Introduction to AdS/QCD and Light-Front Hadron Dynamics Part II

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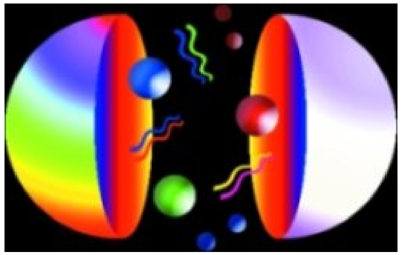

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1 Light-Front Dynamics and Mapping AdS Modes

Light-Front Wave Functions in QCD

- Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n \psi_{n/h} |n\rangle$ of the LF Hamiltonian $H_{LF} = P^2 = P^+ P^- - \mathbf{P}_\perp^2$

$$H_{LF} |\psi_h\rangle = \mathcal{M}_h^2 |\psi_h\rangle,$$

at fixed LF time $\tau = t + z/c$ (Dirac '49; Brodsky, Pauli and Pinsky, Phys. Rept. 1988).

$(\, |P\rangle = |uud\rangle + |uudg\rangle + +|uud\bar{q}q\rangle \dots)$

- Fock components

$$\psi_{n/h}(x_i, \mathbf{k}_{\perp i}) = \left\langle n; \; x_i, \mathbf{k}_{\perp i}, \middle| \psi_h(P^+, \mathbf{P}_\perp) \right\rangle,$$

frame independent and encode hadron properties in high momentum-transfer collisions.

- Momentum fraction $x_i = k_i^+ / P^+$ and $\mathbf{k}_{\perp i}$ are the relative coordinates of parton i in Fock-state n

$$\sum_{i=1}^{n} x_i = 1 \quad \sum_{i=1}^{n} \mathbf{k}_{\perp i} = 0.$$

$P = (P^+, P^-, \mathbf{P}_\perp)$

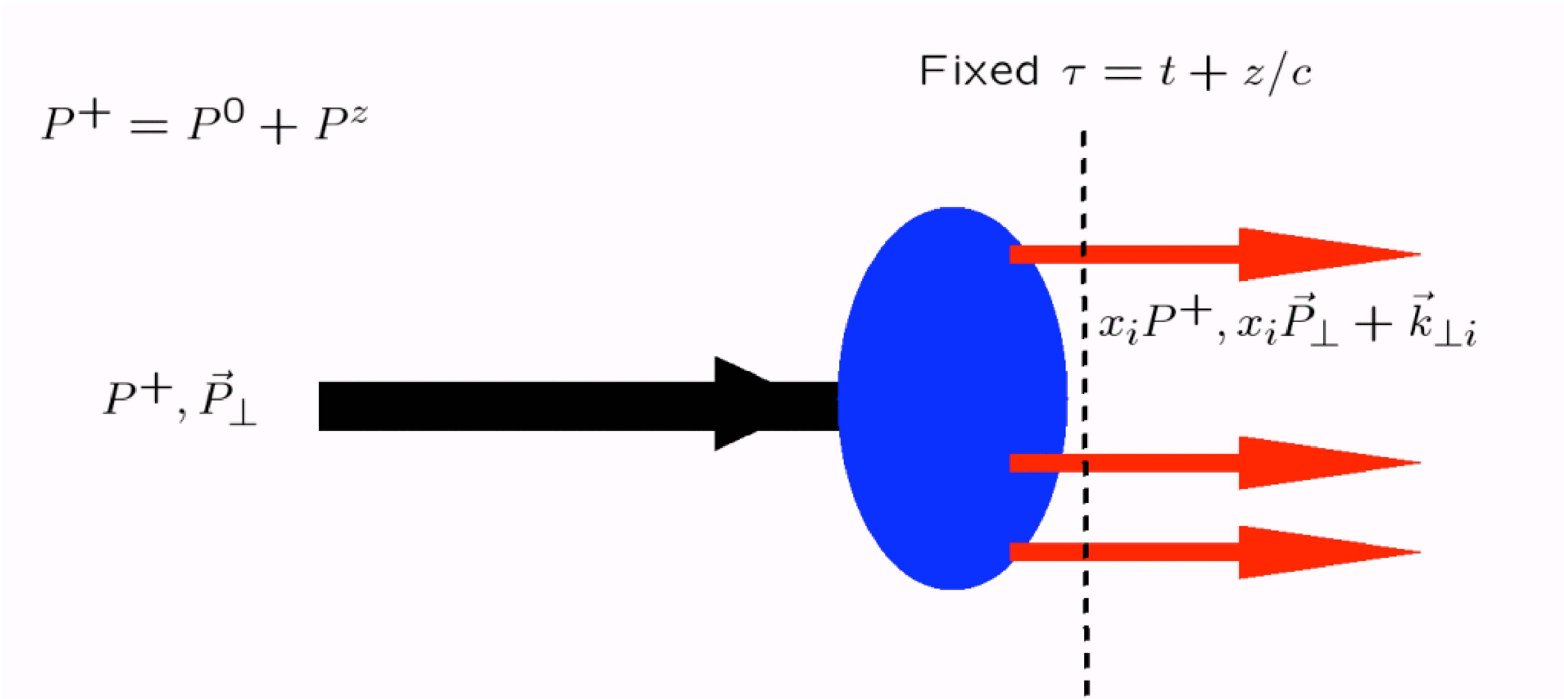


Fig: LFWF: $\psi(x_i, \mathbf{k}_{\perp i}, \lambda_i)$

- The LFWF are boost invariant: independent of total momentum P^+ and $\mathbf{P}_\perp$ of the hadron and depend only on the relative partonic coordinates.

Current Matrix Elements in the QCD Light-Front Frame

- Electromagnetic form factor $(P' = P + q)$

$$\left\langle P' \left| J^+(0) \right| P \right\rangle = 2\left(P + P'\right)^+ F(Q^2).$$

- Drell-Yan-West (DYW) expression for meson form factor integrated over phase-space momentum

$$F(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \sum_j e_j \psi^*_{n/P'}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_{n/P}(x_i, \mathbf{k}_{\perp i}, \lambda_i),$$

where $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} + (1 - x_i)\, \mathbf{q}_\perp$ for a struck quark and $\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\, \mathbf{q}_\perp$ for each spectator. The formula is exact if the sum is over all Fock states n.

- Normalization of LFWFs

$$\sum_n \int \left[dx_i\right] \left[d^2\mathbf{k}_{\perp i}\right] \left|\psi_{n/h}(x_i, \mathbf{k}_{\perp i})\right|^2 = 1,$$

- Transverse position coordinates $x_i \mathbf{r}_{\perp i} = x_i \mathbf{R}_\perp + \mathbf{b}_{\perp i}$

$$\sum_{i=1}^{n} \mathbf{b}_{\perp i} = 0, \quad \sum_{i=1}^{n} x_i \mathbf{r}_{\perp i} = \mathbf{R}_\perp .$$

- LFWF $\psi_n(x_j, \mathbf{k}_{\perp j})$ expanded in terms of $n-1$ independent coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \ldots, n-1$

$$\psi_n(x_j, \mathbf{k}_{\perp j}) = (4\pi)^{\frac{n-1}{2}} \prod_{j=1}^{n-1} \int d^2\mathbf{b}_{\perp j} \exp\left(i \sum_{j=1}^{n-1} \mathbf{b}_{\perp j} \cdot \mathbf{k}_{\perp j}\right) \widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j}).$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} |\widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j})|^2 = 1.$$

- The form factor has the exact representation (DYW)

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \exp\left(i\mathbf{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\right) |\widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j})|^2,$$

corresponding to a change of transverse momentum $x_j \mathbf{q}_\perp$ for each of the $n-1$ spectators and elementary coupling to the struck parton.

- Define effective single particle transverse density (Soper '77)

$$F(q^2) = \int_0^1 dx \rho(x, \vec{q}_\perp)$$

with

$$\rho(x, \vec{q}_\perp) = \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp).$$

- From DYW expression for FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \, \delta\big(1 - x - \sum_{j=1}^{n-1} x_j\big) \delta^{(2)}\big(\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} - \vec{\eta}_\perp\big) \left|\widetilde{\psi}_n(x_j, \mathbf{b}_{\perp j})\right|^2$$

- Integration over the $n-1$ spectator partons, and $x = x_n$ is the coordinate of the active quark.

- $\vec{\eta}_\perp = \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}$ is the x-weighted transverse position coordinate of the $n-1$ spectators.

Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \; x^\ell = (x^\mu, z)$

$$ig_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x,z) \Phi^*_{P'}(x,z) \overleftrightarrow{\partial}_\ell \Phi_P(x,z).$$

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

$$A(x,z)_\mu = \epsilon_\mu e^{-iQ\cdot x} J(Q,z), \quad A_z = 0.$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2 \partial_z^2 - z \, \partial_z - z^2 Q^2\right] J(Q,z) = 0,$$

  subject to boundary conditions $J(Q=0,z) = J(Q,z=0) = 1.$

- Solution

$$J(Q,z) = zQK_1(zQ).$$

- Substitute hadronic modes $\Phi(x,z)$ in the AdS EM matrix element

$$\Phi_P(x,z) = e^{-iP\cdot x} \, \Phi(z), \quad \Phi(z) \to z^\Delta, \;\; z \to 0.$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $J(Q,z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int_0^{\Lambda_{\rm QCD}^{-1}} \frac{dz}{z^3}\, \Phi(z) J(Q,z) \Phi(z).$$

- Since $K_n(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x}$, the external source is suppressed inside AdS for large Q. Important contribution to the integral is from $z \sim 1/Q$, where $\Phi \sim z^\Delta$.
- For large Q^2

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\Delta - 1},$$

and the power-law ultraviolet point-like scaling is recovered [Polchinski and Susskind (2001)]

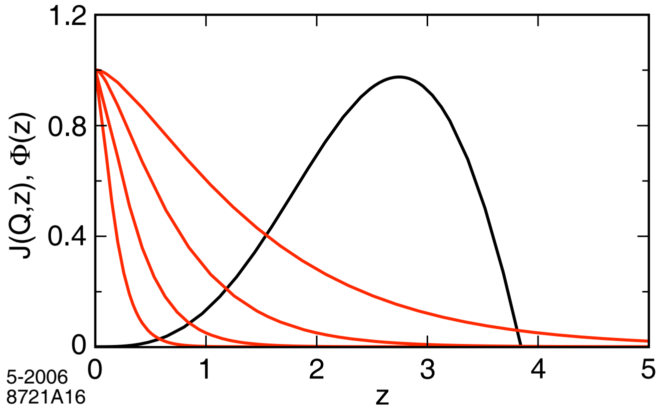


Fig: Suppression of external modes for large Q inside AdS. Red curves: $J(Q,z)$, black: $\Phi(z)$.

Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution: bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x, \zeta),$$

with $\widetilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|.$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

- Hadronic QCD transverse density $\tilde{\rho}$ is identified with the string mode density $|\Phi|^2$ in AdS space!

$$\boxed{\tilde{\rho}(x,\zeta) = \frac{R^3}{2\pi} \, \frac{x}{1-x} \, \frac{|\Phi(\zeta)|^2}{\zeta^4}}$$

- Precise relation between string modes $\Phi(\zeta)$ in AdS space and the QCD transverse density $\tilde{\rho}(x,\zeta)$

  SJB and GdT (2006)

- The variable ζ represents the invariant separation between point-like constituents and it is also the holographic variable: $\zeta = z$.

- For two-partons $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$

$$\tilde{\rho}(x,\zeta) = \frac{1}{(1-x)^2} \left|\widetilde{\psi}\,(x,\zeta)\right|^2.$$

- Two-parton holographic bound state LFWF

$$\left|\widetilde{\psi}(x,\zeta)\right|^2 = \frac{R^3}{2\pi} \, x(1-x) \, \frac{|\Phi(\zeta)|^2}{\zeta^4}.$$

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathbf{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathbf{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\overline{q}q/\pi}(x,\mathbf{b}_\perp) = \kappa^{L+1}\sqrt{\frac{2n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathbf{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathbf{b}_\perp^2\right).$$

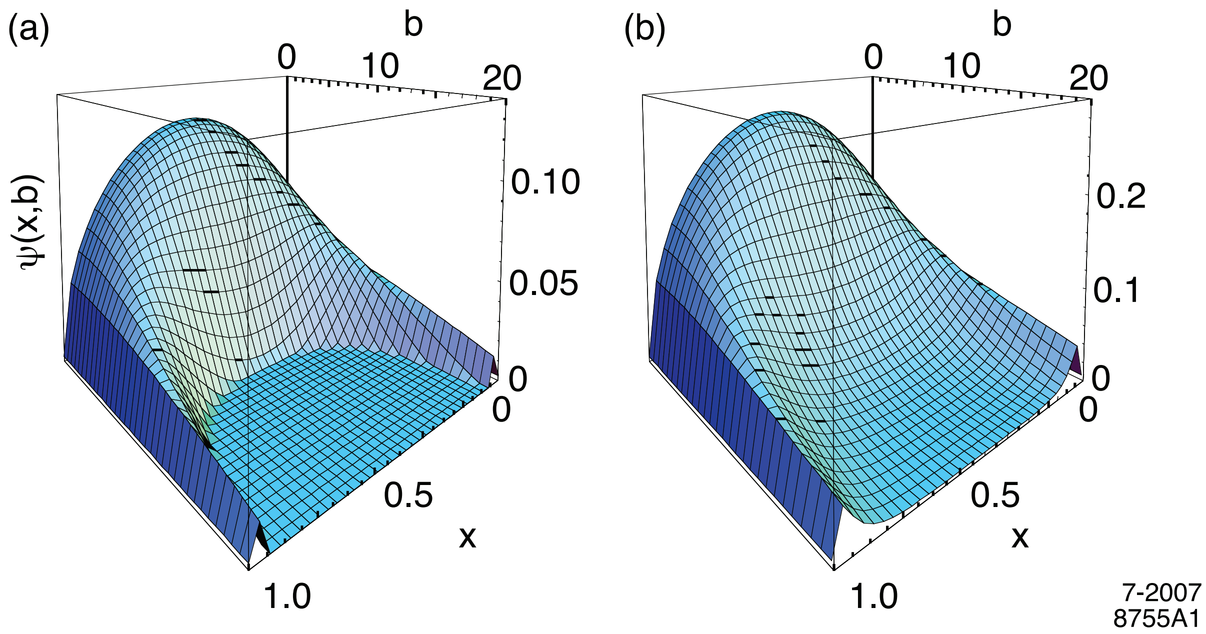


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Space and Time-Like Pion Form Factor

- Hadronic string modes $\Phi_\pi(z) \to z^2$ as $z \to 0$ (twist $\tau = 2$)

$$\Phi_\pi^{HW}(z) = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2} J_1(\beta_{0,1})} z^2 J_0\left(z\beta_{0,1}\Lambda_{QCD}\right),$$

$$\Phi_\pi^{SW}(z) = \frac{\sqrt{2}\kappa}{R^{3/2}}\, z^2.$$

- F_π has analytical solution in the SW model $F_\pi(Q^2) = \frac{4\kappa^2}{4\kappa^2+Q^2}$.

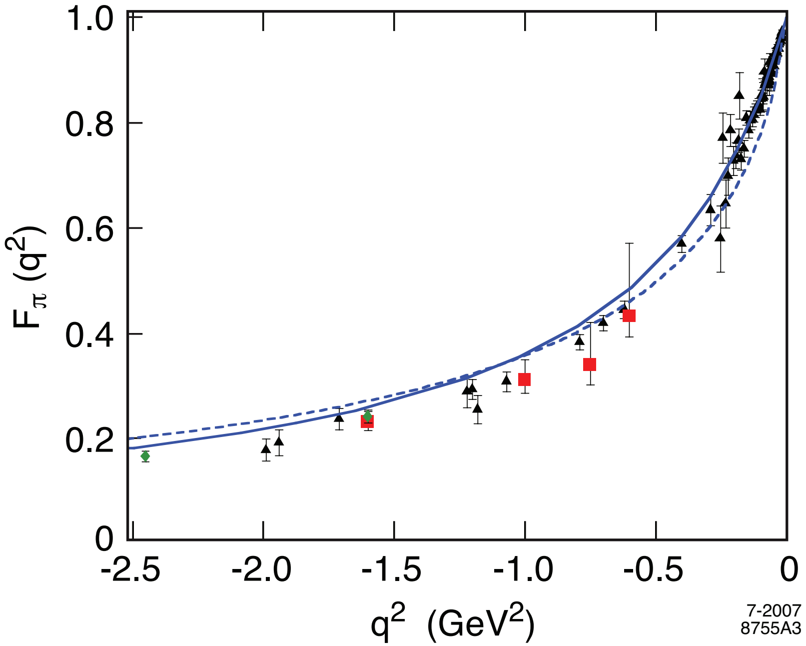


Fig: $F_\pi(q^2)$ for $\kappa = 0.375$ GeV and $\Lambda_{QCD} = 0.22$ GeV. Continuous line: SW, dashed line: HW.

- Scaling behavior for large Q^2: $Q^2 F_\pi(Q^2) \to \text{constant}$ $\boxed{\text{Pion } \tau = 2}$

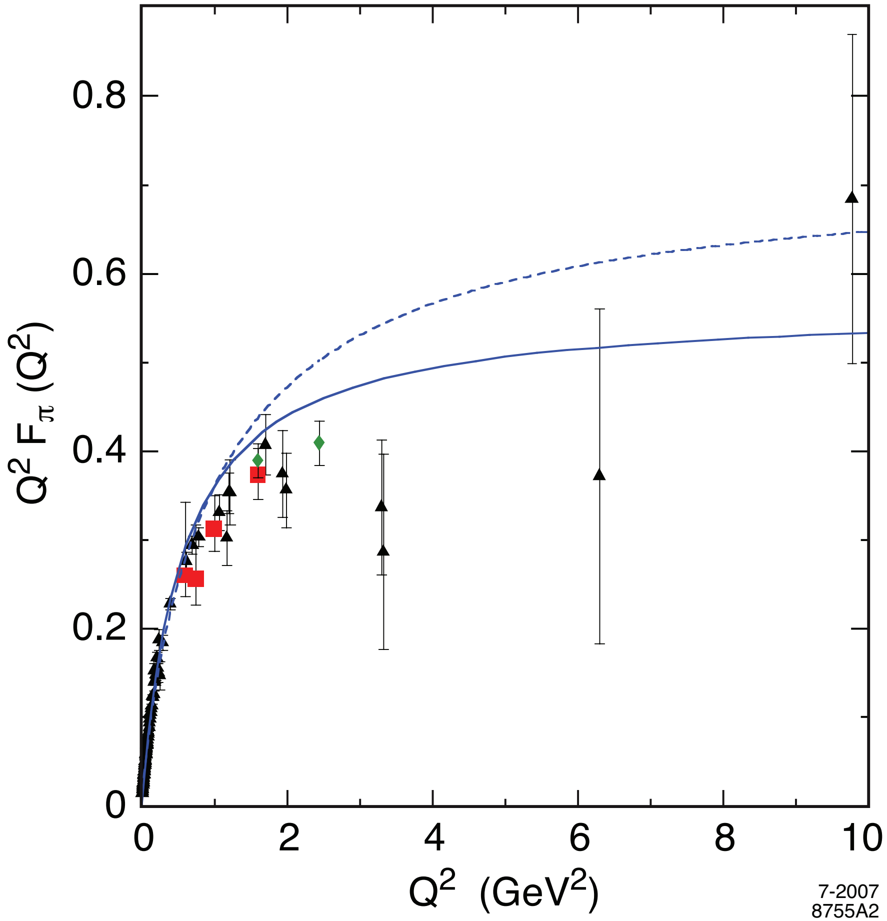
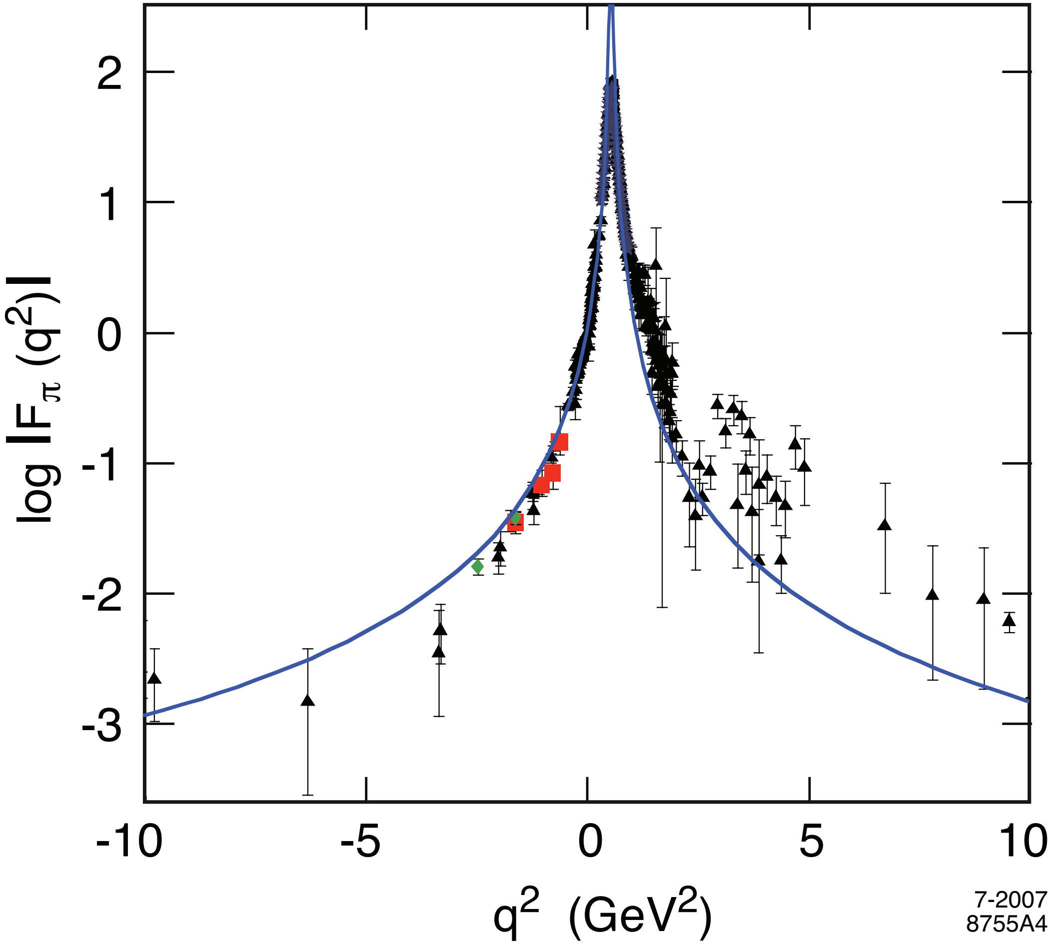


Fig: Continuous line: SW model for $\kappa = 0.375$ GeV. Dashed line: HW model for $\Lambda_{QCD} = 0.22$ GeV.

- Analytical continuation to time-like region $q^2 \to -q^2 \quad (M_\rho = 4\kappa^2 = 750$ MeV)
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).



Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

- Extract the value of the mean pion charge radius

$$F_\pi(Q^2) = 1 - \frac{1}{6}\langle r_\pi^2\rangle\, Q^2 + \mathcal{O}(Q^4), \qquad \langle r_\pi^2\rangle = -6\frac{dF_\pi(Q^2)}{dQ^2}\Big|_{Q^2=0}.$$

- Soft wall model

$$\langle r_\pi^2\rangle_{SW} = \frac{3}{2\kappa^2} \simeq 0.42\,\mathrm{fm}^2,$$

compared with the PDG value $\langle r_\pi^2\rangle = 0.45(1)\,\mathrm{fm}^2$.

- Hard-wall model with non-confined electromagnetic current expand $J(Q^2, z)$ for small values of Q^2

$$J(Q^2, z) = 1 + \frac{z^2Q^2}{4}\left[2\gamma - 1 + \ln\left(\frac{z^2Q^2}{4}\right)\right] + \mathcal{O}^4,$$

where $\gamma = 0.5772\ldots$. Since there is no scale in $J(Q^2, z)$, value of $\langle r_\pi^2\rangle$ diverges logarithmically.

- Problem in defining $\langle r_\pi^2\rangle$ does not appear if one uses Neumann boundary conditions for the HW model:

$$\langle r_\pi^2\rangle_{HW} \sim 1/\Lambda_{\mathrm{QCD}}^2.$$

(Grigoryan and Radyushkin (2007))

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle .$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}} R^{3/2} \lim_{\zeta\to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{\rm QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})} \Lambda_{\rm QCD} = 91.7 \, {\rm MeV}, \;\; f_\pi^{SW} = \frac{\sqrt{3}}{8} \kappa = 81.2 \, {\rm MeV},$$

Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\quad \Gamma(N + z) = (N - 1 + z)(N - 2 + z) \dots (1 + z)\Gamma(1 + z)$.

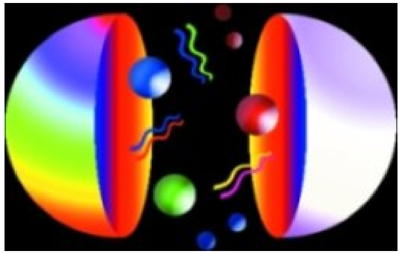
- Form factor expressed as $N - 1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2, \\
F(Q^2) &= \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3, \\
& \cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \cdots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

- For large Q^2:

$$F(Q^2) \to (N-1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

2 Fermionic Modes



From Nick Evans

- Baryons Spectrum in ”bottom-up” holographic QCD
  GdT and Brodsky: hep-th/0409074, hep-th/0501022.

- Conformal metric $x^\ell = (x^\mu, z)$:

$$
\begin{aligned}
ds^2 &= g_{\ell m} dx^\ell dx^m \\
&= \frac{R^2}{z^2}(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).
\end{aligned}
$$

- Action for massive fermionic modes on AdS$_{d+1}$:

$$
S[\overline{\Psi}, \Psi] = \int d^{d+1}x \, \sqrt{g} \, \overline{\Psi}(x,z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x,z).
$$

- Equation of motion: $\left(i\Gamma^\ell D_\ell - \mu\right) \Psi(x,z) = 0$

$$
\left[i \left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0.
$$

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(z)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(z)$ are solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where $H_{LF} = \alpha\Pi$ and the operator

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi_\nu^\dagger(\zeta)$ satisfy the commutation relations

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \frac{2\nu + 1}{\zeta^2}\gamma_5.$$

- Supersymmetric QM between bosonic and fermionic modes in AdS?

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C \sqrt{\zeta} \left[J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right].$$

  Baryonic modes propagating in AdS space have two components: orbital L and $L + 1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm \left(\zeta = 1/\Lambda_{\rm QCD} \right) = 0,$$

  given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k} \Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k} \Lambda_{\rm QCD},$$

  with a scale independent mass ratio.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

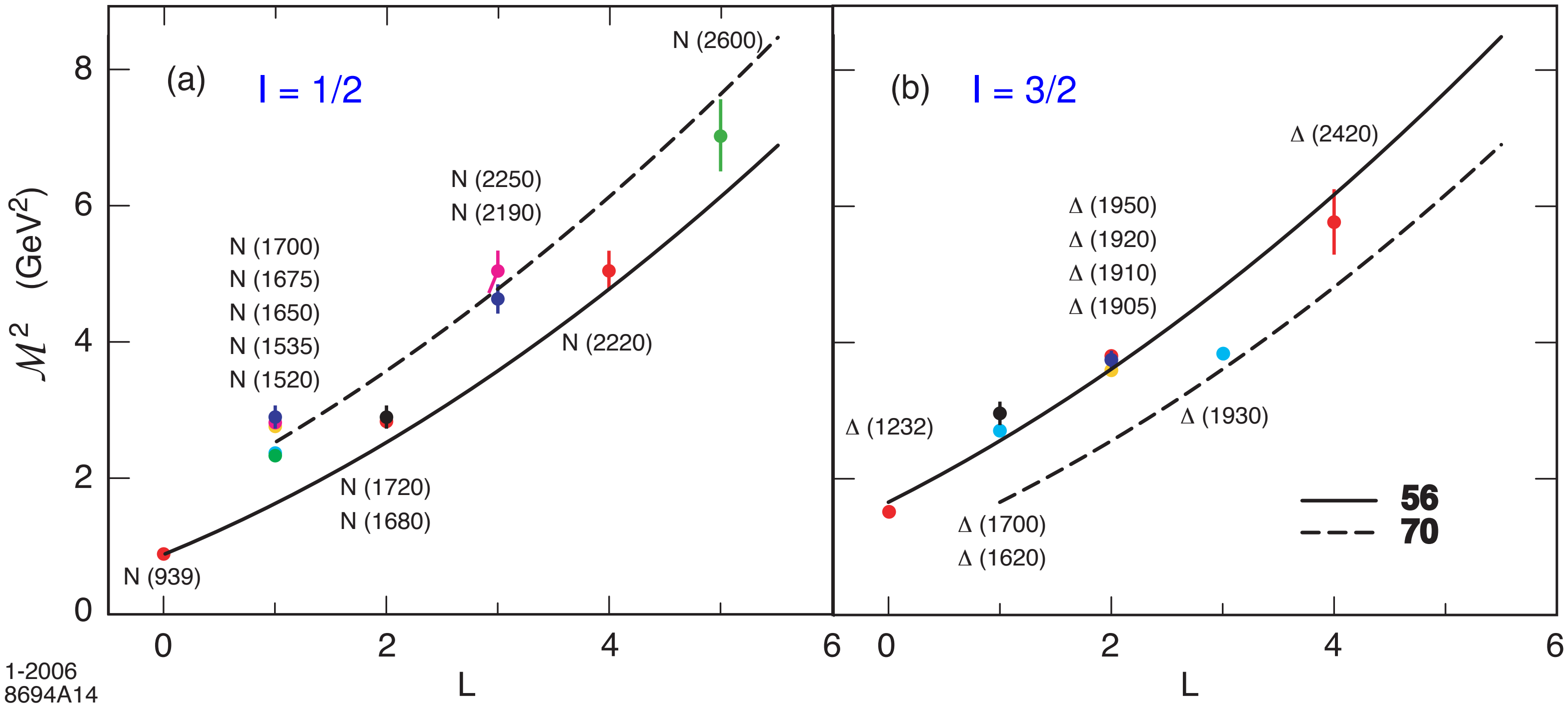


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\,\psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π_ν

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right),$$

- Commutation relations for fermionic generators

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta)\right] = \left(\frac{2\nu+1}{\zeta^2} - 2\kappa^2\right)\gamma_5.$$

- Solutions to the Dirac equation

$$\begin{aligned}\psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^\nu(\kappa^2\zeta^2),\\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2).\end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$$

Linear Holographic Confinement

- Compare with usual Dirac equation in AdS space $\left(x^\ell = (x^\mu, z)\right)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + V(z)\right]\Psi(x^\ell) = 0.$$

in presence of a linear confining potential $V(z) = \kappa^2 z.$

- Upon substitution $\Psi(x,z) = e^{-iP\cdot x} z^2 \psi(z),\ z \to \zeta$ we find

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\,\psi(\zeta)$$

with

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right), \quad \mu R = \nu + \frac{1}{2},$$

our previous result.

- Soft-wall model for baryons corresponds to a linear confining potential in the LF transverse variable ζ!

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2 (n + L + 1).$$

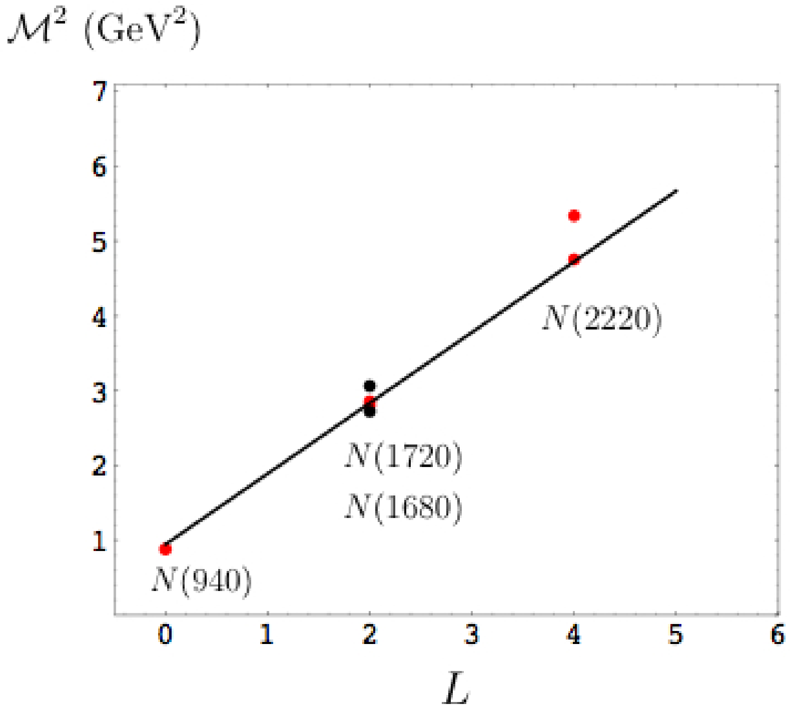


Fig: Proton Regge Trajectory $\quad \kappa = 0.49$ GeV

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta\, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

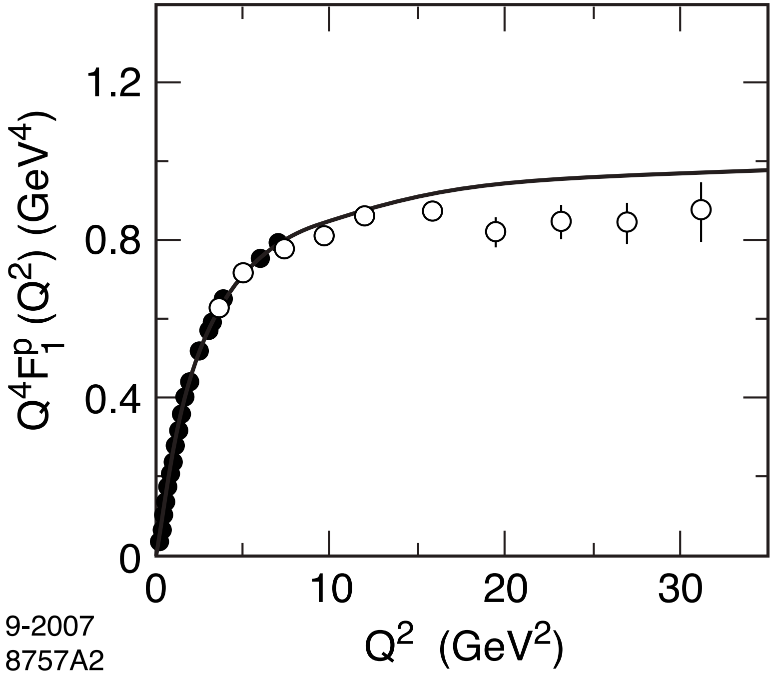
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta\, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta\, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],$$

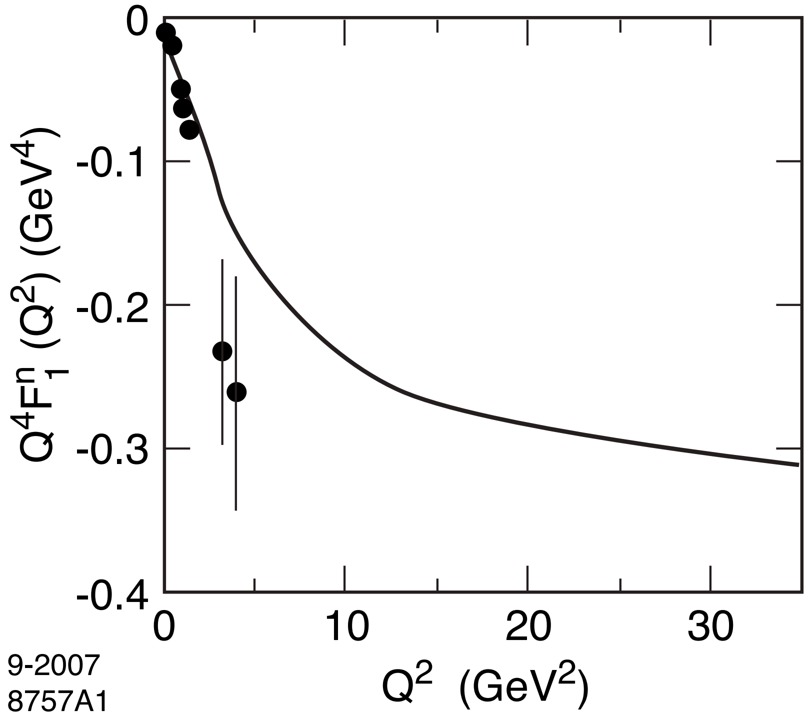
where $F_1^p(0) = 1,\ F_1^n(0) = 0$.

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

A Few References: Bottom-up-Approach

- Derivation of dimensional counting rules of hard exclusive glueball scattering in AdS/CFT:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic scattering in AdS/CFT:

  Polchinski and Strassler, hep-th/0209211.

- Unified description of the soft and hard pomeron in AdS/CFT:

  Brower, Polchinski, Strassler and Tan, hep-th/0603115.

- Hadron couplings and form factors in AdS/CFT:

  Hong, Yoon and Strassler, hep-th/0409118.

- Low lying meson spectra, chiral symmetry breaking and hadron couplings in AdS/QCD (Emphasis on axial and vector currents)

  Erlich, Katz, Son and Stephanov, hep-ph/0501128,

  Da Rold and Pomarol, hep-ph/0501218, hep-ph/0510268.

- Counting rules, low lying meson and baryon spectra and form factors in AdS/CFT, holographic light front representation and mapping of string amplitudes to light-front wavefunctions, integrability and stability of AdS/CFT equations (Emphasis on hadronic quark constituents)

  Brodsky and GdT, hep-th/0310227, hep-th/0409074, hep-th/0501022, hep-ph/0602252, 0707.3859 [hep-ph], 0709.2072 [hep-ph].

Thanks !