REQUIREMENTS FOR A POSSIBLE COAXIAL DRIVE
SYSTEM FOR PROJECT M

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NOTE: The remarks presented in this memorandum are designed to
provide a basis for discussion. They are by no means final in
character character.
Two possible drive systems for Project M are presently under consideration: the hollow waveguide and the coaxial drive line. This memorandum deals exclusively with the latter. In the remarks that follow, the results of R. B. Neal's, Project M Report 105, Dec. 22, 1953, are assumed.

**General Remarks:**

As seen from the above reference, the relative phase shift between the r.f. drive wave and the electron bunches is given by:

\[
\frac{\delta \phi}{\phi_e} = \frac{\delta \phi}{\phi} = \frac{\delta z}{z} - \frac{c}{v_c} \frac{\delta z}{z} + \left(1 - \frac{c}{v_c} \right) \frac{\delta \omega}{\omega} + \frac{c}{v_c} \frac{\delta v}{v_c}
\]

(1)

where subscripts \( e \) and \( w \) refer to r.f. wave and electron variations, respectively, \( v_c \) is the velocity of the r.f. wave in the coaxial drive line, \( \omega \) is the radial frequency of the drive signal and \( c \) is the velocity of light.

The basic design criterion adopted here is:

\[
\frac{\delta \phi}{\phi} < \frac{1}{600,000} = \frac{\zeta}{\phi}
\]

(2)

i.e., for a total phase shift in cycles of \( \frac{\phi}{2\pi} = \frac{\beta_0 L}{2\pi} \approx 30,000 \) cycles over the total length \( L \) of the accelerator, \( \frac{\delta \phi}{\phi} \) should be less than 0.005 cycles. Variations due to temperature are regarded as slow in nature whereas variations due to frequency can be sudden. For design purposes, one may consider two methods of operation:

1) Fixed frequency operation:

\[
\frac{\delta \omega}{\omega} = 0
\]

2) Frequency adjusted for maximum energy. In this case, the frequency is tuned to compensate for temperature changes entailing changes in the phase velocity of the accelerator pipe. Thus, for a change \( \delta T_e \)
in the accelerator pipe, the frequency must be retuned as shown by the relations:

\[
\frac{\delta \lambda_0}{\lambda_0} = g \frac{\delta T_e}{e} = \frac{\delta \omega}{\omega}
\]

(3)

where \( g \) is the expansion coefficient of the accelerator pipe.

A variation \( \delta T_e \) of 2° C for a copper structure \((g = 1.6 \times 10^{-5}/^\circ C)\) corresponds to a \( \delta f \) of \( 1/10 \) Mc/sec at \( f = 2858 \) Mc/sec.

Temperature variations along the 2-mile accelerator will, of course, differ from section to section, and it will be impossible to obtain the frequency corresponding to \( \gamma_p = c \) for all sections. In spite of this difficulty, it still seems desirable to have frequency tuning as an extra degree of freedom in operation to compensate for both short and long-term variations. It can be shown that the present phasing technique derived by D. Goerz is still satisfactory for departures from synchronism corresponding to frequency variations of \( 1/10 \) Mc/sec.

In order to somewhat relax temperature tolerances between the drive line and the accelerator structure, R. B. Neal has shown that the most satisfactory scheme is that illustrated by Fig. 1. The accelerator has expansion bellows and its expansion is governed by a rigid drive line mechanically tied to it. The logical consequence of such a set-up is that the drive line must be close to the accelerator structure and thus in the same tunnel. If the drive line is a coax, it is unlikely that it be simultaneously used as a vacuum manifold.

In this case \( \left( \frac{\delta z}{z} \right)_e = \left( \frac{\delta z}{z} \right)_w = g \frac{\delta T_w}{T_w} \)

and since \( \frac{c}{\nu} \approx 1 + \epsilon \) where \( \epsilon \) should be very small, Eq. (1) becomes

\[
\frac{\delta \phi}{\phi} = -\epsilon \frac{\delta \omega}{\omega} + \frac{\delta \nu}{\nu_c} + \frac{\epsilon \nu}{\nu_c}
\]

(4)

Even if \( \epsilon \) is of the order of \( 10^{-2} \), the first term in Eq. (4) is
negligible for a 1/10 Mc/sec variation:
\[ \frac{\delta \omega}{\omega} \approx 3 \times 10^{-7} \]

Thus, expression (1) reduces to
\[ \frac{\delta \Phi}{\Phi} = \frac{\delta v}{v} \] (5)

This is a dispersion relation and it must be determined experimentally as a function of \( \omega \) and \( T \) for a given coaxial line. Information is presently being gathered on this point.

**Particular problems of the coaxial drive line:**

The coaxial drive line has two main disadvantages as compared to waveguides:

1) High loss (of the order of 1 db per 100 ft).
2) Low power handling capacity (of the order of 3 kW, cw).

Even if it were possible to get couplers to feed all klystrons directly from the coaxial line, a simple computation shows that at the rate of 300 watts of drive power per klystron (present requirements), at the very most nine klystrons could be driven before a series power booster became necessary along the drive line. Such a scheme would require 107 boosters for Stage II. Booster failure would stop machine operation and possibly require rephasing even if stand-by boosters were provided. It is seen that unless much higher drive powers can be used, this scheme is not satisfactory. The alternative scheme is illustrated by Fig. 2.

The coaxial drive line still provides the main drive power but r.f. is periodically abstracted and fed to the klystrons, in the adjacent tunnel, by a conventional waveguide hereafter called sub-drive line. Each sub-drive line is provided with a booster hereafter called sub-booster. Gain and power requirements of these sub-boosters will be considered below.
With this composite set-up, the total phase error is a superposition of coax and waveguide errors. R. B. Neal has shown that waveguide phase shifts are governed by relation:

\[ \frac{\delta \varphi}{\varphi} = \left( \frac{\delta z}{z} \right) - \frac{c}{v} \left( \frac{\delta z}{z} \right) + \frac{\delta v}{v} - 1 - \frac{\lambda}{c} \left[ 1 - \frac{(c \varphi)^2}{v^2} \right] \quad (6) \]

where subscript \( v \) now refers to waveguide variations, \( \lambda_0 \) is the free space wavelength, \( \lambda_c \) the cut-off wavelength and \( v_w \) the phase velocity in the waveguide.

Assuming the accelerator and waveguide both have the same expansion coefficient \( g \), relations

\[ \left( \frac{\delta z}{z} \right) = g \frac{\delta T_e}{e} \quad (7) \]

\[ \frac{\delta \lambda}{\lambda} = \left( \frac{\delta z}{z} \right) = g \frac{\delta T_w}{w} \]

lead to

\[ \left( \frac{\delta \varphi}{\varphi} \right)_{\text{wave guide}} = g \frac{\delta T_e}{e} - \frac{v_w}{c} g \frac{\delta T_w}{w} + \left( \frac{v_w}{c} - 1 \right) \frac{\delta \lambda_0}{\lambda_0} \quad (8) \]

and

\[ \left( \frac{\delta \varphi}{\varphi} \right)_{\text{Total}} = \left( \frac{\delta \varphi}{\varphi} \right)_{\text{coax}} + \left( \frac{\delta \varphi}{\varphi} \right)_{\text{wave guide}} \quad (9) \]

We may now derive general tolerance expressions as a function of:
\( n \) = length of sub-drive line
\( n \) = number of sub-drive lines = number of sub-boosters

1) Fixed frequency:

\[
\begin{pmatrix}
\delta v \\
\delta c \\
\delta c
\end{pmatrix} = 0
\]

in this case

\[
\frac{\delta \lambda}{\lambda} = \frac{\delta w}{w}
\]

so that

\[
\frac{\delta \phi}{\phi} = \frac{\delta \phi}{\phi} = \frac{\delta T_v}{T_v} - \frac{\delta T_w}{T_w}
\]

(10)

Condition (2) is now relaxed because \( \phi \) is reduced by a factor of \( n \): 

\[
\frac{\delta \phi}{\phi} \ll \frac{n \lambda}{\delta \lambda} = \frac{n \delta \lambda}{\lambda}
\]

(11)

For copper we obtain

\[
\delta T_v = \frac{\lambda}{c} \delta T_w \ll \frac{n \lambda}{10}
\]

(12)

For standard rectangular waveguides,

\[
\lambda_0 = 15.52 \text{ cm}
\]

\[
\lambda_c = 14.42 \text{ cm}
\]

\[
\frac{\lambda}{\lambda_c} = \frac{\lambda_0}{\lambda_c} = 1.060
\]

and assuming \( n = 30 \)
condition (12) becomes

\[ \delta T_e - 1.46 \delta T_w \leq 3.12^\circ C \]

Such tolerances seem acceptable.

2) Optimized frequency:

\[ \frac{\delta \lambda_0}{\lambda_0} = -\frac{\delta \omega}{\omega} = g \delta T_e \]

When 960 sections are considered, this relation only indicates an upper frequency tuning limit. It is meaningless to apply it to any one section since expansions are unlikely to be uniform along the machine. Equation (8) now becomes

\[ \frac{\delta \Phi}{\Phi} = \frac{v}{c} \left( \delta T_e - \delta T_w \right) \]

(13)

and condition (2) must be revised:

\[ \frac{\delta \Phi}{\Phi} \leq \frac{n v}{\cos} \left( \frac{v - c}{v_c} \right) = n \delta \]

(14)

Then our tolerance condition becomes:

\[ \delta T_e - \delta T_w < \frac{\beta n}{v} \]

(15)

Assuming that \( \frac{\delta v}{v_c} \rightarrow 0 \) and a rectangular waveguide is used as in case (1),

\[ v_c \]

with \( n = 30 \), relation (15) becomes:

\[ \delta T_e - \delta T_w < 2.14^\circ C \]
Such tolerances also seem acceptable. If the phase shift were to become too large, it may be possible to have a few phase-shifters along the coax drive line to compensate for dispersion.

**Booster and sub-booster requirements:**

We may now determine booster and sub-booster requirements. The present klystron specifications call for inputs of about 300 watts per klystron. The following calculations are made for Stage II (960 klystrons).

Let:

- \( L \) = total length of accelerator = 10,000 ft.
- \( s \) = length of sub-drive line = \( \frac{10,000}{n} \) ft.
- \( n \) = no. of sub-drive lines = no. of sub-boosters.
- \( \frac{960}{n} \) = no. of different waveguide couplers.
- \( P_s \) = power needed out of sub-booster in kw = \( \frac{960 \times 300}{n} \) kw.
- \( G \) = gain of main booster (series) amplifier.
- \( g_s \) = gain of sub-booster amplifier.
- \( n_1 \) = number of sub-drive lines per main booster.
- \( \frac{L}{n_1^s} \) = number of main boosters.
- \( P_c \) = maximum coax power (out of booster)
- \( \alpha \) = loss in nepers per ft. for coax line.

A simple derivation shows that the number \( n_1 \) of main boosters is given by
\[
(n_1 - 1) = G \left[ e^m \left( 1 - \frac{300}{n g_s P_0} \right) \right] \quad (13)
\]

**Example:**

For \( n = 30 \) sub-drive lines,

32 different types of couplers

\( P_s = 10 \) kw

\( G = 20 \) db

\( g_s = 30 \) db

\( a = \frac{L}{30} \) 330 ft. (33 klystrons per sub-drive line)

\( P_c = 3 \) kw

\( \alpha = 0.011 \) db/ft = \( 2.52 \times 10^{-3} \) nepers/ft.

we obtain \( n_1 = 6 \).

Thus, **five** main boosters would be needed along the coaxial drive line. The phase stability of these main boosters will have to be of at least one order of magnitude greater than all other tolerance factors along the line. Thus, if \( \frac{1}{\phi} < \frac{1}{600,000} \), the total phase shift variations of all five boosters put together should be less than \( 1.8^\circ \) or, for each amplifier, of the order of \( 0.36^\circ \). A scheme should be devised in order to permit automatic switching of a stand-by booster unit in case of main or sub-booster failure.
FIG. 1

- \( n \) total no. of sub-drive lines
- \( n_1 \) number of sub-drive lines per main booster

FIG. 2