Supersymmetry: the Next Spectroscopy?

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In the previous lecture, I discussed the phenomenology of dark matter

how we know it is there

how we propose to detect it

I described the model of dark matter as composed of WIMPs produced in the early universe.
In this lecture, I would like to discuss:

What kind of an elementary particle theory would give rise to WIMPs?

I will present the most beautiful example:

Supersymmetry
Supersymmetry is not a theory of one particle; rather, it is a theory of a whole new spectroscopy.

We will see later that this is typical of theories of WIMP dark matter.

To understand the properties of the WIMP, we will need to understand the larger picture of new particles of which the WIMP is a part.
Supersymmetry begins from the following question:

In Nature, we see two types of particles:

- fermions - particles that compose matter
- bosons - particles that mediate forces

Is it possible to unify these two general particle types?

More specifically, can we find a symmetry $Q$ for which fermions and bosons can live in the same multiplet?
Fermions have half-integer spin.
Bosons have integer spin.

So $Q$ must change angular momentum and must carry spin-1/2.

This means that $Q$ must transform under rotations and Lorentz transformations, a nontrivial constraint.

Anyway, postulate

$$Q_\alpha |b\rangle = |f\rangle \quad Q_\alpha |f\rangle = |b\rangle$$

and see where this takes us.
This structure looks innocent, but it is not.

Consider the quantity: \( \{ Q_\alpha, Q^\dagger_\beta \} \)

This object has the following properties:

- it is a 4-vector: \( \{ Q_\alpha, Q^\dagger_\beta \} = 2\gamma^{m}_{\alpha\beta} R_m \)  
  *\n- it is conserved: \( \{ \{ Q_\alpha, Q^\dagger_\beta \}, H \} = 0 \)
- it is nonzero:

\[
\langle \psi | \{ Q_\alpha, Q^\dagger_\alpha \} | \psi \rangle = \langle \psi | Q_\alpha Q^\dagger_\alpha | \psi \rangle + \langle \psi | Q^\dagger_\alpha Q_\alpha | \psi \rangle
= \| Q^\dagger_\alpha | \psi \| ^2 + \| Q_\alpha | \psi \| ^2
\]

which vanishes only if \( Q_\alpha \) and \( Q^\dagger_\alpha \) annihilate all states.

So, \( R_m \) is a nontrivial conserved 4-vector charge.

* This is a pure 4-vector, with no scalar piece.
In a relativistic theory, conservation of energy and momentum already imposes severe restrictions on scattering,

e.g., $2 \leftrightarrow 2$ scattering amplitudes depend only on 1 parameter, the CM polar angle.

Coleman and Mandula proved that, if there is a second conserved vector, the 2-particle scattering is actually forbidden.

So we have no choice: $R^\mu = P^\mu$. 
That is,
\[ \{Q_\alpha, Q^\dagger_\beta\} = 2\gamma_{\alpha\beta} P^\mu \]
where \( P^\mu \) is the energy-momentum of everything!

This means that every particle has a partner created by \( Q_\alpha \):

\[ \gamma \rightarrow \tilde{\gamma} \quad W^+ \rightarrow \tilde{w}^+ \]
\[ e_L^− \rightarrow \tilde{e}_L^- \quad e_R^- \rightarrow \tilde{e}_R^- \]

even:
\[ G \rightarrow \tilde{G} \]

so we will also need to fundamentally modify the structure of space-time.
There is no direct evidence that any of these new particles exist.

But, the presence of these particles can address some of the important open problems of elementary particle physics.
To discuss this, review our present picture of elementary particle interactions.

Particle physicists speak of a ‘Standard Model’ of strong, weak, and electromagnetic interactions. Here are the ingredients:

Electromagnetism is a force mediated by

a vector field $A_\mu$ or a spin-1 particle $\gamma$

The equations have a natural symmetry:

$U(1)$ phase rotation of the Schrödinger wavefunction

Generalize this to a larger symmetry group:

$SU(3) \times SU(2) \times U(1)$

with one vector field/particle for every generator of this group.
Choose $H$ so that its ground state has an orientation with respect to $\text{SU}(2) \times \text{U}(1)$ (‘spontaneously broken symmetry’)

Then:

$\text{SU}(3) \triangleright \text{strong interactions}$, mediated by 8 gluons

$\text{SU}(2) \times \text{U}(1) \triangleright \text{electromagnetic and weak interactions}$,
mediated by: $\gamma, W^+, W^-, Z^0$

with the properties:

$\gamma$ is naturally massless

$\gamma = \sin \theta_w A^0 + \cos \theta_w B^0$

parity is violated: $A^+, A^0, A^-$ couple only to $f_L$

masses are generated for $W, Z$ with $m_W/m_Z = \cos \theta_w$
In the 1990’s, experiments at CERN and SLAC studied the Z boson through

Here are a few snapshots from this data set:

The parity violation of the weak interactions is expected to be exhibited as a spin asymmetry of fermions produced in Z decays. Left-handed fermions are preferred. The asymmetry is predicted to be:

for leptons: \[ 8 \left( \frac{1}{2} - \sin^2 \theta_w \right) \approx 15\% \]

for d, s, b quarks: almost maximal 94%
polarization of \( \tau \) leptons from \( Z \) decay:

\[
\tau \rightarrow \pi \nu
\]

in good agreement with the SLD measurement of

\[
e^{-}e^{+}/e^{-}e^{+} \rightarrow Z^{0}
\]
polarization of b quarks from Z decay, reflected in the forward-backward asymmetry in $e_L^-e^+/e_R^-e^+ \rightarrow Z^0$

consistent with $A_b = 0.94$
The line-shape of the Z resonance is complex. Some effects that must be considered are:

- from weak interactions: $W^-$ $W^+$ 2% correction
- from strong interactions: $q$ $g$ $q$ 4% correction
- from electromagnetism: $\gamma$ 25% correction and distortion

The final result agrees with experiment at the 0.1% level!
The precision Z data gives us the values of the basic coupling constants of $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)$:

$$\alpha_3 = 1/8.5 \quad \alpha_2 = 1/29.6 \quad \alpha_1 = 1/59.1$$

where

$$\alpha_i = g_i^2 / 4\pi \hbar c$$

to be compared to the QED coupling

$$\alpha = e^2 / 4\pi \hbar c = 1/137$$

Can we explain this set of values?
Postulate a ‘grand unification’ symmetry group
e.g. SU(5)

for which SU(3) x SU(2) x U(1) is a subgroup. Imagine that, at very short distances, we have the complete grand symmetry.

This model predicts \( \alpha_3 = \alpha_2 = \alpha_1 \)

However, in quantum field theory, couplings constants are not static objects. They depend on the distance scale at which forces are measured.

In QED, we have

vacuum polarization

causing \( e \) and \( \alpha \) to decrease at larger distances, up to \( \hbar / m_e c \)
This leads to a logarithmic dependence of $\alpha$ on $r$.

$$\frac{de}{d\log r} = -\frac{e^3}{12\pi^2} \sum_f Q_f^2$$

It implies $\alpha(r = \frac{\hbar}{m_Z c}) = 1/129$.

which is what is actually needed to fit the precision $Z$ data.

In a model with a larger symmetry group $SU(N)$, there is another effect that has the opposite sign

asymptotic freedom

giving

$$\frac{dg}{d\log r} = +\frac{11N g^3}{48\pi^2} - \ldots$$
This is just what we need. If all of the $\alpha_i$ are equal at very short distances, then at larger distances

$$\alpha_3 \text{ increases}$$

$$\alpha_1 \text{ decreases}$$

to give the observed pattern.

Does it work quantitatively? This depends on what particle spectrum is assumed. Use $\alpha_1, \alpha_2$ to predict $\alpha_3$,

first with only the bosons, quarks, and leptons of the Standard Model

then adding their supersymmetry partners
In a model with supersymmetry, it is easy to arrange that the following quantum number is conserved:

$$R = (−1)^{3B−L+2S}$$

where $3B = 3$ (baryon number) = quark number
$L = $ lepton number
$S = $ spin

Standard Model particles have $R = +1$;
their supersymmetry partners have $R = -1$.

If $R$ is conserved, the lightest supersymmetric particle is absolutely stable.

If this particle is neutral (e.g. $\tilde{\gamma}$), it has all of the properties of a WIMP.
There is one more important consequence of supersymmetry; this involves the origin of spontaneous symmetry breaking of SU(2) \times U(1).

In the Standard Model, there is no explanation for this symmetry breaking.

It is common to parametrize this phenomenon in the following way: assume that there is a scalar field $\varphi$ (the Higgs field) which gives an orientation with respect to SU(2) rotations.

Assume a potential energy function for $\varphi$:

$$ V = \mu^2 |\varphi|^2 + \lambda |\varphi|^4 $$

with $\mu^2 < 0$

Then $\langle \varphi \rangle \neq 0$ and this value breaks the symmetry.
Condensed-matter physicists recognize this as the same strategy as that used in the Landau-Ginzburg theory of superconductivity. We parametrize our ignorance by introducing a new scalar field.

The true (Bardeen-Cooper-Schrieffer) theory of superconductivity is much more interesting. Electron pairs near the Fermi surface of a metal have attractive interactions from phonon exchange. This leads to Bose condensation of there pairs at low temperature.

It is always better to find a physical explanation for an potential energy unstable to symmetry breaking. Can we find this for SU(2) x U(1) ?
There is a constraint: The radiative correction

\[ \begin{array}{c}
  Z \\
  \phi \\
  Z
\end{array} \]

is needed for the precise agreement of theory with the data from the Z experiments.

This requires that the Higgs boson must be light.

That is very difficult to arrange in models in which symmetry breaking comes from strong interactions and fermion pair condensation.

We must find a theory that involves a light scalar particle.
Now there is a second constraint:

We did not explain why $\mu^2 < 0$; we simply assumed it.

But, the value of $\mu^2$ receives large radiative corrections:

$$\mu^2 = \mu_{\text{bare}}^2 + \frac{\lambda}{8\pi^2} \Lambda^2 - \frac{3y_t^2}{8\pi^2} \Lambda^2 + \cdots$$

How large is $\mu^2$? If the model is to be exact up to the scale of grand unification, we need $\Lambda \sim 10^{16}$ GeV but, at the same time $\mu^2 \sim -(100 \text{ GeV})^2$.

It would take a miracle to make this work!
Supersymmetry supplies this miracle.

Add the fermion partners of the Higgs bosons (‘Higgsinos’), boson partners of the top quarks (‘top squarks’).

For each pair, as an exact consequence of supersymmetry:

\[
\phi \ = \ \phi + \psi
\]

\[= 0\]

If the familiar particles are light and the supersymmetry partners have masses of a few times 100 GeV, we naturally find

\[|\mu| \sim 100 \text{ GeV}\]
Exact supersymmetry would imply that the masses of the Standard Model particles and their supersymmetry partners should be identical.

But this cannot be correct. There is no scalar particle with $m_e$.

This is an unattractive aspect of the theory. Supersymmetry must be spontaneously broken. However, this symmetry-breaking can take place at a very high mass scale.
Supersymmetry could be broken by new forces that are coupled very weakly to the particles of the Standard Model. Grand unified theories and superstring theories offer many possibilities:

The weak coupling explains the large hierarchy

\[ m(\tilde{f}) \sim 100 \text{ GeV} \ll 10^{16} \text{ GeV} \]

The value of \( m(\tilde{f}) \) then leads to \( \mu, m_W \sim 100 \text{ GeV} \).
Let’s now look at a sample mass spectrum of supersymmetric particles.

We have supersymmetry partners of the quarks, leptons, vector bosons, and Higgs bosons.

There are 5 Higgs particles in all: $h^0, H^0, A^0, H^\pm$.

The fermionic partners of $W^+, H^+$ mix;
call the mass eigenstates $\tilde{C}^+$ (charginos).

The fermionic partners of $\gamma, Z, h, H$ mix;
call the mass eigenstates $\tilde{N}^0$ (neutralinos).

The left- and right-handed states of $q, \ell$ have independent scalar partners.
for example:
This example is simpler than it looks.

Like the SU(3) x SU(2) x U(1) couplings, the supersymmetry mass parameters change with distance scale.

Parameters associated with particles with SU(3) charge become largest at large distances.

The superpartner masses can be unified at short distances and appear very different at \( r \sim 100 \text{ GeV} \).
We can figure this out - eventually - by measuring the spectrum of new particles and trying to discern its regularities.

There is one more important consequence of the dependence of mass parameters on length scale:
The mass parameters of the top squarks receive corrections from their coupling to Higgs bosons:

\[
\begin{align*}
\frac{dM_t^2}{d \log r} &= -\frac{2}{(4\pi)^2} \cdot 1 \cdot y_t^2 \cdot [M_t^2 + M_t^2 + M_H^2] + \frac{8}{3\pi} \alpha_3 m_{\tilde{g}}^2 + \cdots \\
\frac{dM_H^2}{d \log r} &= -\frac{2}{(4\pi)^2} \cdot 2 \cdot y_t^2 \cdot [M_t^2 + M_t^2 + M_H^2] + \frac{8}{3\pi} \alpha_3 m_{\tilde{g}}^2 + \cdots \\
\end{align*}
\]

This leads to a family of equations:

\[
\begin{align*}
\frac{dM_t^2}{d \log r} &= -\frac{2}{(4\pi)^2} \cdot 1 \cdot y_t^2 \cdot [M_t^2 + M_t^2 + M_H^2] + \frac{8}{3\pi} \alpha_3 m_{\tilde{g}}^2 + \cdots \\
\frac{dM_t^2}{d \log r} &= -\frac{2}{(4\pi)^2} \cdot 2 \cdot y_t^2 \cdot [M_t^2 + M_t^2 + M_H^2] + \frac{8}{3\pi} \alpha_3 m_{\tilde{g}}^2 + \cdots \\
\end{align*}
\]

The three mass parameters race to negative values.

\[
M_H^2 = \mu^2 \quad \text{wins. This gives a physical explanation of} \quad \mu^2 < 0 .
\]
We have now seen that supersymmetry leads naturally to models of new elementary particles with the following properties:

- explanation of the values of the SU(3) x SU(2) x U(1) couplings
- a discrete symmetry that keeps the lightest new particle stable. This particle is a WIMP.
- a sensible context for the light scalar Higgs field
- the prediction $\mu^2 < 0$ by virtue of the heaviness of the top quark

The price (if you call it that) is the prediction of a whole spectrum of supersymmetric particles.
I should note that these properties are not unique to supersymmetry.

Recently, other models of elementary particle physics have been constructed that have most or all of these properties.

In some of these models, the light Higgs boson is composite (‘little Higgs’).

Other models invoke a fifth dimension of space (and more). The new particles are quantum states with momentum in the extra dimensions.

All of these models predicts spectra of new particles. All have a place for a discrete symmetry that would lead to a stable WIMP.
What does supersymmetry predict for the LHC?

At short distances, the proton is an intense source of gluons.

Gluon-gluon collisions can create pairs of the supersymmetry partners of quarks and gluons.

These decay to lighter superparticles, and eventually to the WIMP.

Here is a typical event:
It is complex to evaluate the Standard Model backgrounds. Nevertheless, these events are highly characteristic for large jet activity and large missing energy.
The same qualitative features are found in all of the other models discussed earlier.

In some models, the partners of Standard Model particles are bosons rather than fermions (or vice versa). This is not so easy to check experimentally.
Many other questions remain:

To understand the properties of the supersymmetry model, we must measure the mass spectrum. Can this be done in the situation that some final-state particles are not observed?

If the WIMP is not observed, its properties are not measured directly. Can we obtain enough information from our measurements of the other superparticles to determine the WIMP properties that are important for astrophysics?
I will discuss all of these issues in the third lecture.