TeV Gravity + KK Graviton Excitations in e^+e^-/e^-e^- Collisions

- Examine Two Models on the Market:
  - (i) Arkani-Hamed, Dimopoulos + Dvali (ADD)
  - (ii) Randall + Sundrum I (RS)

- Contrast signatures for graviton exchange in these two models in e^+e^- and e^-e^- collisions

- Very different in e^+e^-, more similar in e^-e^-

TGRizzo
12/99

ADD : Large, compactified extra dimensions

$$M_{Pl}^2 = V_n \, M_*^{n+2} \swarrow$$ n+4 dim. Planck scale

$\uparrow$ n-dimensional compactified volume

e.g., a torus $V_n = (2\pi R)^n$

- Gravity lives in n-dimensional "BULK" + interacts with SM fields constrained to lie on a 3-brane "WALL"

- Gravity becomes as strong other forces ~ M_* ≈ few TeV?

$n = 1$ $R \sim 10^{11}$ m Excluded by Newton!

$n = 2$ $R \sim 1$ mm $M_* > 150$ GeV by astrophysics

$\Rightarrow$ $n > 2$ R down to fm $M_* \gtrsim 1$ TeV

$\Rightarrow$ $F_{grav}(r<R)$ is $\sim \dfrac{1}{r^{n+2}}$!

$$S_{int} \equiv -\frac{1}{M_*^{1+n/2}} \int d^4x \, d^ny \; h^{AB}(x,y) \, T_{AB}(x,y)$$

But
$$\begin{cases} T_{AB} = \eta^{\mu}_{A}\eta^{\nu}_{B}\, T_{\mu\nu}\,\delta(\vec{y}) \\ h_{AB} \equiv \sum_n h^{(n)}_{AB}(x)\cdot \frac{1}{\sqrt{V_n}}\, e^{i n\cdot y/R} \end{cases}$$

sm on wall

KK decomposit

→ decompose h_{AB} + go to unitary gauge
integrate over extra dim's
via δ functions

→ $m_n^2 = n^2/R^2$ $\quad n^2 = 0, 1, \ldots$

a closely spaced distribut
of masses

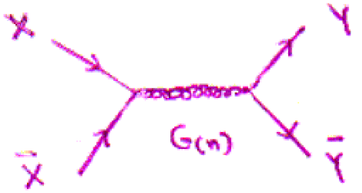
* Graviton tower couples to $T_{\mu\nu}$, $\bar{M}_{Pl}^{-1}$ (spin-2)
Scalar tower couples to T^{μ}_{μ}, $\sim\bar{M}_{Pl}^{-1}$ (spin-0)
Vectors + other scalars decouple 'on wall'

* $T^{\mu}_{\mu} \sim m_{fermion}$ or m^2_{boson} ∴ not interesting
in $e^+e^- \to X\bar{X}$ $\quad \{ W^+W^- \xrightarrow[\phi]{} t\bar{t} \}$

$$\mathcal{L}_{int} = -\frac{1}{\bar{M}_{Pl}}\, T_{\mu\nu}\, G^{\mu\nu}_{(n)}$$ ↔ all modes couple identically

↑ sm fields

At Colliders



$$A = \frac{1}{M_{pl}^2} \sum_n \frac{T^{(X)}_{\mu\nu}\, P^{\mu\nu\alpha\beta}\, T^{(Y)}_{\alpha\beta}}{s - m_n^2 + i\epsilon}$$

Diverges !!

- cut-off
- finite 'fermion' / source size
- wall tension
- etc
- ⋮

allows sum / integral* to be evaluated

$^*\ \sum_n \rightarrow \int \rho(m)\, dm$

$\Rightarrow$ dimension 8 operators

$$M_G^{e^+e^-\to f\bar{f}} \equiv \frac{\lambda}{\Lambda_H^4} \left\{ \bar{f}\gamma_\mu f\ \bar{e}\gamma^\mu e\ (p_{\bar{f}} - p_f)\cdot(\bar{p}_e - p_e) + \bar{f}\gamma_\mu f \cdot \bar{e}\gamma_\nu e\ (p_{\bar{f}} - p_f)^\nu (p_{\bar{e}} - p_e)^\mu \right\}$$

$\boxed{\lambda = \pm 1}$ and Λ_H = (usually) $M_* / M_s / M_D \ldots$

** This normalization fixes couplings for all other processes

* Recall $m_n^2 = n^2/R^2$ and ∴ is almost a continuum for some $\sqrt{s} \gg 1/R$

$$\boxed{e^+e^- \to \bar{f} f}$$

$$\begin{aligned} \frac{d\sigma}{dz} = N_c \frac{\pi\alpha^2}{2s} \Bigg\{ & A(1+z^2) + 2Bz && \text{SM piece} \\ & -\frac{\lambda s^2}{2\pi\alpha M_*^4}\left[\, 2z^3 C + (1-3z^2)D \,\right] && \text{interference} \\ & + \frac{s^4}{16\pi^2\alpha^2 M_*^8}\left[(1-3z^2+z^4)E\right] \Bigg\}_{z=\cos\theta} && \text{pure gravity} \end{aligned}$$

(1) Spin-2 leads to $\underline{\cos^3\theta}$ + $\underline{\cos^4\theta}$ terms
<u>not</u> in SM cross-section !

(2) Int. term <u>vanishes</u> upon $\int d\cos\theta$!

$\rightarrow$ look at distributions + <u>not</u>
$\Rightarrow$ integrated quantities

$\boxed{\text{But}}$ grows like $\boxed{s^2/M_*^4}$ relative to SM !

(3) Spin-2 $\boxed{\text{uniquely}}$ different from other exchanges : $\mathcal{J}$, Z', LQ

Processes

$e^+e^- \to f\bar{f}$ (f-tagged) $f = \mu, \tau, c, b, t$ *

$\to jj$ $[j = u, d\ s + \text{gluons}]$

$$\left.\begin{array}{l} e^+e^- \to \gamma\gamma \\ e^+e^- \to ZZ \end{array}\right\}$$ closely related, new s-channel exchange

$e^+e^- \to W^+W^-$

$\Longrightarrow$ σ_{TOT}, $\frac{d\sigma}{dz}$, various asymmetries

All have been looked at....

*

$$\left.\begin{array}{l} e^+e^- \to e^+e^- \\ e^-e^- \to e^-e^- \end{array}\right\}$$ somewhat special...

Hewett

Angular Distributions for $e^+e^- \to f\bar{f}$

$\sqrt{s} = 500$ GeV

$M_s = 1.5$ TeV

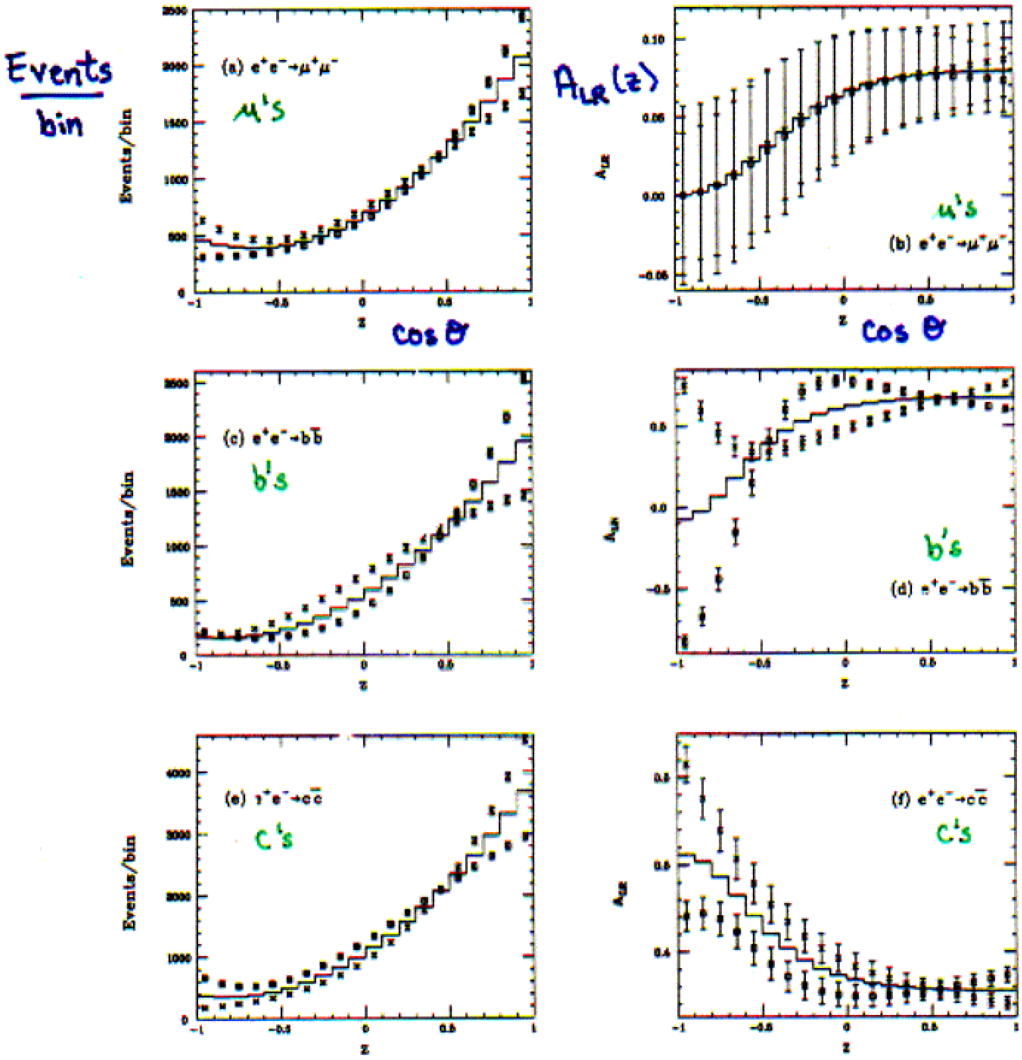
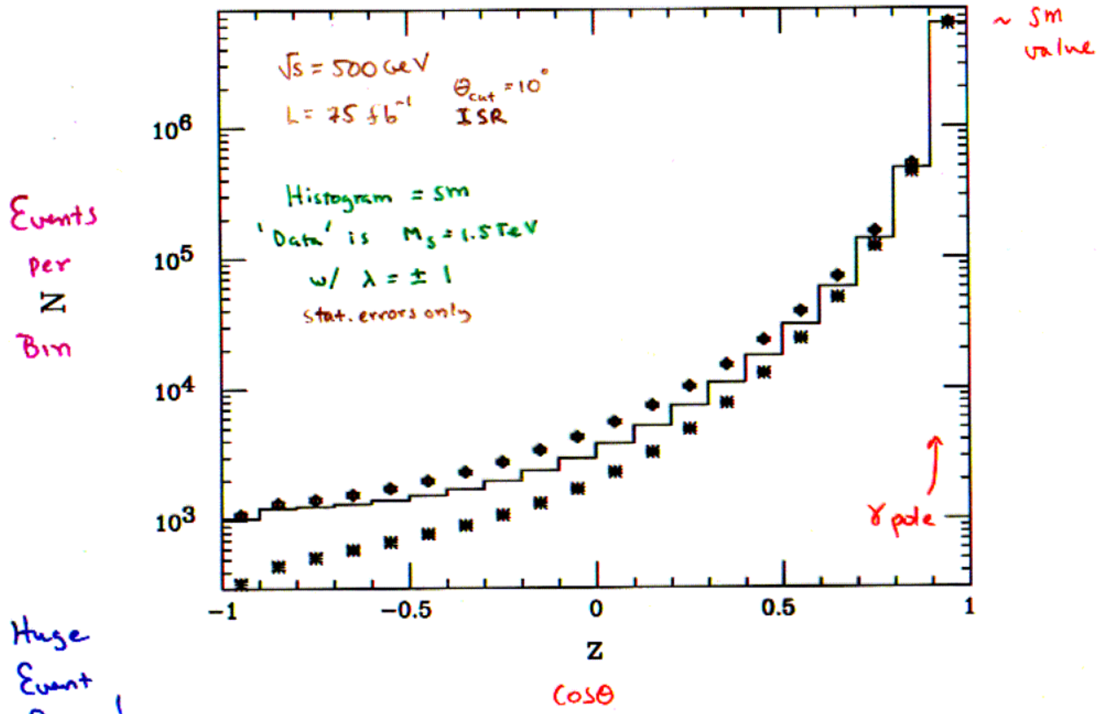


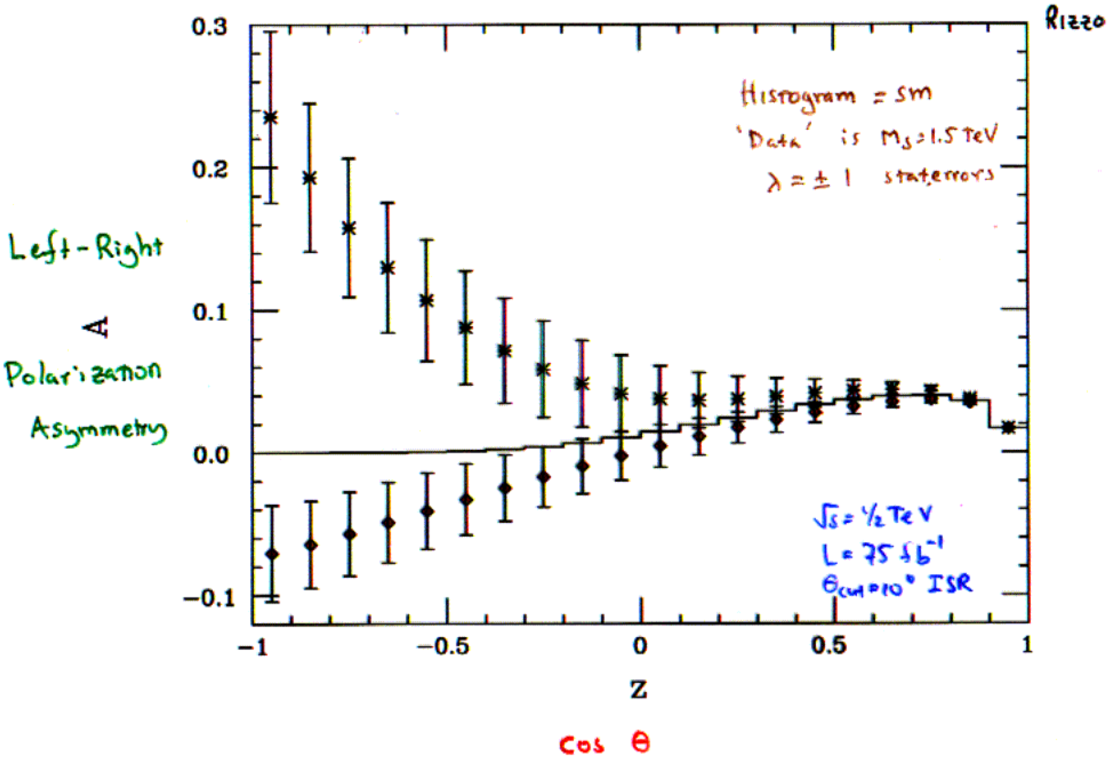
Figure 1: Bin integrated angular distribution and z-dependent Left-Right asymmetry for $e^+e^- \to \mu^+\mu^-, b\bar{b}, c\bar{c}$. In each case, the solid histogram represents the SM, while the 'data' points are for $M_s = 1.5$ TeV with $\lambda = \pm 1$. The error bars correspond to the statistics in each bin.

Bhabha Scattering

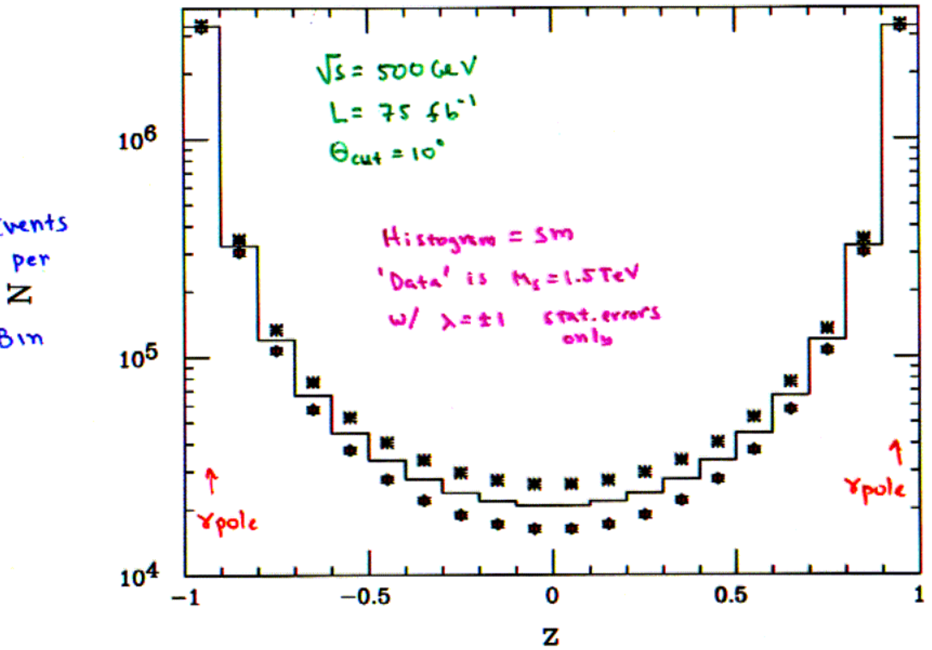


Huge Event Rates!

Bhabha Scattering



Moller Scattering



$\sqrt{s} = 500$ GeV

$L = 75$ fb^{-1}

$\theta_{cut} = 10°$

Histogram = SM

'Data' is $M_s = 1.5$ TeV w/ $\lambda = \pm 1$

stat. errors only

~SM value

↑ γpole

↑ γpole

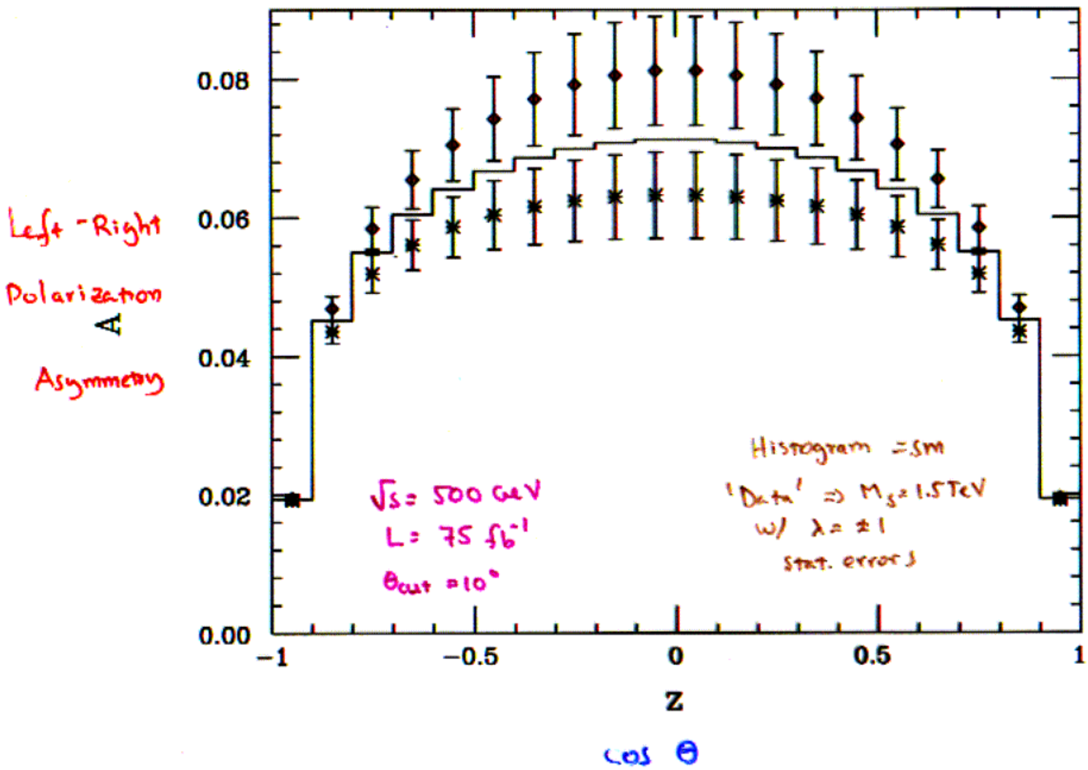
Events per Z Bin

cos θ

Huge Event rates

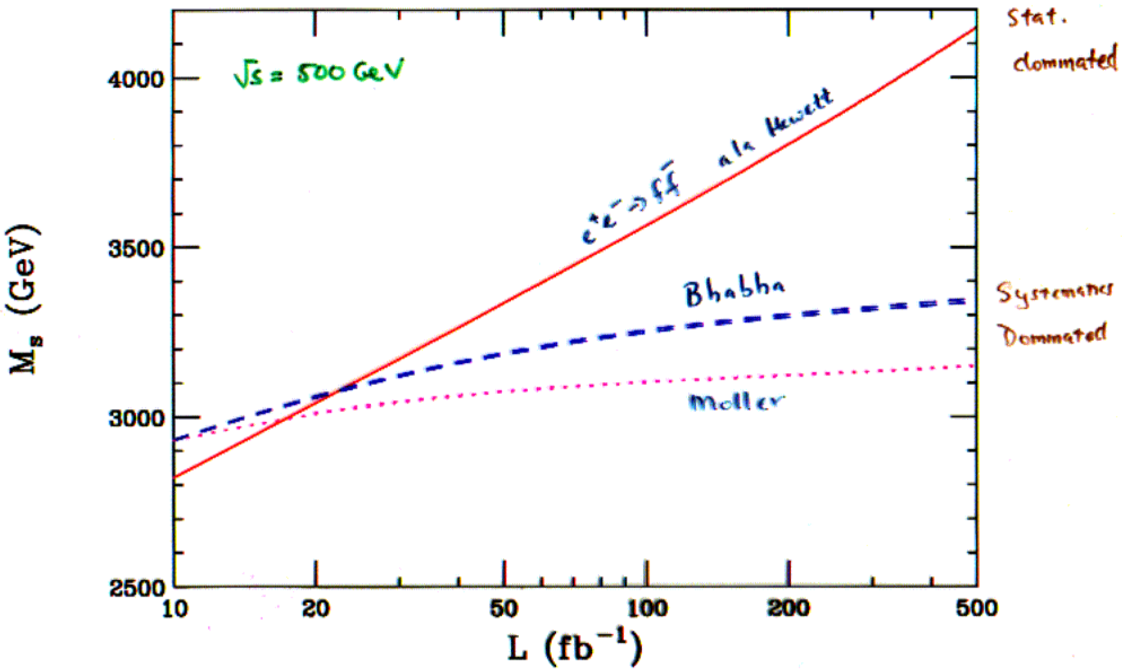
Moller Scattering

Both Beams Polarized



Search Reaches

Rizzo

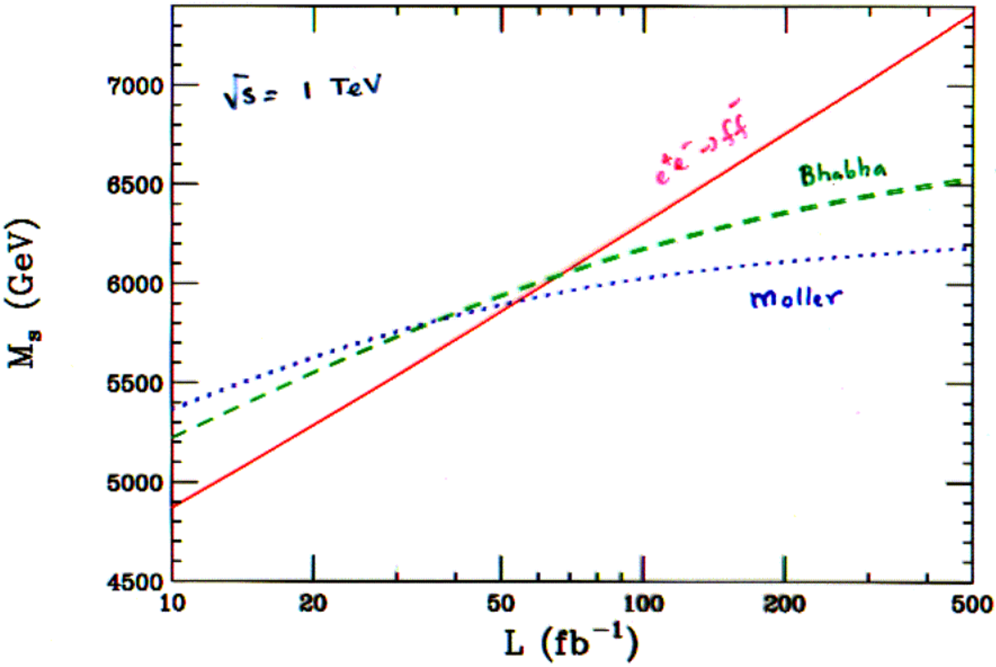


Moller wins at low lumi
< 10 fb^{-1}

$f = \mu, \tau, c, b, t$

Search Reaches

Rizzo

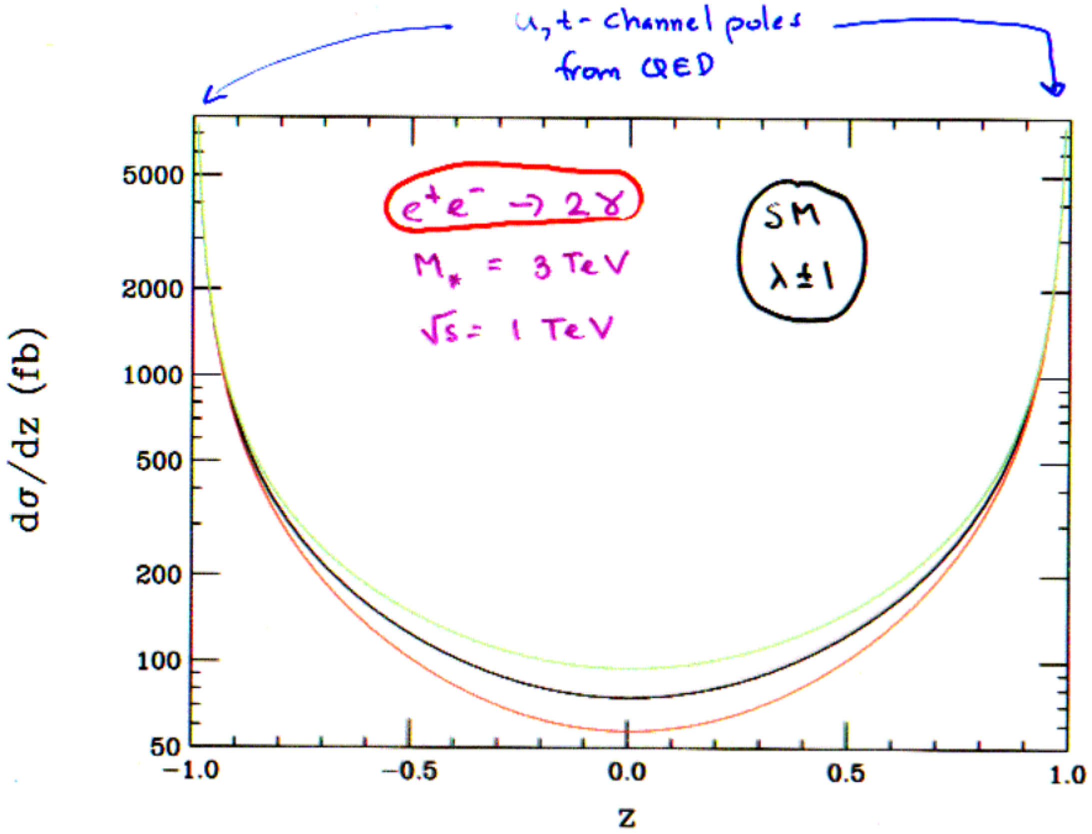


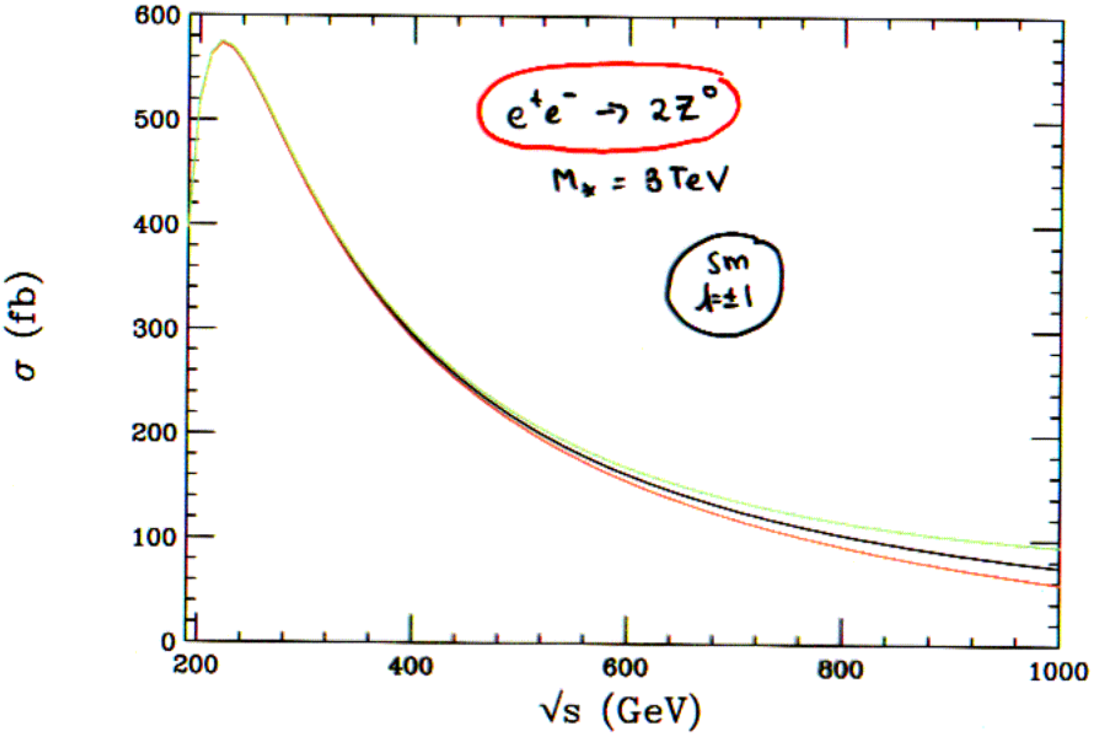
Stat. dominated

} sys. domin.

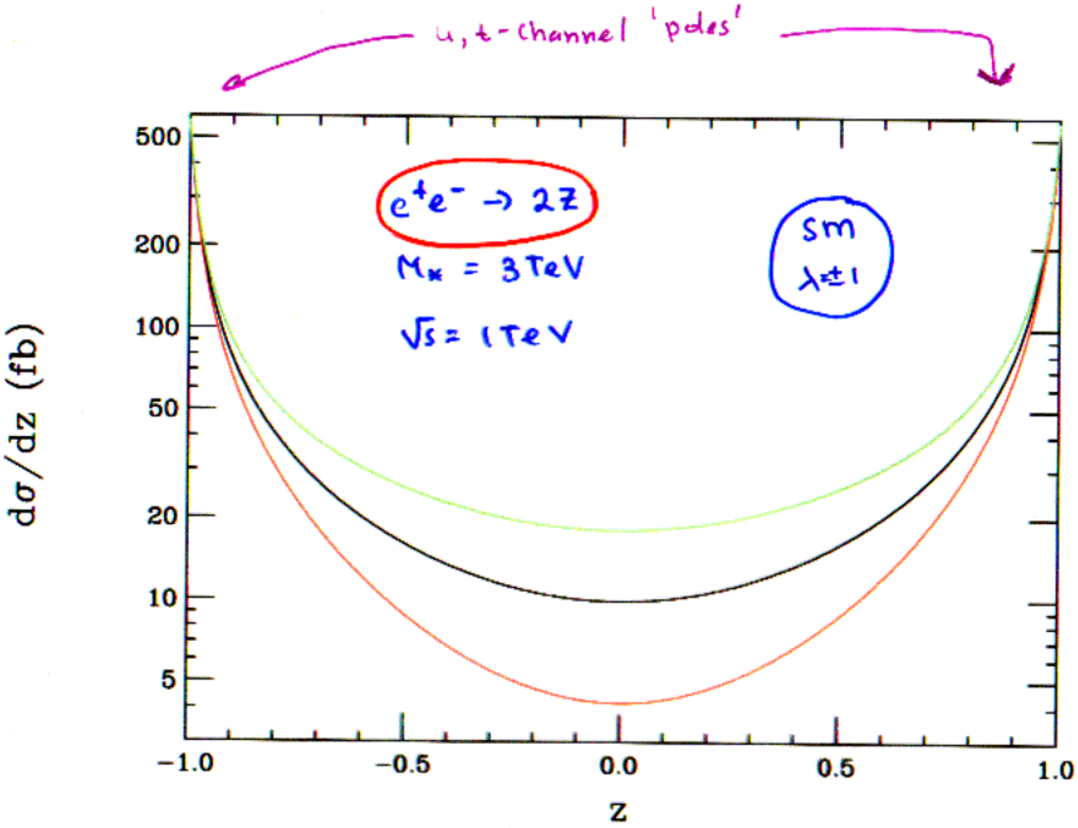
Moller wins at low lumi

$\lesssim 30\ fb^{-1}$


u, t- channel poles
from QED
e+e- → 2γ
M* = 3 TeV
√s = 1 TeV
SM
λ ± 1
dσ/dz (fb)
5000
2000
1000
500
200
100
50
-1.0
-0.5
0.0
0.5
1.0
z



$e^+e^- \to 2Z^0$
$M_* = 3$ TeV
Sm
$\lambda = \pm 1$
600
500
400
300
200
100
0
σ (fb)
200
400
600
800
1000
$\sqrt{s}$ (GeV)



u, t-channel 'poles'
e⁺e⁻ → 2Z
M* = 3TeV
√s = 1TeV
SM
λ=±1
dσ/dz (fb)
500
200
100
50
20
10
5
−1.0
−0.5
0.0
0.5
1.0
z

Reaction	LEP II (2 fb^{-1})	LC (100 fb^{-1})
$e^-e^- \to ff$	1.15	$6.5\sqrt{s}$
$e^+e^- \to e^-e^-$	1.0	$6.2\sqrt{s}$
$e^-e^- \to e^-e^-$		$6.0\sqrt{s}$
$e^+e^- \to \gamma\gamma$	1.4	$3.2\sqrt{s}$
$e^+e^- \to WW/ZZ$	0.9	$5.5\sqrt{s}$
	Tevatron (2 fb^{-1})	LHC (100 fb^{-1})
$\overset{(-)}{p}\overset{(-)}{p} \to \ell^+\ell^-$	1.4	5.3
$\overset{(-)}{p}\overset{(-)}{p} \to t\bar{t}$	1.0	6.0
$\overset{(-)}{p}\overset{(-)}{p} \to jj$	1.0	9.0
$\overset{(-)}{p}\overset{(-)}{p} \to WW$	0.8	
$\overset{(-)}{p}\overset{(-)}{p} \to \gamma\gamma$	1.4	5.4
	HERA (250 pb^{-1})	
$ep \to e+$ jet	1.0	
	$\gamma\gamma$ Collider (100 fb^{-1})	
$\gamma\gamma \to \ell^-\ell^-/t\bar{t}/jj$	$4\sqrt{s}$	
$\gamma\gamma \to \gamma\gamma/ZZ$	$(4-5)\sqrt{s}$	
$\gamma\gamma \to WW$	$11\sqrt{s}$	

Table 1. M_s search limits in TeV for a number of various processes.

* Reach

Summary

(1998); Z. Berezhiani and G. Dvali, hep-ph/9811378; N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353; Z. Kakushadze, hep-th/9811193 and hep-th/9812163; N. Arkani-Hamed *et al.*, hep-ph/9811448; G. Dvali and S.-H.H. Tye, hep-ph/9812483.

[2] J.C. Long, H.W. Chan and J.C. Price, hep-ph/9805217.

[3] For an alternative approach to both large and small extra dimensions with asymmetric compactifications, see J. Lykken and S. Nandi, hep-ph/9908505.

[4] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999); L.J. Hall and D. Smith, hep-ph/9904267; V. Barger, T. Han, C. Kao and R.J. Zhang, hep-ph/9905474; G.C. McLaughlin and J.N. Ng, hep-ph/9909558.

[5] L. Randall and R. Sundrum, hep-ph/9905221 and hep-ph/9906064; W.D. Goldberger and M.B. Wise, hep-ph/9907218 and hep-ph/9907447; H. Davoudiasl, J.L. Hewett and T.G. Rizzo, hep-ph/9909255.

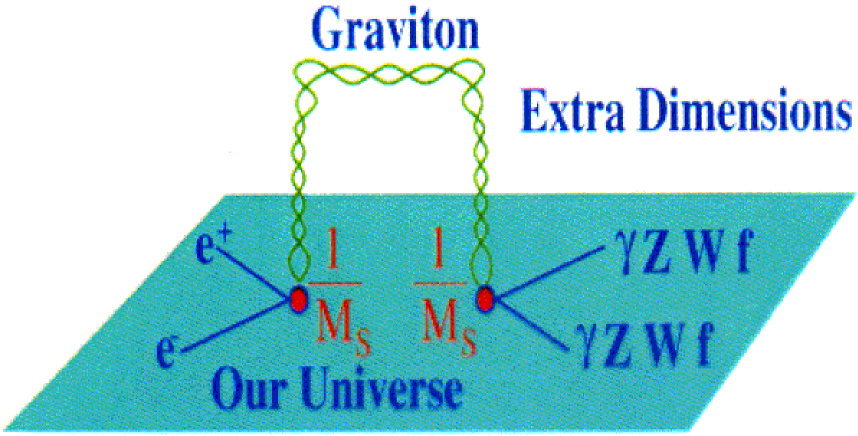
[6] G.F. Giudice, R. Rattazzi and J.D. Wells, Nucl. Phys. **B544**, 3 (1999); T. Han, J.D. Lykken and R. Zhang, Phys. Rev. **D59**, 105006 (1999); E.A. Mirabelli, M. Perelstein and M.E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999); J.L. Hewett, Phys. Rev. Lett. **82**, 4765 (1999).

[7] For an incomplete list, see N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, hep-ph/9811448; N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353; K. Benakli and S. Davidson, hep-ph/9810280;

* An analysis combining all channels will be useful + has not yet been performed.

⇒ A quick estimate is that $M_* \sim 9\sqrt{s}$ may be reachable for ~ 100 fb^{-1} at 500 GeV - 1TeV

Low Scale Gravity



ZZ channel updated from 192 to 200 GeV

95 % C.L. lower limits on M_S:

Final State	$\sqrt{s}$ GeV	M_S (TeV) $\lambda = +1$	M_S (TeV) $\lambda = -1$
ZZ	192-202	1.1	1.1
Fermions+Bosons	≤189	1.1	0.9

Large improvement with the ZZ channel as the energy and luminosity increase

Randall - Sondrum Model I

- One extra dimension
- non-factorizable metric ; 4-d metric depends on extra dimension

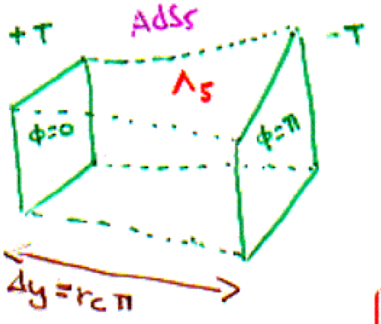
$$ds^2 = \overset{\text{warp factor}\nearrow}{e^{-2\sigma(y)}} \underbrace{\eta_{\mu\nu}\, dx^\mu dx^\nu}_{\text{Poincare' Invariant}} + dy^2 \qquad \boxed{y \equiv r_c \phi}$$

- two 3 branes a distance $r_c\pi$ apart in y

  one with $\oplus$ tension, the other $\ominus$ tension

  $\oplus$ tension $\equiv$ Planck brane ; $\ominus$ tension $\equiv$ SM brane

- A 5-D cosmological constant lives in bulk between branes , Λ_5



→ Einstein's Eq. gives unique solution: AdS5 slice

- $\sigma \equiv \underline{\underline{k}} r_c |\phi|$ (+c)
- Λ_5 related to brane tensions:

$$\boxed{\bar{M}_{Pl}^2 = M_{5D}^3 / k} \qquad \left(k \approx M_{5D} \approx \bar{M}_{Pl} \right)$$

no new hierarchy

- mass m_0 on Planck brane ($\phi = 0$), appears to us ($\phi = \pi$) as $m_0 e^{-kr_c\pi}$ - exponentially smaller!

→ hierarchy generated by exponential

→ Planck scale / weak scale set by $\boxed{kr_c \simeq 11\text{-}12}$

⇛ - stable solution [Goldberger + Wise]

Phenomenology : Davoudiasl, Hewett + TR $\left\{ \begin{matrix} \text{PRL} \\ \text{PLB} \end{matrix} \right.$ +

- $$\mathcal{L} = -\frac{1}{\bar{M}_{Pl}} h^{(0)}_{\mu\nu} T^{\mu\nu} - \frac{1}{\Lambda_\pi} h^{(n>0)}_{\mu\nu} T^{\mu\nu}$$

← stress-energy tensor

usual zero mode coupling

higher ($n>0$) KK mode coupling

$$\boxed{\Lambda_\pi \equiv \bar{M}_{Pl} \, e^{-kr_c\pi}}$$

weak scale coupling

- Spin-2 gravitons have widely separate weak-scale masses:

$$m_n = k x_n e^{-kr_c\pi}$$ where $J_1(x_n) = 0$ Bessel function

∴ roots → masses are not equally spaced

→ • If m_1 is known all others are fixed

→

→ • $\Gamma_n \sim m_n^3 k^2$; widths grow rapidly with n

- Validity of Einstein's Equation :

  Demand curvature of 5D space is less than that set by 5D Planck scale :

$*$ $\quad |R_5| \equiv 20k^2 < M^2 \quad \Longrightarrow \quad \boxed{\frac{k}{\overline{M}_{Pl}} \lesssim 0.1}$

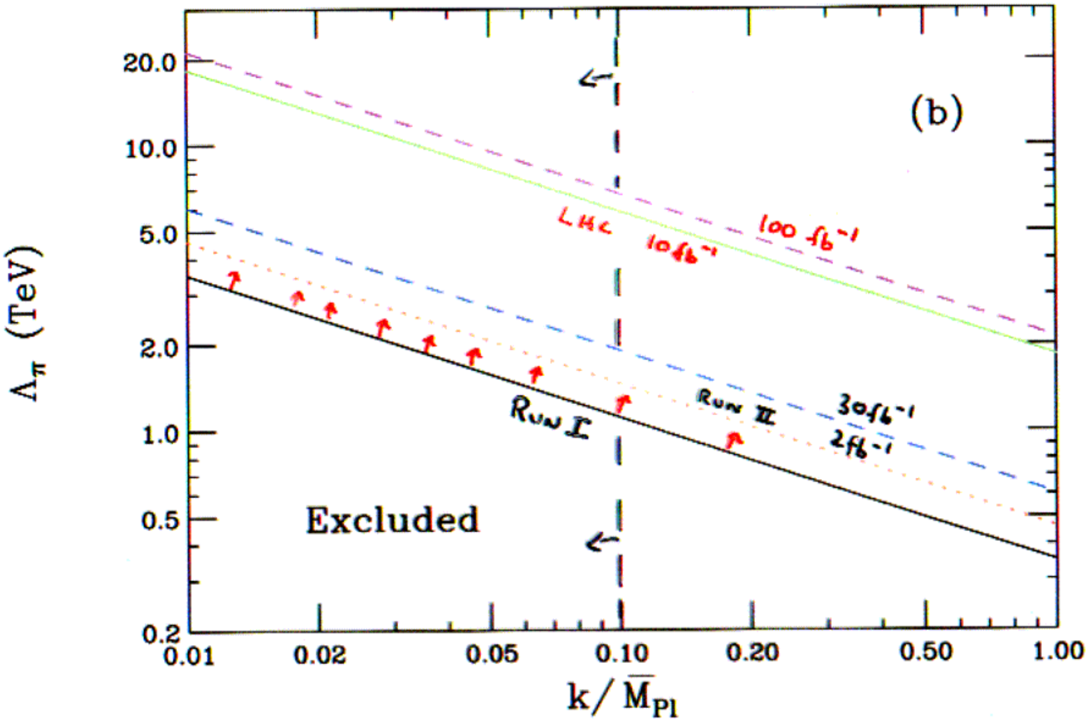
- Searches : below m_1 repeat ADD contact interaction analyses

$$\frac{\lambda}{M_H^4} \to \frac{-1}{8\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s-m_n^2}$$

$$\simeq \frac{1}{64\Lambda_\pi^4} \left(\frac{k}{\overline{M}_{Pl}}\right)^{-2}$$

OR look for weak scale graviton bumps at colliders

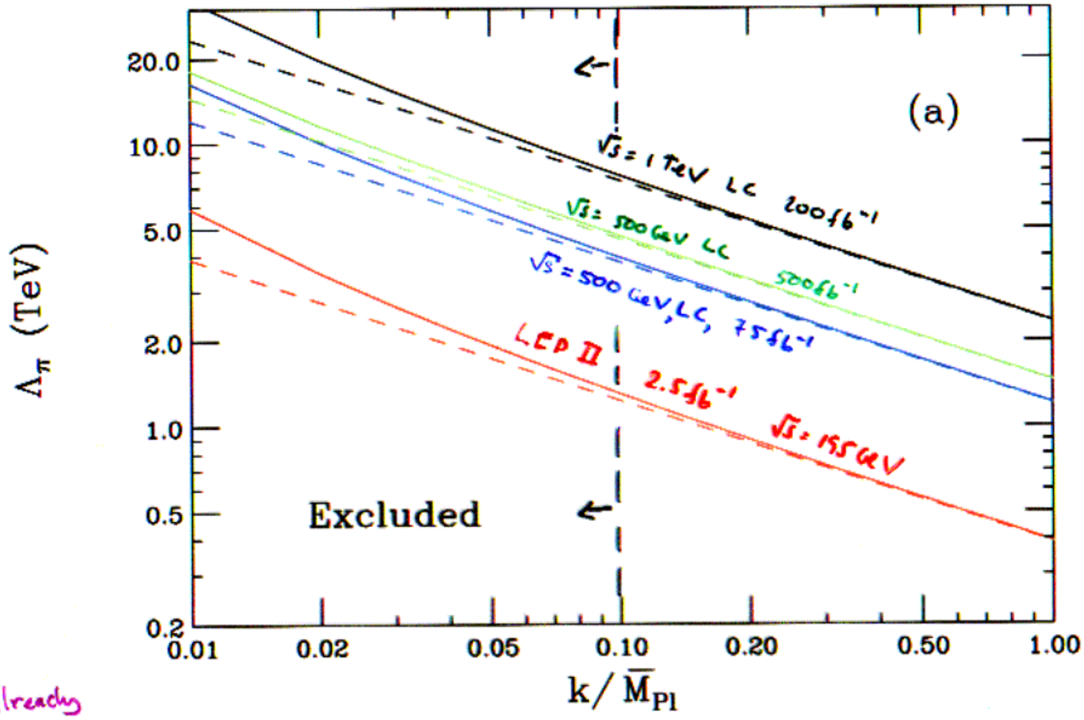
C.I Constraints on Randall-Sundrum Model



At some point identifying Λ_π as not the week scale

→ another hierarchy

CI Constraints on Randall-Sundrum Model

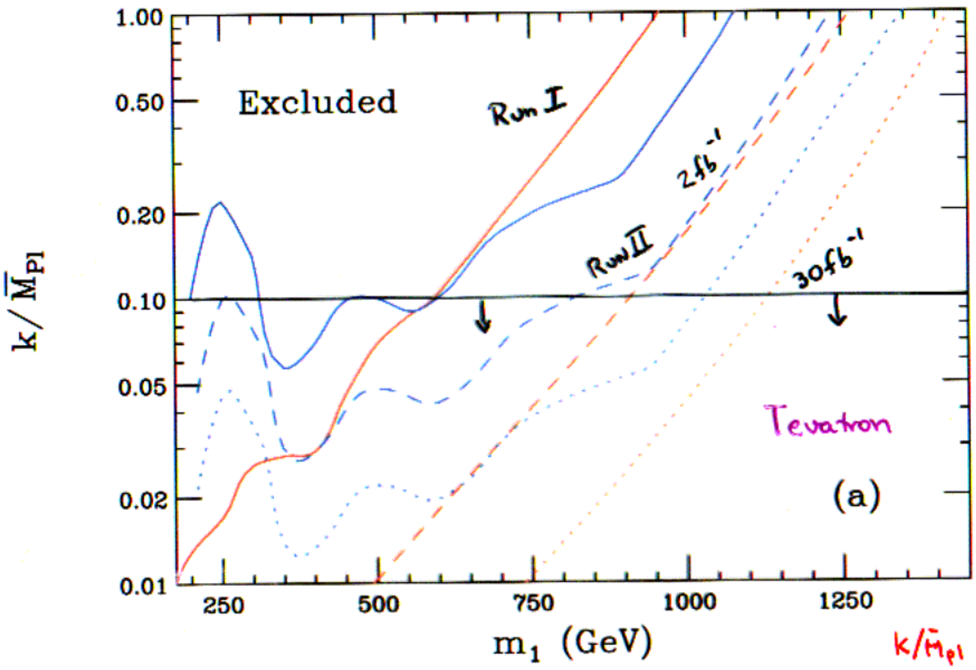


Already $k/\overline{M}_{Pl}$ much smaller than 10^{-2} looks bad for hierarchy problem solution...

Direct Bump Searches

Drell-Yan = red

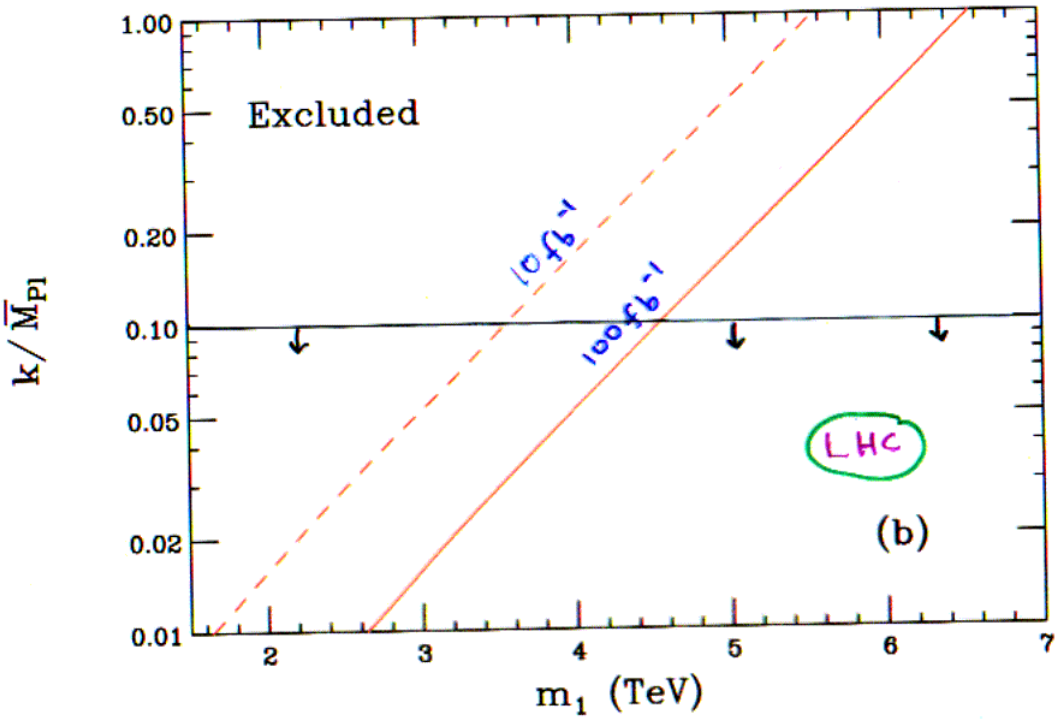
Dijet bumps = blue



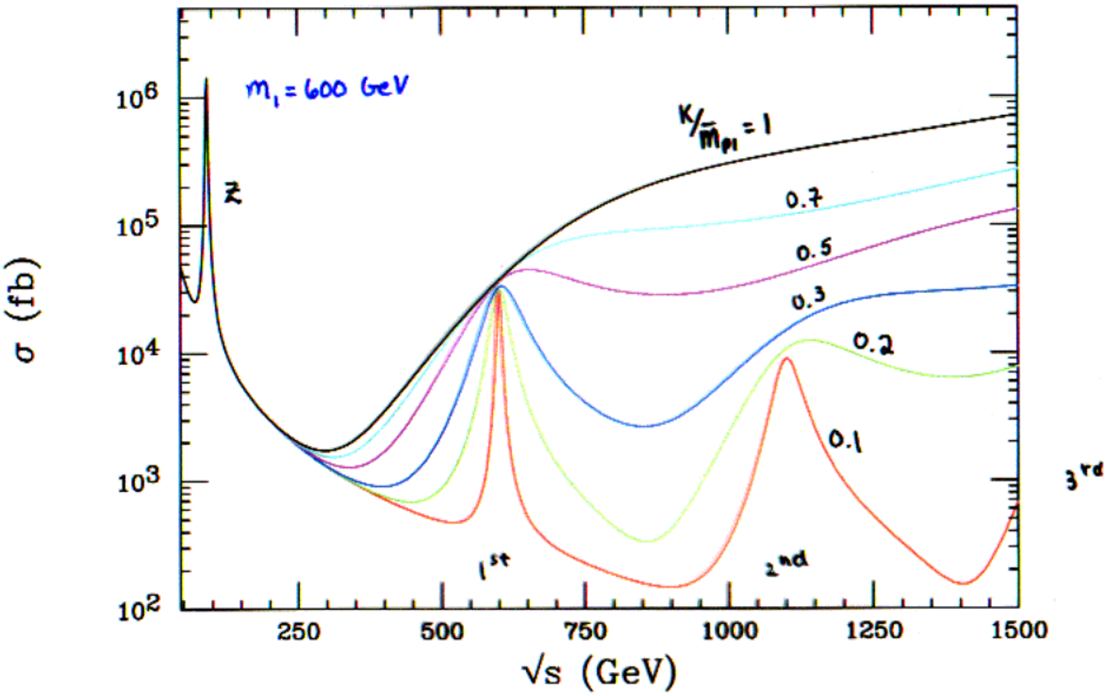
$k/\bar{M}_{Pl} = 0.1$

Randall-Sundrum model parameter constraints → $m_1 \gtrsim 600$ GeV

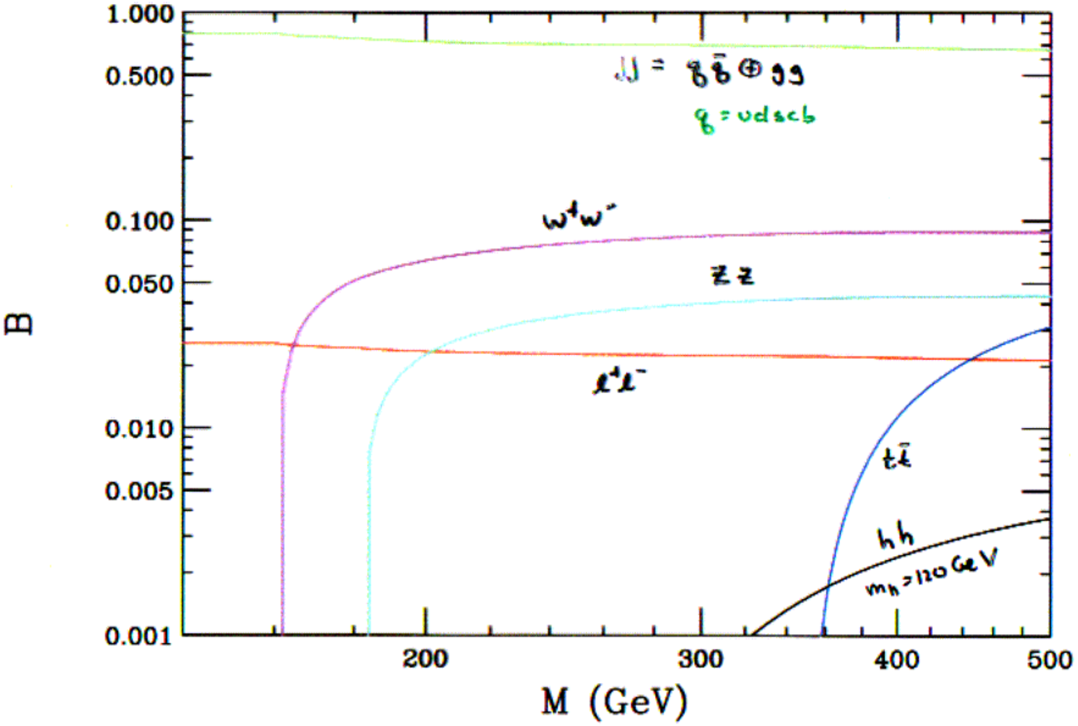
Drell-Yan Constraints on Randall-Sundrum Model



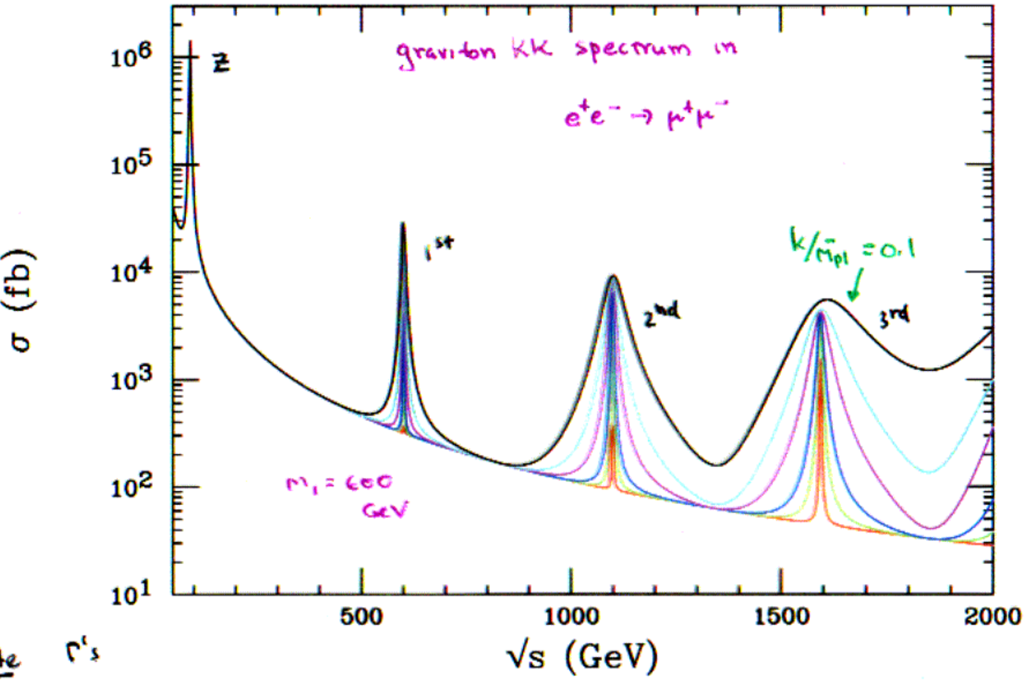
KK spectrum in $e^+e^- \to \mu^+\mu^-$



RS Graviton branching fractions



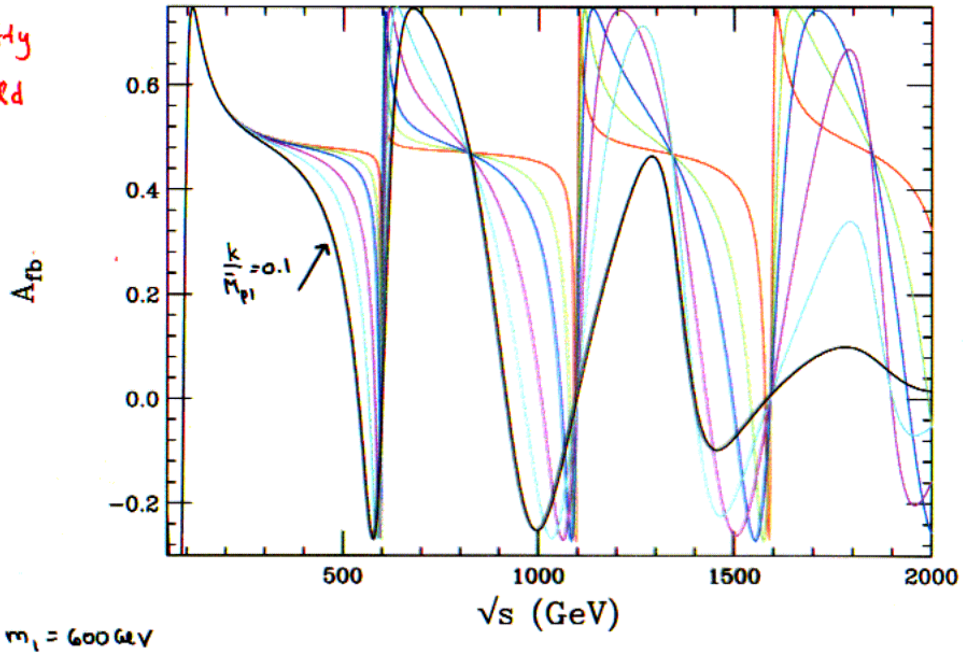
→ $B_{\gamma\gamma} = 2 B_{\ell^+\ell^-}$



⇒ Note Γ's increase with both n and $k/\overline{M}_{Pl}$

A_{FB} in $e^+e^- \to \mu^+\mu^-$ via RS gravitons

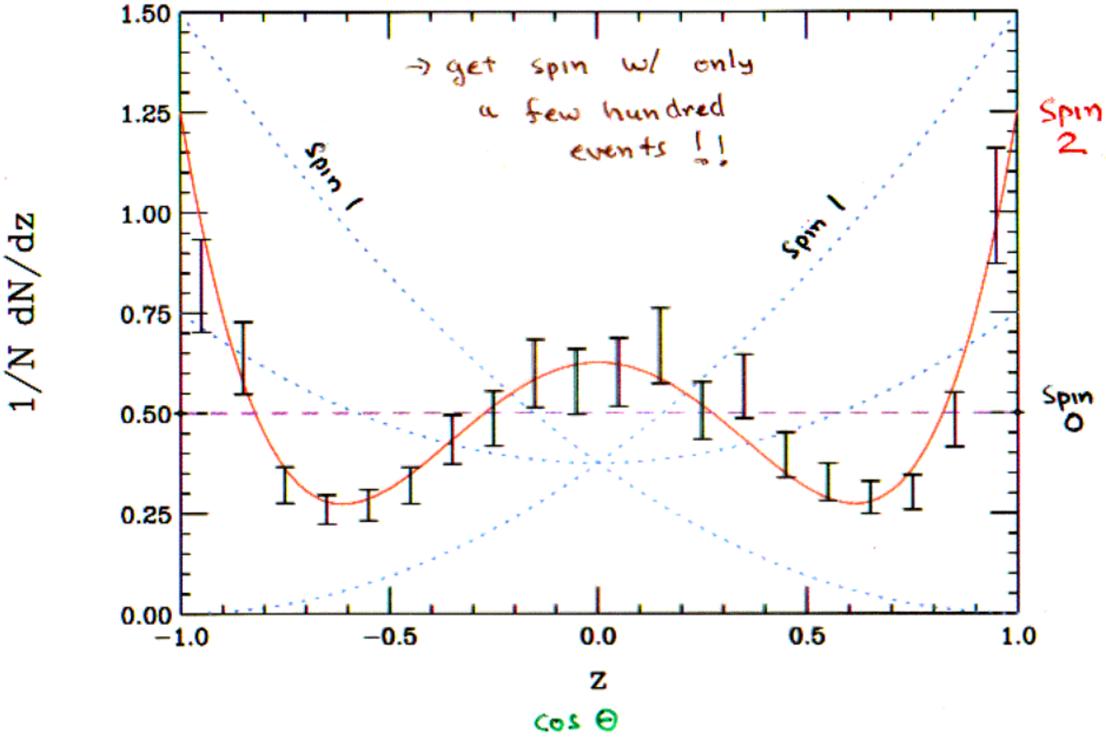
Pretty Wild



Note Damping !!

$m_1 = 600$ GeV

$f\bar{f} \to f'\bar{f}'$ on Graviton resonance



→ get spin w/ only a few hundred events !!

$$\boxed{e^-e^-}$$

$\Rightarrow$ Unfortunately, no graviton resonances in e^-e^- $\therefore$ will look like ADD model....

Conclusions + Summary

- ADD model will be well tested in both e^+e^- and e^-e^- collisions
  - $e^+e^- \to f\bar{f}, W^+W^-, 2Z, 2\gamma, 2g$
  - $e^-e^- \to e^-e^-$

  Parameters

  M_* and a sign λ

- Combining many processes / many observables with various systematics may allow a reach as high as $\approx \boxed{9\sqrt{s}}$!

- RS I offers something completely different!

  "weakly" coupled, spin-2 gravitons with weak scale masses in a non-equal spaced series

  → perfect for colliders !!

  * resonances in almost all channels

  2 parameters

  $k/\bar{M}_{Pl}$ and

  Λ_π or m_1

- Colliders will continue to be the testing ground for new TeV scale gravity ideas