Models of electroweak symmetry breaking produce a spectrum of new particles in the energy range of a few hundred GeV to a TeV.

In this lecture, I will review what these systems of particles look like at the LHC, how we will discover them, and how we will measure their properties.
As I will discuss in a moment, the LHC brings with it an exceptionally difficult environment in which to search for new physics. But this does not mean that new physics must be hard to find. In these lectures, I have discussed two scenarios in which the discovery is straightforward.

- new vector bosons decaying to $\mu^+ \mu^-$

- new stable heavy leptons
At LEP and Tevatron energies, we see the gauge group

\[ SU(3) \times SU(2) \times U(1) \]

It is likely that the true gauge group in Nature is larger. We know that some gauge bosons get mass at the 100 GeV scale; why not others at the TeV scale?

Gauge bosons with mass \( m_Z, m_W \), but obtaining mass from the same physics, appear naturally in

models with extended gauge groups

superstring models

little Higgs models

Flat and Randall-Sundrum extra dimensions predict massive recurrences of \( \gamma, Z, G \) with similar properties.
For states visible in $pp \rightarrow Z' \rightarrow \mu^+ \mu^-$ the LHC has exceptional sensitivity at low values of luminosity.
For low values of mass, these resonances can be discovered even before aligning the muon system.
The second scenario with a distinctive signature observable in early data is the one described at the end of the previous lecture in connection with the idea of super-WIMPs.

A partner of the tau lepton could be the lightest SM partner particle. This state would eventually decay to $\tau + G'$, where $G$ is the super-WIMP dark matter particle, but it would appear to be a \textbf{massive stable particle} on the times scales of collider physics experiments.

Every new physics event would contain two such particles.

For definiteness, I will consider a SUSY model in which the apparently stable particle is the $\tilde{\tau}$ slepton.
Stable sleptons appear as muons which are slow but can still be within the time bucket of the muon system. This is a very easy signature of SUSY compared to the usual ones.

Using $\beta$ vs. $p$, it is possible to measure the mass to 0.1%.
Gauginos decay to the slepton by
\[ \tilde{\chi}_i^0 \rightarrow \tau^+ \tau^- \]
so we can measure the spectrum of gauginos by associating $\tau$ jets with staus. Of course, the $\tau$ is not observed completely. But in hadronic $\tau$ decays, the LHC detectors can see most of the energy.
So we can combine stable $\tilde{\tau}$s with $\tau$ jets and look for resonances. Including detector effects, these appear as kinematic edges.
The super-WIMPs themselves are unobservable and make no signals in direct or indirect dark matter detection.

However, if it is possible to stop the sleptons or observe slepton decay, some interesting checks are possible.

If the sleptons are produced by the thermal freeze-out mechanism described in the previous lecture, then we have a prediction for $\Omega_{DM}$:

$$\Omega_{DM} = \Omega_N \cdot \frac{m_G}{m_{\tilde{\tau}}}$$

If the super-WIMP is the gravitino, there is a definite relation (from the super-Higgs mechanism) between the gravitino mass, the stau mass, and Newton’s constant:

$$t = \frac{6}{G_N} \frac{m_G^2}{m_{\tilde{\tau}}^5} (1 - \frac{m_G^2}{m_{\tilde{\tau}}^2})^{-4}$$

Hamaguchi, Kuno, Nakaya, and Nojiri
For the rest of this lecture, I will concentrate on the more difficult case in which the lightest new particle is the dark matter particle itself, a stable, neutral WIMP.

The direct pair production of the WIMP has a small cross section and is difficult to observe.

However, we can take advantage of the fact that models of EWSB predict a spectrum of new particles, including particles with QCD pair production. In particular, it is an essential feature of most models that there is a partner of the top quark in the new particle sector.

The dominant production mechanism for the WIMP at the LHC by production of a pair of heavy colored particles, each of which undergoes a decay cascade that ends in the WIMP.
The description is generic. It applies to all EWSB models with a new particle spectroscopy and a candidate for WIMP dark matter.

The most detailed studies have been done in SUSY models. So please excuse me that, for the most part, I will use examples from SUSY in the rest of this lecture.
I remind you of the sample SUSY spectrum that I showed in Lecture 3.

At the LHC, the largest cross sections will be those for gluino and squark pair production.

The gluinos and squarks will rapidly decay through a sequence that ends in the lightest neutralino.
The new physics events can be characteristic in having multiple jet production and unbalanced visible momentum. A typical event would have the following form: (Particle labels are for supersymmetry.)

It is expected that events of this kind will appear as a very significant signal above background.

Here are the estimates of Tovey (2003) for supersymmetry models with universal scalar and gaugino masses at the GUT scale.
For squark and gluino masses below 1 TeV, the missing energy signature should be significant with a very small amount of integrated luminosity.
At the same time, many different signatures of new physics should be seen above the Standard Model expectation.
However, the expectation of large signals above Standard Model background does not mean that we can be complacent.

The theoretical background levels must be understood very well both absolutely and in relation to the actual data.
In order to reach the level of new physics signals, we will need to work down through a series of levels dominated by Standard Model processes of different types.

Here is an idea of the hierarchy:

\[
\begin{align*}
\sigma_{\text{tot}} & \quad 100 \text{ mb} \\
\text{jets w. } p_T > 100 & \quad 1 \mu\text{b} \\
\text{Drell-Yan} & \quad 100 \text{ nb} \\
\ttbar & \quad 800 \text{ pb} \\
\text{SUSY (} M < 1 \text{ TeV)} & \quad 10 \text{ pb}
\end{align*}
\]
The first challenge comes with the realization that the processes that we are looking for occur at rates of order

\[10^{-10}\]

of the total pp cross section.

Still, the interesting events have several jets with large values of \(p_T\). To find jets, we can look at the ‘lego plot’ of \(p_T\) deposited in the plane of \(\theta\) and \(\phi\) - or, better, rapidity \(y\) and \(\phi\). If we look for these objects instead of simply searching for large energy deposition, we already win about 6 orders of magnitude.
To go further, we need to search for events that do not belong to the classes generated by QCD. These should be events with multiple jets, plus leptons or unbalanced visible momentum.

QCD will generate unbalanced momentum if jets are mismeasured. To control this effect, it is necessary to understand the detectors, to eliminate noise and electronic signals unrelated to the physics events, and to correct for cracks and geometric inefficiencies.
CMS and ATLAS claim that they can control these effects to the required level. That story is expressed in these figures from the ATLAS TDR.

ATLAS simulation of missing ET in $Z(\rightarrow \mu^+\mu^-) + jet$

$\eta$ of the jet w. the highest ET in events w. ET > 50
In the physics studies of ATLAS and CMS, more difficult backgrounds to new physics come from a different source, heavy particle production within the Standard Model, production of $W, Z, t\bar{t}$ plus jets.

These reactions already offer missing energy, leptons, and hadronic activity. They populate the region of large jet transverse energy associated with new physics to the extent the additional jets are radiated along with the heavy particles.
2 examples of signal and background from recent CMS SUSY studies:

\[ m(\tilde{g}) = 600 \text{ GeV} \]

\[ \mathcal{E}_T \]

\[ M_{\text{eff}} = \mathcal{E} + \sum_i E_{T,i} \]
$m(\tilde{g}) = 1890 \text{ GeV}$

$E_T$

$M_{\text{eff}} = E + \sum_i E_{T_i}$

Yetkin and Spiropulu
To understand heavy particle + multijet backgrounds to new physics, there is a methodology that has been used successfully in the Tevatron, especially in the CDF and DO analyses of top quark production.

Use the fact that new particles appear in events with large numbers of jets and large $H_T = \sum \ E_T^i$

Compute systematically the SM rates for n jet production. The results for fewer jets can be validated against data, both in a general setting and also with the experimental cuts that define the new physics search. Now extrapolate to large numbers of jets and large $H_T$. 

There is an important regularity that the SM cross sections fall off roughly exponentially when plotted against the number of jets, number of leptons, or (more roughly) with increasing ET and HT.

In UA2, this regularity was called “Berends scaling”. The first theoretical calculations of these distributions were done in the 80’s by Ellis, Kleiss, and Stirling.

I like to call this regularity a “staircase”.

The key idea for LHC physics is that we can smoothly descend the staircase and then decide whether there is an anomaly at the bottom that signals new physics.

Let me give some illustrations from the Tevatron data.
systematics of $W + \text{ jets}$

$(W \to e\nu) + \geq n \text{ jets}$

CDF Run II Preliminary

CDF Data $\int dL = 320 \text{ pb}^{-1}$

$W \text{ kin: } E_T^e \geq 20[\text{GeV}]; |\eta^e| \leq 1.1$

$M_T^W \geq 20[\text{GeV/c}^2]; E_T^\nu \geq 30[\text{GeV}]$

Jets: JetClu R=0.4; $|\eta|<2.0$

hadron level; no UE correction

LO Alpgen + PYTHIA

Total $\sigma$ normalized to Data
search for SUSY in acoplanar di-jet events
top quark: require 1 b tagged jet

Here there are staircases both with respect to the number of jets,

**DØ Run II Preliminary**

![Graph showing 1 b-tag events by jet multiplicity]
and with respect to the number of b-tagged jets.

**DØ Run II Preliminary**

2 b-tag events
Tevatron Run II $p\bar{p}$ at $\sqrt{s} = 1.96$ TeV

Cross-Section [pb]

- CDF Preliminary
- CDF Published
- D0 Preliminary
- D0 Published
- Theory

thanks to M. Neubauer
Once we have observed excesses above the levels predicted for the Standard Model, we can move to the next level of analysis, estimating the masses of the new particles and determining their properties.

Now I will review some methods for such analyses. I will discuss the determination of the overall mass scale of the new particles, and specific kinematic observables that sensitive to particle masses and mass differences.
A warning:

It is very dangerous to apply these methods if the backgrounds are not well-understood and correctly subtracted.

Analysis of top–antitop production and dilepton decay events and the top quark mass

R.H. Dalitz a and Gary R. Goldstein b

a  Department of Theoretical Physics, University of Oxford, Oxford OX1 3NP, UK
b  Physics Department, Tufts University, Medford, MA 02155, USA

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If this event really represents top–antitop production and decay, then the top quark mass would be $131^{+22}_{-11}$ GeV.
There are significant difficulties in trying to measure new particle masses at the LHC from features in kinematic distributions.

Any given process involves one quark or gluon colliding with another. We do not know the momenta of these individual particles. So we do not know the momentum of the initial state.

The final state might contain two dark matter particles. We do not observe these particles or measure their momentum. So we have incomplete information about the final state.
As a start, consider the overall transverse energy deposition in the detector. To remove noise from the underlying event, we might alternatively sum the ET of the hardest jets.

Hinchliffe, Paige, Shapiro, Soderqvist and Yao proposed the observable

\[ M_{\text{eff}} = E_T + \sum_{\text{jets}=1}^{4} E_{Ti} \]

and showed that, in a variety of ‘mSUGRA’ models, it correlates well with the smallest of the squark and gluino masses.
That analysis worked because it was applied to models with gaugino universality.

In models with small mass differences between the gluinos and the charginos and neutralinos, much of the transverse energy in the reaction is carried off by neutralinos and is invisible. But still, the quantity $M_{eff}$ is a reasonable indicator of the mass difference between the directly produced and the final SUSY states.
These relations rely on the fact that most new particle production occurs close to threshold. The production cross sections turn on at threshold and then rise only slowly when $\hat{s} \gg 4m^2$, while parton luminosities fall off very rapidly.
Recently, Thaler, Schuster, Toro, et al. (hep-ph/0703088) suggested that we can take advantage of this to do a broader phenomenological analysis. They suggest that we

Choose an **appropriate set of candidate new particles**

Approximate all **production cross sections** by **constants**

Choose **appropriate decay modes** for each particle. These might be 2-body decays or multi-body decays through effective operators. Approximate all **decay matrix elements** by **constants**.

**Fit the data** to obtain the masses, cross sections, and branching fractions.

They refer to this description as an on-shell effective theory (OSET). The program is encoded in a software package called **MARMOSET**.
Here is a fit to the $H_T$ and $\slashed{E}_T$ distributions for a specific SUSY model (red) and the corresponding curves (green, blue) when the gluino mass is shifted by 20%, 40%.
Here is the effect of shifting the gluino mass by 40%, keeping the gluino-neutralino mass difference fixed.

The $H_T$ distribution is unaffected, but the $E_T$ distribution gives a poor fit.
To extract more specific information, we need to perform analyses that rely on special features of the supersymmetry spectrum.

Every spectrum has special features. It is part of the art of experimentation to find and exploit them.

In the discussion to follow, I will pick out a particular feature that has been studied in a number of different analyses and use it to illustrate that level of insight that one could achieve in the hadron collider environment.
It is typical in supersymmetry models that the partners of quarks and gluons are relatively heavy states. These decay to the charginos and neutralinos, the partners of SU(2)xU(1) gauge bosons and Higgs bosons.

A feature of many supersymmetry spectra is the decay chain

\[ \tilde{q} \rightarrow q \rightarrow N_2 \rightarrow N_1 \rightarrow l+l^- \]

The lepton momenta are measured completely, and we can construct their spectrum of invariant masses. From this point, depending on the specific model of the dilepton decay, the analysis can proceed in several different ways.
The decay of the $N_2$ can occur by any of the mechanisms:

\[ N_2^0 \rightarrow \ell^\pm + \tilde{\ell}^\mp, \tilde{\ell}^\mp \rightarrow \ell^\mp + N_1^0 \]

\[ N_2^0 \rightarrow N_1^0 Z^0, Z^0 \rightarrow \ell^+ \ell^- \]

\[ N_2^0 \rightarrow N_1^0 Z^*^0, Z^*^0 \rightarrow \ell^+ \ell^- \]

In a model in which $N_2^0 \sim \tilde{w}^0$, $N_1^0 \sim \tilde{b}^0$, these modes are preferred in the order listed: 2-body decays dominate over 3-body decays, and the $N_2$ coupling to sleptons is larger than the $N_2$ coupling to $Z^0$. 
The decay to an on-shell $Z^0$ is hard to work with, but the other two cases are interesting. To analyze them, consider the Dalitz plot associated with the 3-body system $N_1^0 \ell^+ \ell^-$.

Let
\[ x_0 = \frac{2E(N_1)}{m(N_2)} \quad x_+ = \frac{2E(\ell^+)}{m(N_2)} \quad x_- = \frac{2E(\ell^-)}{m(N_2)} \]
in the frame of the $N_2$.

\[ x_0 + x_+ + x_- = 2 \]

The kinematic boundaries are located at:
\[ x_+ + x_- = 1 - m^2(N_1)/m^2(N_2) \]
\[ (1 - x_+)(1 - x_-) = m^2(N_1)/m^2(N_2) \]

\[ m^2(N_1 \ell) = m^2(N_2)(1 - x_+) \]
We can distinguish the cases of 2-body decay to a slepton and 3-body decay in the following way:

2-body decay populates lines on the Dalitz plot and leads to a sharp endpoint:

3-body decay populates the whole Dalitz plot and gives a slope at the endpoint:
In the 3-body case, the endpoint in $m(\ell^+ \ell^-)$ is exactly

$$m(N_2) - m(N_1)$$

so we obtain a precise measurement of this quantity. The shape of the spectrum has more information. For example, for heavy slepton masses, this shape is different for gaugino-like or Higgsino-like lightest neutralino.
an example where the lightest neutralinos are gaugino:

Hinchliffe et al.
an example where the lightest neutralinos are Higgsino:
The origin of the shape difference is interesting. The neutralino mass matrix has the form

\[
\begin{pmatrix}
    m_1 & 0 & -m_Z c_\beta s_w & m_Z s_\beta s_w \\
    0 & m_2 & m_Z c_\beta c_w & -m_Z s_\beta c_w \\
    -m_Z c_\beta s_w & m_Z c_\beta c_w & 0 & -\mu \\
    m_Z s_\beta s_w & -m_Z s_\beta c_w & -\mu & 0
\end{pmatrix}
\]

so the signs of the eigenvalues \( m_i \) are (+ +) for gauginos, (-+) for Higgsinos.

The decay \( N_i \to N_j \ell^+ \ell^- \) through a virtual Z involves interfering diagrams

The interference term is proportional to \( m_i m_j \).
Hinchliffe et al. noticed that one could go further.

At the endpoint, the unobserved WIMP is at rest in the frame of the l⁺l⁻ pair. If we have an estimate of the mass of the WIMP, we can add back its 4-vector.

Now there is no more missing information. Add observed jets and reconstruct the parent squarks.
At the endpoint, the $N_1$ is at rest in the frame of the $\ell^+ \ell^-$. If we know (or guess) the mass of the $N_1$, we know its 4-vector. Now we have solved the problem of missing momentum; we can add jets and try to reconstruct the parent squarks.

Hinchliffe et al.
The case of a 2-body decay is even nicer. There is a sharp endpoint at

\[
m(\ell^+ \ell^-) = m(N_2) \sqrt{1 - \frac{m^2(\ell)}{m^2(N_2)}} \sqrt{1 - \frac{m^2(N_1)}{m^2(\ell)}}
\]
The decay $\tilde{q} \rightarrow qN_2$ is also 2-body, and so there are also upper and lower kinematic endpoints in combinations $(j\ell)$, $(j\ell\ell)$. From 4 endpoints, one can solve for the 4 unknown masses in the problem.
The case in which the decay involves an on-shell Z is the most difficult. To get definite kinematic information, we need to combine the Z with other jets in the events.

Recently, Butterworth, Ellis, and Raklev have considered the following strategy, directed at squark decay chains including $\tilde{q} \rightarrow qC_1^+ \rightarrow qW^+ N_1$

select SUSY events (jets + missing ET)

look for hadronic jets with $p_T > 200$, 2-jet substructure and mass consistent with the W mass

combine these W candidates with the highest $p_T$ jets and look for kinematic endpoints
$m(jW)$

**Graphs:**

- **Graph α:**
  - $m_{qW}^\text{max} = 527.4 \pm 8.7$

- **Graph β:**
  - $m_{qW}^\text{max} = 650.2 \pm 6.5$

- **Graph γ:**
  - $m_{qW}^\text{max} = 482.2 \pm 4.0$
  - $m_{qW}^\text{min} = 347.0 \pm 2.9$

- **Graph δ:**
  - $m_{qW}^\text{max} = 1219.0 \pm 85.1$

**References:**

Butterworth, Ellis, and Raklev
One more case of an $N_2 \rightarrow N_1$ decay should be mentioned. If the 2-body decays to sleptons are not kinematically allowed, the dominant 2-body decay might be

$$N_2 \rightarrow N_1 + h^0$$

In this case, supersymmetry production can provide a copious source of Higgs bosons.
If we expect that there are two missing particles in the event, we might try to partition the missing momentum into two parts.

The first analysis of this type was done for the process
\[ e^+ e^- \rightarrow \tilde{q} \tilde{q}^* \rightarrow q N_1 \bar{q} N_1 \]
by Feng and Finnell. In this case, the momenta of the quark jets is measured, and the squark energies are known from the beam energy. Then the magnitudes of the neutralino energies are known. The missing momentum is measured. So it is only necessary to find the orientation of the neutralino momentum vectors. These must lie on the circle \( C \) as shown:

We can then find the point on this circle that gives the minimum value of the reconstructed squark mass. This is a lower bound on the actual squark mass.
“...momentum vectors lying on large circles $C$ may give mass minima both close and far from the actual squark mass. However, small circles give only accurate solutions, and thus the calculated minimum masses preferentially lie close the actual underlying squark mass.” - Feng and Finnell
Lester and Summers have suggested a similar analysis in the hadron collider environment. They considered

\[ pp \rightarrow \tilde{l}^+ \tilde{l}^- \rightarrow l^+ N_1 l^- N_1 \]

Because we do not know the frame of the parton-parton collision, work with transverse momenta only.

A mass estimate from transverse momenta is the transverse mass

\[ m_T^2(12) = m_1^2 + m_2^2 + 2(E_{T1} E_{T2} - \vec{p}_{T1} \cdot \vec{p}_{T2}) \]

where \( E_T = (p_T^2 + m^2)^{1/2} \). This is a lower bound to the actual mass

\[ m^2(12) = m_1^2 + m_2^2 + 2(E_{T1} E_{T2} \cosh(\eta_1 - \eta_2) - \vec{p}_{T1} \cdot \vec{p}_{T2}) \]
Assume that we knew the mass of the neutralino. Then if we also knew the transverse momentum of the neutralino, we could estimate

$$m(\tilde{l})^2 \geq m_T^2(\ell N_1)$$

Since we only know the sum of the two neutralino momenta, we need to partition these two momenta in an arbitrary way. Some partition will still give a bound. Then

$$m(\tilde{l})^2 \geq M_{T2}^2 \equiv \min_{\not\!p_T=1+2} \left[ \max(m_T^2(l, 1), m_T^2(l, 2)) \right]$$
The distribution of $m_{T2}$ has a sharp endpoint at the correct value of $m(\tilde{l})$ (assuming that we have input the correct neutralino mass):

Kawagoe, Nojiri, and Polesello and Cheng et al have discussed other methods that partition the measured missing momentum.
The methods I have discussed can be even more powerful in combination. Kitano and Nomura have tried a squark reconstruction by combining the variables:

\[
\min_{i=1,2} \left[ m(\ell\ell j_i) \right]
\]

multibody endpoint

\[
\min_{\not{\not{\not{}}^T=1+2} \left[ m_T^2(1j_1), m_T^2(2j_i) \right]
\]

Lester and Summers
The endpoint positions have a different functional dependence on the squark and neutralino masses. Demand consistency:

\[ m_{N_1} = 169 \pm 17 \text{ GeV} \quad m_{\tilde{q}} = 486 \pm 11 \text{ GeV} \]
In these examples, we are obtaining rather precise particle masses, at the level of

10% or below for WIMP, squark, gluino masses

1% for mass differences in l+l- cascades

It is more subtle to obtain information on the spins and electroweak quantum numbers of new particles discovered at the LHC.

The most important information comes from the value of the pair production cross sections.

In special cases, there are also asymmetries that directly test spin/chirality assignments.
I cannot finish a discussion of new particle mass measurements without discussing the stage in the program after the LHC.

Eventually, the LHC data will be supplemented by data on SUSY particle production in electron-positron collisions from the International Linear Collider.

Here the center of mass energy is fixed, the events are very simple and easy to analyze, and initial-state polarization is available as an incisive probe of spin and of the mixing of weak-interaction eigenstates.
$e^+e^- \rightarrow \bar{\tau}^+\bar{\tau}^-$
\[ e^+ e^- \rightarrow \tilde{C}_1^+ \tilde{C}_1^- \]
\[ \rightarrow e^+ \nu \tilde{N}_1^0 \ q \bar{q} \tilde{N}_1^0 \]
Here are two examples of muon energy distributions from Blair and Martyn presented in the TESLA simulation studies:

\[ e^+ e^- \rightarrow \tilde{\mu}^+ \tilde{\mu}^- \rightarrow \mu^+ N_1 \mu^- N_1 \]

It is expected that these masses could be measured at the ILC to a few hundred MeV (parts per mil).
In these lectures, I have argued that

1. **The Standard Model is incomplete.** It lacks the physics that would explain the spontaneous breaking of its fundamental SU(2)xU(1) symmetry.

2. **Models of EWSB are complex.** They require a new particle spectroscopy at the energy scale of a few hundred GeV, including particles with QCD color and large production cross sections at the LHC.

3. **Cosmic dark matter** probably originates as a relatively light stable neutral particle in this new sector.

4. Despite the difficulties of analyses in hadron-hadron collisions, there are many tools to determine the properties of these new particles at the LHC.

If these conclusions are correct, we have an exciting era ahead.