$$\boxed{\text{Light-Front Formulation of the Standard Model}}$$

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- SU(2) x U(1) GWS Model of EW Interactions
- Non-Abelian Higgs Model in L.C. Gauge ($A^+=0$)

* Unitary, ghost-free!: no Faddeev-Popov ghosts
  no Gupta-Bleuler ghosts
* Renormalizable!
* Perturbative Vacuum plus zero mode higgs (SSB)
* t' Hooft conditions

$$\partial \cdot W^\pm = \pm i\, M_W\, G_W^\pm \quad ,$$

$$\partial Z = M_Z\, G^0 \quad ,$$

$$A^+ = Z^+ = (W^\pm)^+ = 0$$

Simple Example of SSB on LF : U(1) Theory

$$\mathcal{L} = |\partial^\mu \phi|^2 - V(\phi^\dagger \phi)$$

$$= \partial_+ \phi^\dagger \partial_- \phi + \partial_- \phi^\dagger \partial_+ \phi - \partial_\perp \phi^\dagger \partial_\perp \phi - V(\phi^\dagger \phi)$$

$$V(\phi^\dagger\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \qquad [\lambda > 0,\ \mu^2 < 0]$$

Mode separation:

$$\phi(\tau, x^-, x^\perp) = \omega(\tau, x^\perp) + \varphi(\tau, x^-, x^\perp)$$

zero long. mom mode dynamical, c-no.

quantum fluctuations $\langle 0 | \varphi | 0 \rangle = 0$

Constraint Eqn (from Euler-Lagrange)

$$\int dx_\perp dx^- \left[\partial_\perp \partial_\perp \phi - \frac{\delta V}{\delta \phi^\dagger} \right] = 0 \qquad (\text{as } \phi \leftrightarrow \phi^\dagger)$$

For simplicity, assume $\partial_\perp \omega = 0$. Then

$$\frac{\delta V}{\delta \phi^\dagger}\Big|_{\phi=\omega} , \frac{\delta V}{\delta \phi}\Big|_{\phi=\omega^\dagger} = 0$$

$$\Rightarrow \quad \omega = 0 \quad \text{or} \quad \omega^\dagger \omega = -\mu^2/2\lambda^2$$

Broken phase

U(1) invariance:

$$\omega = \frac{v}{\sqrt{2}} , \quad v = + \sqrt{\frac{-\mu^2}{\lambda}}$$

Noether U(1) Symmetry current:

$$J_\mu = i\,[\phi^+ \partial_\mu \phi - \phi\, \partial_\mu \phi^+]$$

$$\partial_\mu J^\mu = 0.$$

- However the U(1) generator current

$$j_\mu = i\,[\varphi^+ \partial_\mu \varphi - \varphi\, \partial_\mu \varphi^+]$$

is not conserved! $[\,\phi = v/\sqrt{2} + \varphi\,]$

$$j_\mu = J_\mu - \frac{iv}{\sqrt{2}}\, \partial_\mu(\varphi - \varphi^+)$$

$$\partial^\mu j_\mu = \frac{iv}{\sqrt{2}}\, \Box(\varphi - \varphi^+) \neq 0!$$

- The U(1) generator is $G(x^+) = \int d^2x^\perp\, dx^-\, j_-$

$$[\varphi(x), G] = \varphi\,, \quad [\varphi^+(x), G] = \varphi^+$$

$$G = \int d^2k_\perp\, dk^+\, \theta(k^+)\, [a^+(k)a(k) - b^+(k)b(k)]$$

$$\boxed{G|0\rangle_{LF} = 0}$$, $|0\rangle_{LF}$ remains invariant indep. of SSB!

although j_μ not conserved.

$\boxed{\text{Simple example of SSB on LF}}$

Abelian Higgs Model in LC Gauge

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi^\dagger \phi)$$

$$D_\mu = \partial_\mu + ieA_\mu$$

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\mu^2 < 0, \quad \lambda > 0$$

$$\phi(x) = \frac{1}{\sqrt{2}} v + \varphi = \frac{1}{\sqrt{2}} [\underset{\substack{\uparrow \\ \text{zero mode}}}{v} + \underset{\substack{\uparrow \\ \text{higgs}}}{h(x)} + i\underset{\substack{\uparrow \\ \text{Goldstone}}}{\eta(x)}]$$

$$v^2 = -\mu^2/\lambda$$

Find $\quad \partial \cdot A - M\eta = 0 \quad$ t'Hooft cond'n

$$H_0^{LF} = \frac{1}{2}(\partial_\perp A_\perp^i)(\partial_\perp A_\perp^i) + \frac{1}{2} M^2 A_\perp A_\perp$$

$$+ \frac{1}{2}(\partial_\perp \eta)(\partial_\perp \eta) + \frac{1}{2} M^2 \eta^2$$

$$+ \frac{1}{2}(\partial_\perp h)(\partial_\perp h) + \frac{1}{2} m_h^2 h^2$$

$$M = ev, \quad m_h^2 = 2\lambda v^2 = -2\mu^2$$

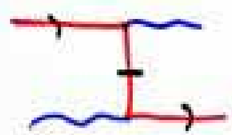
"Instantaneous" L.F. Interactions

$$e^2 \frac{J\cdot n \; J\cdot n}{(k\cdot n)^2}$$ from elimination of A^-

In tree graphs:

$$K^{\mu\nu} \to -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k\cdot n}$$

"text book l.c.g."



$$\bar{u}\,\gamma\cdot\epsilon^* \,\frac{\gamma\cdot n}{k\cdot n}\, \gamma\cdot\epsilon\, u$$

from elimination of $\psi_- = \Lambda^- \psi$

In tree graphs, restores Feynman propagator

$$\not k_{on} + m \Rightarrow \not k_{Feynman} + m$$

Quantize Theory:

$$A^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k_\perp \, dk^+ \, \frac{\theta(k^+)}{\sqrt{2k^+}}$$

$$\sum_{\alpha=1,2,3} \epsilon_{(\alpha)}^\mu(k) \left[a_\alpha(k^+, k^\perp) e^{-ik\cdot x} + a_\alpha^\dagger(k^+, k^\perp) e^{ik\cdot x} \right]$$

$$\left[a_\alpha(k), a_\beta^\dagger(l) \right] = \delta_{\alpha\beta} \, \delta^2(k_\perp - l_\perp) \, \delta(k^+ - l^+)$$

$$\epsilon_{(\alpha)}^{\mu=+} = 0 \quad , \quad \epsilon_{1,2}^\mu = \left(0, 2\frac{\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k^+}, \vec{\epsilon}_\perp\right)$$

$$\epsilon_3^\mu = -\frac{M}{k^+} n^\mu = -\frac{M}{k^+}(0, 2, 0_\perp)$$

↖ null norm

$$\langle 0 | T(A_\mu(x), A_\nu(y)) | 0 \rangle$$

$$= \frac{i}{(2\pi)^4} \int d^4k \, \frac{K_{\mu\nu}(k)}{(k^2 - M^2 + i\epsilon)} \, e^{-ik\cdot(x-y)}$$

$$K^{\mu\nu} = -g^{\mu\nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{n\cdot k} - \frac{(k^2 - M^2)}{(n\cdot k)^2} n^\mu n^\nu$$

Good high-energy behavior unlike

Proca : $\quad -g^{\mu\nu} + \dfrac{k^\mu k^\nu}{M^2}$

More interactions [Abelian Higgs Theory]

also fermion coupl.
to h, $\eta = G$

$$-\mathcal{H}_{int} = \mathcal{L}_{int} =$$

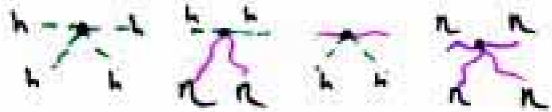
$$e M A_\mu A^\mu h$$

$$-e^2 \frac{m_h^2}{2M}(\eta^2+h^2)h$$

$$+ e(h\partial_\mu \eta - \eta \partial_\mu h)A^\mu$$

$$+\frac{e^2}{2}(h^2+\eta^2)A^\mu A_\mu$$

$$-\frac{\lambda}{4}(\eta^2+h^2)^2$$



$$-\frac{e^2}{2}\left(\frac{1}{\partial_-}J^+\right)\left(\frac{1}{\partial_-}J^+\right)$$

$$J^+ = h\partial^+\eta - \eta\partial^+ h$$

In tree graphs $\quad E^\mu_{(3)\,eff} = E^\mu_{(3)} - \frac{k^\mu\, k\cdot E_{(3)}}{k^2}$

$$k\cdot E_{(3)\,eff} = 0\,, \quad E^2_{(3)\,eff} = -1$$

Goldstone
provides part
of long. mode -

PPS
PPS + SSB

Light-Front Quantization of Standard Model

$\Rightarrow$ New insight into Higgs mechanism

$$\Phi_i(\tau, x^-, \vec{x}_\perp) = \omega_i(\tau, x_\perp) + \varphi_i(\tau, x^-, \vec{x}_\perp)$$

isospin multiplet — zero mode — quantized field

Condition for SSB: (tree level)

$$V_i'(\omega) - \partial_\perp \partial_\perp \omega_i = 0$$

Construct generators:

$$G_a = -i \int dx^\perp dx^- (\partial_- \varphi_i)(t_a)_{ij} \varphi_j$$

SSB $\rightarrow$ Current not conserved; $[H_{LF}, G_a] \neq 0$

However $G_a |0\rangle_{LF} = 0$!

* LF Vacuum retains symmetry.

$\Rightarrow$ Speculation: Interpret $\omega_i \neq 0$ as "external field"
remnant of Higgs field from early cosmology

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Light-Front Quantization of Standard Model
$SU(2) \otimes U(1)$

* $n \cdot A = A^+ = 0$ gauge

  $\Rightarrow$ { physical gauge quanta
  unitary
  renormalizable

* SSB from zero mode of scalar field

$$\phi^{(0)} = \frac{v}{\sqrt{2}}$$

$$\phi(x) = \frac{1}{\sqrt{2}}\left(v + h(x) + i\eta(x)\right)$$

$$\partial \cdot A = M\eta \quad , \quad M = ev$$

* Goldstone Field $\eta(x)$ restores ϵ_L :

$$\epsilon^{(L)}_\mu = \frac{n_\mu M}{n \cdot k} - \frac{k_\mu M}{k^2} \qquad (\epsilon \cdot k = 0)$$

* LC vacuum remains trivial

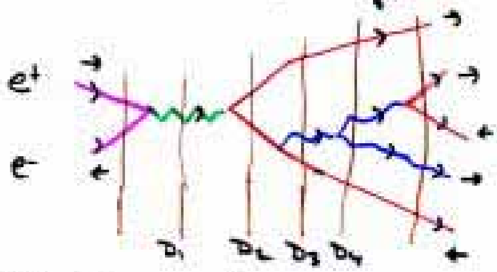
* Amplitude Event Generator

  $\Rightarrow$ Renormalized Amplitudes from LF T-O PTh
  Ghost-Free, $\Pi d^2k_\perp dx$, DLCQ discret.

"Event Amplitude Generator"

Generate amplitude from LF TO PTh $\left\{ \begin{array}{l} \text{Tree +} \\ \text{Virtual} \end{array} \right.$

$$\mathcal{M} = \sum_{\substack{\text{time-}\\ \text{orderings}}} \mathcal{M}_\alpha \quad (\text{Specific spins } S_z)$$



$$\mathcal{M}_\alpha = H_I \frac{1}{D_1} H_I \frac{1}{D_2} \cdots$$

$\sum k^+, \sum k_\perp, \sum J_z$ conserved

$k^+ > 0$: Few surviving LF time-orderings

Physical polarization sums: $\sum_{(i)} e_\mu^{(i)} e_\nu^{(i)}$

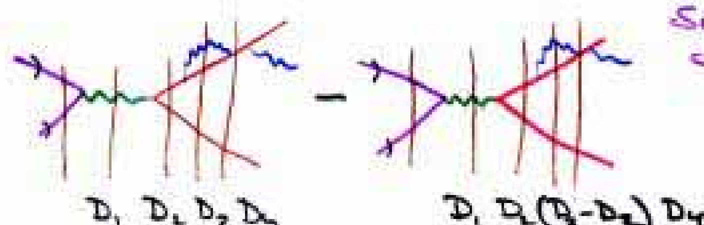
$(i) = 1, 2, 3$

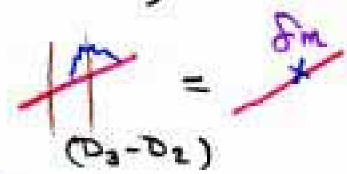
Standard Model W

Compute renormalized amplitude

– "alternating denominator" method

Roskies
Suaya
SJB

Example: 

equivalent to subtracting mass counterterm! 

$\pi \, d^2k_\perp dx$, Unitary, no ghosts.

Other Applications of LF Quantization

Light-Front Thermodynamics

Set boundary conditions at fixed $\tau = t + z/c$
not t

$$Z_{LF} = \sum_n \exp - \frac{M_n^2}{T_{LF}} \qquad M_n^2 = P_n^- P^+ + P_\perp^2$$

SJB
Das et al

Light-Front Lippmann-Schwinger

$$T = H_I + H_I \frac{1}{M^2 - \sum_i \frac{m_\perp^2}{x} + i\epsilon} T$$

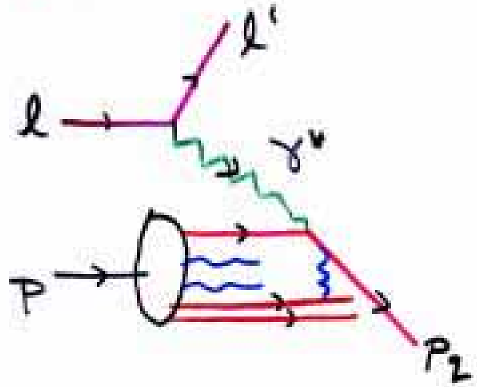
Variational Solutions to Bound-State Probs:

minimize $\langle \psi_{trial} | H_{LF} | \psi_{trial} \rangle$

Construct $\langle \psi_{trial} | n \rangle = \psi_n^{trial}(x_i, k_{\perp i}, \lambda_i)$

using numerators from LF
ladder relations

Unexpected Role of Final State Interactions in Deep Inelastic Scattering



gluon exchange after photon acts

not in LF WF

* Single-spin asymmetry $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$ Bjorken-scaling
* Diffraction at Leading Twist
* Nuclear Shadowing (interference of diff channels)
* Energy Loss , P_T Broadening

Diffraction, Nuclear Shadowing, Pomeron not in nuclear wavefunction!

In general,

initial, final state interactions

will produce single-spin asymmetries

$$\vec{S} \cdot \vec{p}_1 \times \vec{p}_2$$

wrt virtually any production or

scattering plane!

Perturbatively calculable at large $\vec{r}_\perp, \vec{q}_\perp$

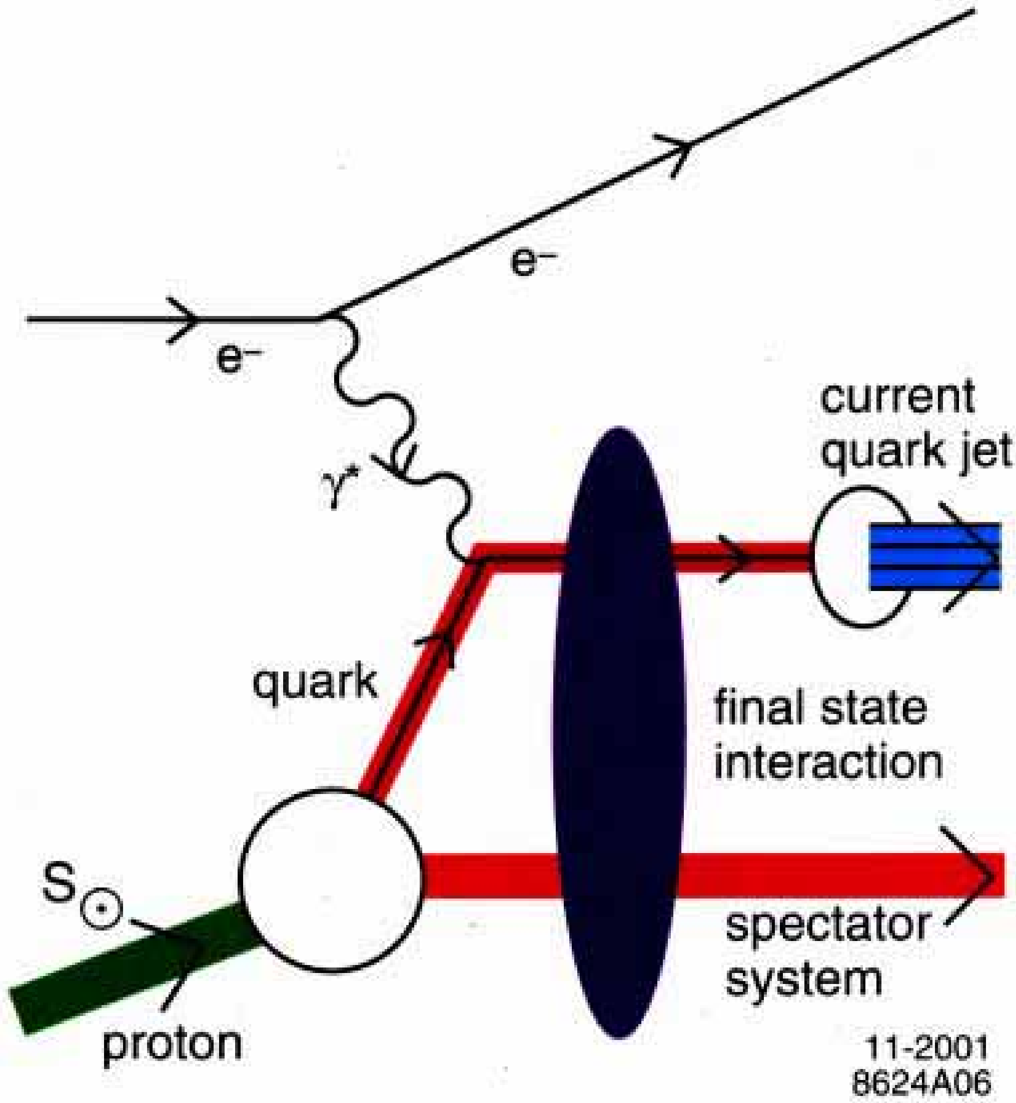
New measure of $\alpha_s(r_\perp^2)$

Application to Drell-Yan: $\vec{S}_p \cdot \vec{p}_\pi \times \vec{q}$
$\pi p_\uparrow \to \mu^+\mu^- X$

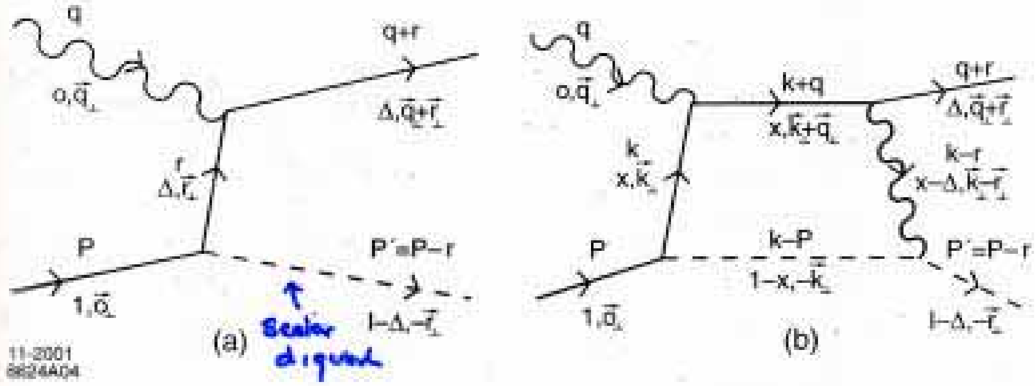
$e^+e^- \to \Lambda_\uparrow X$: $\vec{S}_\Lambda \cdot \vec{p}_\Lambda \times \vec{q}$

A_N: $p_\uparrow p \to \pi X$: $\vec{S}_p \cdot \vec{q}_\pi \times \vec{p}_p$

$pp \to \Lambda_\uparrow X$: $\vec{S}_\Lambda \cdot \vec{p}_\Lambda \times \vec{p}_p$



e⁻
e⁻
current
quark jet
γ*
quark
final state
interaction
S
proton
spectator
system
11-2001
8624A06



q
o,$\vec{q}_\perp$
q+r
$\Delta,\vec{q}_\perp+\vec{r}_\perp$
r
$\Delta,\vec{r}_\perp$
P
1,$\vec{0}_\perp$
P′=P−r
$1-\Delta,-\vec{r}_\perp$
(a)
Scalar diquark
11-2001
8624A04
q
o,$\vec{q}_\perp$
k+q
$x,\vec{k}_\perp+\vec{q}_\perp$
q+r
$\Delta,\vec{q}_\perp+\vec{r}_\perp$
k
$x,\vec{k}_\perp$
k−r
$x-\Delta,\vec{k}_\perp-\vec{r}_\perp$
P
k−P
$1-x,-\vec{k}_\perp$
P′=P−r
1,$\vec{0}_\perp$
$1-\Delta,-\vec{r}_\perp$
(b)

Explicit calculation of FSI SSA

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hep/ph/0201296

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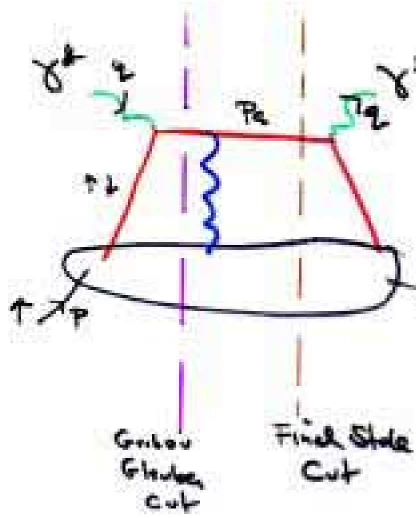


Overlap of wavefunctions with $\Delta L_z = 1$

$$\left[e^{i(\chi_1 - \chi_2)}\right]$$

$\chi_1 - \chi_2$: IR Finite

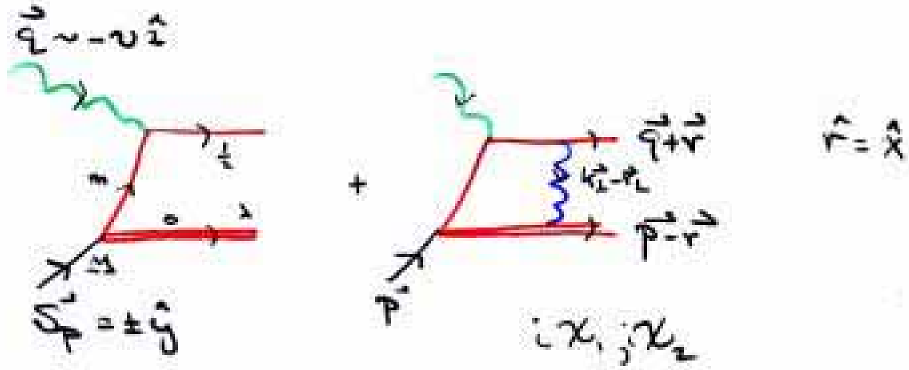
$$i\,\vec{S}_p \cdot \vec{q} \times \vec{p}_q = i\,\vec{S}_p \cdot \vec{q} \times \vec{r}$$

$$\vec{p}_q = \vec{q} + \vec{r}$$

$$P_y = A_n \cong \frac{\alpha_s(r_\perp^2)\; x_{Bj}\; M\,|\vec{r}_\perp|\, \ln r_\perp^2}{r_\perp^2}$$

Bjorken scaling for finite $r_\perp$

Same matrix elements as $a_p = F_2(0)$



$$\sigma \propto \epsilon^{\mu\alpha b c} P_\mu S_a q_b r_c = M \vec{S} \cdot \vec{q} \times \vec{r}$$

$$A_N = P_y = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

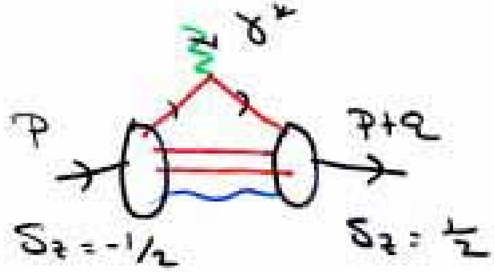
$$= \frac{C_F \alpha_s(\mu^2)(\Delta M + m)\, r_x}{(\Delta M + m)^2 + r_\perp^2}$$

$$\otimes \left[r_\perp^2 + \Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta}\right) \right]$$

$$\otimes \left[\frac{1}{r_\perp^2} \ln \frac{r_\perp^2 + \Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta}\right)}{\Delta(1-\Delta)\left(-M^2 + \frac{m^2}{\Delta} + \frac{\lambda^2}{1-\Delta}\right)} \right]$$

$\Delta = x_g$, $\qquad \mu^2_{MI} = \langle e^{-5/3} (\vec{k}_\perp - \vec{r}_\perp)^2 \rangle$

$\boxed{\text{Pauli Form Factor } F_2(q^2)}$ $\quad \kappa = F_2(0)$



$\boxed{\text{Requires overlap of LCWFs with } \underline{\Delta L_z = 1}}$

e.g. (3q) $\quad \psi^{1/2}_{-1/2\,-1/2\,-1/2} \otimes \psi^{-1/2}_{-1/2\,-1/2\,-1/2}$

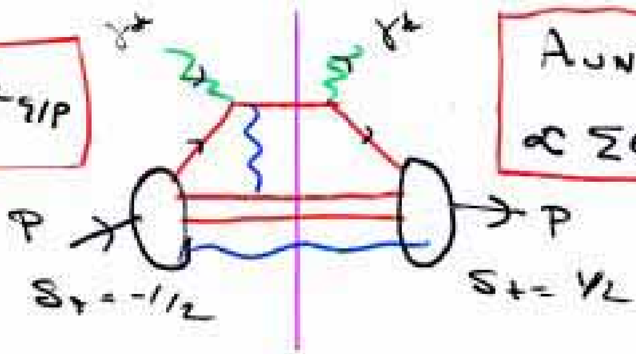
$L_z = +2 \qquad L_z = +1$

$\psi^{1/2}_{1/2\,1/2\,-1/2} \otimes \psi^{-1/2}_{1/2\,1/2\,-1/2}$

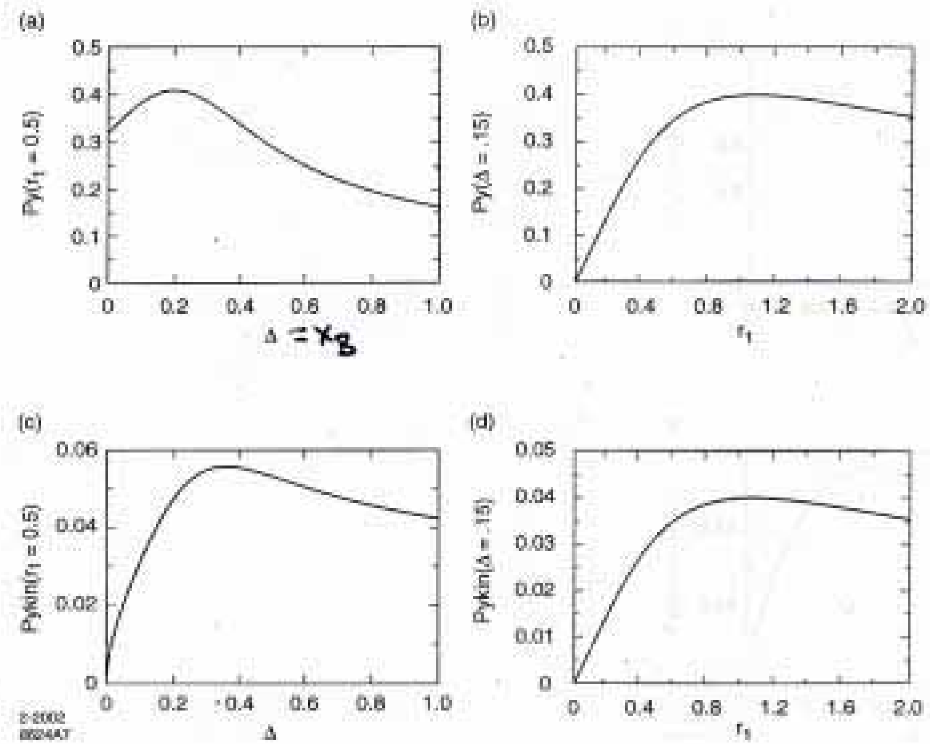
$L_z = 0 \qquad L_z = -1$

<u>Same matrix elements appear in SSA</u>

$\boxed{\kappa_p = \sum_{q \in p} e_q \kappa_{q/p}}$



$\boxed{A_{UN} = SSA \propto \sum e_q^2 \kappa_{q/p} \alpha_s}$


(a)
Py(r1 = 0.5)
Δ = x_B
(b)
Py(Δ = .15)
r1
(c)
Pykin(r1 = 0.5)
Δ
(d)
Pykin(Δ = .15)
r1
0.5
0.4
0.3
0.2
0.1
0
0.06
0.04
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0.05
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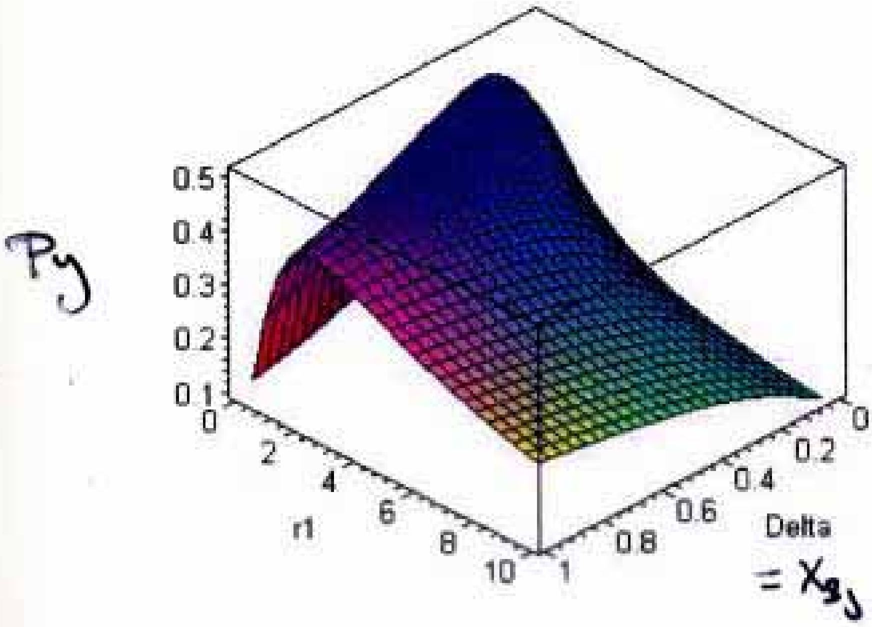
Py
0.5
0.4
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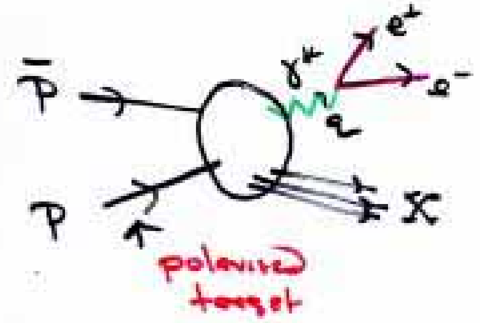
0.2
0.1
0
2
4
6
8
10
r1
0
0.2
0.4
0.6
0.8
1
Delta
= X_{g_j}

Hwang, Schmidt, SJB

Collins, X. Ji + Yuan

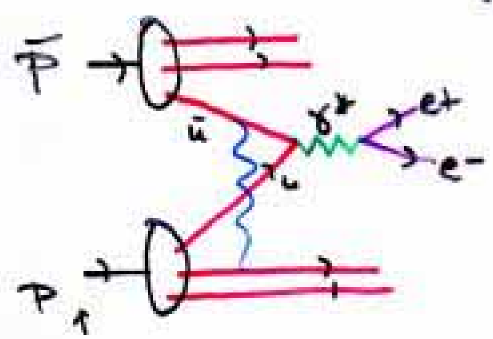
Single-Spin Asymmetries in $\bar{p}p$ collisions



$\vec{S}_p \cdot \vec{p}_{\bar{p}} \times \vec{q}_\perp$

"T-odd" observable

* New theory due to initial state gauge ints. for SIDIS



Interference of amplitudes produces phase gauge-indep

Same interference of $\Delta L_z = 1$ states prod. μ_A^p

* $A_{UN} \sim \alpha_s \frac{M r_\perp}{r_\perp^2 + M^2} \mu_A^p$ $\qquad \vec{q} = \vec{p}_{\bar{p}} + \vec{r}_\perp$

* Scales in DY limit at fixed $r_\perp$!

* opposite in sign to SIDIS SSA

References

"Structure functions are not parton probabilities"

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"Final-state interactions and Single-spin Asymmetries in Semi-inclusive Deep Inelastic Scatt."

S.J.B., Dae Sung Hwang, I. Schmidt

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* J.C. Collins hep-ph/0204004

* X. Ji + F. Yuan hep-ph/0206057

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