The Quantitative Analysis of Invisible Particles

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In the previous lecture, I presented the idea of supersymmetry as a basis for physics beyond the Standard Model.

We saw that this model gave interesting solutions to many of the current questions in elementary particle physics. It naturally incorporated explanations of

- the spontaneous symmetry breaking needed for the weak interactions
- the values of the Standard Model coupling constants
- the existence of a WIMP to make up dark matter.

I argued also that this model led to large cross sections for characteristic processes at the LHC.
Now another set of questions arise.

Once we find that supersymmetric particles are being produced at the LHC, we will want to learn about their masses and properties.

There are serious difficulties for this sort of study.
For the high-energy processes we are considering, a proton is a bag of quarks and gluons. Any given process involves one quark or gluon colliding with another. We do not know the momenta of these individual particles.

If the collision produces supersymmetry, the final state contains two WIMPs. We do not observe these particles or measure their momentum.
We need methods for determining the masses of supersymmetry particles that do not require this information.
First, we would like to have a general idea of the scale of the masses of the superparticles.

It seems likely that a measure of the total activity in the event would tell us this.

For example:

\[ m_{\text{eff}} = E_T + \sum_{i=1}^{4} E_{Ti} \]

Use transverse projections of the measured energy deposition; these are insensitive to the momentum imbalance of the initial particles.
Now focus in on the details of the spectrum.

Here we will use tricks that take advantage of special properties of each scenario.

Every spectrum has its own special features; we need to recognize and make use of them.
A feature of many supersymmetry spectra is the decay chain

\[ \tilde{q} \rightarrow q \rightarrow N_2 \rightarrow l^+l^- \]

The lepton moment are measured completely, and we can construct their spectrum of invariant masses. If

\[ m(N_2) - m(N_1) < m_Z \]

this spectrum terminates at

\[ m(l^+l^-)_{max} = m(N_2) - m(N_1) \]
Hinchliffe et al.
Hinchliffe et al. noticed that one could go further.

At the endpoint, the unobserved WIMP is at rest in the frame of the $l^+l^-$ pair. If we have an estimate of the mass of the WIMP, we can add back its 4-vector.

Now there is no more missing information. Add observed jets and reconstruct the parent squarks.
Information on the properties of the WIMP is also available.

If the WIMP is the lightest neutralino, it can in principle be any linear combination of

$$\tilde{\gamma}, \tilde{z}, \tilde{h}, \tilde{H}$$

The shape of the lepton mass distribution can distinguish gaugino- and Higgsino-like WIMPs.
Again we can go further, following the analysis of Kitano and Nomura. Find the two hardest jets, and try to combine one with the lepton pair.

Some useful variables are:

\[ \min_{1,2}\{m(\ell\ell j)\} \]

\[ M_T^2 = \min \{p_{T1} + p_{T2} = p_T\} \max \{m_T^2(p_1 \not{p}_1), m_T^2(p_2 \not{p}_2)\} \]

Lester and Summers
$m_{N_1} = 169 \pm 17 \text{ GeV} \quad m_{\tilde{q}} = 486 \pm 11 \text{ GeV}$
With these and other tricks, one can determine masses at the level of

10% or below for WIMP, squark, gluino masses

1% for mass differences in l+l- cascades
We could obtain an even higher level of precision by building a different type of particle accelerator.

By colliding $e^+e^-$ instead of $pp$, we eliminate major difficulties of the LHC environment:

- electrons are elementary particles, so the initial momenta are known
- the Standard Model annihilation cross sections are small and can be computed precisely, so backgrounds are small and controlled
- the CM energy can be adjusted, so we can concentrate on the lightest superparticles with the simplest decay processes

The unobservability of the WIMP is still a problem.
A major new e+e- collider is now under design.

the International Linear Collider (ILC)

The design CM energy is 500 GeV, with the potential for upgrade to 1000 GeV.

The ILC will be a global project. The design team is drawn from laboratories in the US, Europe, and Japan.

Argonne, Brookhaven, Cornell, DESY, Fermilab, Frascati, KEK, Novosibirsk, Orsay, and SLAC are among the labs with major involvement in this project.
the International Linear Collider (ILC)
Here is a typical supersymmetry event at the ILC:
$e^+e^- \rightarrow \tilde{\tau}^+\tilde{\tau}^-$
The first goal of the ILC will be to definitively establish the overall model in which the WIMP is embedded.

Here we use the fact that

\[ e^+e^- \rightarrow XX \]

has a cross section of characteristic shape and normalizations determinated only by the spin and electroweak quantum numbers of X.
It is also possible to dramatically improve the precision of supersymmetry spectrum measurements.

Consider the reaction $e^+ e^- \rightarrow \tilde{e}^- \tilde{e}^+$ with $\tilde{e}^- \rightarrow e^- N_1$

The $\tilde{e}$ are spin-0 particles and decay isotropically.

The boost of an isotropic distribution is a flat distribution in energy between endpoints that are determined by kinematics. Locating the endpoints, we can solve for $m(\tilde{e}), m(N_1)$.
In this setting, $m(\tilde{e})$, $m(N_1)$ can be found to 0.1% accuracy.
How does this feed back into the question of dark matter?

Recently, Baltz, Battaglia, Wizansky, and I studied this in detail for several models of supersymmetric dark matter.

For LHC, ILC-500, ILC-1000, and for each model

We started from the set of measurements of supersymmetry masses and cross sections that would be done at these machines.

We generated $10^6$ supersymmetry parameter sets consistent with these measurements within the errors.

We computed the properties of the WIMP for each parameter set and studied their distributions.
Let’s now go through the analysis for one of these points (LCC2). You can read about the others in hep-ph/0602187.

LCC2 is a point at which squarks and sleptons are very heavy, and the WIMP annihilates through

\[ N_1 N_1 \rightarrow W^+ W^-, Z^0 Z^0 \]

The annihilation cross section depends on the neutralino masses but also on the neutralino content and the gaugino-Higgsino mixing angles, which are subtle to determine.
The important observables are associated with transitions among the light neutralinos and charginos.

Cross sections from polarized $e^+e^-$ initial states also play an important role.
$e^+e^- \rightarrow \tilde{C}_1^+ \tilde{C}_1^-$

$\rightarrow e^+ \nu \tilde{N}_1^- q \bar{q} \tilde{N}_1^0$
$m(\tilde{N}_2) - m(\tilde{N}_1) = 58.7^{+0.2}_{-0.1}$ GeV

$m(\tilde{N}_3) - m(\tilde{N}_1) = 82.0^{+0.4}_{-0.1}$ GeV

The detailed shape of the distribution is predicted by supersymmetry.
For this model, the spectrum constraints from the LHC alone give multiple solutions with different WIMP properties.
The ILC-500 cross section measurements resolve the ambiguity.
Microscopic prediction of the cosmic dark matter density
Microscopic prediction of the neutralino annihilation cross section at threshold.
Models of the dark matter distribution near the galactic center vary widely:

Gamma ray observations might distinguish these models.
Here are some objects that we might wish to observe in gamma rays:

The galactic center:

For definiteness, assume the NFW profile shown earlier. Observe gammas in a circle of area $1.5 \times 10^{-4}$ sr. Add the background from Bergstrom, Ullio, Buckley.

A subhalo dark matter clump:

Pick a typical object from the simulation data of Taylor and Babul:

$$M = 10^6 M_\odot \quad \text{scale size} = 500 \text{ pc} \quad \text{distance} = 6 \text{ kpc}$$

NFW profile. Observe in a circle of $1.5 \times 10^{-4}$ sr.
galactic center

LCC2

sub-halo clump

\[
\text{probability density } \frac{dP}{dx}
\]

\[
\langle J(\Omega) \rangle \text{ (0.4° radius, galactic center)}
\]

\[
\langle J(\Omega) \rangle \text{ (0.4° radius, clump)}
\]

LHC+ILC-1000

LHC+ILC-500

LHC
Microscopic prediction of the direct detection cross section dominated by exchange of the Higgs boson $h$. 
Super-CDMS should see 67 events. Using this number (with its statistical error) and the cross section just determined, we can directly evaluate the flux of dark matter impinging on the CDMS detector. Here is the likelihood distribution:
In this and the other examples,

The LHC and the ILC give precision data on the spectrum of new particles that is in its own right important information about the fundamental interactions.

In addition, these data constrain the WIMP properties so that astrophysicists can use these to determine the distribution of WIMPs in the galaxy.

In the next 5 - 10 years I expect major developments both in elementary particle physics and in the astrophysics of dark matter.

Be there!