\pi} \right)^2 \frac{1}{(2\pi)^2} \frac{1}{4\pi} \frac{\sigma_{p+P, \text{total}}(k-E)}{(M/E)^4} \]

\[ \times \left\{ \frac{(2-(k/E))(1-1.79)^2(M/E)^2 + (-1.79)^2(M/E)^2 [2-(kE/2M^2)(M/E)^2]}{(M/E)^4} \right\} \]

where \( E \) is the energy of the fast antiproton, \( k \) is the energy of the \( \gamma \)-ray, \( M \) is the antiproton rest mass, \((-1)\) is the charge of the antiproton, and \((-1.79)\) is the anomalous magnetic moment of the antiproton.

Note: Equations (1) and (2) are valid only in the regions \( \omega > k/2 \) and \( E > k/2 \), respectively. In the calculations done here, it has been assumed that \( \sigma_{p+P}(k-E) = \text{const} \approx 30 \text{ mb} \), for \( 5 \text{ Bev} < (k-\omega) < 25 \text{ Bev} \).

C. Muon production

For muon production, both vertices are electromagnetic, and therefore the usual Bethe-Heitler formulas apply:

\[ \frac{d^2\sigma_{\mu}(k,E,\theta)}{d\Omega \, dE} = \frac{\alpha}{2\pi} \left( \frac{2\pi}{4\pi} \right)^2 \frac{1}{4\pi} \]

\[ \times \left\{ \frac{[k^2/E(k-E)]([\mu/E]^2 + \theta^2]^2 + ([\mu/E]^4 + \theta^4]}{([\mu/E]^2 + \theta^2)^4/16} \right\} \]

\[ \times \frac{2\pi}{E^2} \left[ \frac{E(k-E)}{k^3} \right] \left( \frac{\theta}{\mu} \right) \]

For the mean angle \( \theta = \mu/E \) this reduces to

\[ \frac{d^2\sigma_{\mu}(k,E,\theta)}{d\Omega \, dE} = \frac{\alpha}{2\pi} \left( \frac{2\pi}{4\pi} \right)^2 \frac{1}{4\pi} \frac{1}{E^2} \left[ \frac{E(k-E)}{k^3} \right] \left( \frac{\theta}{\mu} \right) \]
where $Z$ is the atomic number of the target nucleus, $\mu$ is the rest energy of the muon, $E$ is the energy of the fast muon, and $k$ is the energy of the $\gamma$-ray.

If the anomalous magnetic moments are ignored, then Eq. (4) also applies to the ordinary electromagnetic pair production of proton-antiproton pairs with the nucleon mass $M$ substituted for $\mu$.

D. Photon spectrum

In order to calculate the yield of particles for an incident electron beam, it is necessary to use an appropriate bremsstrahlung spectrum due to the electron. We compute this by using a simple approximation to the first-generation $\gamma$-ray spectrum.

If $E_0$ is the energy of the incident electron, then the number $S$ of electrons of energy $(E_0 - E)$ in the interval $dE$ remaining after the initial beam has penetrated $t$ radiation lengths can be approximated by

$$S(E, E_0) \, dE = N/(E_0 - E)^{1-t} \, dE, \text{ for } t < 1 , \quad (5)$$

where $N$ is a normalizing factor.

Normalizing to unit incident beam,

$$1 = N \int_0^{E_0} \frac{dE}{(E_0 - E)^{-t}} = \frac{N \, E_0^t}{t} ; \quad (6)$$

therefore,

$$S(E, E_0) \, dE = \frac{[1 - (E/E_0)]^t}{-\frac{1}{E_0} (E - E_0)} \, dE ,$$

or

$$S(\varepsilon) \, d\varepsilon = \frac{t \, d\varepsilon}{1 - \varepsilon} (1 - \varepsilon)^{t} ,$$
where
\[ E = \frac{E}{E_0}. \]

The γ-ray spectrum \( N_\gamma(\epsilon, k', t) \) produced at a depth \( t \) by an electron of energy \( \epsilon \) then becomes (letting \( k' = k/E_0 \)),

\[
\frac{d^3N}{dt \, d\epsilon} = \begin{cases} 
S(\epsilon) \frac{dk'}{k} & \text{for } k' < \epsilon, \\
0 & \text{for } k' > \epsilon.
\end{cases}
\]

Integrating over \( \epsilon \) from \( \epsilon = k' \) to \( \epsilon = 1 \), and over \( t \) from \( t = 0 \) to \( t = t \), the photon spectrum becomes

\[
\frac{dN(k')}{dk'} = \frac{1}{k} \left[ \frac{1}{-\ln(1 - k')} \right]^{t} \quad \text{(7)}
\]

The final cross section \( \Sigma \) per incident electron is then obtained by numerical integration, giving

\[
\Sigma = \int_{\omega/E_0}^{2\omega/E_0} \frac{d\sigma(E, k, \theta)}{d\omega} \frac{dN(k')}{dk'} \, d\omega, \quad \text{for } (2\omega/E_0) < 1. \quad \text{(8)}
\]

For \( (2\omega/E_0) > 1 \), the limits are between \( k' = \omega/E_0 \) and \( k' = 1 \), where \( \omega \) is the energy of the outgoing pion or antiproton.

Since all the photons and electrons considered in these calculations are high energy, the pair-production and bremsstrahlung mean free paths are considered constant for a given \( Z \).

3. Specific Assumptions Made in Computations

A. Assumptions made in calculating the pion production from Eq.(1)

1. The total pion-nucleon cross section remains constant at 30 mb for pion energies between 2.5 and 25 Bev.
2. The yield is calculated from hydrogen, although in practice a low-Z target would be used.

3. Production is calculated for a target of \( \frac{1}{2} \) radiation length, even though a thicker target might actually be used. The yield is calculated by using the photon spectrum as calculated from Eq. (7) to \( t = \frac{1}{2} \), but using \( t/4 \) as the thickness in which pion production occurs. This will introduce a small error in the direction of underestimating the high-energy yield.

4. Proper edge targeting which takes advantage of the small beam diameter of the accelerator is assumed to avoid nuclear absorption of the produced pions in the target.

5. The yields at low energies are underestimated because of the nature of the photon spectrum used in these calculations.

6. For purposes of comparison with the Hagedorn statistical model of pion production at \( 1^0 \) in proton-proton collisions, the pion yield from electrons is calculated for \( \theta = 1^0 \). The mean angle for pions is actually \( \mu/\omega \) which varies from \( 1/3 \) to \( 2/3 \) from \( \omega = 25 \) to \( 12.5 \) Bev. A calculation using the mean angle would then give a yield per electron roughly a factor of 4 times the yield at \( 1^0 \) at these energies.

Figures 1 and 2 show the results of these calculations. Figure 1 compares the yield per electron with that per proton of the statistical model, while Fig. 2 compares the total yield per second of a linear accelerator producing \( 3.6 \times 10^{14} \) electrons per second (60 \( \mu \)A) with an existing proton accelerator producing \( 10^{11} \) protons/second (CERN PS machine), both at 25 Bev.

B. Assumptions made in calculating the antiproton production from Eq. (2).

1. Assumptions 1, 2, 3, 4, and 5 considered above for the pions are also used here.

2. The calculation was done for \( \theta = M/E \), which averages about \( 3^0 \) for the high energies, and this is compared with Hagedorn's calculations at \( 3^0 \).

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Figure 1:

PION YIELD PER 25 BEV PARTICLE

NO. OF $\pi$'S / INCIDENT PARTICLE STER$^{-1}$ BEV$^{-1}$

BY PROTONS $\theta=1^\circ$

BY ELECTRONS $\theta=1^\circ$
(DRELL CALC.)

BY ELECTRONS $\theta=0^\circ$
(PHOTO-PRODUCTION)
(THICK TARGET)

ENERGY (BEV) OF PRODUCED $\pi$ MESON

$10^0$ $10^1$ $10^2$ $10^3$ $10^4$ $10^5$ $10^6$ $10^7$ $10^8$

0 5 10 15 20 25 30

FIG. 1
PIGION YIELDS / SEC. FOR 25 BEV PARTICLES

BY ELECTRONS
(3.6 x 10^{14} / SEC.)
\( \theta = 1^\circ \)

BY PROTONS
(10^{11} / SEC.)
\( \theta = 1^\circ \)

ENERGY (BEV) OF PRODUCED \( \pi \) MESON

FIG. 2
3. The Hagedorn calculations are normalized to experimental results at CERN in which the antiproton-to-pion production ratio is about 1% at 15 Bev. The shape of the distribution was retained, however.

4. It is necessary to point out that Eq. (2) is essentially an upper limit for this process, since a diagram in which several pions accompany the interacting proton will not materially change the angle of emission of the fast proton and is likely to reduce the total yield by interference. This effect has not been estimated. However, this uncertainty is not significant for the pion calculations.

The results are shown in Figs. 3 and 4 for yield per particle and yield per second, respectively.

C. Assumptions made in the calculation of the spectra of muons, pions and antiprotons from Eq. (4).

1. The angle of emission is \(M/E\), where \(M\) is the mass of the emitted particle and \(E\) is its energy.

2. The target is one-half radiation length of Be (40 g/cm\(^2\)), considered to be a point charge of magnitude \(Ze\). (This is fully justified since the momentum transfers to the nucleus are small.)

3. There is no attenuation due to nuclear interactions in the target for the pions and antiprotons.

4. In the pion case, the number has been reduced from that of the muons by a factor \((1/7)(m_\mu/m_\pi)^2\). The factor of \(1/7\) is due to the spin-zero character of the \(\pi\) meson.

5. In the antiproton case, no account has been taken of the anomalous magnetic moment.

The results of the calculations using Eq. (4) are shown in Fig. 5.

A general remark applying to all the processes calculated in Section 3 is that the yields are for one emerging particle, not for a pair.
\( \bar{p} \text{ YIELDS PER 25 BEV PARTICLE} \)

**Figure 3**

Energy (BEV) of Produced \( \bar{p} \)

- No. of \( \bar{p} \)'s / Incident Particle ster. BEV
- By Protons \( \theta = 3^\circ \)
- By Electrons \( \theta = \frac{M}{E} \)
\( \bar{p} \) YIELDS / SEC. FOR 25 BEV PARTICLES

NO. \( \bar{p} \)'S / SEC. STER. \( \text{BEV}^{-1} \)

BY ELECTRONS
\( 3.6 \times 10^{14} / \text{SEC.} \)
\[ \theta = \frac{M}{E} \]

BY PROTONS
\( 10^{11} / \text{SEC.} \)
\[ \theta = 3^\circ \]

ENERGY (BEV) OF PRODUCED \( \bar{p} \)

FIG. 4
YIELD PER 25 BEV ELECTRON VIA ORDINARY PAIR PRODUCTION

\[ \theta = \frac{M}{E} \]

\[ \mu \]
\[ \pi \]
\[ \bar{\rho} \]

NO. OF PARTICLES/INCIDENT ELECTRON STER. \per BEV

ENERGY OF PARTICLES (BEV)

FIG. 5
4. Previous Calculations of Pion Yields from High-Energy Photoproduction

Calculations of secondary pion yields produced in an electron-induced shower have been made previously by Schiff\(^3\) and Dedrick.\(^4\) Schiff based his estimate on the product integral over energy of the \(\gamma\)-ray track length times an arbitrary power-law energy dependence of a photopion production cross section; no theoretical model of this cross-section dependence was assumed. The result gave a ratio of pion yields for electrons to protons in beryllium of about \(3.4 \times 10^{-3}\) for all pion energies. In the light of the previous discussion this is clearly a very substantial underestimate for large pion energies, but is probably a good estimate in the 1 - 3 Bev region.

The calculations of Dedrick\(^4\) are based on the assumption that photons in a shower are absorbed by nucleons with a cross-section which varies as the inverse square of the photon energy in the nucleon-photon center of mass. Other power laws were also computed with comparable results. Disintegration of the target nucleus follows photon absorption with the distribution of fragments governed by the statistical model. The yields computed are given in Fig. 6 for comparison with the results of the calculations based on the specific pair-production diagrams. These yields are computed for an infinitely thick target in which the full track length of the shower can be developed and are computed for \(0^\circ\) pion angle. It is seen that, similar to the estimates of Schiff,\(^3\) the yields per electron fall two to three orders of magnitude below the yields per proton according to the statistical model calculations of Hagedorn and Behr.\(^2\)

5. Gamma-Ray Yields from Bremsstrahlung

It is possible to compare the gamma yield from a linear accelerator with that from \(\pi^0\)'s made in a proton accelerator. For the latter we use the Hagedorn and Behr calculation for a 25-Bev machine. For the

\(^3\)L. I. Schiff, Stanford Preprint No. M-158 (1960).

\(^4\)K. Dedrick, private communication. The results of Dedrick have been outlined by Panofsky in M-157 (1960).
γ YIELDS PER 25 BEV PARTICLE

BY ELECTRONS
θ = 0°

BY PROTONS
θ = 1°

FIG. 6
former we use Eq. (7) which gives the gamma spectrum for bremsstrahlung from a fairly thin target. The angle of emission of the gammas is given by the mean scattering, which for high energies is

\[ \theta = \left( \frac{0.021}{E} \right) t^{\frac{1}{2}} \]

where \( E \) is the energy of the electron in Bev, and \( t \) is the number of radiation lengths. For \( t = \frac{1}{2} \) and \( E = 25 \) Bev, \( \theta \) is \( 6 \times 10^{-4} \) radians, and the corresponding solid angle becomes \( 1.1 \times 10^{-6} \) steradians. In figures 6 and 7 are plotted the relative \( \gamma \)-ray yields.

6. Summary and Conclusions

1. Secondary yields per particle of pions and kaons of energy larger than about one half of the energy of an incident electron beam are comparable to the yields due to an incident proton beam.

2. Secondary yields per particle of antiprotons of high energy are one or two orders of magnitude larger for electrons than for protons.

3. Secondary yields per particle of \( \mu \)-mesons of high energy are three to four orders of magnitude larger for electrons than for protons.

4. Secondary yields per particle of secondary pions and antiprotons in the lower energy (1 - 3 Bev) range are two to three orders of magnitude smaller for electrons than for protons.

5. The proposed Stanford linear electron accelerator has a particle flux of about \( 4 \times 10^3 \) times that of the CERN proton synchrotron and comparable to that considered in FFAG studies. The above yields should be considered in this light.

ACKNOWLEDGMENT

The writer is indebted to S. Drell, W. K. H. Panofsky, and D. Ritson for helpful discussions, and to Miss Eleanor River for aid in the computations.
\( \gamma \) YIELDS / SEC. FOR 25 BEV PARTICLES

\begin{figure}
\centering
\includegraphics[width=\textwidth]{gamma-yields-25-bev-particles.png}
\caption{\( \gamma \) Yields for 25 BEV Particles}
\end{figure}

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The yields of secondary particles (pions, kaons, antinucleons, and hyperons) predicted by Drell's theory\textsuperscript{1} and calculated by Ballam\textsuperscript{2} for a 25-Bev machine are such that some bubble-chamber experiments become possible without the necessity of using complicated magnetic beam optics.

In this note we shall discuss experimental setups for neutral hyperon and pion beams, to be used with a 6-ft hydrogen bubble chamber operating in a field of 20 kilogauss. The yields of hyperons used in these discussions are optimistic in that the theory\textsuperscript{1} does not include all possible diagrams and assumes that cross-sections such as $\Sigma^-$-N remain constant at 30 mb for $\Sigma^-$ energies between 5 and 25 Bev.

We assume that the accelerator delivers $10^{12}$ electrons per pulse over 1 $\mu$sec, and that the chamber operates once every 3 sec.

1. **20-Bev Neutral Hyperon and Antihyperon Beams (Fig. 1)**

To calculate the number of 20-Bev particles going through the chamber per pulse, we consider the following:

(a) The mean hyperon decay distance is 6 ft.

(b) Initial flux of particles: There are \(3 \times 10^8 \frac{\text{m}^2}{\text{ster} \cdot \text{Bev}}\) per pulse per steradian per Bev produced by the full-intensity electron beam in a Be target \(\frac{1}{2}\) radiation length in thickness (40 g/cm\(^2\)), leaving the target with a mean angle of 3° with respect to the electron beam. Since it will turn out that this is too small an angle for a practical experiment, we will take the flux at 7°, which will reduce the number by roughly 5\(^1\)

(c) Beam setup: It is important to keep the chamber as close to the target as possible in order to keep the ratio of high-energy neutrons to hyperons reasonable. It is therefore proposed to place a clearing magnet of aperture one inch by one inch and length 9 ft at a distance of 2\(\frac{1}{2}\) ft from the target and at an angle of 7° to the electron beam. With a 20-kg field, the highest energy particles will strike the vertical walls of the gap less than half way up the magnet. The rest of the magnet is used to bend scattered charged particles out of the way. The solid angle is then determined by the exit opening of the magnet channel and is about \(5 \times 10^{-5}\) ster. The main electron beam, plus high-energy muons and pions (mean angle of 5-Bev muons is 1.5°), will continue forward and eventually be dumped, and the lower energy muons will be stopped in the shielding.

(d) Momentum resolution: We will calculate the rate for a 1-Bev momentum spread (5%) since the particles will have a mean momentum determined (for small angles) by the angle of emission. The channel restricts this angle to 7° ± 0.3° and the mean momentum to roughly ± 5%. A certain gain in momentum resolution is achieved by the decay of the slower \(\pi^+\). For example, at 7° the most favorable energy for the \(\pi^+\) is 10 Bev. These have a factor 2 less time dilation than at 20 Bev and have a mean decay distance of 3 feet. Their attenuation is a factor 20 greater than for a 20-Bev particle.
(e) Attenuation of hyperons due to decay: Taking a distance of 19 ft from target to chamber center (7 ft from the end of the clearing magnet to the center of the chamber), we have a total attenuation of 24.

Thus the number \( N \) of hyperons \((\Lambda_c, \Sigma_c, \Xi^0, \Xi^\pm)\) through the chamber per pulse is

\[
N = 3 \times 10^8 \times (1/5.5) \times 5 \times 10^{-5} \times (1/24) \approx 100 .
\]

The flux of \( n \) and \( \bar{n} \) at the same energy will be 24 times greater. However, the number of neutrons at a mean energy corresponding to 7° (i.e., 8 Bev) will be roughly 125 times greater.

Assuming that a useful yield of particles is one per picture through the chamber, we can reduce the intensity by a factor of 100; this leaves per picture:

- 1 \( \Xi^0 + \Xi^\pm \), \( \bar{E} = 20 \text{ Bev} \)
- \( 1/2 \) \( \Lambda_c + \Sigma_c \), \( \bar{E} = 17 \text{ Bev} \)
- 20 \( n^0 + n^\pm \), \( \bar{E} = 14 \text{ Bev} \)
- 100 \( n^0 + n^\pm \), \( \bar{E} = 8 \text{ Bev} \)

A 6-ft hydrogen chamber is about 1/5 of a mean free path \( (\sigma = 30 \text{ mb}) \). This gives about 12 neutron interactions per picture, and one antihyperon interaction every 10 pictures. This is on the limit of scanning, but it should be pointed out that, especially in the case of \( \Xi^0 \) interactions, a stringent selection is made in regard to conservation of strangeness. Furthermore, for all anti-particle events the baryon number is zero. For example, unique interactions with \( \Xi^0 \) are:

\[
\Xi^0 + p \begin{cases} 
\rightarrow D^+ \text{ (if it exists)} & (1) \\
\rightarrow \phi^0 + K^+ & (2) \\
\rightarrow \Lambda_c + p + \phi^0 & (3)
\end{cases}
\]
and with \( \Xi^0 \) are:

\[
\Xi^0 + p \rightarrow \Lambda^c + \Sigma^+ \quad (4)
\]
\[
\rightarrow \Sigma^0 + \Sigma^+ \quad (5)
\]
\[
\rightarrow \theta^0 + \theta^0 + n + p \quad (6)
\]
\[
\rightarrow \Lambda^c + p + \theta^0 \quad (7)
\]
\[
\rightarrow D^- + p + p . \quad (8)
\]

The success of such experiments depends to a large degree on the ability to measure momenta and angles in the laboratory. Assuming that in the two-body final states like (2), (4) and (5) above, the energy is shared, we have to discuss the feasibility of measuring 10-Bev momenta in 20-kg fields in a 6-ft chamber. If the track points can be located to 0.2 mm, then the measurement error is about one part in 30 assuming an average track length of one meter. It thus appears reasonable to have 10\% or better measurements of 10-Bev particles. Combined with a 1\% error in angle measurements, distinctions between \( \Xi^0, \Lambda^c \), and \( \theta^0 \) becomes fairly routine.

Knock-on protons from low-energy neutrons made in the target and coming down the channel will be a large part of the background.

To estimate this effect we assume all neutrons come from the giant resonance at about 20 Mev \( \gamma \)-ray energy. Assuming an integrated \((\gamma,n)\) cross-section of 40 Mev mb and a \(dk/k\) bremsstrahlung spectrum produced by 25-Bev electrons, the neutron production in \( \frac{1}{2} \) radiation length of beryllium is \( 10^{-3} \) neutrons per electron. At the full intensity of the accelerator, 5000 neutrons come through the \( 5 \times 10^{-5} \) steradian channel per pulse, assuming an isotropic neutron distribution. At 1\% of full intensity this background should be tolerable, provided that about 2 mean free paths of boron are placed in the beam hole.

The cross section of boron for giant resonance (1-2 Mev) neutrons is about 3 barns. The cross section for fast neutral hyperons' is \( 9 \times 30 \) mb per nucleus, or about 10 times less. Thus, while the giant resonance neutrons are attenuated by a factor of 8, the hyperons are attenuated by about 20\%.
An alternate scheme to reduce the slow neutron background is to produce bremsstrahlung in a $\frac{1}{2}$ radiation length target. After a sweeping magnet to remove the electrons, a long beam hardener of low-Z material is used to reduce the number of 20-Mev $\gamma$ rays relative to the number of multi-Bev $\gamma$ rays. For example, the attenuation coefficient for 20-Mev $\gamma$ rays in hydrogen is $0.021 \text{ cm}^2/\text{gm}$, while at very high energies it is $0.0091 \text{ cm}^2/\text{gm}$. Thus $500 \text{ gm/cm}^2$ of liquid hydrogen attenuate the multi-Bev $\gamma$ rays by a factor of 100, while attenuating the 20 Mev $\gamma$ rays by a factor of $4.5 \times 10^{-5}$. The slow neutron background in the bubble chamber is thus reduced to less than one per pulse. Clearly 500 $\text{gm/cm}^2$ of hydrogen is impractical, but lithium hydride or lithium metal would be just about as good. Higher Z materials would not be useful.

It should be understood that the discussion above represents feasibility arguments, indulged in mainly because of the unique character of such experiments.

2. Experiments with Charged Pions (Fig. 2)

The intensity of pions at the mean angle of $0.4^\circ$ is $4 \times 10^{10}$ per burst at 20 Bev. This means that one could go to $3^\circ$ and the intensity would be down by something like 100. However, the attenuation caused by pion decay is negligible. It thus appears feasible to place a bending magnet some 40 ft down from a $1/4$-radiation-length target and at an angle of $3^\circ$ with respect to the electron-beam direction. With a 20-kg, 17-ft magnet, this would give a lateral bend of $10^\circ$ for 20-Bev particles. The magnet gap should be 1-in. high and 9-in. wide. The dispersion is 1% for $0.1^\circ$. For the chamber to intercept $0.1^\circ$ laterally, it has to be (for a 2-ft-wide chamber) 600 feet away. Thus the solid angle intercepted by the chamber is about $10^{-5}$. Therefore the rate of pions in the chamber is

$$4 \times 10^{10} \times \frac{1}{100} \times 10^{-5} \times 0.25 = 10^3 \text{ pions/pulse}$$

into the chamber having a momentum of $20 \pm 0.2$ Bev for a full-intensity electron beam.
FIG. 2--20 Bev pion bubble chamber experiment.
Assuming 20 particles per pulse as a useful number, and a background of muons equal to that of pions, the accelerator could be operated at $1/50$ of full intensity. A sketch of the experimental layout is shown in Fig. 2. With 660 ft between chamber and target, the shielding problem is not difficult.
Electron-proton scattering at multi-Bev energies has been discussed in a recent paper by Cassels. It is pointed out that reasonable counting rates can be expected for both elastic and inelastic scattering with up to 10-Bev incident electrons if a spectrometer with about $5 \times 10^{-3}$ sr solid angle is employed.

We have designed a spectrometer with solid angle of $5 \times 10^{-3}$ sr for energies up to 10 Bev. The instrument is based on the design principles discussed in our recent paper on large-solid-angle spectrometers. The spectrometer consists of two conventional quadrupole magnets, two Panofsky-type quadrupoles, and a deflecting magnet, as shown in Fig. 1. Detectors are located at the unity-magnification image of the scattering target. The dispersion of the system is more than 1% per inch. The obtainable energy resolution depends upon the aberrations of the system. Assuming a circle of confusion of 1-in. radius at the detector, the resolution is 2% (full-width at half-maximum). If this resolution is not sufficient, it can be improved either by increasing


3. This is based on the qualitative estimate that the aberration at the image is about 10% of the magnet aperture size.
the length of the deflecting magnet, or by reducing the solid angle (thus reducing the aberrations). It can be observed in Fig. 1 that at scattering angles in the 10°-30° range, the unscattered electrons may strike the coils or return yoke of the first magnet. We believe this is a minor problem which can be solved by bending the coils slightly and boring small holes (1-in. diameter) in the iron.

The parameters of the spectrometer, in the notation of reference 2, are:

\[ \alpha_1 l_1 = 1.8, \quad \alpha_2 l_1 = 1.67 \]

\[ l_1 = 16 \text{ ft.}, \quad l_2 = 8 \text{ ft. (Class IIIb)} \]

\[ B_1 = 10 \text{ kg at } \alpha_1 = 8.65 \text{ in.} \]

\[ a_2^- = 13.1 \text{ in.}, \quad a_2^+ = 7.3 \text{ in.} \]

\[ \theta_+ = 0.0766 \text{ rad}, \quad \theta_- = 0.0211 \text{ rad} \]

\[ d_0 = 37 \text{ in.}, \quad F_+ = 70 \text{ in.}, \quad F_- = 643 \text{ in.} \]

\[ \beta = 5 \times 10^{-3} \text{ sr} \]

The actual magnets are designed with apertures somewhat larger than the values listed above. Actual dimensions of the magnets are listed in Table I. Rough cost estimates are given in Table II. The conclusion is that a 5 \times 10^{-3} \text{ sr 10-Bev spectrometer with a resolution of about 2\% can be built for about $500,000 and will consume about 4 Mw of electrical power.}
FIG. 1--10 Bev electron scattering spectrometer.
### TABLE I. Magnet parameters. The values are not necessarily optimized with respect to cost and power consumption.

<table>
<thead>
<tr>
<th></th>
<th>Conventional quadrupole (2 each)</th>
<th>Panofsky-type quadrupole (2 each)</th>
<th>Deflecting magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall length</td>
<td>16 ft</td>
<td>8 ft</td>
<td>10 ft</td>
</tr>
<tr>
<td>Pole length</td>
<td>14.5 ft</td>
<td>6.75 ft</td>
<td>9 ft</td>
</tr>
<tr>
<td>External cross-section dimensions</td>
<td>4 ft x 4 ft</td>
<td>71 in. x 42.5 in.</td>
<td>98 in. x 56 in.</td>
</tr>
<tr>
<td>Aperture dimensions</td>
<td>18-in. diam.</td>
<td>30 in. x 15 in.</td>
<td>34 in. x 12 in.</td>
</tr>
<tr>
<td>Magnetic field at poletips</td>
<td>10.4 kg</td>
<td>16.7 kg</td>
<td>18.0 kg</td>
</tr>
<tr>
<td>Power consumption</td>
<td>500 kw</td>
<td>1,260 kw</td>
<td>790 kw</td>
</tr>
<tr>
<td>Iron weight</td>
<td>79,000 lb</td>
<td>42,000 lb</td>
<td>146,000 lb</td>
</tr>
<tr>
<td>Copper weight</td>
<td>12,700 lb</td>
<td>16,000 lb</td>
<td>8,900 lb</td>
</tr>
<tr>
<td>Total weight</td>
<td>46 ton</td>
<td>29 ton</td>
<td>77 ton</td>
</tr>
</tbody>
</table>
TABLE II. Cost estimate. These are crude values based on $0.50/lb for iron and $4.50/lb for copper. The total power consumption of the system is 4350 kw.

<table>
<thead>
<tr>
<th></th>
<th>Conventional quadrupole</th>
<th>Panofsky-type quadrupole</th>
<th>Deflecting magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>iron</td>
<td>iron</td>
<td>iron</td>
</tr>
<tr>
<td></td>
<td>$ 39,500</td>
<td>21,000</td>
<td>73,000</td>
</tr>
<tr>
<td></td>
<td>57,000</td>
<td>72,000</td>
<td>40,000</td>
</tr>
<tr>
<td></td>
<td>$ 96,500</td>
<td>93,000</td>
<td>113,000</td>
</tr>
<tr>
<td></td>
<td>x 2 = $193,000</td>
<td>x 2 = 186,000</td>
<td>113,000</td>
</tr>
</tbody>
</table>

Subtotal - Magnet costs   492,000

Power - 1000 hr/yr at $0.01/kwh, 5 yrs 218,000

Total - $ 710,000
Recent calculations\textsuperscript{1,2} show that copious beams of high-energy pions can be produced by electrons. In particular, the yield of 20-Bev pions produced by 25-Bev electrons incident on a 1/3-radiation-length target is \(6.9 \times 10^{-4}\) pion per Bev per steradian per electron. The characteristic production angle is \(\theta_1 = \frac{m_\pi}{E_\pi} \approx 7 \times 10^{-3}\) radian. At angles larger than this the yield per steradian falls off as \((\theta_1/\theta)^2\).

To obtain good secondary pion beams we want a magnet system with an acceptance solid angle much greater than \(\pi \theta_2^2 = 1.5 \times 10^{-4}\) sr, and a momentum resolution of a few percent. A major difficulty is the fact that the pions are in a very sharp forward cone around the electron beam. Since the electron beam has been degraded in energy in the target, we should stop the electrons (and as many of the \(\mu\)-mesons as possible) before allowing the beam to enter a magnetic field. If we do not follow this procedure, the unwanted beam will be dispersed in the field, making the shielding problem very much more difficult.

A solution to this problem is to use a spectrometer centered on the beam direction. The angular spread of the electron beam due to multiple scattering is given by

\begin{enumerate}
\end{enumerate}
for \( E_e = 25 \text{ Bev} \) and \( t = 1/3 \) radiation length. We can thus put a plug on the spectrometer axis which will stop the electron beam and have very little effect on the pion beam. We have previously shown that we can stop the electron beam effectively in a stopper composed of 1/16-in. copper plates separated by 1/8-in. water spaces. A beam-stopper of this type, 4-ft long, contains 30 radiation lengths, thus completely stopping the forward-going electromagnetic shower. The diameter of the beam-stopper is determined by the following criteria: It should subtend an angle much greater than \( 0.5 \times 10^{-3} \text{ rad} \), but much less than \( 7 \times 10^{-3} \text{ rad} \) at the primary target. It should be reasonably large in diameter because of construction problems (particularly the need for about 1000 gal/min water flow), but fairly close to the target in order that the spectrometer not be too far away. A beam-stopper diameter of 1 in., with the front end 15-ft from the target, seems to be a reasonable compromise. In this configuration, the half-angle subtended at the target is \( \theta_0 = 0.5/(15 \times 12) = 2.8 \times 10^{-3} \text{ rad} \). The stopper must be supported, and cooling water must be supplied. Clearly, this should be done in a way which results in subtending the smallest possible solid angle at the target. We will not consider the details of the beam-stopper here.

The flux of 20-Bev \( \mu \)-mesons generated in the beam stopper will be comparable to the flux of pions coming from the target. To reduce this background we can fill the space directly behind the beam-stopper with absorber. A 12-ft long copper absorber tapered from 1-in. diameter at the front to 2-in. diameter at the back will not absorb any pions coming from the target, because it is in the shadow of the beam-stopper, but it represents 6 Bev of energy loss to the \( \mu \)'s coming from the beam-stopper. Since the \( \mu \) cross section drops quite abruptly above 24 Bev (about 1-Bev below the initial electron energy), any \( \mu \)'s penetrating the absorber will have less than 18 Bev remaining and can be easily separated from the 20-Bev pions by momentum analysis. The back end of the \( \mu \) absorber subtends a half-angle of \( 5.2 \times 10^{-3} \text{ rad} \) at the beam-stopper. Since the characteristic production angle \( m_\mu / E_\mu \) of 20-Bev mesons is nearly equal to this angle, this absorber is really quite effective in reducing the \( \mu \)-meson background.
We have designed a magnet system with a solid angle acceptance of $10^{-3}$ sr for a 20-Bev/c pion beam. The design is based on the principles set forth in our paper on large-solid-angle spectrometers \cite{penner1954} with the following additional considerations: (1) The source-to-first-magnet distance must be quite large in order to leave space for the beam-stopper and $\mu$-meson absorber; and (2) other factors being equal, we are to prefer the system with the most nearly equal acceptance angles in the two transverse directions, because the pion flux falls off as the inverse square of the angle to the axis. These considerations imply that we cannot use an optimized design, in the sense of reference \cite{penner1954}.

The spectrometer we have designed is a unity-magnification, symmetric conventional quadrupole quartet with a deflecting magnet in the center. In the notation of reference \cite{penner1954}, the quadrupole parameters are:

\begin{align*}
\alpha_1 l_1 &= 1.0, & \alpha_2 l_2 &= 0.836 \text{ (Class IIa)} \\
l_1 = l_2 &= 12 \text{ ft}, & d_0 &= 23 \text{ ft} \\
B_m &= 10 \text{ kg at } a_1 = 7.9 \text{ in. and at } a_2 = 11.3 \text{ in.} \\
\theta_+ &= 0.0254 \text{ rad}, & \theta_- &= 0.0126 \text{ rad} \\
F_+ &= 16.5 \text{ ft}, & F_- &= 71.8 \text{ ft} \\
\Omega &= 1.01 \times 10^{-3} \text{ sr for 20 Bev/c momentum}
\end{align*}

We have made a crude design and cost estimate for a set of magnets to provide the above-listed parameters. The magnet specifications are listed in Table I. No particular attempt was made to optimize the design with respect to power and cost: the numbers listed in Table I are intended only as a first estimate of space and power requirements.

A deflecting magnet is placed in the center of the quadrupole system. A 10-ft long magnet provides a deflection of $4.7^\circ$ at a field of 18 kg. A dispersion of 1.4% per inch is obtained at the focal spot. Since we may guess at a minimum 1-in. circle of confusion at the focal point (due entirely to aberrations), we can expect a pion energy resolution of about 3%. The specifications of a deflecting magnet designed to do this job are also given in Table I. The gap dimensions

\begin{itemize}
\item \cite{penner1954} S. Penner, "Quadrupole focusing systems for very large momenta," Hansen Laboratories Report No. M-200-12.
\end{itemize}
TABLE I. Specifications of magnets. The values are first estimates. No attempt has been made to optimize with respect to cost and power consumption.

<table>
<thead>
<tr>
<th></th>
<th>First quadrupole</th>
<th>Second quadrupole</th>
<th>Deflecting magnet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall length</td>
<td>12 ft</td>
<td>12 ft</td>
<td>10 ft</td>
</tr>
<tr>
<td>Pole length</td>
<td>10.5 ft</td>
<td>10 ft</td>
<td>9 ft</td>
</tr>
<tr>
<td>External cross-section dimensions</td>
<td>4 x 4 ft</td>
<td>5 x 5 ft</td>
<td>50 x 85 in.</td>
</tr>
<tr>
<td>Aperture dimensions</td>
<td>18-in. diam.</td>
<td>24-in. diam.</td>
<td>12 x 28 in.</td>
</tr>
<tr>
<td>Magnetic field at poletip</td>
<td>11.9 kg</td>
<td>10.6 kg</td>
<td>18.0 kg</td>
</tr>
<tr>
<td>Power consumption</td>
<td>460 kw</td>
<td>470 kw</td>
<td>760 kw</td>
</tr>
<tr>
<td>Iron weight</td>
<td>57,000 lb</td>
<td>84,000 lb</td>
<td>113,000 lb</td>
</tr>
<tr>
<td>Copper weight</td>
<td>9,000 lb</td>
<td>13,000 lb</td>
<td>8,600 lb</td>
</tr>
</tbody>
</table>
of 12-in. x 28-in. are chosen just large enough to have no effect on the system solid angle. The entire system is shown in Fig. 1.

We have integrated the angular distribution of the pions over the acceptance region of the spectrometer, taking account of the losses due to the beam-stopper in the center and the \((\theta_1/\theta)^2\) falloff of the pion distribution for angles greater than \(\theta_1 = 7 \times 10^{-3}\) rad. Assuming a flat angular distribution at angles less than \(\theta_1\), the effective solid angle of the system is \(4.1 \times 10^{-4}\) sr. Using this value, a pion flux of \(6.9 \times 10^{-4}\) pion/Bev-sr-electron, a 3% momentum acceptance (0.6 Bev/c), and an initial electron beam of 60 \(\mu\)a, we calculate a pion flux of \(6.5 \times 10^7\) pion/sec. This number is for 20-Bev pions produced by 25-Bev electrons.

Crude cost estimates are given in Table II. The numbers are based on costs of $0.50/lb of iron and $4.50/lb of copper. It should be remarked that if one desires to conduct the pion beam to a second focal point in order to improve the background situation, a second magnet system whose size and cost are comparable to those of the system described in this report is required. Another important point is that if we did not require the large source distance (in order to put in a beam-stopper), we could have designed a different system providing \(10^{-3}\) sr for 20-Bev/c for only about half the cost of the present system.

The spectrometer design discussed in this report evolved largely from discussions with J. Ballam and R. Mozley.
TABLE II. Cost estimate. These are very crude values based on $0.50/lb for iron and $4.50/lb for copper. The total power consumption of the system is 2620 kw.

<table>
<thead>
<tr>
<th>Type</th>
<th>Iron</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-inch quadrupole</td>
<td>$28,500</td>
<td>43,000</td>
</tr>
<tr>
<td>$71,500 x 2 = $143,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24-inch quadrupole</td>
<td>42,000</td>
<td>60,000</td>
</tr>
<tr>
<td>102,000 x 2 = 204,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Deflecting magnet</td>
<td>57,000</td>
<td>39,000</td>
</tr>
<tr>
<td>96,000</td>
<td>96,000</td>
<td></td>
</tr>
<tr>
<td>Subtotal - magnet costs</td>
<td>$443,000</td>
<td></td>
</tr>
<tr>
<td>Power - 1000 hr/yr at $0.01/kwh, for 10 years</td>
<td>262,000</td>
<td></td>
</tr>
<tr>
<td>Total -</td>
<td>$705,000</td>
<td></td>
</tr>
</tbody>
</table>
QUADRUPOLE FOCUSING SYSTEMS FOR PARTICLES OF VERY LARGE MOMENTA

By

S. Penner

INTRODUCTION

We are considering the problems of designing systems for focusing and momentum-analyzing particles with multi-Bev/c momenta. We shall show that, after making a number of simplifying assumptions, we can specify the parameters of quadrupole magnet systems which typically provide solid angles of a few millisteradians at momenta of several Bev/c. These parameters can be optimized with respect to the total cost of the system and other important considerations (such as the length of the system in the case of short-lived particles). We present here a set of graphs from which all the important parameters of such optimized systems can be directly obtained.

Calculations of properties of quadrupole systems are usually made with digital computers because of the tedious nature of the mathematics. For example, Blewett\(^1\) has calculated the focal properties of several quadrupole configurations; and Cooper and Zombeck\(^2\) have considered optimum quadrupole design, subject to certain restrictions, for 4 Bev/c momentum. On the other hand, we have attempted to pursue the problem as far as possible without resorting to computers. While some accuracy is lost in our method, qualitative features of good magnet system design, such as the dependence of solid angle on cost and momentum, are more easily seen.

The complexity of the problem can be seen in Fig. 1, in which is shown the most general quadrupole doublet which provides (astigmatic) imaging of a point source. The system is completely specified by 13 parameters when we allow the possibility that both quadrupoles have rectangular apertures\(^3\) ("Panofsky-type quadrupoles"), and that the image

\(^{1}\) All references are found at the end of this report.
FIG. 1--General quadrupole doublet for astigmatic imaging.

(a) Initially focusing plane

(b) Initially defocusing plane
distances are different in the two planes. The parameters are the ten dimensions shown in Fig. 1, the magnetic field gradients of the two magnets, and the particle momentum. There are only two conditions imposed on the parameters: that imaging in the two planes is achieved at the appropriate positions. Even this system is not complete, because we have not included any deflecting magnets, which are of course necessary if momentum analysis is to be achieved; nor have we considered the possibility of including more than two quadrupoles in the system. In the following, in addition to doublets, we shall consider quadrupole triplets and quartets under certain simplifying symmetry restrictions.

SIMPLIFICATIONS

In order to reduce the number of variables, we shall make the following simplifications throughout this paper:

1. We consider only systems in which the distance $d_1$ between quadrupoles is zero. The justification for this is that if $d_1$ is not zero for some desirable solution, we can extend the length of the two quadrupoles until they fill this space. This can always be done in such a way as to effectively put an element with positive focusing in the space $d_1$ and thus increase the solid angle.

2. We insist that the first quadrupole be of "conventional" (as opposed to Panofsky-type) design. This is not essential, but in any case we shall make at least one of the two magnets of conventional design because the Panofsky type uses much more power than the conventional type with the same aperture size.

3. We consider only doublet systems for which $d_2^+$ and $d_2^-$ are both infinite, i.e., we achieve parallel-beam focusing from a point source. This is not as drastic as it might at first appear, because systems in which $d_2^+$ and $d_2^-$ are much larger than $d_0$ (large magnification) may be thought of as minor perturbations. Similarly, systems with small solid angle and magnification much less than unity are obtained by using the mirror image of the given system. Probably of most interest, however, is the symmetric quartet system obtained by following the doublet which gives parallel-beam focusing by its mirror image, as shown in Fig. 2, thus obtaining a system having unity
FIG. 2--Symmetric quartet quadrupole system providing anastigmatic unity magnification. All dimensions are the same in (b) as in (a), except \( a_2^- \neq a_2^+ \) if a Panofsky-type quadrupole is used.
magnification in both planes and no astigmatism. Since the particle flux from a point source is transformed to a parallel beam in the space \( D \) between the doublets, the magnitude of \( D \) is arbitrary. If \( D = 0 \), we have a symmetric triplet. To obtain momentum resolution, a deflecting magnet can be inserted in the space \( D \). (In this case, of course, one would line up the second doublet along the deflected beam.) The only disadvantage of large values of \( D \) is a loss of solid angle from off-axis source points.

By means of these three simplifications, we eliminate four of the 13 variables of the problem. We can find two relations between the variables, expressing the conditions for parallel-beam focusing in two planes. We are left with a system of seven degrees of freedom. These seven can be chosen from the nine parameters: source distance \( d_0 \), aperture radius of first quadrupole \( a_1 \), two magnet lengths \( l_1 \) and \( l_2 \), two field gradients, the aperture dimensions \( a_2^+ \) and \( a_2^- \) of the second quadrupole, and the momenta. Associated with each set of values of the seven degrees of freedom is a value of acceptance solid angle. We have succeeded in specifying the solid angle as a function of all the degrees of freedom in a readily usable form. Thus, subject to the simplifications given above, we have essentially solved the problem of obtaining large solid angle at large momenta in an optimum way.

**CLASSES OF SOLUTIONS**

The solutions to this problem naturally fall into three classes, according to the types and relative sizes of quadrupoles used:

I. **Two conventional quadrupoles of equal aperture.** One of the two quadrupoles (and in special cases both) is operated at maximum field. In this case, the first magnet defines the angular acceptance \( \theta_+ \) in the initially focusing plane, and the second magnet defines \( \theta_- \).

II. **Two conventional quadrupoles of unequal aperture.** This class of solutions is obtained from Class I by increasing the aperture of the quadrupole that is not operated at maximum field in Class I
until either (a) it is at maximum field, or (b) further increase of size does not increase the solid angle of the system.

III. A Panofsky-type quadrupole as the second magnet. The aperture $a^+_2$ of the second (Panofsky-type) magnet in the initially converging plane is chosen to be the smallest value which does not reduce the acceptance angle $\theta_+; \text{ and its aperture } a^-_2 \text{ in the initially diverging plane is either (a) the value at which its magnetic field attains a limiting value, or (b) a value which if further increased does not increase the acceptance angle } \theta_- \text{. The aperture } a^-_2 \text{ is the smaller of the two values (a) or (b) above. In both cases the first (conventional) quadrupole is operated at maximum field.}

We shall begin our discussion of the actual solutions by first considering only Class I solutions. Extension to the other classes is easily made from this starting point. In Class I, $a^+_2 = a^-_2 = a_1$, and therefore the total number of degrees of freedom is five, including the momentum.

APPROXIMATIONS

The approximations made in all solutions are as follows:

A. We assume paraxial ray optics. This means that we retain only first-order terms in $\theta_+$ and $\theta_-$. As we shall see, at multi-Bev/c momenta the largest practical values of $\theta_+$ and $\theta_-$ are of the order of 0.1 rad. Neglecting second-order terms thus results in uncertainties of the order of 10%.

Spherical aberrations, that are the result of the paraxial-ray approximation, can be determined either with the aid of digital computers or by model construction. For very expensive systems, one or both of these procedures should be employed. Our solutions are not valid for solid angles much greater than 0.03 sr because of the dominance of spherical aberrations.

B. We shall not consider in detail the change of trajectories associated with small changes of momenta. This is justifiable for momentum spreads $\Delta p/p$ of the order of a few percent, and is consistent with the paraxial-ray approximation.
C. We neglect all effects of the fringing fields at the ends of the quadrupoles, except to note that the effective lengths \( l_1 \) and \( l_2 \) are larger than the lengths of the magnet poles by about twice the aperture radius. Because of this effect, it is realistic to allow the spacing between magnets (\( d_1 \) in Fig. 1) to go to zero. In large-momentum systems the ratio of aperture to length is small, so that all other fringing-field effects can be safely neglected.

D. We assume that the magnetic fields in the magnets are pure quadrupole fields. It has been shown\(^4\) that any two-dimensional magnetic field of the type shown in Fig. 3, which is antisymmetric about the \( xz \) and \( yz \) planes and symmetric about the \( x'z \) and \( y'z \) planes, must have the form

\[
\begin{align*}
B_r &= h_2 r \sin 2\phi + h_6 r^5 \sin 6\phi + h_{10} r^9 \sin 10\phi + \cdots \\
B_\phi &= h_2 r \cos 2\phi + h_6 r^5 \cos 6\phi + h_{10} r^9 \cos 10\phi + \cdots
\end{align*}
\]

(1)

The constants \( h_2, h_6, h_{10}, \ldots \) are determined by the conditions at the boundaries of the system. The approximation of pure quadrupole fields amounts to neglecting all but the leading terms in Eq. (1), in which case

\[
\frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = \text{const} = h_2
\]

(2)

The effect of the presence of the terms in \( h_6 \) and \( h_{10} \) in actual magnets results in imperfect imaging. However, since \( h_6/h_2 \) and \( h_{10}/h_2 \) are usually small (a few percent), we can neglect their effects. If difficulties occur, the imaging can be improved substantially by reducing the aperture slightly, because the relative effect increases as the fourth power of the aperture radius.

**SOLUTION OF THE EQUATIONS OF MOTION**

We now proceed to solve the equations of motion of charged particles in the magnetic fields of the quadrupoles represented by Fig. 2.
FIG. 3.--Definition of coordinate system of a quadrupole magnet. The point P has rectangular coordinates $(x, y)$ and cylindrical coordinates $(r, \phi)$. The z-direction is perpendicular to both $x$ and $y$. The $x'y'$ axes are inclined $45^\circ$ to the $xy$ axes. The planes $x'z$ and $y'z$ are planes of symmetry; $xz$ and $yz$ are planes of magnetic antisymmetry.
We use the matrix method of trajectory calculation, in which the position \( x(z) \) and slope \( \theta_+ = \frac{\partial x}{\partial z} \) of the trajectory projected on the \( xz \) plane are the components of a vector giving the particle position in the \( x,p_x \) phase space (where \( p_x = \theta_+ p \)). In this space the coordinates at the entrance of the first magnet of a particle originating from the source are \((d_0 \theta_+, \theta_+)\). We wish to find a condition between the variables of the problem such that the coordinates of the particle upon leaving the second magnet are \((x_f, 0)\). That is, all particles leaving the source point on the axis are to be moving parallel to the axis after leaving the quadrupole doublet. The equation connecting these two states of the particle is

\[
\begin{pmatrix}
  x_f \\
  0 \\
\end{pmatrix} = \begin{pmatrix}
  \cosh \alpha_2 l_2 & \frac{1}{\alpha_2} \sinh \alpha_2 l_2 \\
  \alpha_2 \sinh \alpha_2 l_2 & \cosh \alpha_2 l_2 \\
\end{pmatrix} \begin{pmatrix}
  \cos \alpha_1 l_1 & \frac{1}{\alpha_1} \sin \alpha_1 l_1 \\
  -\alpha_1 \sin \alpha_1 l_1 & \cos \alpha_1 l_1 \\
\end{pmatrix} \begin{pmatrix}
  \frac{d_0 \theta_+}{p} \\
  \theta_+ \\
\end{pmatrix},
\]

where

\[
\alpha_1 = \left( \frac{1}{p} \frac{\partial B_1}{\partial x} \right)^{\frac{3}{2}},
\]

and

\[
\alpha_2 = \left( \frac{1}{p} \frac{\partial B_2}{\partial x} \right)^{\frac{3}{2}}.
\]

The subscripts 1 and 2 refer to the first and second quadrupoles, respectively. The \( \alpha \)'s have dimensions of reciprocal lengths. Equation (3) applies to the initially focusing plane. Performing the indicated matrix multiplications and solving for the lower (angular) element of the vector on the left-hand side of Eq. (3), we obtain
0 = d_0 (\alpha_2 \cos \alpha_1 l_1 \sinh \alpha_2 l_2 - \alpha_1 \sin \alpha_1 l_1 \cosh \alpha_2 l_2) \\
+ \cos \alpha_1 l_1 \cosh \alpha_2 l_2 + \left[ \left( \frac{\alpha_2}{\alpha_1} \right) \sin \alpha_1 l_1 \sinh \alpha_2 l_2 \right]. 
\tag{4}

Solving for \( d_0 \) yields
\[
\frac{d_0}{l_1} = 1 + \left[ \left( \frac{\alpha_2}{\alpha_1} \right) \tan \alpha_1 l_1 \tanh \alpha_2 l_2 \right] / (\alpha_1 l_1 \tan \alpha_1 l_1 - \alpha_2 l_2 \tanh \alpha_2 l_2) .
\tag{5}
\]

In the initially diverging plane, the equation of motion is obtained by interchanging circular and hyperbolic functions in Eq. (3). (The algebraic sign of the lower left-hand elements of both matrices must also be changed.) The equation analogous to (4) is

\[
0 = d_0 (\alpha_1 \cos \alpha_2 l_2 \sinh \alpha_1 l_1 - \alpha_2 \sin \alpha_2 l_2 \cosh \alpha_1 l_1) \\
+ \cos \alpha_2 l_2 \cosh \alpha_1 l_1 - \left[ \left( \frac{\alpha_2}{\alpha_1} \right) \sin \alpha_2 l_2 \sinh \alpha_1 l_1 \right]. 
\tag{6}

Solving Eq. (6) for \( d_0 \) yields
\[
\frac{d_0}{l_1} = 1 - \left[ \left( \frac{\alpha_2}{\alpha_1} \right) \tanh \alpha_1 l_1 \tan \alpha_2 l_2 \right] / (\alpha_2 l_1 \tan \alpha_2 l_2 - \alpha_1 l_1 \tanh \alpha_1 l_1) .
\tag{7}
\]

Equations (5) and (7) contain two relations between the five variables \( d_0, \alpha_1, \alpha_2, l_1, \) and \( l_2, \) and in effect constitute the solution to the focusing problem. We have solved this system numerically by the following procedure: values of \( \alpha_1 l_1 \) and \( l_2/l_1 \) are chosen. For each set of values the right-hand members of Eqs. (5) and (7) are evaluated as a function of \( \alpha_2 l_1. \) By numerical methods, a value of \( \alpha_2 l_1 \) is found for which the right-hand members of Eqs. (5) and (7) are equal. In Fig. 4 we have plotted the dependence of that value of \( \alpha_2 l_1 \) which satisfies this condition as a function of \( \alpha_1 l_1, \) with \( l_2/l_1 \) as a parameter. In Fig. 5 the corresponding values of \( d_0/l_1 \) are graphed. It is seen that \( \alpha_1 l_1 \approx 1.8 \) is an upper limit to the range of interest because \( d_0 \) rapidly approaches zero as \( \alpha_1 l_1 \) increases. Values of \( \alpha_1 l_1 \) less than about 0.6 are of little interest because of rapidly decreasing solid angle.
FIG. 4.—The value of $\alpha_2 \ell_2$ which provides focusing in both planes, as a function of $\alpha_1 \ell_1$. Values are obtained by simultaneous solution of Eqs. (5) and (7).
FIG. 5.—Source distance $d_0/l_1$ as a function of $\alpha_1 l_1$, calculated by means of Eq. (5).
SOLID-ANGLE CALCULATION

The calculations of the previous section give us sets of parameters which solve the problems of parallel-beam focusing with doublets and unity-magnification systems of symmetric triplets or quartets. We are now in a position to calculate the angular acceptance of these systems in terms of these sets of parameters.

From Fig. 2 we see that the maximum angle $\theta_+$ accepted in the initially focusing plane is determined by the aperture radius $a_1$ of the first quadrupole if the apertures of all quadrupoles are equal. We may state the criterion which determines the extreme ray as follows: the ray which enters the first quadrupole a distance $d_0 \theta_+$ from the axis at an angle $\theta_+$ with the axis attains a displacement $a_1$ at a distance $Z$ inside the quadrupole. At this point the ray is parallel to the axis of the system. This statement is represented by the following equation:

$$
\begin{pmatrix}
a_1 \\
0
\end{pmatrix} = 
\begin{pmatrix}
\cos \alpha_1 Z & \frac{1}{\alpha_1} \sin \alpha_1 Z \\
-\alpha_1 \sin \alpha_1 Z & \cos \alpha_1 Z
\end{pmatrix} 
\begin{pmatrix}
d_0 \theta_+ \\
\theta_+
\end{pmatrix}.
$$

(8)

We solve this equation for $Z$ and $\theta_+$, finding

$$
\tan \alpha_1 Z = \frac{1}{\alpha_1 d_0},
$$

and

$$
\theta_+ = \frac{a_1/d_0}{\sqrt{1 + (\alpha_1 d_0)^{-2}}^2}.
$$

(9)

Calculation of acceptance angle in the initially defocusing plane is done in an analogous manner. Referring to Fig. 2, we see that the extreme ray which enters the first quadrupole with coordinates $(d_0 \theta_-, \theta_-)$ leaves the second quadrupole with coordinates $(a_1, 0)$ for the case of equal-aperture quadrupoles. We thus have the condition
\[
\begin{pmatrix}
  a_1 \\
  0
\end{pmatrix}
= \begin{pmatrix}
  M \\
  \theta
\end{pmatrix}
\begin{pmatrix}
  d_0 \\
  \theta
\end{pmatrix}.
\tag{10}
\]

Here \(M\) is the transport matrix from the entrance of the first magnet to the exit of the second magnet:

\[
(M) = \begin{bmatrix}
  \cos \alpha_2 l_2 & \frac{1}{\alpha_2} \sin \alpha_2 l_2 \\
  -\alpha_2 \sin \alpha_2 l_2 & \cos \alpha_2 l_2
\end{bmatrix}
\begin{bmatrix}
  \cosh \alpha_1 l_1 & \frac{1}{\alpha_1} \sinh \alpha_1 l_1 \\
  \alpha_1 \sinh \alpha_1 l_1 & \cosh \alpha_1 l_1
\end{bmatrix}.
\tag{11}
\]

Equation (10) gives the two linear equations

\[
a_1 = m_{11} d_0 \theta_- + m_{12} \theta_-, \tag{12}
\]

and

\[
0 = m_{21} d_0 \theta_- + m_{22} \theta_. \tag{13}
\]

where \(m_{ij}\) are the matrix elements of \(M\). Note that Eq. (13) is identical to Eq. (6).

Equation (12) is solved for \(\theta_-\), eliminating \(d_0\) by means of Eq. (13):

\[
\theta_- = \frac{a_1}{m_{12} - (m_{11} m_{22}/m_{21})}. \tag{14}
\]

We can simplify Eq. (14) by noting that the determinant of \(M\) must be unity to preserve phase space (Liouville's theorem\(^6\)). Thus, \(m_{11} m_{22} - m_{12} m_{21} = 1\), so that \(\theta_- = -a_1 m_{21}\) Applying Eq. (13) again, we obtain

\[
\theta_- = \left(\frac{a_1}{d_0}\right) m_{22}
= \left(\frac{a_1}{d_0}\right) \left[\cosh \alpha_1 l_1 \cos \alpha_2 l_2 - \left(\frac{\alpha_2}{\alpha_1}\right) \sinh \alpha_1 l_1 \sin \alpha_2 l_2\right]. \tag{15}
\]

In order to calculate the solid angle of the system, we assume that all particles which remain within a circle of radius \(a_1\) throughout the system are accepted. It is then plausible that the acceptance region.
is an elliptical cone with semi-major and semi-minor angles of $\theta_+$ and $\theta_-$. It has been shown by ray-tracing that this estimate of the solid angle is conservative (by a few percent). In this approximation, the solid-angle acceptance is

$$\Omega = \pi \theta_+ \theta_-$$

$$= \pi \left( \frac{a_1}{d_0} \right)^2 \frac{\left[ \cosh \alpha_1 l_1 \cos \alpha_2 l_2 - (\alpha_2/\alpha_1) \sinh \alpha_1 l_1 \sin \alpha_2 l_2 \right]}{\left[ 1 + (\alpha_1 d_0)^{-2} \right]^{1/2}} \quad . \quad (16)$$

Since we have previously succeeded in expressing $\alpha_2$ and $d_0/l_1$ in terms of $\alpha_1 l_1$ and $l_2/l_1$, Eq. (16) is an expression for solid angle in terms of the variables $\alpha_1 l_1$, $l_2/l_1$, and $a_1$. It will be convenient to eliminate $a_1$ from Eq. (16) and use the ratio $B_1/p$ as the third variable, where $p$ is the particle momentum and $B_1$ is the magnetic field at the poletips of the first quadrupole. The field gradient then is $\partial B_1/\partial x = B_1/a_1$, so that

$$\alpha_1 l_1 = (B_1/p a_1)^{1/2} l_1 \quad . \quad (17)$$

Expressing $a_1$ and $l_1$ in inches, $B_1$ in kilogauss, and $p$ in Mev/c, we find that

$$a_1 = (0.762) \frac{B_1}{p} \frac{l_1^2}{(\alpha_1 l_1)^2} \quad . \quad (18)$$

We now return to Eq. (16), and re-express $\Omega$ as a function of $\alpha_1 l_1$, $l_2/l_1$, and $B_1/p$ as primary variables, with $\alpha_2$ and $d_0/l_1$ to be obtained from Figs. 4 and 5 [or analytically by means of Eqs (5) and (7)]. The result is
This equation is one of the main results of the present paper; it gives the obtainable solid angle as a function of momentum, magnetic field, and magnet length. In Fig. 6 we have plotted \((p/B_1 l_1)^2 \Omega\) against \(p a_1 / B_1 l_1^2\) [obtained from Eq. (18)], with \(l_2 / l_1\) as the parameter. This single graph thus gives the obtainable solid angle as a function of the pertinent variables—momentum, magnetic field, and magnet dimensions—for conventional quadrupole doublets and triplets with all apertures equal.

Several important considerations should be borne in mind when Fig. 6 is used to design a quadrupole system. First, for certain values of \(p a_1 / B_1 l_1^2\) (and hence certain values of \(a_1 l_1\), \(a_2\) is greater than \(a_1\). These regions are determined by referring to Fig. 4, in which the line \(a_2 = a_1\) has been drawn for reference. Thus, on the dotted portions of the curves of Fig. 6, the obtainable solid angle is less than the value shown because the limiting field gradient is attained in the second magnet. The obtainable solid angle in these regions is calculated by dividing the values in Fig. 6 by the corresponding value of \((a_2/a_1)^4\). After this is done, \(B_1\) in the scale factor for the ordinate of Fig. 6 can be set equal to the maximum obtainable value (about 10 kg). Second, different regions of the curves of Fig. 6 represent greatly different magnet costs. For a given value of \(a_1\) and \(l_1\), the relative costs of the systems \(l_2 = \frac{1}{2} l_1 : l_2 = \frac{2}{3} l_1 : l_2 = l_1\) are in the ratio \(\frac{3}{4} : \frac{5}{6} : 1\) (i.e., in the ratio of the total magnet length). Thus, for any given value of \(p a_1 / B_1 l_1^2\) less than about unity, the greatest solid angle per dollar is obtained with \(l_2 = \frac{1}{2} l_1\) systems. Finally, the greatest solid angle per dollar occurs at a value of \(p a_1 / B_1 l_1^2\) considerably less than the value at which \((p/B_1 l_1^2 \Omega\) attains a maximum, because for fixed length the cost of a magnet increases with aperture. Considering magnet systems with
FIG. 6.—Solid angle as a function of aperture, length, field, and momentum, for systems of equal-aperture quadrupoles. Lengths are in inches, fields in kilogauss, momenta in Mev/c, and solid angle in steradians. The dotted portions of the curves represent "unattainable" regions in the sense that the magnetic field limit is reached in the second magnet rather than the first. The curves are calculated from Eqs. (19) and (28) and Figs. 4 and 5.
equal apertures only, therefore, the greatest solid angle per unit cost is obtained with systems having \( l_2 = \frac{1}{2} l_1 \), and \( p_{a_2} / B_{l_1} l_2^2 \) in the region of 0.23 to 0.3, corresponding to \( \alpha_1 l_1 \) in the range 1.8 to 1.6.

**CLASS II SYSTEMS**

We now proceed to evaluate systems composed of conventional quadrupole magnets in which the apertures of the first and second magnets are unequal. For the cases \( l_2 = l_1 \), \( l_2 = \frac{2}{3} l_1 \) with \( p_{a_1} / B_{l_1} l_2^2 < 0.575 \), and \( l_2 = \frac{1}{2} l_1 \) with \( p_{a_1} / B_{l_1} l_2^2 < 0.30 \), the second magnet may be increased in aperture since at equal aperture the field in the second magnet is less than the maximum value. From Fig. 2 it is seen that this will increase the acceptance angle only in the initially defocusing plane. For equal poletip fields in the two magnets, the aperture radius of the second magnet can be as large as \( (\alpha_2 / \alpha_1)^2 a_1 \). However, from Fig. 2 we see that there is a point beyond which no solid-angle increase is obtained by increasing the size of the second magnet. This point occurs when the aperture of the second magnet is such that the particle that leaves the second magnet a distance \( a_2 \) away from the axis (in the initially defocusing plane) and parallel to the axis has a coordinate \( a_1 \) at the exit of the first magnet. This condition is given by the matrix equation

\[
\begin{pmatrix}
    \theta_2 \\
    0
\end{pmatrix} =
\begin{bmatrix}
    \cos \alpha_2 l_2 & \frac{1}{\alpha_2} \sin \alpha_2 l_2 \\
    -\alpha_2 \sin \alpha_2 l_2 & \cos \alpha_2 l_2
\end{bmatrix}
\begin{pmatrix}
    \theta_1 \\
    \theta_2'
\end{pmatrix}.
\]

(20)

Eliminating \( \theta' \), we find that the value of \( a_2 \) beyond which no solid-angle increase can be achieved is

\[
a_2 = a_1 / \cos \alpha_2 l_2.
\]

(21)

Class II systems are thus obtained from Class I systems in the following way: (a) If \( \alpha_1 > \alpha_2 \) and \( (\alpha_1 / \alpha_2)^2 < 1 / \cos \alpha_2 l_2 \), then
\[ a_2 = \left(\frac{a_1}{a_2}\right)^2 a_1, \] and the solid angle is improved over the Class I case by a factor of \( \left(\frac{a_1}{a_2}\right)^2 \). (b) If \( a_1 > a_2 \) and \( 1/\cos a_2 l_2 < \left(\frac{a_1}{a_2}\right)^2 \), then \[ a_2 = \frac{a_1}{\cos a_2 l_2}, \] and the solid-angle improvement factor is \( 1/\cos a_2 l_2 \).

When \( a_1 = a_2 \), Class I and II solutions obviously become identical. We could consider Class II solutions in which \( a_2 < a_1 \), that is, \( \left(\frac{a_1}{a_2}\right)^2 \) is less than unity. This type of system is uninteresting, however, because it is generally far from optimum on a basis of solid angle per unit cost.

**CLASS III SOLUTIONS**

The use of Panofsky-type quadrupoles allows increasing the aperture of the second magnet, as in Class II solutions, with the additional advantage that in practice maximum magnetic fields in Panofsky-type quadrupoles are about 1.5 times those obtained with conventional quadrupoles. Because of the rectangular geometry, however, the maximum field occurs at a distance \( \left[ (s_2^+)^2 + (a_2^+)^2 \right]^{1/2} \) from the axis. Combining these factors we find that the maximum allowable aperture for a Panofsky-type quadrupole in the initially defocusing plane is

\[ a_2^- = \left[ \left(1.5\right)^2 \left(\frac{a_1}{a_2}\right)^4 a_1^2 - (a_2^+)^2 \right]^{1/2}. \]

We shall consider only the case where \( a_2^+ \) is so chosen that the extreme trajectory in the initially focusing plane is simultaneously limited by \( a_1 \) and \( a_2^+ \) as shown in Fig. 2. The equations which specify this condition are Eq. (8) and the analogous statement referring to the exit of the first magnet:

\[ \begin{pmatrix} a_2^+ \\ \theta_n^+ \end{pmatrix} = \begin{bmatrix} \cos a_1 l_1 & \frac{1}{a_1} \sin a_1 l_1 \\ -a_1 \sin a_1 l_1 & \cos a_1 l_1 \end{bmatrix} \begin{pmatrix} d_0 \theta_+ \\ \theta_+ \end{pmatrix}. \]

From Eq. (23),

\[ a_2^+ = \left[ d_0 \cos a_1 l_1 + (1/a_1) \sin a_1 l_1 \right] \theta_+ ; \]

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but from Eq. (8),

\[ a_1 = \left[ d_0 \cos \alpha_1 z + \frac{1}{\alpha_1} \sin \alpha_1 z \right] \theta_+ \tag{25} \]

Taking the ratio of these last two equations, and using the fact that \( \tan \alpha_1 z = (\alpha_1 d_0)^{-1} \), as is shown under Eq. (8), we obtain

\[ \frac{a_2^{+}}{a_1} = \frac{\cos \alpha_1 l_1 + (\alpha_1 d_0)^{-1} \sin \alpha_1 l_1}{\left[ 1 + (\alpha_1 d_0)^{-2} \right]^{\frac{3}{2}}} \tag{26} \]

Substitution of (26) in (22) yields the ratio \( \left( a_2^{-}/a_1 \right) \). As in the case of Class II solutions, increasing \( (a_2^{-}/a_1) \) to values greater than \( (1/\cos \alpha_2 \ell_2) \) does not increase the solid angle of the system. Thus, we distinguish two cases: (a) \( (a_2^{-}/a_1) < 1/\cos \alpha_2 \ell_2 \), and the solid angle is improved over Class I by the factor \( (a_2^{-}/a_1) \). (b) \( (a_2^{-}/a_1) > 1/\cos \alpha_2 \ell_2 \), in the case we set \( a_2^{-} = a_1 / \cos \alpha_2 \ell_2 \), and the solid angle is improved over Class I by a factor \( 1/\cos \alpha_2 \ell_2^{2} \). For Class III solutions we shall consider only cases where \( a_2^{-} > a_1 \). In fact, Class III solutions will be economically satisfactory only when \( a_2^{-} \) is considerably larger than \( a_1 \) (and hence \( a_1^{+} \)), because of the large power consumption of the Panofsky-type magnet.

SUMMARY OF SOLID ANGLE RESULTS

Figures 7, 8, and 9 give the final results on solid angle for the cases \( \ell_2 = \frac{1}{2} \ell_1, \ell_2 = \frac{2}{3} \ell_1, \) and \( \ell_2 = \ell_1 \), respectively. The abscissa scales are \( p \alpha_1 / B_m \ell_1^{2} \), where \( p \) is the particle momentum in Mev/c, \( B_m \) is the poletip field in kilogauss in the conventional quadrupole which has the higher field \( (B_m = 10-12 \text{ kg} \) is a good working value), \( \ell_1 \) is the length in inches of the first magnet, and \( a_1 \) is the aperture radius in inches of the first magnet. The ordinate scales are \( (p/V_m \ell_1)^{2} \Omega \), where \( \Omega \) is in steradians.

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FIG. 7.— Solid angle as a function of aperture, length, field, and momentum, for systems of quadrupoles with unequal apertures. Lengths are in inches, fields in kilogauss, momenta in Mev/c, and solid angle in steradians. For detailed explanation, see text, especially p. 24.
FIG. 8.—Solid angle as a function of aperture, length, field, and momentum, for systems of quadrupoles with unequal apertures. Lengths are in inches, fields in kilogauss, momenta in Mev/c, and solid angle in steradians. For detailed explanation, see text, especially p. 24.
FIG. 9.— Solid angle as a function of aperture, length, field, and momentum, for systems of quadrupoles with unequal apertures. Lengths are in inches, fields in kilogauss, momenta in Mev/c, and solid angle in steradians. For detailed explanation, see text, especially p. 24.
The labels on the curves distinguish the following classes of solutions:

I. Conventional quadrupoles of equal aperture
   a. The first magnet has a poletip field $B_1 = B_m$, and the second magnet has a poletip field $B_2 = (\alpha_2/\alpha_1)^2 B_m$.
   b. Here $B_2 = B_m$, and $B_1 = (\alpha_1/\alpha_2)^2 B_m$. In this case only the apertures are not correctly given by the abscissae in Figs. 7, 8, and 9: the correct aperture is $(\alpha_1/\alpha_2)^2$ times the value implied by the abscissa scale.

II. Conventional quadrupoles of unequal aperture
   a. The case $B_1 = B_2 = B_m$, and $a_2 = (\alpha_1/\alpha_2)^2 a_1$.
   b. The case $a_2 = a_1/\cos \alpha_2/\alpha_2$. Here $B_1 = B_m$, but
      $B_2 = (\alpha_2/\alpha_1)^2 (a_2/a_1) < B_m$.

III. Systems employing Panofsky-type quadrupoles
   a. The maximum field in the Panofsky magnet is $1.5 B_m$ where $B_1 = B_m$. The aperture dimensions of the Panofsky quadrupole are given by Eqs. (26) and (22).
   b. The base $a_2 = a_1/\cos \alpha_2/\alpha_2$. The value of $a_2^+$ is given by Eq. (25). The maximum field in the Panofsky magnet is

   \[
   B_p = B_m \left( \frac{\alpha_1}{\alpha_2} \right)^2 \left[ \left( \frac{a_2^-}{a_1} \right)^2 + \left( \frac{a_2^+}{a_1} \right)^2 \right]^{1/2} < 1.5 B_m .
   \]  

   Note that the solid angles for Class IIIb and Class IIb are identical.

   All parameters of a system corresponding to any point of Figs. 7, 8, or 9 are obtained by first calculating

   \[
   \alpha_1 l_1 = \left[ \frac{1}{0.762}(p_{a_1}/B_{m_1})^{2/3} \right]^{-1/3}. \]  

   Having obtained $\alpha_1 l_1$, values of $\alpha_2 l_1$ and $d_0/l_1$ are readily obtained.
from Figs. 4 and 5. Then $a_2$ (or $a_-^2$ and $a_+^2$) and $B_1$ and $B_2$ are calculated as indicated in the preceding paragraph.

The various classes of solutions, their range of validity, relative apertures, magnetic fields, and solid angles, are summarized in Table I.

COST CONSIDERATIONS

We shall now attempt to estimate the relative costs of the quadrupole systems considered in this report. We must emphasize that these cost considerations are at best very approximate and might in some cases be totally misleading. Before settling on final designs for large systems, where the cost might well approach a million dollars, very careful cost analyses should be made.

We shall estimate the cost of the coils for quadrupole magnets by assuming that the cost is proportional to the weight. The number of ampere-turns required to provide a given poletip field in a quadrupole is approximately proportional to the aperture radius. If the current density in the coil is held constant, independent of aperture, the cost is proportional to the product of aperture radius and length. (To obtain this result, we neglect the small difference between the sum of the physical length of the poles plus the width of the pole, and the magnetic length of the magnet.) Actually the cost probably increases more slowly than linearly with the aperture radius because as the aperture radius increases the magnetic reluctance of the return path decreases relative to the reluctance of the gap.

In large magnets, the cost of providing electrical power for the life of the magnet is usually comparable to the initial cost. Using the above-given assumption of constant current density, the power required to operate a magnet also scales as the product of length and aperture radius for fixed poletip field.

Estimating the variation in the cost of the iron with aperture is very difficult. Assuming that the ratios of pole radius and pole width to aperture radius are kept constant, and also that the poletip field and current density are constant, the cross-sectional area of iron contains
TABLE I. Summary of the solutions

<table>
<thead>
<tr>
<th>Class</th>
<th>Range of validity</th>
<th>$B_1/B_m$</th>
<th>$B_2/B_m$</th>
<th>$a_2/a_1$</th>
<th>$a_2^+a_1$</th>
<th>$\Omega/\Omega_{Ia}^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I a</td>
<td>$\frac{a_2}{a_1} \leq 1$</td>
<td>$1$</td>
<td>$(a_2/a_1)^2$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>I b†</td>
<td>$\frac{a_2}{a_1} \geq 1$</td>
<td>$(a_1/a_2)^2$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
<td>$(a_1/a_2)^4$</td>
</tr>
<tr>
<td>II a</td>
<td>$1 \leq \frac{(a_2/a_1)^2}{\cos \alpha_2 l_2}$</td>
<td>$1$</td>
<td>$(a_2/a_1)^2$</td>
<td>$1$</td>
<td>$(a_1/a_2)^2$</td>
<td>$(a_1/a_2)^2$</td>
</tr>
<tr>
<td>II b</td>
<td>$\frac{(a_2/a_1)^2}{\cos \alpha_2 l_2}$</td>
<td>$1$</td>
<td>$\frac{(a_2/a_1)^2}{\cos \alpha_2 l_2}$</td>
<td>$1$</td>
<td>$\frac{1}{\cos \alpha_2 l_2}$</td>
<td>$\frac{1}{\cos \alpha_2 l_2}$</td>
</tr>
<tr>
<td>III a</td>
<td>$\frac{a_2}{a_1} \leq 1$</td>
<td>$1$</td>
<td>$1.5$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>III b</td>
<td>$\frac{a_2}{a_1} \geq 1$</td>
<td>$1$</td>
<td>$\frac{1}{\cos \alpha_2 l_2}$</td>
<td>$1$</td>
<td>$\frac{1}{\cos \alpha_2 l_2}$</td>
<td>$\frac{1}{\cos \alpha_2 l_2}$</td>
</tr>
</tbody>
</table>

† $\Omega_{Ia}$ is the value given by Eq. (19)

‡ In this case, $a_1$ must be reduced by a factor of $(a_1/a_2)^2$ from the value given by the abscissa scale in Figs. 7, 8, or 9.
terms in the first, three-halves, and second powers of the aperture radius. We can state with certainty only that the iron cost scales somewhat faster than linearly with aperture radius if the cost is just proportional to the weight. The iron cost should, of course, scale linearly with magnet length.

It is easy to see that all cost factors are approximately proportional to length. On the other hand, the dependence of total cost (including power) is a very complicated function of aperture. Since, however, the power and copper costs added together probably exceed the cost of iron in an economical design, we will assume that total cost varies linearly with aperture radius.

The dependence of costs on poletip magnetic fields is a highly non-linear function because of saturation effects. We shall therefore make all cost comparisons on the basis of constant poletip fields, because we fear that to do otherwise would probably be very misleading.

It is reasonable to assume that the cost of Panofsky-type quadrupoles varies linearly with the product of length and major half-aperture \( a_2 \). However, the coefficient by which \( a_2 \) is multiplied to obtain the cost is probably quite different from the corresponding coefficient for conventional quadrupoles. There are several reasons for this. First, we are assuming that Panofsky quadrupoles can be operated at higher maximum fields (by a factor 1.5) than conventional quadrupoles. Second, the cost of iron in Panofsky quadrupoles is usually quite small compared to the copper and power costs. Third, because of the more complicated coil shape of the Panofsky quadrupole, the copper cost per pound will be more than in the case of conventional quadrupoles.

In light of the discussion above, we shall use the following cost-estimating formulas for all systems:

\[
C = k\left( a_1 l_1 + a_2 l_2 \right)
\]  \hspace{1cm} (29)

for systems of Classes I and II, where \( C \) is the total cost and \( k \) is a constant; and

\[
C = k a_1 l_1 + k' a_2 l_2
\]  \hspace{1cm} (30)
for Class III systems. The constant $k'$ may be different from $k$, and is probably somewhat greater. We must emphasize again that these estimates are at best rough approximations.

Using Eqs. (28) and (30), and referring to Figs. 7, 8, and 9 where necessary, we arrive at the following conclusions:

1. For fixed $p$, $B_m$, and $l_1$, and for Class I ($a_2 = a_1$) systems, the most economical system (i.e., greatest solid angle per unit cost) occurs at that value of $a_1$ for which $o/a_1$ is a maximum. For all values of the ratio $l_2/l_1$, this economic condition for Class I systems occurs near $pa_1/B_m l_1^2 = 0.3$ ($pa_1/B_m l_2^2 = 0.24$ is better in the case $l_2 = \frac{1}{2} l_1$).

2. For fixed $p$, $B_m$, and $l_1$, for Class I systems, small ratios of $l_2/l_1$ are economically favorable when the value of $a_1$ is also optimized; $l_2 = \frac{1}{2} l_1$ is 25% better than $l_2 = \frac{2}{3} l_1$, which in turn is 25% better than $l_2 = l_1$. Values of $l_2$ less than half of $l_1$ might be somewhat better, but the optimization appears to be a relatively slowly varying function of $l_2/l_1$.

3. For fixed $p$, $B_m$, and $l_1$, and with $a_1$ optimized, Class II systems provide more solid angle per dollar than Class I systems. The improvement is 25% for $l_2 = l_1$ and $l_2 = \frac{2}{3} l_1$, and 10% for $l_2 = \frac{1}{2} l_1$.

4. For fixed $p$, $B_m$, and $l_1$, and with $a_1$ near its optimum value, Class III systems provide more solid angle per dollar than Class I systems if $k' \leq 1.5 k$ for $l_2 = l_1$ and $l_2 = \frac{2}{3} l_1$, or if $k' \leq 1.7 k$ for $l_2 = \frac{1}{2} l_1$. Class III systems provide more solid angle per dollar than Class II systems only if $k' \leq k$ for $l_2 = l_1$ and $l_2 = \frac{2}{3} l_1$, or if $k' \leq 1.4 k$ for $l_2 = \frac{1}{2} l_1$.

5. The most economical systems are obtained with the parameters $l_2 = \frac{1}{2} l_1$ and $pa_1/B_m l_1^2 \approx 0.24$. If $k' \leq 1.4 k$, Class III systems are most economical; but if $k' \geq 1.4 k$, Class II systems are preferred.

6. For fixed $p$ and $B_m$, under optimum conditions the length of a system increases as the square root of the solid angle, and the aperture radius increases as the square of the length. Thus, the cost of a
system increases as the 3/2 power of the solid angle for fixed momentum. The apparent conclusion that two small systems would cost less than one large system is incorrect because of such peripheral costs as deflecting magnets, detecting equipment, and shielding.

7. The cost of providing a given amount of solid angle increases as the square of the momentum, assuming fixed $B_m$ and optimized design parameters.

SAMPLE CALCULATIONS

In this section we shall give two examples illustrating the use of the material in this report. In the first example we shall "design" an optimum quadrupole system, and in the second we shall calculate the solid angle obtainable with a hypothetically pre-existing set of quadrupoles.

Suppose we are given the problem of designing a quadrupole system to image a small source of 10-Bev/c particles with unity magnification and a solid angle of $5 \times 10^{-3}$ sr. We assume the correctness of the tentative conclusions reached in the preceding section on cost considerations, and calculate the parameters of a quartet magnet system of the type shown in Fig. 2, with $l_2 = \frac{1}{2} l_1$, and the interior magnets of the Panofsky type. Using the operating point $\frac{p a_1}{B_m l_1^2} = 0.235$, we see from Fig. 7, curve IIIb, that $(p/B_m l_1^2)^2 \Omega = 0.136$. In this expression, we substitute $\Omega = 5 \times 10^{-3}$ sr, $p = 10^4$ Mev/c, and $B_m = 10$ kg. Then,

$$l_1 = \frac{p}{B_m} \left( \frac{\Omega}{0.136} \right)^{\frac{1}{2}} = \frac{10^4}{10} \left( \frac{5 \times 10^{-3}}{0.136} \right)^{\frac{1}{2}} = 192 \text{ in.},$$

and

$$a_1 = 0.235 \frac{B_m l_1^2}{p} = \frac{0.235 \times 10 \times (192)^2}{10^4} = 8.65 \text{ in.}$$

We next use Eq. (28) to calculate

$$\alpha_1 l_1 = (0.235/0.762)^{-\frac{1}{2}} = 1.8.$$ 

We consult Figs. 4 and 5 to find that $\alpha_2 l_1 = 1.67$ and $a_0/l_1 = 0.194,$

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from which \( d_0 = 37.2 \text{ in.} \). Since the operating point is in region IIIb, we calculate the larger aperture dimension of the Panofsky quadrupole from the relation \( a_2^2/a_1 = 1/\cos \alpha_2 \ell_2 \). Using \( \ell_2 = \frac{1}{2} \ell_1 \) and the previously obtained value of \( a_1 \), we obtain

\[
a_2^+ = 13.1 \text{ in.}
\]

Equation (26) is used to obtain the other aperture dimension:

\[
a_2^- = 7.3 \text{ in.}
\]

The system is now completely defined, and detailed estimates of cost, weight, power consumption, and so forth, can be made.

We now show how the present work can be used to answer the question: Given a set of magnets, how much solid angle can I get? Let us assume that we have available four conventional quadrupoles, each 48-in. long, having 12-in. bore and a maximum field gradient of 2 kg/in. In our notation, \( \ell_1 = 48 \text{ in.}, a = 6 \text{ in.}, \) and \( B_m = 12 \text{ kg}. \) The question is: How much solid angle can we achieve at 2 Bev/c momentum and in what configuration?

We shall have to compute two cases: \( l_2 = l_1 \), which uses all four magnets; and \( l_2 = \frac{1}{2} l_1 \), which uses only three of the magnets. The first step is to calculate the values of \( pa_1/B_m l_1^2 \) which can be achieved. At 2 Bev/c,

\[
\frac{pa_1}{B_m l_1^2} = \frac{2 \times 10^3 \times 6}{12 \times (48)^2} = 0.434
\]

[Clearly we can operate at any value of \( pa_1/B_m l_1^2 \) which is larger than 0.434 simply by reducing the field in the magnets. This would be advantageous only if \( (p/B_m l_1)^2 \Omega \) increased more rapidly than \( (pa_1/B_m l_1^2)^2 \), because the latter function can be increased only by decreasing \( B_m \). Such a condition does not occur in the example.] From Fig. 9 we find that for Class I systems with \( l_2 = l_1 \), at the given operating point \( (p/B_m l_1)^2 \Omega = 0.123 \). Putting in \( p = 2 \times 10^3 \), \( l_1 = 48 \), \( B_m = 12 \text{ kg}, \) yields

\[
\Omega = 0.041 \text{ sr.}
\]
Before calculating other parameters of this system, we should consider the possibility of using \( l_2 = \frac{1}{2} l_1 \) systems. From Fig. 7 we see that for \( l_2 = \frac{1}{2} l_1 \) and \( p_{1/2}/B_{m1} = 0.434 \), we are in region Ib; that is, the second quadrupole has higher field than the first. Therefore \( B_m \) must be reduced. This clearly puts us in a region of Fig. 7 where \( (p/B_m l_1)^2 \) is much less than the obtainable value for \( l_2 = l_1 \) systems. Thus, with the equipment that is (hypothetically) available, we do better by using all four magnets. However, if we could design a new set of quadrupoles specifically for this momentum and solid angle, they would cost far less than the hypothetical set of four equal magnets, if we used the design principles employed in the first example.

**EFFECTS OF FINITE SOURCE SIZE AND MOMENTUM SPREAD**

We have thus far considered only the solid-angle acceptance for a monoenergetic point source located on the axis of the system. Clearly the acceptance solid angle will decrease for off-axis and off-momentum source points.

The difficulty in calculating acceptance angles for the systems we have considered is that in most cases there are several potential defining apertures. In systems of Classes I, IIa, and IIIa there is only one defining aperture in each plane for a parallel-beam focusing doublet. In Class IIb and IIIb systems, however, there are two simultaneous defining apertures in the doublet in the initially defocusing plane. In the symmetric quartet unity-magnification systems the number of apertures is twice as great as in the doublet case. For an off-axis point, any one of these potential apertures may actually define the acceptance, and for large displacements from the axis, still other points may become defining apertures. The only statement of general validity that we can make is that increasing the inter-doublet spacing \( D \) (in Fig. 2) decreases the solid angle for off-axis source points. The amount of decrease will depend on the ratio of \( D \) to the system focal lengths. We do not propose to investigate the solid-angle properties of the various systems in detail. However, to aid in estimating reasonable values of the spacing \( D \), and to allow simple ray-tracing calculations to be made, we shall next obtain the focal lengths of the various quadrupole combinations.

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It turns out that the negative reciprocal of the focal length of any system is given by the lower left-hand element of its transport matrix. For example, to calculate the focal length $F_+$ in the initially focusing plane of an arbitrary quadrupole doublet, we multiply together the two matrices in Eq. (3). The lower left-hand element of the resulting matrix is the desired result:

$$\frac{1}{F_+} = \alpha_2 \sinh \alpha_2 l_2 \cos \alpha_1 l_1 - \alpha_1 \cosh \alpha_2 l_2 \sin \alpha_1 l_1 .$$  \hspace{1cm} (31)

Similarly, we obtain the focal length $F_-$ in the initially defocusing plane from the matrix in Eq. (11):

$$\frac{1}{F_-} = -\alpha_2 \sin \alpha_2 l_2 \cosh \alpha_1 l_1 + \alpha_1 \cos \alpha_2 l_2 \sinh \alpha_1 l_1 .$$  \hspace{1cm} (32)

The functions $F_+/l_1$ and $F_-/l_1$ are plotted against $\alpha_1 l_1$ in Fig. 10. Only the cases $l_2 = l_1$ and $l_2 = \frac{1}{2} l_1$ are shown, since the focal lengths are not strongly dependent on the ratio $l_2/l_1$.

In the case of parallel-beam-focusing doublets, we know that the source is located at the entrance focal point (a distance $d_0$ in front of the entrance to the first magnet). Clearly the entrance principal plane is located a distance $F$ after this focal point. In order to ray-trace through the system, we also need to know the location of the exit principal plane. It can be shown that the distance from the exit of the second magnet to the exit principal plane is given by

$$g = F (m_{11} - 1) ,$$  \hspace{1cm} (33)

where $F$ is the corresponding focal length, and $m_{11}$ is the upper left-hand element of the transport matrix for the doublet. Our sign convention is that the exit principal plane is after the exit of the magnet if $g$ is a positive quantity. The matrix element $m_{11}$ can be obtained from Eq. (3) for the initially defocusing plane. The results are
FIG. 10.—Focal lengths as a function of $\alpha_1 \ell_1$. The case $\ell_2 = \frac{2}{3} \ell_1$ is not shown, but can be calculated from Eqs. (31) and (32).
and
\[ g_+ = F_+ \cosh \alpha_2 l_2 \cos \alpha_1 l_1 - (\alpha_1 / \alpha_2) \sinh \alpha_2 l_2 \sin \alpha_1 l_1 - 1 \]  \hspace{0.5cm} (34)

and
\[ g_- = F_- \cos \alpha_2 l_2 \cosh a_1 t_1 + (\alpha_1 / \alpha_2) \sin \alpha_2 l_2 \sinh a_1 t_1 - 1 \]  \hspace{0.5cm} (35)

Curves of \( g_+/l_1 \) and \( g_-/l_1 \) are plotted against \( \alpha_1 l_1 \) in Fig. 11.

Two more quantities that are of considerable interest in determining the properties of a magnet system are the acceptance angles \( \theta_+ \) and \( \theta_- \). These are given by Eqs. (9) and (15), respectively, and are plotted in Fig. 12 as a function of \( \alpha_1 l_1 \) for the cases \( l_2 = l_1 \) and \( l_2 = \frac{1}{2} l_1 \).

It should be noted that the determining aperture for \( \theta_+ \) is always \( a_1 \), and for \( \theta_- \) is \( a_2 \).

From Fig. 12 we see that the acceptance angles \( \theta_+ \) and \( \theta_- \) tend to be quite different. This asymmetry can be very useful. For example, in a scattering experiment, if the initially defocusing plane of the magnet system is in the scattering plane, the angular resolution for a given solid angle may be as much as an order of magnitude greater than if the initially focusing plane is in the scattering plane.

Another way to make use of the natural asymmetries of the quadrupole systems is in the design of a deflecting magnet to go in the space \( D \) between the two doublets of a quartet magnet. The dimensions of the deflecting magnet gap must be at least \( 2a_2^- \) in the initially defocusing plane and \( 2F_+ \theta_+ \) in the initially focusing plane. Since \( F_+ \theta_+ < a_1 \) and \( a_1 \leq a_2 \), a smaller-gap-height magnet may be used if the deflection is in the initially defocusing plane. This turns out also to be the advantageous direction for deflection in terms of momentum resolution, which can be seen as follows: Suppose the source is a circle of radius \( r_0 \).

Then after the first doublet, particles originating at the edge of the source are at angles to the axis \( \theta'_+ = r_0 / F_+ \) in the initially focusing plane and \( \theta'_- = r_0 / F_- \) in the initially defocusing plane. Suppose now that we have a deflecting magnet which deflects the particles through an angle \( \psi \). The best obtainable momentum resolution is proportional to the ratio \( \theta'_+ / \psi \) or \( \theta'_- / \psi \), depending on the direction of deflection. Since \( \theta'_- \) is always much smaller than \( \theta'_+ \) (\( F_- >> F_+ \), according to Fig. 10), better resolution is achieved if the deflection is in the initially defocusing plane.
FIG. 11.—Exit principal plane location of doublets as a function of $\alpha_1 l_2$.

The distances $g_+$ and $g_-$ are measured from the exit of the second magnet. Distances measured to the right (in Fig. 1) are positive. Note that $g_+$ is always negative. The case $l_2 = \frac{2}{3} l_1$ is not shown, but can be obtained from Eqs. (34) and (35).
FIG. 12.—Acceptance angles in radians as a function of $\alpha_1 \ell_1$. The case $\ell_2 = \frac{3}{2} \ell_1$ is not shown, but can be obtained from Eqs. (9) and (15).
We now complete our discussion of equivalent-plane optics by considering the equivalent planes of a symmetric triplet or quartet. Clearly the focal lengths of such a system are just half the corresponding values \( F_+ \) and \( F_- \) for the doublets of which the quartet is composed. Since the magnification is unity, the source and image points are each one (quartet) focal length from the entrance and exit focal planes, respectively (the entrance and exit principal planes are by definition one focal length from the corresponding focal planes). Thus the optics of a symmetric quartet system is completely defined by the doublet focal lengths \( F_+ \) and \( F_- \) and the object distance \( d_0 \).

The behavior of quadrupole systems for particles of slightly different momentum than the design value can be evaluated by means of the equivalent plane approach. Equation (28) is used to determine the variation of \( \alpha_1 \ell_1 \) with momentum. An analogous expression gives the variation in \( \alpha_2 \ell_2 \). Equations (5) and (7) can then be used to calculate the "correct" source distances for this momentum (the correct source distance is generally different in the two planes). Doublet focal lengths for the off-momentum particles are calculated by means of Eqs. (31) and (32). Knowing the correct source distances and focal lengths for the off-momentum particles, we can construct the equivalent planes and trace rays through the system. It will be found that the system is astigmatic for off-momentum particles. This will result in a finite-sized image from a point source with a broad momentum spread. Obviously this procedure is very complicated and tedious; we will not pursue it farther. We only want to point out that the method described here can be used to determine the momentum acceptance of a quadrupole system, and to calculate corrections to the simple-minded resolution calculation that is indicated two paragraphs above.
REFERENCES


5. The matrix method is discussed by a number of authors, e.g., P. A. Sturrock, Static and Dynamic Electron Optics (Cambridge Press, 1955); E. D. Courant and H. S. Snyder, Ann. Phys. 3, 1 (1958); S. Penner, Rev. Sci. Instr. (to be published).

ELECTRON BEAM DEFLECTION SYSTEMS FOR THE MONSTER

by

S. Penner

INTRODUCTION

In order to use the electron beam from the Monster effectively, an elaborate beam switchyard will be needed. The primary function of the switchyard is to guide the electron beam into a number of well-separated target areas. There are several other important functions, such as momentum analysis of the beam, which we will discuss presently.

The parameters of the accelerator which are important for switchyard design are: electron energy in the range 15-45 Bev, energy spread of $\pm 0.5\%$, a beam diameter of about $\frac{1}{2}$ in., angular divergence of $\pm 10^{-4}$ rad, pulse length about 2 $\mu$sec, repetition rate 360 pulses/sec, beam current of 60 $\mu$A, and rf frequency of 3000 Mc/sec. All of these parameters are tentative, or in some cases only rough estimates, but they will serve for the discussion to follow.

Most of the ideas discussed in this report grew out of discussions among the members of the Summer Group. We feel it is worthwhile to collect all the ideas relating to the beam switchyard in one place to serve as a basis for further study.
PULSED DEFLECTION SYSTEMS

It now appears that bubble chambers can be used very effectively with the Monster. Since the maximum cycling rate of bubble chambers is much lower than the accelerator's repetition rate, it will be desirable to have the ability to deflect an occasional beam pulse into a bubble-chamber channel, as shown schematically in Fig. 1. An important parameter for laying out the switchyard area is the required drift distance D. Suppose that the pulsed magnet is an iron-free device operating at a peak field of 50 kg. The corresponding bending radius for 45-Bev electrons is 100 ft. If the pulsed magnet is 5-ft long, the deflection angle is 0.05 rad (2.8°). For reasons of economy the dc magnet system which finally guides the deflected beam to the bubble-chamber area should employ H-configuration magnets. To insure that the undeflected beam not hit the return leg of the first dc magnet, the pulsed magnet should provide a deflection of at least 2 ft. The distance D must therefore be at least 2/0.05 = 40 ft. In this

distance, the beam size increases by \( 2 \times 10^{-4} \text{ rad} \times 40 \text{ ft} \approx 0.1 \text{ in.} \), so that the gap height of the dc magnet need not be very large. Owing to its 1% energy spread, the horizontal beam size at the deflecting magnet is \( 0.01 \times 2 \text{ ft} \approx 1/4 \text{ in.} \), which is negligible. It thus appears that pulsed defectors of this type are feasible and highly desirable. If longer drift distances are allowable, a longer magnet with an iron return path can be used, with considerable savings.

**RADIOFREQUENCY DEFLECTORS**

The radiofrequency bunching of the electron beam means that discrete pulses of electrons are delivered at intervals of 0.33 \( \mu \text{sec} \). Since this is considerably less than the resolving time of present-day electronics, one can conceive of situations in which the useful counting rate in some types of experiments is practically unchanged if only every second rf bunch is delivered to the experiment. In other words, if the time between delivered bunches is larger than the resolving time of the electronics, chance coincidences can occur only between events originating in the same rf bunch. We can make use of this fact if we can design an rf deflector which operates at a subharmonic of the accelerator frequency, sending alternate rf bunches to different experimental areas. The setup of Fig. 2 shows a situation in which a
deflector operating at \( \frac{1}{2} \) the accelerator frequency deflects alternate bunches to left and right.

The difficulty with this scheme is the smallness of the deflection angles that the rf deflector is capable of providing. Suppose the deflector is 20-ft long and has an electric field gradient of 4.5 MeV/ft. Then the angular deflection of a 45-Bev electron riding the crest of the rf wave is only \( 2 \times 10^{-3} \) rad. At the entrance to the dc deflector, assumed to be 100 ft away, the beam has been deflected by only 2.4 in. This is obviously not enough space to allow installation of conventional deflecting magnets, but a quadrupole magnet can be located here. A quadrupole with a focal length of -20 ft provides an additional deflection of \( 10^{-2} \) rad, so that the deflected beam emerges from the quadrupole at an angle \( \theta = 1.2 \times 10^{-2} \) rad. Therefore, 150 ft past the quadrupole the beam has been deflected by 2 ft, which is then enough to begin an H-magnet deflection system. To obtain a -20 ft focal length in the quadrupole is not too difficult. If the magnet is 5-ft long, the gradient required is 4.2 kg/in. Since the beam is less than 3 in. from the axis at the exit of the quadrupole, a maximum field of 13 kg is ample. This value is quite reasonable for a Panofsky-type quadrupole.

This scheme as described above will not work, because the initial angular divergence of the beam of \( \pm 10^{-4} \) rad means that the beam has been fanned out to a width of about 3 in. at the entrance of the H magnet. It will therefore be necessary to install one or more additional quadrupoles in the system to hold the beam together. The magneto-optics of this system require careful study, but the scheme is workable in principle. The main point at present is that such a system requires a space of at least 250 ft along the undeflected beam axis.

**DC DEFLECTION SYSTEMS**

The pulsed and rf deflectors described serve to get the electron beam headed into various channels. Whether these schemes are employed or not, it will be necessary to use large dc deflection systems to provide deflections of \( 10^0-30^0 \) in order that the various experimental
areas can be reasonably well shielded and separated from each other. For many experiments it will also be necessary to provide some momentum analysis. The deflecting systems should be of the zero-dispersion ("achromatic") type, because one may wish to conduct the beam several hundred feet from the deflection system to an experiment. For the same reason, it is important that very high quality magnet optics be used. A deflection system which satisfies these criteria is shown in Fig. 3.

In this system, the electron beam incident on the deflecting magnet \(M_1\) is brought to a focus (both horizontally and vertically) on the symmetry axis \(SS\) in the center of the quadrupole \(Q\). This quadrupole focuses in the horizontal plane, serving the function of recollecting electrons of different momenta, so that after the beam passes through deflecting magnet \(M_2\) it is achromatic. It can be shown that if the angular divergence of the beam is very small compared to one radian, and if the diameter of the beam is very small compared to the bending

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2. This system was first proposed by K. L. Brown (private communication). The properties of the system are discussed in S. Penner, "Calculations of properties of magnetic deflection systems," National Bureau of Standards Internal Report, 12 November 1958 (unpublished), and to be published in Rev. Sci. Instr.
radii in $M_1$ and $M_2$, the beam out of the system has zero dispersion no matter how large its momentum spread is. Momentum analysis is achieved by a defining collimator located inside $Q$ on the symmetry axis. Thus $M_2$ serves the additional important function of separating the electron beam from the background generated in the collimator.

In this system it can be shown that if vertical focusing is achieved by using wedge focusing at the exit edge of $M_1$, then

$$D = 2 \rho \cot \theta,$$  \hspace{1cm} (1)

where $\rho$ is the radius of curvature of $M_1$. Since $\rho = 250$ ft for 45-Bev electrons in a 20-kg field, $D$ must be greater than 865 ft if $\theta$ is less than 30°. This is clearly impractical, so that vertical focusing must be obtained by means other than wedge focusing. Non-uniform fields in $M_1$ and $M_2$ (i.e., $n$ values) could be used to provide the vertical focusing, but this would substantially increase the cost of the magnets. The obvious solution is to use auxiliary quadrupoles to provide the focusing, as shown in Fig. 4. This system is the same as that shown in Fig. 3, except that the quadrupole pair $F_1,F_2$ and their mirror image $F_2,F_1$ have been added for the purpose of making $D$ much less than the value given by Eq. (1). We now
proceed to estimate the performance of this system under the following approximations: (a) All focusing effects of the deflecting magnets are negligible. This is satisfactory if \( D \ll \rho \cot \theta \). (b) We employ the small-angle approximation \( \theta = \sin \theta = \tan \theta \), and \( \cos \theta = 1 \).

(c) We regard \( F_1,F_2 \) as a thin lens located at the entrance of \( M_1 \), with equal horizontal and vertical focal lengths. This is justified because the lengths of \( F_1 \) and \( F_2 \) can be much less than the length of \( M_1 \) or the distance \( D \).

Using these approximations it is easy to see that the focal length \( F \) of the combination \( F_1,F_2 \) must be

\[
F = \rho \theta + D .
\]  

Then the radius \( r \) of the electron beam at the focal point on the symmetry axis is

\[
r = \phi F ,
\]  

where \( \phi \) is the angular divergence of the beam \( (10^{-4} \text{ rad}) \).

Different momentum components of the beam, after leaving \( M_1 \), appear to be diverging from the center of \( M_1 \). The horizontal displacement \( x \) at the symmetry axis of electrons with \( \phi = 0 \) and momentum \( P \pm \Delta P \) is

\[
x = \frac{\Delta P}{P} \left( D + \frac{\theta}{2} \right) ,
\]  

where \( P \) is the central momentum. The best momentum resolution that can be achieved by this system is given by the value of \( \Delta P/P \) for which \( x = r \). Thus,

\[
\frac{\Delta P}{P_{\text{min}}} = \frac{\phi F}{\theta[D + (\theta/2)\rho]} ,
\]

or

\[
\frac{\Delta P}{P_{\text{min}}} = \frac{\phi[\theta + (D/\rho)]}{\theta[(\theta/2) + (D/\rho)]} .
\]  

- 7 -
As trial parameters, let us choose \( D = 150 \text{ ft}, \rho = 250 \text{ ft}, \theta = 0.175 \text{ rad (10°)}, \) and \( \phi = 10^{-4} \text{ rad}. \) For these values, \((\Delta P/P)_{\text{min}} = 0.065\% .\) This value is the half-width at half-maximum of the minimum momentum acceptance, so that the parameters given above represent a satisfactory system. We now proceed to estimate some of the other parameters of the system.

It turns out that a pair of quadrupoles like \( F_1, F_2 \) have equal focal lengths in the two transverse planes when the lengths \( l \) and field gradients \( \partial B/\partial x \) of the two quadrupoles are equal. In thin-lens approximation, the field gradient (in kg/in.) required to obtain focal length \( F \) is

\[
\frac{\partial B}{\partial x} = \frac{P}{l\left[P(l + t)\right]^{\frac{3}{2}}},
\]

where \( P \) is the particle momentum (kg-in.), \( t \) is the spacing between \( F_1 \) and \( F_2 \), and all dimensions are in inches.\(^3\) Using the parameters listed above, and choosing \( t = 0, \ l = 36 \text{ in.} \), we find that \( \partial B/\partial x = 6 \text{ kg/in.} \) for 45-Bev electrons. This is not an unreasonable value since the aperture radius of these quadrupoles can be very small (\( \sim 1 \text{ in.} \)).

The focal length \( F_Q \) of the central quadrupole \( Q \) must be half the distance from \( Q \) to the center of \( M_1 \), in order to recollect the off-momentum components of the beam. That is,

\[
F_Q = \frac{1}{2}\left[D + \left(\frac{\theta}{2}\right)\rho\right].
\]

With the values of \( D, \theta, \) and \( \rho \) given above, \( F_Q = 86 \text{ ft}. \) Its field gradient is given by

\[
(\partial B/\partial x)_Q = \frac{P}{F_Q^3} l_Q.
\]

---

\(^3\) It can be shown that because of a peculiar cancellation in the approximate thin-lens formula, the actual value of \( \partial B/\partial x \) required is \((3/2)^{\frac{3}{2}} \) times the value obtained from Eq. (6). This was pointed out by K. L. Brown (private communication). The values of \( \partial B/\partial x \) quoted in the report have not been corrected by this factor.
where $l_Q$ is the length of $Q$. Choosing $l_Q = 3$ ft, we find $(\partial B/\partial x)_Q = 1.5$ kg/in. If the horizontal aperture dimension of $Q$ is 18 in., then by Eq. (4) the momentum acceptance of the system is a generous 4.5%, and the maximum magnetic field is 13.5 kg. This field value is reasonable since $Q$ should be a Panofsky-type quadrupole. The vertical size of the beam in $Q$ as given by Eq. (3) is $2r < \frac{1}{16}$ in.

We now proceed to calculate the aperture-size requirements for the deflecting magnets. This will essentially determine the cost of the system, since all quadrupole magnets are comparatively small. At the entrance of $M_1$, the electron beam has a diameter of $\frac{1}{2}$ in. At the exit of $M_1$ the horizontal beam size $w$ is, to first order,

$$w = d_0[1 - (\theta_0/F)] + 2\theta_0 \rho,$$

where $d_0$ is the initial beam diameter. Evaluating this expression with $d_0 = \frac{1}{2}$ in., $\theta = 0.175$ rad, $\phi = 10^{-4}$ rad, and $\rho = 250$ ft, we find from Eq. (2) that $F = 194$ ft, and $w = 3/8$ in. Thus the gap width in $M_1$ can be very small. The next important point to notice is that in the horizontal plane the central quadrupole $Q$ acts as a field lens, i.e., the beam is focused at $Q$, and $Q$ images the aperture of $M_1$ on $M_2$. Thus, the horizontal beam dimension in $M_2$ is very nearly the same as in $M_1$. In the vertical plane the situation is not so good. The vertical beam size decreases uniformly from $\frac{1}{2}$ in. at the entrance to $M_1$ to $1/4$ in. at $Q$. Since $Q$ is defocusing vertically, it tends to blow the beam up. The angular deflection given the electrons by $Q$ is $\Delta \phi = y/F_Q$, where $y (= \theta F)$ is the vertical displacement of the beam at $Q$. It can be shown from this that the vertical size of the beam at the exit of $M_2$ is

$$h = 2\phi F[2 + (F/F_Q)] + d_0.$$  

For the numerical values previously chosen, $h = 2.5$ in. This is an unfortunately large value because the cost of the deflecting magnets $M_1$ and $M_2$ is approximately proportional to their gap height. The value of $h$ may be reduced in several ways. If it should turn out that $\phi$ is considerably less than $10^{-4}$ rad, then the problem is resolved. (The momentum resolution would also be improved.) The value
of \( h \) can be reduced by decreasing \( F_1 \), but this would require that the quadrupoles \( F_1,F_2 \) be longer and would hurt the momentum resolution.

Note that Eq. (5) can be expressed as

\[
\left( \frac{\Delta P}{P} \right)_{\text{min}} = \frac{\phi}{\theta} \frac{F}{F - (\theta/2)\rho}.
\]

A good way of decreasing \( h \) is to change the vertical focal length of \( Q \) without changing its horizontal focal length. This can be done, for example, by replacing the single quadrupole \( Q \) by a symmetric triplet having positive focal length \( F_Q \) in both planes. If this is done then \( Q \) acts as a field lens in the vertical as well as the horizontal plane, so that the vertical beam size in \( M_2 \) is the same as in \( M_1 \) (i.e., \( \frac{1}{2} \)-in. maximum). It would then be conservative to build \( M_1 \) and \( M_2 \) with 1-in. gaps. A practical solution is to build the deflection systems according to this procedure. If, when the machine is built, it turns out that the angular spread of the beam is much less than \( 10^{-4} \) rad, the central triplet can be replaced by a single quadrupole, thus simplifying the system and making its operation much easier.

When the Monster is first built, it will probably operate at 20 Bev or less, but eventually 45 Bev could be reached. We suggest that the length of the deflection system and all of the quadrupoles be designed for the higher energy. Since the deflecting magnets are so long (4.4 ft per degree of bend at 45 Bev), they will be built in relatively short (say, 10-ft) straight sections. Thus when higher energies are reached, it is only necessary to add more sections to the deflecting magnets.

It should be realized that the discussion above is only semi-quantitative. Precise calculations of the magnetic optics must be completed before construction is undertaken. We believe, however, that the deflection-system dimensions shown in Fig. 5 can serve as a basis for determining the layout of the target area.
Dimension $A = 250 \text{ ft} \times \tan \left( \theta / 2 \right)$
GENERAL DESIGN PRINCIPLES OF THE EXPERIMENTAL AREAS
FOR PROJECT M

By

B. Cork

The design of the experimental areas should be made so that steering magnets, targets, focusing magnets and shielding can be easily rearranged as the experimental requirements change, and as more experimental equipment becomes available. This of course requires good coverage of the various areas by cranes, magnet power, and cooling water facilities.

The number of interesting high-energy-physics experiments is so great that the experimental areas should be designed so that at least several experiments can be set up simultaneously and others can be in the process of assembling, rearranging, debugging, or disassembling while the machine is operating. This of course requires rather thorough and continuous planning and scheduling, as well as good radiation shielding between all beam areas. Since many experiments will not be able to use a thick target and all of the beam, the areas should be designed for multiple simultaneous use. This is especially true for experiments using scintillation and Cerenkov counters because of the low duty cycle of the machine.

Simultaneous use of the beam can be achieved by combinations of several methods:

1. The beam can go through a thin target for the first experiment and then through succeeding thin targets for other experiments. If one or more of the experiments require a thick target in order to get a high enough counting rate, these experiments can be located near the end of the channel and scheduled to take most, or all, of the beam at the required time.

2. A second method of sharing the beam is by allowing the electron beam to go through a "small" target, by steering the
electron beam so that only a fraction of the beam strikes the first target, some more strikes the second target, etc. This requires care in the design of the target, steering, and monitoring.

3. To maintain a favorable duty cycle, a third method of sharing the beam is by the use of a radiofrequency beam-splitter. This is a device similar to an rf spectrometer except that it will operate at a sub-harmonic of the radiofrequency of the accelerator. Thus, during each 2 μsec beam pulse a part of each pulse will be incident on either of two or more channels. The requirements for such a beamsplitter are rather severe in peak power, length of separator and in drift distance. Since photomultipliers will count at rates that are considerably higher than a megacycle, but not as high as 3000 megacycles per second, this device will be useful both for multiple-beam experiments where a long duty cycle is important, and for experiments where the interesting particles are counted "after" the pulsed beam from the accelerator, i.e., muon experiments. These beams will also be useful with bubble chambers and spark chambers. If methods are developed for modulating the gain of a photomultiplier at a high frequency, then time-of-flight experiments can be done using the coherent electron beam from the accelerator.

Other Design Considerations

Since the experimental areas will be large and rather discontinuous, the design of each area should be such that magnets, shielding, and experimental equipment can be moved and arranged rapidly. For many of the areas a beam height of 5 to 6 feet appears most reasonable. The network of facilities, in trenches, could be well grouped so that if an unusual clearance below the beam line is required, it would be possible to remove sections of the floor by means of a crane in, say, 100-square-foot panels, and then excavate to obtain the required clearance. This is much less expensive than buying and moving the added shielding and platforms required for an area with a higher beam height.
Areas which are in continuous use for experiments should be well covered with a 50-ton crane. Other relatively permanent areas, reached by beam steering with dc or pulsed magnets, could be built so that access will be only with portable cranes. Special buildings will be necessary for these areas so that the crane can have access.

The centipede type of device will be useful for these areas too. This is a system of hydraulic (air) driven feet that allows large loads of several tons per square foot, to be picked up a fraction of an inch. Then a horizontal component of force allows the load to be moved in the desired direction. Rates of a few feet per minute appear feasible with this method, and it should be possible to move loads that are as large as $10^3$ tons. An advantage of this device is that heavy loads can be moved over rough surfaces. Actually it appears quite reasonable to build an entire experiment on a platform that is moved by means of the centipede type of device. Thus, much of the "set-up" of an experiment could be done before the apparatus was moved into the final beam area. In some cases the air-film type of platform will be useful too.

The really difficult shielding problem is the high-energy muons, most of which are produced by pair production, at a rate of $10^{-3}$ per electron. Since 30-Bev muons have a range of 150 feet of concrete, care must be taken in collimating the electron beam a considerable distance before the exit end of the accelerator, and the slits for energy selectors will have to be located so that the muons will not interfere with the experiments and experimenters.

Fortunately it is possible to deflect the lower momentum electrons away from the axis of the accelerator so that the pair-produced muons need not be directed into the experimental areas.
SOME CONSIDERATIONS IN THE DESIGN OF A TARGET AREA
FOR PHOTON-BEAM EXPERIMENT

By

R. W. Kenney

I. INTRODUCTION

The end station for photon-beam experiments requires an intense beam of small diameter and free of charged particles; and, ideally, the experimental area should be free of background of all sorts. The 45-Bev, 60-μa electron beam must be ditched after traversing the photon radiator. The photon-beam diameter should be limited by collimation with minimum sacrifice of total photon flux. The electron beam incident upon the photon radiator is assumed to have already been analyzed, with the beam spot-size 1/4 inch in diameter and a divergence angle of \( 10^{-4} \sqrt{E} \) radians, where \( E \) is in Bev.

Three possible arrangements are discussed, each having identical radiator and photon collimation geometry and therefore the same photon-beam characteristics. The basic shielding considerations outlined in the Project M proposal (April 1957) have been used in this rough design of end-station geometries; each proposal has satisfactory nucleon and electron-proton shielding but somewhat different \( \mu \)-meson background flux conditions.

The primary electron beam ditching technique and the photon beam characteristics are common to all three proposals, and will be discussed first.

After passing through the radiator, the electron beam must be ditched in an effective manner to suppress background arising from interactions with the beam-stopping medium. In each alternative layout below, an 18 kilogauss magnet bends the electron beam downward and directs it into a water beam-stopper. A high-field cryogenic (not superconducting) ditching magnet was considered in an effort to minimize the length of the
end station. The minimum field for a sensible advantage is 50 kg, and
a 35° bend would be desirable. R. F. Post estimates the cost between
2.5 and 4 million dollars, varying approximately as $B^3$. In comparison
with the $350,000 cost of an adequate conventional magnet, it is clear
that cryogenic magnets are of no help in this end station design.

A water beam-stopper 26 radiation lengths long (31 feet) and of
adequate diameter to contain the shower is inexpensive and easy to build.
Nearly all the beam energy is dissipated in the electron-photon shower,
so that only this part of the beam interaction is considered. Water is
chosen because of its high thermal capacity, reasonable radiation length,
ease of handling and extraction of heat, and attractive radioactivity
properties. The high-flux, high-energy bombardment of the beam stopper
should produce $T, C^{14}, Be^7, Be^{10},$ and $F^{18}$ on $H_2O$ by electro-
magnetic- and nucleon-initiated processes. Yields have not been
estimated. Each species (except $T$) can be easily handled by appropriate
chemical extraction techniques on a continuous-flow basis. Excessive
$T$ contamination will require a change of water. $C^{14}$ will probably exist
as CO and will pass into the vapor phase if exposed to air as a fine
spray. Be will exist as Be(OH)$_2$ and may be extracted by solubility
or ion-exchange techniques. F may be easily reacted. (Dr. Amos
Newton of URL has been very helpful in discussing these problems.)
Impurities in the water may also be activated.

If the beam-stopping geometry were to be ideally efficient, a
cooling-water flow of $> 600$ gal/min would be required. A reasonable
design is only ~20% efficient, and cooling-water flow of 3000 gal/min
is required to achieve nearly complete absorption of the shower. A
possible design is shown in Fig. 1. A closed circulating system with
$H_2$ and $O_2$ recombining catalyzer will be required to contain the
radioactive products.

Target corrosion problems which are pertinent to the beam stopper
design have arisen at the 60" Crocker cyclotron at Berkeley. Certain metal
foils which are in contact with $H_2O$ have been damaged in bombardments
by 45-Mev a-particle currents of 100 $\mu$A/cm$^2$ lasting more than 5 hours.
Probably $H_2O_2$, produced by ionization, attacks the metal target
containers. This problem should be investigated further.
The photon radiator should be approximately 0.1 radiation lengths (of Ta) in order to convert a reasonable fraction of the 2.7 MW electron beam energy into photons without undue angular spreading of the photon beam resulting from electron multiple scattering preceding the radiation events. The electron beam deposits only 70 watts of thermal energy in the 0.1 radiation length radiator (which can be air-cooled).

It appears wise to collimate the photon beam in order to clip off the angular tail arising from multiple electron scattering in the radiator. Various alternatives place the downstream limit of the shielding wall from 105 to 160 feet from the radiator. A 1/2"-diameter, 10'-long collimator placed at 50 feet gives a reasonable beam spot and penumbra size and passes ~95% of the beam (see Fig. 2). It is 13 nuclear-absorption mean-free paths in length to attenuate pions. Thirteen kilowatts of photon beam power is dissipated in the water-cooled copper collimator. Cooling-water flow of 10 gpm is adequate and no heating problems arise in this case, where the incident flux is distributed over a rather large area. The intensity in the photon beam as a function of radius is shown in Fig. 3 for the position 150' from the radiator. The total flux in the penumbra is 18% of the beam for 1/2" collimator. The flux from a 1/4" collimator is 85% penumbra.

A photon beam stopper of approximately 20 radiation lengths of water should be employed in order to contain radioactive products arising from disposal of the beam. It must dissipate approximately 270 kw and should be buried in an adequate earth shield for the attenuation of muons and the nucleon cascade.

The various proposals differ in the shielding design and in the placement of the beam stopper. The most serious shielding problem arises from the μ mesons, which have only Coulomb (ionization) interactions with matter. If the in-line shielding for μ mesons is adequate one has more than adequate in-line shielding for all other particles. Those reactions giving products at angles greater than 10° to 15° will require shielding in addition to that provided for μ mesons; a 35' concrete forward shield is proposed in all cases to attenuate the nucleon cascade adequately. Transverse shielding depends upon the details of
FIG. 2--Total transmitted flux \( \propto \int_0^r I(r) \, r \, dr \) through a hole 50 ft from a point source.
Fraction of total collimated flux passed within circle of given radius $\propto \int_0^\theta I(\theta) \, \theta \, d\theta$

**FIG. 3**--Beam radius in inches at 150 feet from radiator
the particular geometry under consideration. In all cases, the photon showers which arise in the 10'-long collimator are clearly attenuated and may be neglected. Their products (muons and nucleon cascades) are, however, the most penetrating components of the secondary-particle fluxes.

The following processes lead to muon production:

1. Electromagnetic muon pair-production.

\[ \sigma \propto l^2 \]

\[ \mu \]

This process is easily calculable in detail from Bethe-Heitler pair theory. Its total yield is given by Drell to be 11 times that of pion pair-production.

2a. Pion pair-production, giving muons as decay products.

\[ \sigma \propto 1 \]

\[ \pi \]

2b. Drell pion production, also giving muons as decay products with a total yield \( \sigma \propto 20 \).
2c. Multiple pion production, as given in the Stanford Proposal, pg. II-C-15, also yielding muon decays.

The first three processes give closely collimated pion and muon yields. Multiple pion production gives its high-energy yield in a cone of \( \approx 10^\circ \) half-angle.

The basis for muon-shield calculations is as follows:

1. Absolute yields are calculated by Bethe-Heitler theory for \( \mu \) pairs, and in those cases where no pion absorption can be achieved, yields are multiplied by 3 to include Drell production of pions, all of which are assumed to decay.

2. Angular limits of production are taken as \( 100/E_{\mu,\text{MeV}} \) for \( \mu \) pairs, and \( 150/E_{\pi,\text{MeV}} \) for Drell pion production.

3. The multiple pion process is taken directly from the Stanford Proposal, pg. II-C-15, (adjusted for 60 \( \mu \)a beam) and its angular limits are taken to be \( 10^\circ \).

4. Pion pair-production \( (\sigma \approx 1) \) is ignored.

It is proposed first that complete containment of the \( \mu \) flux can be nearly achieved by extensive in-line shielding with only minimal sweeping of secondary particles. The \( \mu \) shield cost can be as low as $300,000 if surplus steel (4 cents/lb.) is available.

A second approach would be to allow the \( \mu \) mesons to flood the experimental area, knowing that no nuclear interactions would occur and that the counters could be located at great distances transverse to the beam and remain out of the \( \mu \) flux.
A third geometry calls for strong sweeping of the μ mesons and all other secondaries to create a clean experimental area. Vertical sweeping and beam bending is desirable (from the crane-cost standpoint, it is favorable to build deep, narrow magnet facilities); however, some μ flux may pass into the atmosphere. Extensive sweeping magnets will be required to direct all μ flux into the earth.

II. GEOMETRY TO CONTAIN μ FLUX COMPLETELY WITHIN EXTENSIVE SHIELDING

The highest energy μ mesons are those electromagnetically produced in pairs. Their production angle is $\theta \sim m_\mu \cdot c^2/E_\mu$. Hopefully, one can contain these within a long cylindrical iron shield whose diameter must exceed the probable transverse displacement from multiple scattering and whose length must equal the range of the highest energy μ. Minimal sweeping after the collimator can succeed in keeping long-range muons within the shielding while assuring a clean photon beam in the experimental area. (See Fig. 4)

Shielding Length Estimate

μ-meson yields based on (1) Bethe-Heitler pair theory, (2) 60 μamps of 45 Bev electrons incident upon a 0.1 radiation length radiator, (3) 10% of the photon beam lost upon the collimator edges, and (4) the requirement of $< 7$ particles/cm² sec transmitted by the shield show that only muons in the energy interval between 43.9 and 45 Bev may pass through the shield. We simplify matters by requiring that all μ mesons (45 Bev = 115' Fe) be stopped in the shield.

Shielding Diameter Estimate

The transverse displacement of μ mesons in scattering with energy loss is estimated from Eyges, Phys. Rev. 74, 1534 (1948). A 45 Bev μ meson has only an rms displacement of 22 cm at 85' depth in the shielding. Other considerations dictate the shield diameter, however. Shielding for muons from multiple-pion production and decay has already been estimated in the Stanford Proposal for a pion free decay length of 3' before any attenuation occurs. The results adjusted for
60 \, \mu \text{a} \, \text{beam intensity and using } S = 1 \, \text{(pg. II-C-15)} \text{ require that the shield be } 3' \, \text{diameter along its entire length. This estimate is based on a } 1'' \, \text{diameter axial hole, and a transmitted flux of } 10 \, \text{muons} / \text{cm}^2 \, \text{sec coming out of the shielding along its length at } \approx 10^0 \, \text{to the axis and of energies extending from 0 up to } 45 \, \text{Bev with a spectrum decreasing with energy (enfolding energy } = 2.6 \, \text{Bev). This process then dictates the shield diameter (apart from our consideration on the ditching magnet below) rather than the } \mu \, \text{pair production.}

It is proposed that the 3' (see below) diameter shield begin at the radiator with an axial hole of 1'' diameter through which the photon beam will pass. The electron beam ditching magnetic field should be as close to the radiator as possible in order to deflect small-angle charged pions into the shield where they will interact before decaying. (A 10-Bev pion requires only } 5.10^{-4} \, \text{of its dilated lifetime to transverse one nuclear absorption mean free path in iron.) Copper shielding can be used within the magnet gap to make the shielding as complete as possible.

**Beam Ditching**

Vertical bending places the primary beam stopper at the end of 80' of 18 kilogauss field where the electron beam is 12' below the beam line and deflected through 15.5'. Nucleon cascade secondaries must traverse the full forward shield to reach the experimental area, and additional concrete shielding should be placed closely about the stopper since no undue increase in the total shielding mass results. Long range (> 10 Bev) muons which can reach the experimental area must be produced at the unlikely angle of 22'.

The sweeping magnet which follows the collimator should be just long enough (10', < 20 kg) to clear the photon beam of charged particles, care being exercised to minimize the transverse displacement of the \( \mu \) mesons in the shielding. A small axial hole for photons should be the only opening in the final shielding. The usual 35-foot concrete neutron shield at the end is required, with an additional 4' cladding of ferrite concrete surrounding the Fe shield to assure the complete
FIG. 4—Extensive shielding geometry
absorption of small angle muons at the end of their range where scattering becomes serious. A roof shield is necessary, such as 13' concrete, to augment the steel shield in attenuating neutrons to the levels required for the table of transverse shields in the Stanford Proposal, pg. II-C-16. This should surround the Fe core on three sides, leaving the bottom face of the Fe core open to the earth up to the point where the beam stopper is located.

Secondary in the Shielding

In attempting to shield the experimental area from all radiation, the forward 35' concrete shield adequately attenuates the nucleon cascade neutrons, particularly with the cylindrical Fe shield as additional shielding. Secondary muons, pions, and electrons constitute the principal sources which must be attenuated. Negative particles from the radiator and their forward secondaries are deflected into the earth by the beam-ditching magnet. The positive particles from the radiator are deflected upward and pass out of the magnetic field deflected at angles $\theta$ which increase as the particle range $R$ decreases. The maximum transverse displacement of $\mu$ mesons in iron at the end of their ranges is given by

$$y = \theta R = 7 \sqrt{\frac{f}{1 + f}}$$

where $f = \text{secondary energy}/45 \text{ Bev}$, and $y$ is greatest for $f = 1$ where it equals 5' (no multiple scattering is considered and the effective ditching field is 12' wide). The 3'-radius shield thus seems inadequate for these particles. It is desirable to increase the thickness of the Fe shield on top to ~6'.

Secondary arising from the main collimator are swept from the photon beam by a 10' magnet running at 8 kg. Considering 45 Bev muons at 0°, for example, a 10'' lateral displacement is achieved at a point (75' from the collimator) where their multiple scattering lateral spread $< y > = 5''$. This would appear to be minimal sweeping, and the
magnetic field can be raised so that this displacement can be nearly tripled if so required. The Fe shield is adequate to contain the muons of highest energy. The soft electron-photon shower is very completely contained in the collimator itself. The cost of an Fe shield 6' radius along the entire length, is $340,000 at 4 cents/lb. for surplus iron.

Axial Hole

The axial beam hole through the shielding should be large enough so that precise alignment is not required, yet small enough to allow secondaries to be deflected into the shield in a minimum distance. A diameter of 1" to 1.5" is suggested between the radiator and the 10'-long, 1/2"-diameter collimator. The downstream diameter can then rise in steps from 1" to 3" at the downstream limit of the shielding.

Crane

A minimum crane span of 50' would be required to service this shielding and magnet arrangement adequately. A 25-ton crane of this span would cost of the order of $75,000.

III. MINIMUM SHIELDING GEOMETRY

If the extensive shielding required for complete containment appears to be too expensive, one may choose the opposite approach by allowing the muons to flood the experimental area while shielding against only the nucleon cascade. The beam-ditching magnet need only be 40' in length so that the beam-stopper is located off the beam axis, as shown in Fig. 5.

The axial hole should be ~1" diameter after the sweeping magnet (10' at 20 kg) in order to deflect the pions into the shielding in a minimum distance along the axis so that their nucleon cascades may be adequately attenuated in the 35' shield wall. This approach seems inadequate since no attempt is made to shield against \( \mu \) mesons and
FIG. 5.—Minimum shielding geometry.
severe handicaps may be placed on certain experimental arrangements under these conditions.

IV. STRONG SWEEPING GEOMETRY

A third type of geometry utilizes strong sweeping in order to deflect all $\mu$ mesons away from the experimental area. Containment of $\mu^+$ is achieved by re-bending them into the earth as shown in Fig. 6. Muon-pair yields were calculated above for the photon beam traversing an average radiator thickness of 1/20 radiation length. A crude estimate of the muon-pair yield from the collimator can be made by assuming no shower development and complete photon absorption in one radiation length. The collimator intercepts only $<10\%$ of all photon flux, so that the muon-pair yields from the collimator are $20/10$ of the radiator yield. It was seen that the radiator yield constituted a serious shielding problem, so it will be required that all radiator and collimator muons of energy greater than 8 Bev be adequately swept into the earth. The forward and upper shields will adequately attenuate the 8 Bev muons.

Charged secondaries from the radiator will be deflected vertically by the electron beam ditching magnet. As in the other geometries, the photon beam is closely surrounded by shielding so that pions can be deflected into the shielding as quickly as possible to avoid muon production by decay. Copper shielding will occupy the magnet air gap wherever it is not needed to pass photons or the electron beam. Sweeping magnets for muons will be constructed of solid-magnetized iron in order to conserve copper and power. Multiple muon scattering is small enough that extensive bending is possible in reasonable magnet structures. Figure 7 shows the expected lateral displacement of 45 Bev muons in Fe, calculated from Eyges, Phys. Rev. 74, 1534 (48), to include energy loss as well as the muon pair production angle.

Those positive muons which are deflected upward from the radiator will enter a solid iron magnet of proper shape and be deflected downward into the earth. This magnet's pole shape is shown in Fig. 6 (1' X 30', 15 kg).
FIG. 7--Lateral displacement due to emission angle plus multiple scattering for 45 Gev $\mu$-mesons in iron. (Range for 45 Gev $\mu$-mesons is 115 ft)
Charged secondaries from the collimator will be handled somewhat differently because the secondaries are produced outside the photon beam on the collimator edges. No attempt is made to deflect the secondaries within the collimator; rather, the absorption is more effective in attenuating the numbers of pions in the collimator wall. The forward secondaries are adequately deflected by the sweeping fields which follow the collimator. Multiple pion production flux, within a $10^\circ$ cone, will be attenuated by allowing the pions to traverse the 10'-long Fe collimator wall and absorb by nuclear interactions rather than decay. (The mean decay distance for 15-Bev pions is 750 meters, so 0.4% of the pions decay in 10 feet.) 150 pions/sec are expected to traverse the complete collimator shield. Some small-angle leakage occurs through the collimator hole, but the maximum intensity occurs at the collimator edge at an angle near $4 \times 10^{-3}$ radians and these pions are deflected by the sweep fields along with the other charged particles in the transmitted beam.

The photon-beam sweeping magnet is adequately long (10', 20 kg) but not excessively so, because the charged secondaries which it deflects upward must be rediflected downward by a solid steel magnet of the same type as that used for the radiator secondaries (see Fig. 8). The cost of the steel alone, at 4 cents/lb., for both collimator and radiator solid magnets is approximately $30,000, and only 3000 ampere-turns are required to magnetize the magnet. The magnet cost is unrealistic—probably low by factor $>5$ because machining is required for smooth fits to eliminate air gaps. No attempt has been made to design the magnets in detail. Basic roof-shielding consists of a concrete roof 15' thick to completely attenuate muons of energy $<8$ Bev.

The downstream face of the forward shielding wall is 140' from the radiator. This geometry is 20' shorter than the heavily shielded configuration already discussed. A saving in magnet (steel) costs can probably be effected, with somewhat less certain containment of all muon flux and with the added complication of having to excite and interlock sweeping magnets.
FIG. 8—Photon beam sweeping magnet
The entire end station should be depressed below grade or hidden behind adequate earth shielding to protect outside areas from scattered radiation. The details of this shield will depend upon the experimental plans; it should be expected that \( \sim 10\% \) of the photon beam flux (27 kw) might be absorbed in the experimental area. Heavier targets should be provided with auxiliary shielding placed closely about the target.

It is important that complete coverage by crane be available for the experimental area; 50-ton capacity is probably adequate.
A PROPOSED EXPERIMENT TO MEASURE THE MAGNETIC MOMENT OF THE ANTIPROTON

By

B. Cork

In view of the importance and complexity of a properly designed target area for the Monster, it is useful to consider the design of various possible experiments. A specific experiment, measurement of the magnetic moment of the antiproton, is considered here because of the problems of secondary-beam intensity and shielding.

The antiprotons are produced by high-energy electrons incident on a Be target. (See Fig. 1.) Fairly low energy antiprotons produced in the forward direction are deflected and then focussed by strong-focussing magnetic quadrupoles. It is assumed that antiprotons have to be scattered from a second target in order to become polarized and that these antiprotons will not be depolarized by the magnetic quadrupoles. Polarized antiprotons are then precessed in an axially symmetric magnetic field that is produced by a long solenoidal magnet. These antiprotons are then scattered from a third target and the left-right asymmetry is measured as a function of the magnitude and direction of the magnetic field. From the precession frequency, the magnetic moment is deduced.

The following assumptions are made: an electron beam of 20 microamperes and 25 Bev is incident on a target that is 1/2 of a radiation length in thickness. The yield of $\bar{p}$ for 25 Bev/c incident electrons has been calculated by Ballam$^1$ using the model of Drell$^2$. The peak yield is at about 10 Bev/c but decreases only slightly at 2 Bev/c where the yield is $5 \times 10^{10}$ per (sec x sr x Bev/c). There are $4 \times 10^5$ antiprotons per second accepted by a magnet system with momentum acceptance $\pm$4% and solid angle $10^{-3}$sr.

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$^1$J. S. Ballam, M-200-8, August 1960

$^2$S. Drell, M-200-7, August 1960
The differential cross section for 2 Bev antiprotons from complex nuclei are not known, but the calculated and observed elastic cross-sections of 900 Mev/c antiprotons from beryllium and carbon are known (UCRL 8746) to be 1 barn per steradian at 10° (center of mass). The calculated polarization of the scattered \( \bar{p} \) is approximately 50%.

The yield of polarized \( \bar{p} \) is then (assuming a 0.7-cm-thick beryllium polarizer, a cross section of one barn per steradian, and a strong-focussing quadrupole system with a solid angle of \( 5 \times 10^{-3} \) steradian)

\[
\frac{d\sigma}{d\Omega} = N n \frac{d\Omega}{d\Omega} = 4.3 \times 10^{-4} N
\]

or for \( 4 \times 10^6 \bar{p} \) per second incident on the polarizer, a yield of \( 1.75 \times 10^{3} \) polarized \( \bar{p} \) per second.

To precess the polarized antiprotons a solenoidal field is used. The experiment actually measures the precessional frequency \( \omega \) which is related to the gyromagnetic ratio \( g \) by the relation

\[
\omega = g \frac{eH}{2Mc}
\]

where \( M \) is the mass of the antiproton, and \( H \) is the axial magnetic field. The magnetic moment \( \mu \) is related to \( g \) and the spin \( I \) by

\[
\mu = gI
\]

If \( g \) for the antiproton is the same as that for the proton, and the solenoidal field strength is \( 10^4 \) gauss, the average frequency of rotation of the antiproton spin is \( 4.3 \times 10^7 \) turns per second.

If \( \beta \simeq 1 \), to precess one turn an antiproton must travel in the magnetic field a distance
\[ d = \frac{3 \times 10^{10}}{4.3 \times 10^7} = 7 \text{ meters.} \]

The efficiency for detecting left-right scattering can be made high by surrounding the scatterer with scintillation counters, with each angular interval divided into two or more sections.

Since the elastic-scattering cross-section in the angular interval from 5 to 38 degrees C.M. is approximately one fourth of the total cross-section and the polarization is 50%, the yield of left-right scattering is of the order of a few hundred per second. This is adequate to make a measurement of \( g \) to a statistical accuracy of 1% in a few hours.

**Additional Comments**

1. Although the yield of \( \bar{p} \) is greatest in the forward direction, the background due to electrons may well be less if the \( \bar{p} \) beam is taken off at an angle of perhaps 5 degrees.

2. The time-of-flight system for identifying \( \bar{p} \) and rejecting \( \pi^-, K^- \) and electrons will have to be approximately 25 meters long and include a gas Cerenkov counter, using the present photomultiplier coincidence circuits (WRL-8851). The counting rate after the polarizer is of the order of 15 \( \bar{p} \) per accelerator pulse (1.5 microseconds).

3. There are several sources of charged-particle background. The number of electrons scattered at a 5 degree laboratory angle from a 1/2 radiation length beryllium target is
   a. Multiple Scattering.

\[ \theta_s \approx \frac{21}{E} \frac{1/2}{t} \]

If \( t = 1/2 \), and \( E = 10,000 \text{ Mev} \), then \( \theta_s \approx 0.1^0 \). Thus for \( \theta = 5^0 \), the multiple scattering contributes very little background.
b. Bremsstrahlung.

For incident electron energy $E_1 = 25,000$ MeV and an electron of $E_2 = 2,000$ MeV produced by bremsstrahlung, an approximate formula is:

$$\frac{d^2\sigma}{d\Omega dE} \approx -\frac{\alpha^2}{4\pi} \times \frac{(E_1^2 + E_2^2)^2}{E_1 E_2 (E_1 - E_2)} \times \ln \left( \frac{E_1}{\mu} \right) \frac{\cos^2 \theta/2}{\sin^4 \theta/2}$$

where $r_0$ is the classical electron radius, and $\mu$ is the electron mass. At $5^\circ$, $d\sigma/d\Omega = 10^{-32}$, and the yield of electrons is

$$Y/sec = 1000 \text{ electrons/sec}$$

due to bremsstrahlung, and at $2^\circ$ the background goes up an order of magnitude.

4. The cross-section for production of pions at these angles is best estimated from the statistical model calculations of Hagedorn and is of the order of 10 times the $\bar{p}$ production. The cross section for scattering of 2 Bev/c electrons and pions at $10^\circ$ from target $T_2$ is small compared to the scattering of $\bar{p}$. At 2 Bev/c, the root-mean-square scattering angle is $\pm 1/5$ degree. The nuclear scattering of 2 Bev/c pions at $10^\circ$ is approximately 10% of the $\bar{p}$ cross section.

5. Muon background should be even smaller because the pions have not had time to decay before reaching the region of the final detector. At 2 Bev the relative yield of pair-produced muons is small compared to the pion yield. However, care should be taken that muons produced by the direct primary beam do not get into the final detector.

6. The assumed solid angle of $10^{-3}$ steradian from target $T_1$ is small. It should be possible to increase this solid angle an order of magnitude for momenta of 2 Bev/c. It is desirable that the solid angle be kept as large as possible so that background due to neutrons and room background be small.
7. The thickness of target $T_2$ can be increased, especially if the primary electron beam is low. The best compromise will depend upon the available beam intensity.

8. This general layout of experiment can be modified slightly to investigate the production of particles of mass intermediate to that of the pion and proton, or to investigate the production of deuterons or heavier mass particles.

Summary

It is concluded that a 20 $\mu$A beam of 25 Bev electrons is very favorable for producing a sufficient flux of antiprotons so that the magnetic moment of the antiproton can be measured. Because the duty cycle of the machine is short, special care will be required to use scintillation counter and time-of-flight techniques for identification of antiprotons.

The design of an experimental area and the experimental apparatus for this experiment is a rather modest extrapolation of present techniques used in high-energy-physics experiments.
ESTIMATES OF NEUTRINO FLUXES FROM M

By

W. K. H. Panofsky

In this note we give the neutrino flux estimates resulting from the \( \pi \)-\( \mu \) decay only, ignoring the \( K \)-neutrino yields which might be equal to a substantial fraction of the pion-decay neutrinos.

Let

\[
\begin{align*}
    m_\pi &= \text{pion rest mass}, \\
    p_\pi &= \text{pion laboratory momentum} = \beta m_\pi c, \\
    \gamma &= \frac{1}{(1 - \beta^2)^{1/2}}, \\
    p_0 &= 29.4 \text{ MeV/c} = \text{momentum of muon and neutrino in pion rest frame}, \\
    \theta_0 &= \text{decay angle of neutrino in pion rest frame}, \\
    p_\nu &= \text{neutrino laboratory momentum}, \\
    \tan \varnothing &= \frac{1}{\gamma} \frac{1}{2} \tan \theta_0 = \text{tangent of laboratory angle of neutrino decay cone about direction of pion}, \\
    \theta_\pi &= \text{lab. emission angle of pion relative to direction of initial electron beam}, \text{and} \\
    \tau_0 &= \text{proper mean lifetime of pion}.
\end{align*}
\]

The kinematical relations are shown in Fig. 1.
FIG. 1--The kinematical relations.

Analysis of the decay kinematics and the appropriate relativistic transformations shows that the laboratory spectrum $P(p_\nu)dp_\nu$ of the neutrinos is given by

$$P(p_\nu)dp_\nu \begin{cases} = 1 & \gamma p_0 (1 - \beta) < p_\nu < \gamma p_0 (1 + \beta), \\ = 0 & p_\nu < \gamma p_0 (1 - \beta), \\ = 0 & p_\nu > \gamma p_0 (1 + \beta). \end{cases}$$

We can neglect the lower bound of this rectangular spectrum; numerically we thus put

$$P(p_\nu)dp_\nu \begin{cases} = \frac{\beta m c}{2p_\pi p_0} dp_\nu - \frac{1}{0.425 p_\pi} dp_\nu & 0 < p_\nu < 0.425 p_\pi, \\ = 0 & p_\nu > 0.425 p_\pi. \end{cases}$$

Let

$$S' = \frac{d^2Y}{d\Omega dp_\pi}$$

be the yield of secondary pions per steradian-Bev per incident electron according to the calculations of Drell\(^1\) and Ballam.\(^2\) We shall use the

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\(^1\)S. D. Drell, Report M-200-7A.
\(^2\)J. Ballam, Report M-200-8.
values of $S$ at $0^\circ$ and add the thick-target photoproduction and Drellcalculation contributions. This is possibly a small underestimate for low-energy pions since in the Ballam$^2$ calculations we ignore production by second- and higher-generation photons. Using this model we obtain for the neutrino flux $dN/dA$ per unit area at a distance $L$ meters from the target,

$$\frac{dN}{dA} = \frac{dp_v}{L^2} \int_{(p_v/0.425)}^{\infty} S \left( \frac{L_0}{c\tau_0} \right) \cdot \frac{1}{\gamma} \left( \frac{dp_{\pi}}{0.425 p_{\pi}} \right) = 0.0416 \frac{dp_v}{L^2} \times I L_0 \quad (3)$$

where $L_0$ is the distance during which a decay can take place, and $I$ is given by

$$I(p_v) = \int_{(p_v/0.425)}^{\infty} S \frac{dp_{\pi}}{P_{\pi}} \quad (4)$$

where momenta are in Bev/c and dimensions are in meters. This integral, which is proportional to the neutrino energy spectrum, is shown in Fig. 2, for a primary energy of 25 Bev. Note that the spectrum highly favors low-energy neutrinos. The total flux $\phi_v$ at a distance $L$ meters is

$$\phi_v = \frac{3.6 \times 10^{-4}}{L^2} L_0 \text{ v/m}^2 \text{ per incident electron} \quad (5)$$

A shielding distance of 15 m of iron ($1.34 \times 10^4 \text{ g/cm}^2$) is required to stop the muons and the nuclear cascade; optimum flux is obtained if the decay distance equals the shield thickness. Hence the usable flux at the detector position (Fig. 3) for $L = 30$ m, $L_0 = 15$ m is

$$\phi_v = 2.5 \times 10^4 \text{ cm}^{-2} \text{ sec}^{-1} \quad (6)$$

With an interaction cross section of $2 \times 10^{-38} \text{ cm}^2$/nucleon, this gives an interaction rate of

$$25 \text{ interactions/ton-day} \quad (7)$$

\[ I(p_\nu) = \int_{(p_\nu/0.425)}^{25} \frac{S}{p_{\pi}^2} dp_{\pi} \]

Spectrum at distance \( L \) meters is

\[ \frac{dN}{dA} = \frac{0.041 L_0}{L^2} I \frac{dp_\nu}{d^2} \text{ neutrinos} \]

\[ \text{in Bev} \]

\[ \text{FIGURE 2} \]

\[ - 4 - \]
where the ton refers to the quantity of interaction material in the detector.

This calculation ignores the potential enhancement of the neutrino fluxes by the use of lenses to collimate the pions prior to decay. The expected gain here is small since both the mean production angle of the pions and the laboratory decay angle of the neutrino relative to the pion direction is of order $1/\gamma = m_\pi/p_\pi$. Hence, even if the pion beam is collimated by a lens system, the improvement in rate will not be large.

We note that the high-energy pions contribute relatively little to the neutrino flux, since the pion spectrum is weighted by $p_\pi^{-2}$ to account for the decay time dilation and the neutrino energy spread. Therefore this calculation is sensitive to the low-energy pion spectrum, which is only poorly known. In particular, the conclusion that no order-of-magnitude gain in neutrino intensity is possible by the use of lenses is based on the Drell\(^1\) mechanism; in the lower-energy region more substantial gains may be possible.

\(^1\)S. D. Drell, Report M-200-7A.