Physics of Electroweak Symmetry Breaking

1. The Standard Model, the Higgs Boson, and the Problem of EWSB

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This summer, the LHC at CERN will begin operation.

There are many reasons to be excited about the LHC.

The LHC opens a new regime in energy, realizing ‘supercollider’ physics for the first time.

The LHC is optimized for the discovery of the hypothesized Higgs boson.

Many models of physics beyond the standard model will be tested at the LHC.
But I feel that the LHC has an importance that goes beyond these statements.

The Standard Model is incomplete as a model of physics. It has a major gap that must be remedied by new physics at the energy scale of a few hundred GeV.

This new physics ought to be discovered at the LHC.

What we are looking for at the LHC is more than a new particle or group of particles. We are looking for a new spectroscopy.

In this set of lectures, I will explain these statements and provide concrete examples of models that might fill this gap.
Here is an outline of the lecture series:

1. The Standard Model, the Higgs Boson, and the Problem of EWSB
2. The Rise and Fall of Technicolor
3. Supersymmetry as a Model of EWSB
4. Extra Dimensions: Flat, Warped, and Dual
5. The EWSB Connection to Dark Matter
6. Models of EWSB at the LHC
This lecture will introduce the problem of EWSB.

To begin, I will describe the Standard Model (SM). I will use a notation that may be unfamiliar but will be useful to us in later parts of this course.

The SM consists of three parts:

- the vector bosons (photon, W, Z, gluons)
- the fermions (quarks and leptons)
- the Higgs boson or other symmetry-breaking mechanism
Let me first describe the orthodox theoretical description of the Standard Model.
The vector bosons are organized by the principle of Yang-Mills Theory:

Specify a Lie group $G$. This dictates the equations of motion of the vectors, including all nonlinear interactions. There is one independent coupling constant for each simple factor of $G$.

The right group to describe the Standard Model is

$$G = SU(3) \times SU(2) \times U(1)$$

There are three independent couplings: $g_s$ $g$ $g'$

The non-Abelian factors have an absolute normalization. However, for the Abelian factor, we are free to rescale the coupling and the charges as long as $g'Q_i$ remain fixed.

With an eye to Grand Unification, it is convenient to rescale

$$g_1 = \sqrt{5/3} g'$$

then I will refer to: $g_3$ $g_2$ $g_1$
The basic fermions in 4 dimensions are 2-component Weyl fields. These give massless spin 1/2 particles, with a left-handed particle and a right-handed antiparticle (or vice versa).

A massive fermion has both left- and right-handed spin states. Massive fermions are obtained by mixing left- and right-handed Weyl particles.

In these lectures, I will regard quarks and leptons as fundamentally massless. I will treat the left-handed states as the basic fields or particles.

Then the fermion content of one generation of the Standard Model is:

\[ Q = \begin{pmatrix} u \\ d \end{pmatrix} \quad \bar{u} \quad \bar{d} \quad L = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad \bar{e} \quad + \quad \text{antiparticles} \]
In QED we assign
\[ e : Q = -1, \ L = +1 \] \[ \bar{e} : Q = +1, \ L = -1 \]
Then we are allowed to add a term
\[ \delta \mathcal{L} = -m_e \bar{e} \cdot e = -m_e [\epsilon_{\alpha\beta} \bar{e}_\alpha e_\beta + h.c.] \]
which is the form, in this notation, of the familiar mass term
\[ \delta \mathcal{L} = -m_e \bar{e} e \]
So in QED there is no problem in having a fermion mass.

It is worth noting that \[ \delta \mathcal{L} = -m f_1 \cdot f_2 \]
is the most general form of a fermion mass term. This form incorporates both Dirac and Majorana masses.
In the Standard Model, there is no combination of fermions for which a mass term

$$\delta \mathcal{L} = -mf_1 \cdot f_2$$

is allowed by the gauge symmetry. For example,

$$\delta \mathcal{L} = -m_e \bar{e} \cdot e$$

has \( I = 1/2 \) under SU(2), \( Y = +1/2 \) under U(1).

Also, gauge symmetry prohibits all vector bosons from obtaining mass.

It is an attractive idea that we can have very large mass scales in physics (e.g. the Planck scale), but nevertheless we can have particles at the GeV - TeV scale, because of this property.

A singlet neutrino \( \bar{\nu} \) has \( I = Y = 0 \), so

$$\delta \mathcal{L} = -M \bar{\nu} \cdot \nu$$

is allowed in the Standard Model. Presumably, this is why there are no light singlet (sterile) neutrinos.
This is a compelling story, but do we know that it is correct?

There are three key predictions of this model:

1. The couplings of quarks and leptons to the weak and electromagnetic interactions are described by universal couplings $g$ and $g'$. 

2. The left- and right-handed components of massive quarks and leptons have different quantum numbers.

3. The nonlinear interactions of vector bosons are of the form dictated by Yang-Mills Theory.

All are well supported experimentally.
The universality of couplings is tested in neutrino scattering and, more accurately, in the precision electroweak experiments at the Z resonance.

We can summarize the SM description of the Z couplings by the formula

$$\Gamma(Z \rightarrow ff^*) = \frac{(g^2 + g'^2) m_Z}{24\pi} \cdot Q_Z^2 \cdot N_f$$

where

$$Q_Z = I^3 - s^2_w Q$$

Here $I^3 = \pm \frac{1}{2}$ or $0$, $Q$ is electric charge, and

$$s^2_w = \frac{g'^2}{g^2 + g'^2}$$

We can ask whether the data supports a universal, consistent value of the prefactor and of $s^2_w$. 

$$Q_Z = I^3 - s^2_w Q$$
In the SM, the prefactor is predicted from $\alpha, m_Z, s_w^2$.

The values of $Q_Z$ span a wide range:

$$Q_Z(f) = Q_Z^2(f) + Q_Z^2(\bar{f})$$

<table>
<thead>
<tr>
<th>$f$</th>
<th>$Q_Z(f)$</th>
<th>$Q_Z(\bar{f})$</th>
<th>$S_f$</th>
<th>$A_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>$\frac{1}{2}$</td>
<td>$-$</td>
<td>0.25</td>
<td>1</td>
</tr>
<tr>
<td>$e$</td>
<td>$-\frac{1}{2} + s_w^2$</td>
<td>$-s_w^2$</td>
<td>0.126</td>
<td>0.15</td>
</tr>
<tr>
<td>$u$</td>
<td>$\frac{1}{2} - \frac{2}{3}s_w^2$</td>
<td>$\frac{2}{3}s_w^2$</td>
<td>0.144</td>
<td>0.67</td>
</tr>
<tr>
<td>$d$</td>
<td>$-\frac{1}{2} + \frac{1}{3}s_w^2$</td>
<td>$-\frac{1}{3}s_w^2$</td>
<td>0.185</td>
<td>0.94</td>
</tr>
</tbody>
</table>

where

$$S_f = Q_Z^2(f) + Q_Z^2(\bar{f})$$

$$A_f = \frac{Q_Z^2(f) - Q_Z^2(\bar{f})}{Q_Z^2(f) + Q_Z^2(\bar{f})}$$

gives the contribution to the Z width

$S_f = Q_Z^2(f) + Q_Z^2(\bar{f})$ gives the Z polarization asymmetry
This works extremely well.

For example, for the value of $s_w^2$ above, the ratio of $Z$ decays to $\ell^+\ell^-$ to that to hadrons should be 20.73. The best SM fit to the $Z$ data, from the LEP Electroweak Working Group, predicts 20.744. The LEP experiments give

$$R_\ell = 20.767 \pm 0.025$$

I’ll give two more, less simple examples.
The total width of the $Z$ resonance can in principle be determined from this $Z$ line shape. This is somewhat nontrivial, since the shape of the $Z$ resonance is distorted by initial-state photon radiation. To extract the width, it is necessary to measure the detailed shape and to compare to QED calculations with logarithms resummed to two-loop order (NNLL).

It is amusing to note that all three of the Standard Model interactions - QED, QCD, and of course $SU(2) \times U(1)$ contribute to the $Z$ line-shape.

The result from LEP is: $\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$

which is to be compared to $2.4957 \text{ GeV}$ from the SM fit.
Total Hadronic Cross-Section (nbarn) vs. Center-of-Mass Energy (GeV)

OPAL
composite of the four LEP experiments, showing the effect of ISR
My other example is $\Gamma(Z \rightarrow b\bar{b})$. The diagrams contribute a correction to the $b_L$ $Z$ charge,

$$Q_{ZbL} = -\left(\frac{1}{2} - \frac{1}{3} s_w^2 - \frac{\alpha}{16\pi s_w^2} \frac{m_t^2}{m_W^2}\right)$$

This is a -2% correction to the partial width. It is easier to measure the quantity

$$R_b = \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadrons})}$$

which is almost independent of $s_w^2$ with the SM charge assignments for $b$ and $\bar{b}$.
OPAL

1994 data
Monte Carlo b
Monte Carlo c
Monte Carlo uds

rate vs tagging variable B
The final result is:

\[ R_b = 0.21629 \pm 0.00066 \quad (\pm 0.3\%) \]

to be compared to 0.21586 from the SM fit, confirming the -2% shift due to the t-W diagrams.
A part of this study is the measurement of the longitudinal polarization of fermions in Z decay. The observation of longitudinal polarization in Z decay (an excess of $f_L$ over $f_R$) indicates that the Z couplings to $f$ and $\bar{f}$ are different.

Since $Q_Z = I_3 - s_w^2 Q$, the difference comes from $I_3$ and is part of the more famous story that the charge-changing weak interactions couples only to doublets, not to singlets.
Energy distribution of tau decay products

\[ A = 15\% \quad \text{for leptons} \]
Angular distribution of b jets using polarized electrons

A = 94% for b quarks

SLD
This is the famous V-A structure of weak interactions.

There are many sharp tests of V-A, including

- polarization of electrons in $\beta$-decay. \[ P(e^-) = -\nu(e^-)/c \]
- $\gamma$-distributions in neutrino deep inelastic scattering
- electron spectrum in $\mu$ decay.

All require different SU(2) quantum numbers for $f$ vs. $\bar{f}$.

M. Bardon et al., w. J. Lee-Franzini
Finally, the LEP measurements of $e^+e^- \rightarrow W^+W^-$ allow measurement of the $WW\gamma$ and $WWZ$ couplings.

The SM makes a specific prediction, for example:

$$W^+ \rightarrow e^{+} e^{-} \rightarrow W^+ W^- \rightarrow WW\gamma$$

$$W^+ \rightarrow e^{+} e^{-} \rightarrow W^+ W^- \rightarrow WWZ$$

$W$ pair production involves the three diagrams:

1. $WW\gamma$
2. $WWZ$
3. $W$ pair production

and thus is sensitive to these vertices.
There is a famous subtle feature in this analysis.

Estimate:

\[
\gamma \sim e^{2\bar{u}\gamma^\mu u} \frac{1}{s} (k_+ - k_-)^\mu \epsilon_+^* \cdot \epsilon_-^*
\]

For W bosons with longitudinal polarization

\[
\epsilon^\mu = \frac{1}{m_W} (|k|, E\hat{k})^\mu \approx \frac{k^\mu}{m_W}
\]

then

\[
\epsilon_+^* \cdot \epsilon_-^* \approx \frac{s}{2m_W^2}
\]

But, multiplying the amplitude by this factor violates unitarity in the l=1 partial wave. All three diagrams have this improper behavior.
However, with the Yang-Mills vertices of the SM, the improper terms exactly cancel among the three diagrams (Buras and Gaemers)

What does experiment say?
This cross section measurement and measurements of the $W$ polarizations in pair production allow the LEP experiments to put strong constraints on deviations from the Yang-Mills form.
Thus, experiment supports the universality of weak interaction couplings and the charge assignments for the electroweak symmetry group $\text{SU}(2) \times \text{U}(1)$.

Experiment also supports the Yang-Mills structure for the interaction of electroweak bosons.

This makes it inescapable that Nature is constructed from a gauge theory with $\text{SU}(2) \times \text{U}(1)$ symmetry.

However, that symmetry forbids the generation of masses for quarks, leptons, and gauge bosons. What can we do?
The problem of generating masses is fixed if we add to this structure one elementary scalar field with $I = 1/2$, $Y = 1/2$ under $SU(2) \times U(1)$.

$$\varphi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}$$

We postulate for this field a potential

$$V = \mu^2 |\varphi|^2 + \lambda |\varphi|^4$$

with $\mu^2 < 0$. The potential is minimized at a nonzero value of $\varphi$. All values with the same $|\varphi|$ are equivalent; we can pick $\varphi^0 \neq 0$ and real and expand about this choice:

$$\varphi = \begin{pmatrix} \pi^+ \\ \frac{1}{\sqrt{2}}(v + h + i\pi^0) \end{pmatrix}$$

$\pi^+, \pi^-, \pi^0$ can be removed by an $SU(2)$ gauge transformation. This leaves one scalar field $h$.

$\varphi$ is the **Higgs field**. $h$ creates a scalar particle, the **Higgs boson**.
These postulates do allow a beautiful theory of mass generation. If we ignore the Higgs boson and replace

$$\varphi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$

1. The Higgs field kinetic terms

$$|D_\mu \varphi|^2 = |(\partial_\mu - igA_\mu \cdot \tau - ig' \frac{1}{2} B_\mu)\varphi|^2$$

automatically reduce to mass terms for $W$ and $Z$. These give

$$m_W = \frac{gv}{2} \quad m_Z = m_W / c_w \quad m_\gamma = 0$$

2. Terms allowed by symmetry reduce to quark and lepton mass terms. For example:

$$\delta \mathcal{L} = -y_e \bar{e} \cdot \varphi^\dagger L$$

gives the $e$ mass term with

$$m_e = \frac{y_e v}{\sqrt{2}}.$$
I will call this theory the **Minimal Standard Model (MSM)**.

The **MSM** contains one particle that has not yet been discovered, the Higgs boson. The mass of the Higgs boson is free, since it depends on $\lambda$, which has not yet entered our formulae. However, once the mass of the Higgs boson is known, all other properties of this particle are predicted.

For example,

\[
\begin{align*}
\frac{\lambda}{v} &= -i \frac{m_f}{v} \\
\int_{W} h &= 2i \frac{m_W^2}{v} g^{\mu\nu} \\
\int_{Z} h &= 2i \frac{m_Z^2}{v} g^{\mu\nu}
\end{align*}
\]

The interplay of these couplings gives the complex pattern of Higgs boson branching ratios.
The MSM has no flavor-changing Higgs interactions. This is an elegant property of the model.

If we begin with the most general quark-Higgs coupling

\[ \delta \mathcal{L} = Y_{u}^{ij} \bar{u}^{i} \varphi \cdot Q^{j} - Y_{d}^{ij} \bar{d}^{i} \varphi^\dagger Q^{j} \]

we can diagonalize \(Y_{u}, Y_{d}\) using unitary transformations \(W, V\).

\[ Y_{u} = W_{u} y_{u} V_{u}^\dagger \quad Y_{d} = W_{d} y_{d} V_{d}^\dagger \]

\(W_{u}, W_{d}\) can be absorbed into \(\bar{u}, \bar{d}\). This change of variables puts the \(W\) into the \(\bar{u}, \bar{d}\) couplings to the U(1) gauge field, but there we find \(W_{a} W_{a}^\dagger = 1\). In a similar way, the \(V\)'s can be absorbed into \(u,d\). The charge-changing weak interaction acquires a matrix

\[ \frac{g}{\sqrt{2}} W_{\mu}^{-} u^\dagger \gamma^{\mu} (V_{u}^\dagger V_{d} \, d \]

The effect is nontrivial but desirable; we identify:

\[ V_{u}^\dagger V_{d} = V_{CKM} \]
The MSM is a beautiful model, but an empty one. It is a pure phenomenology. The “reason” that the gauge symmetry is broken is
\[ \mu^2 < 0 \]
and nothing more.

If we would like to have an explanation for the breaking of electroweak symmetry, we need to add more structure to the model.
Before leaving the Standard Model, though, I would like to discuss one more tantalizing notion that it suggests, **Grand Unification**.

The SM contains many different representations of the gauge group G. It is attractive to think that we could organize these into one or a few representations of a **larger group** that is broken to G at some high energy.

Georgi and Glashow found the choice SU(5). The assignment

\[
\begin{align*}
\bar{5} &= \begin{pmatrix} \bar{d} \\ \bar{d} \\ \bar{d} \\ e \\ \nu \end{pmatrix} \\
10 &= \begin{pmatrix} 0 & \bar{u} & \bar{u} & u & d \\ 0 & \bar{u} & u & d \\ 0 & u & d \\ 0 & \bar{e} \\ 0 \end{pmatrix}
\end{align*}
\]

gives the correct SU(3) x SU(2) x U(1) quantum numbers for all Standard Model fermions.
Groups that contain SU(5) could also be used. Pati and Salam had suggested SO(10). This group has a 16-dimensional spinor representation that decomposes under SU(5) as

\[ 16 \rightarrow \overline{5} + 10 + 1 \]

The 1 is a singlet neutrino, which, as we saw, could gain a mass as a result of the grand symmetry breaking. $E_6$ and SU(6) are other possible larger groups.
The Yang-Mills theory of the grand unified group has a single coupling \( g_U \). After symmetry breaking, we predict

\[
g_3 = g_2 = g_1 = g_U
\]

This does not seem right. The three couplings were measured to high accuracy in the LEP experiments

\[
\alpha_3^{-1} = 8.50 \pm 0.14 \quad \alpha_2^{-1} = 29.57 \pm 0.02 \quad \alpha_1^{-1} = 59.00 \pm 0.02
\]

where \( \alpha_a = g_a^2 / 4\pi \).

But actually it is not so bad. Coupling constants run with energy scale according to the renormalization group. The values above are given at the scale \( m_Z \). At higher energies, \( \alpha_3 \) will be weaker and \( \alpha_1 \) will be stronger.

We should analyze this more quantitatively.
The 1-loop renormalization group equation

\[ \frac{d g_a}{d \log Q} = - \frac{b_a}{(4\pi)^2} g_a^3 \]

integrates to

\[ \alpha^{-1}(Q) = \alpha^{-1}(M_U) - \frac{b_a}{2\pi} \log \frac{M_U}{Q} \]

Thus, if the couplings are equal at \( M_U \) and have the known values at \( m_Z \), this must give a constraint on the \( b_a \). The precise quantity constrained is

\[ B = \frac{b_3 - b_2}{b_2 - b_1} \]

If the \( b_a \) were all equal, the three couplings would not diverge.
From the measured values of the couplings, we find

\[ B = 0.716 \pm 0.005 \pm 0.03 \]

where the second error is my estimate of the effect of 2-loop and threshold corrections, omitted in this general analysis.

On the other hand, the MSM predicts

\[
\begin{align*}
b_3 &= 11 - \frac{4}{3} n_g \\
b_2 &= \frac{22}{3} - \frac{4}{3} n_g - \frac{1}{6} n_\varphi \\
b_1 &= -\frac{4}{3} n_g - \frac{1}{10} n_\varphi
\end{align*}
\]

giving \( B = 0.53 \).

So the MSM is in qualitative agreement with Grand Unification but quantitatively it is in error.
If you think that Grand Unification is an attractive feature of a more complete model of physics, then you might ask that our model of physics above $m_Z$ should repair this difficulty.

This is something to look for as we study explicit models of electroweak symmetry breaking.

We will begin that study in the next lecture.