Light-Front Wavefunctions and QCD

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Generalized Parton Distributions
and
Hard Exclusive Processes

INT

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Outline of Talk

- Light-front quantization of QCD

  Light-Front Fock-state wavefunctions $\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$ provide a frame-independent representation of hadrons at the amplitude level in terms of their quark and gluon degrees of freedom, as well as an exact representation of the local matrix elements needed for calculating form factors, distribution amplitudes, and generalized parton distributions.

- Angular momentum on the light front and the construction of light-front wavefunctions (Based on recent work with John Hiller, Dae Sung Hwang, Volodya Karmanov and myself.)

  The known non-perturbative solutions of the Wick-Cutkosky model are used to illustrate the construction of general light-front wavefunctions with specific angular momentum. A kinematic definition of angular and orbital angular momentum is discussed. The invariances of the light-front wavefunctions constrains the analytic dependence of hadronic form factors as functions of Q^2, in agreement with the perturbative QCD form calculated by Belitsky, Ji, and Yuan.

- LF quantization of QCD and the Standard Model (Based on papers with Prem Srivastava,)

  The LF quantization of QCD and the Standard Model utilizes a $k^+ = 0$ zero mode of the Higgs field for spontaneous symmetry breaking. The perturbative vacuum is unchanged. The resulting theory is renormalizable, unitary, and ghost-free.

- Phenomenological evidence for IR-fixed point behavior of the QCD coupling (Based on a paper with Sven Menke, Carlos Merino, Johan Rathsman.)

  The effective charge $\alpha_\tau(s)$ can be determined for $s < m_\tau^2$ from hadronic τ decay data. The remarkably flat dependence of this effective coupling supports recent findings by Zwanziger, Alkhofer, and others showing that QCD coupling has infrared fixed-point conformal-like behavior and leads to an understanding of the near-conformal aspects of hard QCD processes. The behavior of this QCD coupling does not support the notion of "infrared slavery" as the origin of color confinement.

- Conformal behavior of QCD (Based on work with G. De Teramond.)

  The underlying mechanisms for dimensional counting rules are discussed from the perspective of LFWFs as a way to understand the implications of the ADS/CFT correspondence and conformal QCD results of Polchinski and Strassler for QCD phenomenology. The quark interchange amplitude emerges as the dominant large N_C scattering mechanism.

- The effects of final-state interactions in deep lepton-proton inelastic scattering. (Based on collaborations with Dae Sung Hwang, Ivan Schmidt and with Paul Hoyer, Nils Marchal, Stephane Peigne, and Francesco Sannino.)

  The final state interactions of the scattered quark in lead to leading-twist single-spin asymmetries, diffraction, nuclear shadowing, and a new understanding of the $\beta-$dependence of the pomeron structure function as determined from diffractive lepton-proton deep inelastic scattering. The final-state corrections to deep inelastic scattering imply that there will be analogous corrections to DVCS predictions based on the handbag approximation.

PPS + SdB
Kogut Soper
Lepage, SJB
Pauli, SJB

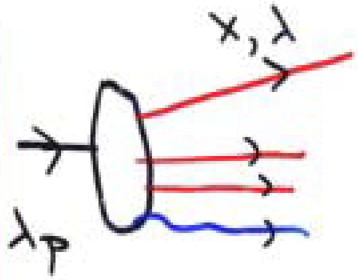
Light-Front Quantization of QCD in light-cone gauge

Advantages:

- Frame-independent wavefunctions
- Current matrix elements simple
- Perturbative vacuum remains eigenstate
- No vacuum processes
- Physical degrees of freedom
  - no ghosts, unitary
- Simple spin properties
- SSB → zero modes
- Renormalizable

Light-Cone Wavefunctions

encode all helicity, transversity distributions

$$q_{\lambda/\lambda_p} = \sum_f \Bigg|$$  $$\Bigg|^2$$

$q_{\lambda/\lambda_p}(x, \Lambda)$ — { transversity: density matrix; light-cone helicity }

$$= \sum_{n,q} \int \left| \psi^{(\Lambda)}_{n,\lambda_p}(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2 \prod_{j=1} dx_j \prod_{j=1} d^2 k_{\perp j}$$

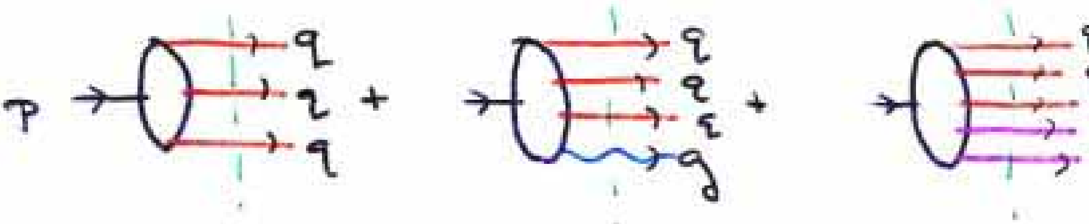
$$\delta\Big(\sum_i x_i - 1\Big)\, \delta\Big(\sum_i \vec{k}_{\perp i}\Big)$$

$$\delta(x - x_q)\, \delta_{\lambda, \lambda_q}$$

$$\Theta(\Lambda^2 - \mathcal{M}_n^2)$$

Factorization
Light-cone Scheme

Light-Cone Fock Representation of Hadrons



$$|p\rangle = \sum_n |n\rangle \, \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum x_i = 1 \, , \, \sum \vec{k}_{\perp i} = 0$

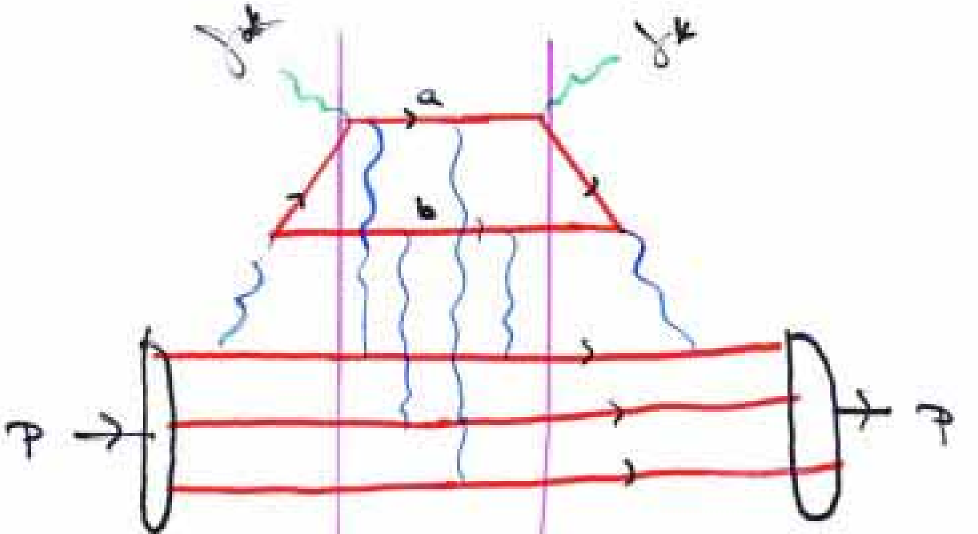
- Explicit solutions using "DLCQ" — QCD(1+1), "collinear" QCD
  SJB, Pauli, Hornbostel, Antonuccio, Dalley

- Calculate structure functions modulo FSI — $q(x)$, $g(x)$, $Q(x)$, Spin-dependence

- Calculate Regge behavior using "ladder relations" — $x \to 0$, BFKL, Spin-dependence
  Mueller, SJB, Antonuccio, Dalley

- $x \to 1$ constraints — Lepage, SJB, Burkardt, Schm

- Properties of heavy quark sea — $s(x) \neq \bar{s}(x)$
  extrinsic vs intrinsic
  physics of $\Delta\Sigma$, anomaly
  Ball
  Kogan, Ma, Schmidt, SJB, Schlumpf



$\psi_{LF} \Leftarrow$ FSIs $\Rightarrow \psi_{LF}(x_i, \vec{k}_{\perp i}, \lambda_i)$

$W = \frac{g^2}{2\pi} \log \frac{|\vec{R}+\vec{r}|}{|\vec{R}|}$

phase $e^{iW_a} e^{iW_b} = e^{iW}$

subtract tree gluon

Augment LFWF

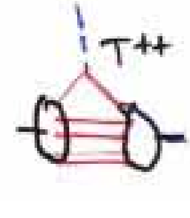
$$\psi_{LF} \Rightarrow \psi_{LF} \frac{e^{iW}-1}{iW}$$

$$F_2 \Rightarrow F_2 \left| \frac{\sin W/2}{W} \right|^2$$

Wilson lines!: like external field. (BJY)

* Shadowing not contained in ψ_{LFWF}

- not in LGTh or ψ_{BS}
- not in local operators



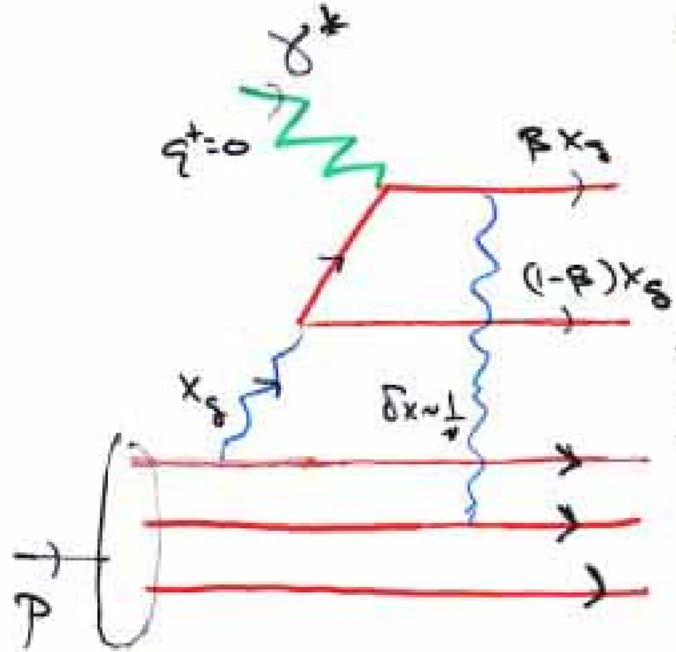
In progress

Hoyer
Ingelman
SJB

Structure Function of Pomeron

$\gamma^* p \to M_{q\bar{q}} + p'$

Diffraction DIS

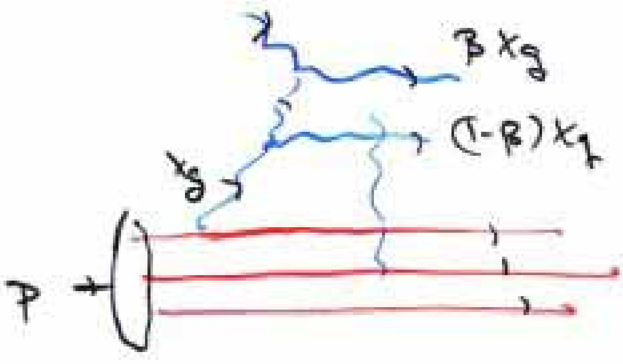


} rap gap

β-distribution same as $g \to q\bar{q}$

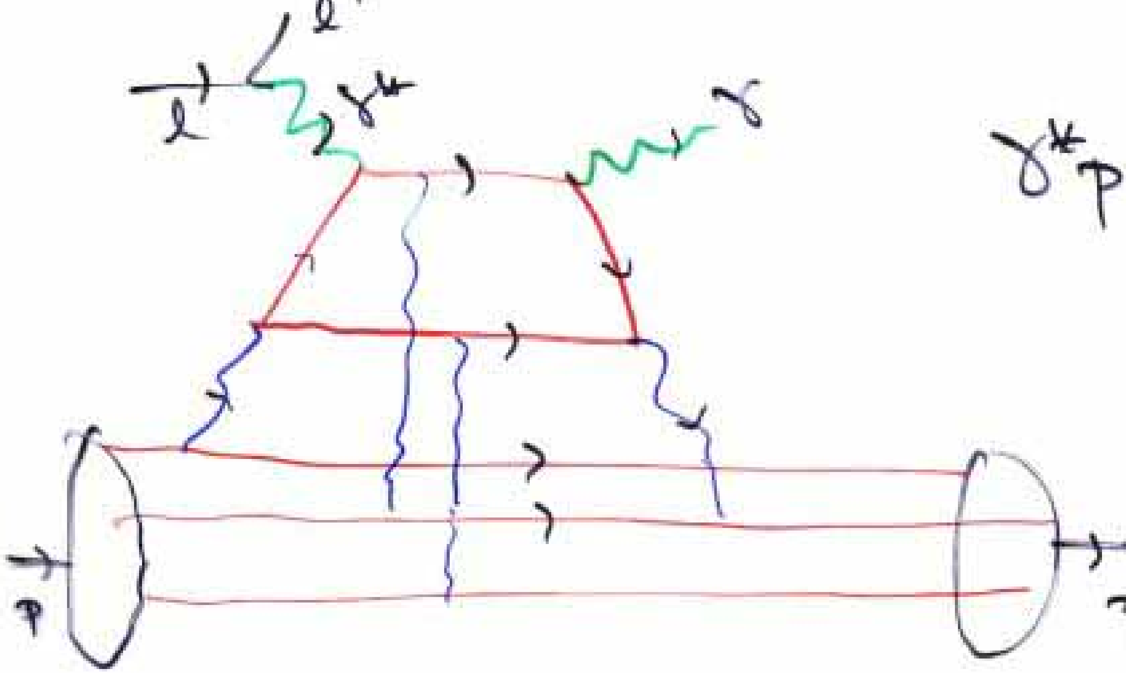
$$F_{2\,pomeron}^{(\beta)} = F_{2\,g/p}(\beta)$$

Different for hadron collisions!



β-Dist. same as $g \to gg$

DVCS



$\gamma^* p \to \gamma p'$

Final State Correct to DIS

$\Rightarrow$ corrections to "handbag" in DVCS !

- Single-spin asymmetries $\quad \vec{S}_p \cdot \vec{q} \times \vec{k}$
- Nuclear corrections $\leftarrow \begin{cases} \gamma^* A \to \gamma p'(A-1) \\ \gamma^* A \to \gamma A' \\ \gamma^* A \to \gamma p X \end{cases}$
- Phase structure

Explicitly Covariant Light-Front Formalism

Karmanov, Smirnov
Hiller, Kuang, Karmanov, SJB

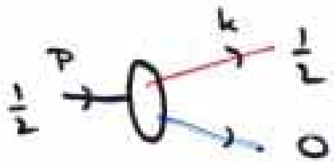
n^μ : arbit direction $\quad n^2 = 2$

$n = (1, 0_\perp, -1)$ standard

Lorentz Scalar LFWF

$$\Phi = \phi(x_i, M^2)$$

$$x_i = \frac{k_i \cdot n}{p \cdot n}, \quad M^2 = \left(\sum_i k_i^\mu\right)^2 = \sum_i \left(\frac{k_\perp^2 + m^2}{x}\right)_i$$



$$\Psi_{1/2} = \bar{u}(k)\left[\varphi_1 + \frac{M\chi}{n \cdot p}\varphi_2\right]u(p)$$

$$\varphi = \varphi(x_i, m^2)$$

In "const. rest frame" $\sum \vec{k}_i = 0$

$$m = \sum_i \sqrt{\vec{k}_i^2 + m_i^2}$$

$$\Psi_{1/2} = \chi_k^+ \left[f_1 + \frac{i}{k}(\hat{n} \times \hat{k}) \cdot \vec{\sigma}\, f_2\right]\chi$$

$$f = f(m^2, \vec{n} \cdot \vec{k})$$

LF angular momentum (in CRF)

$$\vec{J}_{LF} = -i\left(\vec{k} \times \frac{\partial}{\partial \vec{k}}\right) - i\left(\vec{n} \times \frac{\partial}{\partial \vec{n}}\right) + \vec{S}$$

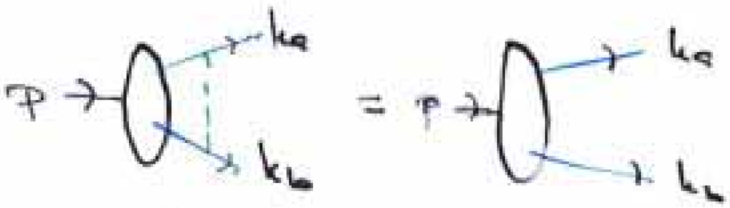
kinematic ! $\vec{n} \times \frac{\partial}{\partial \vec{n}}$ generates rotations

Testing ground for non-perturbative LFWF

Wick-Cutkosky Model

$\phi^2\chi$: Bethe-Salpeter Ladder Approx.

J=0, 1 Solutions known [Karmanov-Smirnov]



$q = \frac{1}{2}(k_a - k_b)$

$\left.\begin{array}{l} J=1 \\ J_z = \mu = \lambda \end{array}\right\}$ $\psi^{J=1}_{BS}(P,q) = \epsilon^{(\lambda)}_\mu(P)\, q^\mu\, \Phi(q^2, P\cdot q)$

project $t=0$ or $\tau = t + z/c = 0$

$\tau = 0$. $\quad \psi_{LF} = \epsilon^{(\lambda)}_\mu(P)\, \psi^\mu_{LF}$ $\quad k^\mu = k^\mu_q$

$\psi^\mu_{LF} = k^\mu \varphi_1 + \frac{n^\mu}{n\cdot P} \varphi_2$

$\varphi_i = \varphi_i(M_0^2, x)$

Same result $\quad P \to \infty \quad \psi_{ET} \Rightarrow \psi_{LF}$!

Wick-Cutkosky Solutions

Eigenfunctions of J^2, J_z

B.S. function: tensor structure

E.T. Ψ : CRF - ordinary $\vec{J}, \vec{L}$

L.F. Ψ : $\vec{J}_{LF} = -i\, \vec{k} \times \frac{\partial}{\partial \vec{k}} - i\, \vec{n} \times \frac{\partial}{\partial \vec{n}}$

(or $\vec{P} \to \infty$)

Form factors. $J=0$, $J=1$

$$F_i = F_i(q^2).$$

In general: $\varphi_B = \varphi_B(M^2, x_i = \frac{n \cdot k_i}{n \cdot p})$

$$\Rightarrow \boxed{F_i = F_i(q^2)}$$

Example: PQCD

$$\boxed{\frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \sim \log^2 Q^2}$$

Belitsky, Ji, Yuan

vs

$$\frac{Q F_2(Q^2)}{F_1(Q^2)} \sim \text{const}$$

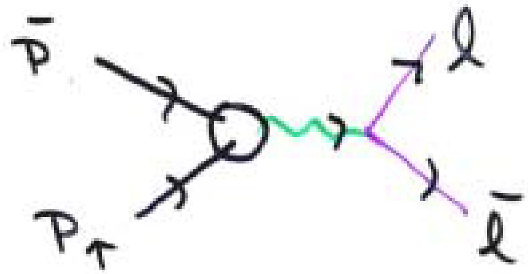
Frank Miller

From Coester + Chung, Sallurf LFWF.
$M \to M_0$.

* New results for $e p_\uparrow \to e p_\uparrow$ from JLAB

$$\frac{G_E(t)}{G_M(t)} \text{ decreasing } \Rightarrow \sqrt{-t}\,\frac{F_2(t)}{F_1(t)} \sim \text{const}$$

Need higher t, timelike $p\bar{p} \to \ell\bar{\ell}$, $\ell\bar{\ell} \to p\bar{p}$ data!



measure pol. of final-state μ ?

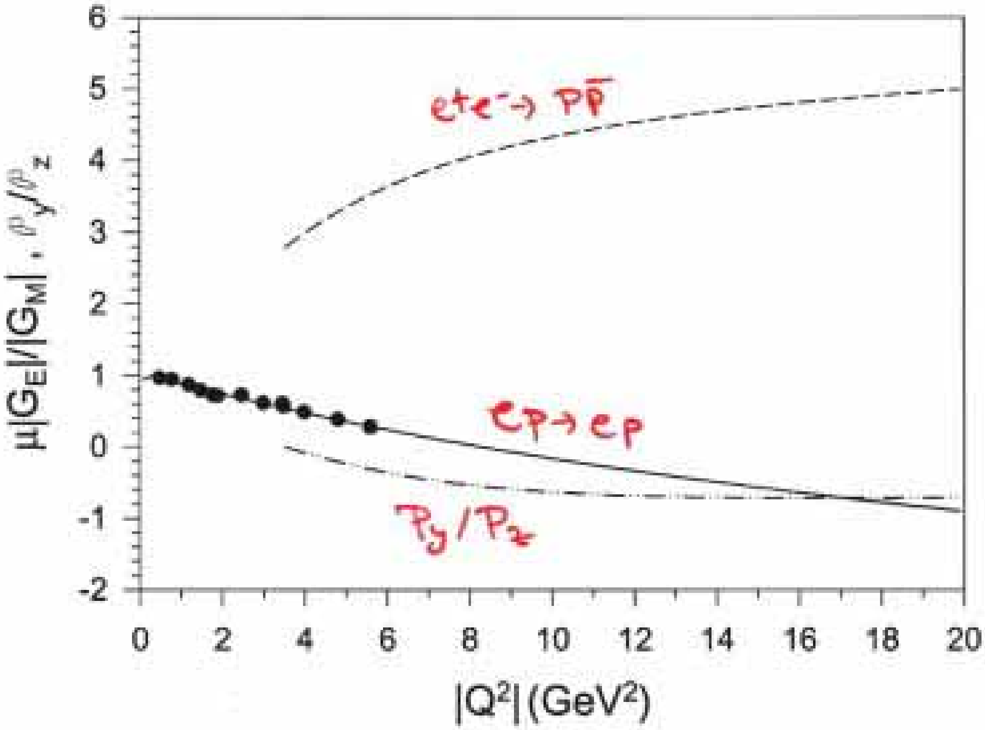
$$\frac{d\sigma}{d\Omega} \propto |G_M(s)|^2(1+\cos^2\theta_{cm}) + \frac{4m^2}{s}|G_E(s)|^2\sin^2\theta$$

SSA
$$P_y = \frac{-\sin 2\theta_{cm}\ \mathrm{Im}\, G_E^* G_M \frac{2M}{\sqrt{s}}}{|G_M|^2(1+\cos^2\theta) + \frac{4m^2}{s}|G_E|^2\sin^2\theta}$$

Pol transfer
$$P_x = P_\ell\,\frac{2\sin\theta_{cm}\ \mathrm{Re}\, G_E^* G_M \frac{2M}{\sqrt{s}}}{|G_M|^2(1+\cos^2\theta) + \frac{4m^2}{s}|G_E|^2\sin^2\theta}$$

does G_E change sign?

does $\frac{d\sigma}{d\Omega} \sim (1+\cos^2\theta)$ PQCD HHC.



$$\frac{F_2(Q^2)}{F_1(Q^2)} = \kappa_p \frac{(1+Q^2/0.791)^2 \ln^{7.102}(1+Q^2/4m_\pi^2)}{(1+Q^2/0.380)^3 \ln^{5.102}(1+Q^2/4m_\pi^2)}$$

$$\sim \frac{\ln^2 Q^2}{Q^2}$$

Hiller, Carlson
Hwang, Karmanov
SJB

Counting Rules

String Theory

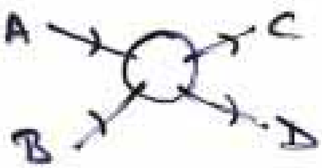
Supergravity dual of conformal QFT $AdS_5 \times X_5$

QCD

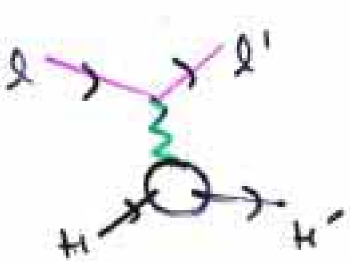
PQCD Factorization

$$\frac{d\sigma}{dt}(AB \to CD) \cong \frac{F_{AB\to CD}(t/s)}{s^{n-2}}$$

$-t \gg \Lambda^2$



$$n = n_A + n_B + n_C + n_D$$



$$F_{\Delta\lambda=0}(t) \cong \left[\frac{\Lambda^2}{t}\right]^{n_i-1}$$

n_i = minimum # Fock constituents

= dim of lowest twist interpolating op.

Farrar + SJB

Matveev, Muradian, Tavkhelidze } conformal PQCD fixed coupling

$\Lambda \sim f_\pi \sim$ wf at origin

Polchinski
Strassler

AdS/CFT correspondence

$\Rightarrow$ power law scaling SQCD ($\mathcal{N}=4$)

$$\mathcal{M}_{2\to 2}(Q) \sim \frac{\left(g_{YM}^2 N_c\right)^{\frac{1}{2}(\Delta-2)}}{N_c^2} \left(\frac{\Lambda}{Q}\right)^{\Delta-4}$$

Λ : mass of lightest glueball

$\Delta = \sum_{i=1}^{4} n_i$, $n_i = (\text{dim} - \text{spin})$ [twist] of lowest-twist interpolating operator

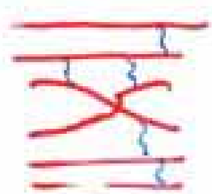
Identify $\sqrt{g_{YM}^2 N_c} = \alpha_s N_c$

Agrees with Dimensional Counting + PQCD

Rey + Yee
Maldacena

Example

$pp \to pp$



$\Delta = 12$

$\frac{1}{2}(\Delta-2) = 5$

$\Delta - 4 = 8$

$$\mathcal{M}_{pp\to pp} = \alpha_s^5 \frac{F(\theta_{cm})}{s^4}$$

conformal FT
BF
MMT

Physics of Hard Scattering

Effective static potential

$$V(r) = \sigma r - \frac{\alpha_{eff}}{r}$$

("soft" ← σr)

QCD Strings + LCQ

$$[M^2 - H_{soft}]\,\psi_{soft} = 0.$$

Perturbation from $-\frac{\alpha_{eff}}{r}$ or gluon exchange

$$\delta\psi_n = \frac{1}{M^2 - H_{soft}}\,\delta H \cdot \psi_{soft}$$

$$\sim \left[\frac{\alpha_{eff}\,\sigma}{m_0^2}\right]^{n-1} \psi_{soft}$$

dom. at high $k_\perp^2$

counting rules Bodsky, Ji, Ma, Yuan B+L

$$\delta\psi_n^{L_z=1} \sim \left(\frac{M k_\perp}{m_0^2}\right) \delta\psi_n^{L_z=0}$$

$$\Rightarrow \quad F_1(Q^2) \sim \left[\frac{\alpha_{eff}\,\sigma}{Q^2}\right]^{n-1}, \quad \text{counting rules}$$

BF, MAT

$$F_2(Q^2)/F_1(Q^2) \sim \frac{M^2}{Q^2} \quad (\text{mod log } Q^2)$$

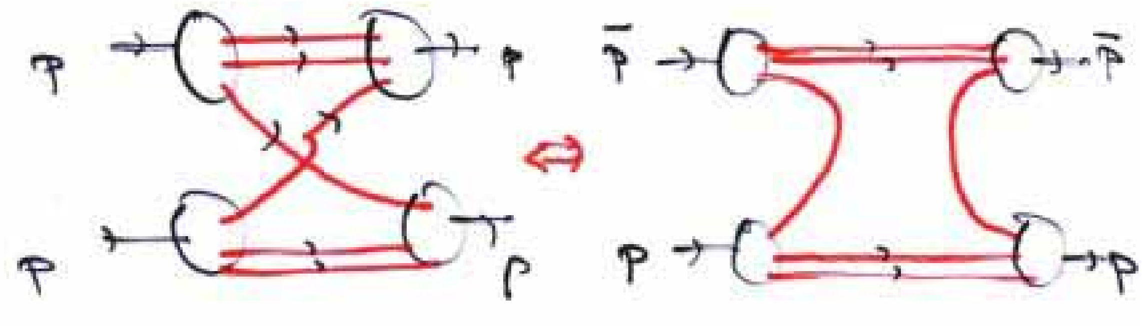
$$F_1(q^2) = \sum_n \int \pi'\, d^2k_\perp\, dx\ \psi_\uparrow'\,\psi_\uparrow$$

$$-\frac{(q_1 - i q_2)}{2M} F_2(q^2) = \sum_n \int \pi'\, d^2k_\perp\, dx\ \psi_\uparrow'\,\psi_\downarrow \qquad \Delta L_z = 1$$

Current Project (with G. de Teramond)

Analyze $pp \to pp$, $\bar{p}p \to \bar{p}p$

ADS/CFT ⇒ Quark Interchange Amplitudes !



→ Scale set by α'_{QCD} : $d\sigma \sim \frac{F(\theta_{cm})}{s^9 \alpha'^9_{QCD}}$

→ Non-leading odderon, pomeron at large θ_{cm}

→ Crossing to $\bar{p}p \to \bar{p}p$ $s \leftrightarrow t$

→ Hadron helicity conservation

→ $M(\uparrow\uparrow \to \uparrow\uparrow)$ $M(\uparrow\downarrow \to \uparrow\downarrow, \downarrow\uparrow)$

$M(\uparrow\uparrow \to \uparrow\downarrow)$ suppressed by $\frac{1}{\sqrt{\alpha'_{QCD}} p_T}$

$M(\uparrow\uparrow \to \downarrow\downarrow)$ " $\frac{1}{\alpha'_{QCD} p_T^2}$

→ $\frac{d\sigma}{dt}$, A_{NN}, A_N well-described

Light-Front Formulation of the Standard Model

P.P. Srivastava + SJB

PRD 66 045019 (2002)

- SU(2) x U(1) GWS Model of EW Interactions
  - Non-Abelian Higgs Model in L.C. Gauge ($A^+=0$)

* Unitary, ghost-free! : no Faddeev-Popov ghosts, no Gupta-Bleuler ghosts
* Renormalizable !
* Perturbative Vacuum plus zero mode higgs (SSB)
* t' Hooft conditions

$$\partial \cdot W^{\pm} = \pm i M_W G_W^{\pm} ,$$

$$\partial \cdot Z = M_Z G^0 ,$$

$$A^+ = Z^+ = (W^{\pm})^+ = 0$$

Light-Front Quantized QCD in l.c. Gauge

P. Srivastava + SJB

Derive Feynman rules in QCD

* Dirac bracket formalism for constrained systems $(A^+ = 0)$

* Dyson-Wick perturbation theory

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{a\mu\nu} F^{a}_{\mu\nu} + B^{a} A^{a}_{-} + \bar{c}^{a} D^{ab}_{-} c^{b} + \bar{\psi}^{i} (i \gamma^{\mu} D^{ij}_{\mu} - m \delta^{ij}) \psi^{j}$$

B : gauge fixing $A^+ = A_- = 0$

$\bar{c}, c$ anti-commuting gauge fields

Light-Front Quantization of Gauge Theory

Bjorken
Kogut
Soper

Lepage, SJB

$$\mathcal{L}_{QCD} \Rightarrow H^{LF}_{QCD} = P^- P^+ - \vec{P}_\perp^{\,2}$$

$$P^- = i \frac{\partial}{\partial \tau}, \quad \tau = t + z$$

Identify independent fields vs constrained fields

constraint $\quad A^+ = 0 \quad$ LC Gauge

Define $\quad \Psi_\pm \equiv \Lambda_\pm \Psi = \frac{1}{2}(1 \pm \alpha^3)\Psi$

$\frac{1}{2}\gamma^0\gamma^\pm$

← l.f. spinors

Ψ_+ dynamical

constraint: $\quad \Psi_- = \frac{1}{2i\partial^+}\left(m\beta - \vec{\alpha}_\perp T^a \cdot \vec{D}^a_\perp\right)\Psi_+$

constraint: $\quad A^-_a = \frac{g}{(i\partial^+)^2} J^+_a$

$$\frac{1}{i\partial^+} \Rightarrow \frac{1}{k^+} = \frac{1}{xP^+}$$

$D^+ = \partial^+$!

dynamical $\quad \vec{A}_\perp$

Derivation of H_{LC}^{QCD} in $A^+=0$ gauge from $\mathcal{L}_{QCD}$

$$P^-P^+ - P_\perp^2 = H_{LC}$$

Define $\Psi_\pm = \Lambda_\pm \Psi = \frac{1}{2}(1 \pm \alpha^3)\Psi$

Since $A^+ = 0$ can eliminate Ψ_-:

$$\Psi_- = \frac{1}{2i\partial^+}\left(m\beta - i\alpha_\perp T^a D_\perp^a\right)\Psi_+$$

Also

$$A_a^- = \frac{g}{(i\partial^+)^2} J_a^+$$

$\frac{1}{i\partial^+} \to \frac{1}{k^+} \to \frac{1}{xP^+}$

Define $\tilde{\Psi} = \Psi(g=0)$, $\tilde{A} = A(g=0)$

Then $P^-_{QCD} = \int dx^- d^2x_\perp \mathcal{P}^-_{QCD}$

where

$$\mathcal{P}^-_{QCD} = \frac{1}{2}\bar{\tilde{\Psi}}\gamma^+\left[\frac{m^2 + (i\vec{\nabla}_\perp)^2}{i\partial^+}\right]\tilde{\Psi} + \frac{1}{2}\tilde{A}(i\nabla_\perp)^2\tilde{A}$$

$$+ g\tilde{J}\cdot\tilde{A} + \frac{g^2}{4}\left[\tilde{A},\tilde{A}\right]^2$$

new from LCQ $\Rightarrow$

$$+ \frac{g^2}{4}\tilde{J}\frac{1}{(i\partial^+)^2}\tilde{J} + \frac{g^2}{2}\bar{\tilde{\Psi}}A\!\!\!/\frac{\gamma^+}{i\partial^+}A\!\!\!/\tilde{\Psi}$$

and $\tilde{J}_a^\mu = \bar{\tilde{\Psi}}\gamma^\mu T_a\tilde{\Psi} + f_{abc}\partial^\mu\tilde{A}_b\tilde{A}_c$

Feynman gauge also possible (P. Srivastava + SJB)

Quantize dynamical fields

$$A^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k_\perp\, dk^+ \; \frac{\theta(k^+)}{\sqrt{2}\, k^+}$$

$$\sum_{\alpha=1,2} \epsilon^\mu_{(\alpha)}(k) \left[a_\alpha(k^+, \vec{k}^\perp) e^{-ik\cdot x} + a^\dagger_\alpha(k^+, \vec{k}^\perp) e^{ik\cdot x} \right]$$

$$\left[a_\alpha(k), a^\dagger_\beta(l) \right] = \delta_{\alpha\beta}\, \delta^2(\vec{k}_\perp - \vec{l}_\perp)\, \delta(k^+ - l^+)$$

$\epsilon_{(\alpha)}$

k $\qquad k^2 = k^+k^- - k_\perp^2 = 0$

$$\epsilon^\mu_{(\alpha)} = (\epsilon^+_{(\alpha)}, \epsilon^-_{(\alpha)}, \vec{\epsilon}_{\perp(\alpha)})$$

$$= \left(0, \frac{2\vec{\epsilon}_{\perp(\alpha)}\cdot\vec{k}_\perp}{k^+}, \vec{\epsilon}_{\perp(\alpha)}\right), \quad \alpha = 1,2$$

$$\vec{\epsilon}_\perp = \frac{\hat{x}+i\hat{y}}{\sqrt{2}}, \; -\frac{\hat{x}-i\hat{y}}{\sqrt{2}}$$

$$\epsilon^+ = 0, \qquad k\cdot\epsilon = 0$$

$$\mathcal{L}_{QCD} \Rightarrow H^{LC}_{QCD}$$

canonical quantization

$A^+ = 0$

Interactions:

$\bar{u}\gamma^\mu u$

$$\frac{\bar{u}\gamma^\nu\gamma^+\gamma^\mu u}{k^+}$$

analogous to

"seagull"

$$\frac{\bar{u}\gamma^+ u\,\bar{u}\gamma^+ u}{(k^+)^2}$$

$k^+ = k^0 + k^3$, $\gamma^+ = \gamma^0 + \gamma^3$

Spinors: eigenstates of $\Lambda_\pm = \frac{\gamma^0\gamma^\pm}{2}$

$k^+ = 0$ sing. cancel in P.T.

$$\mathcal{H}^{LC}_{int} = -g\,\bar{\psi}^i \gamma^\mu A_\mu^{ij}\, \psi^j$$

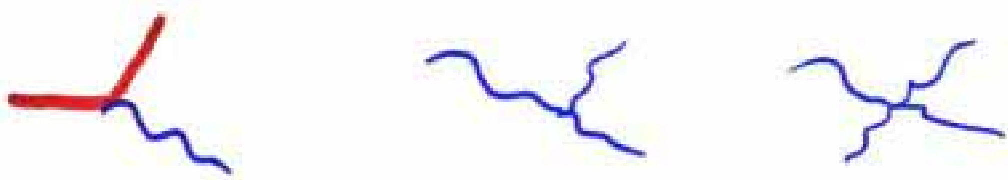
$$+ \frac{g}{2} f^{abc}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) A^{b\mu} A^{c\nu}$$

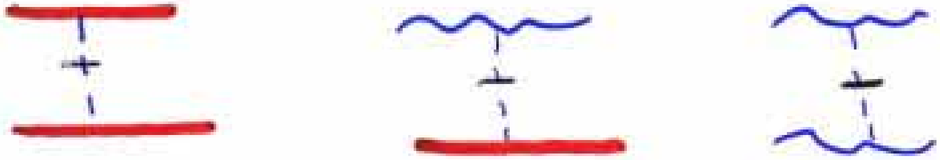
$$+ \frac{g^2}{4} f^{abc} f^{ade} A_{b\mu} A^{d\mu} A_{c\nu} A^{e\nu}$$

$$- \frac{g^2}{2} \bar{\psi}^i \gamma^+ (\gamma^\perp \cdot A_\perp)^{ij} \frac{1}{i\partial_-} (\gamma^\perp A_\perp)^{jk} \psi^k$$

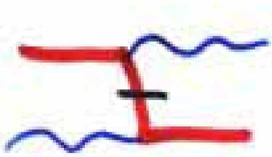
$$- \frac{g^2}{2} J^+_a \frac{1}{(\partial_-)^2} J^+_a$$

$$J^+_a = \bar{\psi}^i \gamma^+ (t_a)^{ij} \psi^j + f_{abc}(\partial_- A_{b\mu}) A^{c\mu}$$





$$\frac{1}{(k^+)^2}$$



$$\frac{1}{k^+}$$

Result:

P. Srivastava + SJB

Gauge propagator

$$\langle 0|T(A^a_\mu(x)\,A^b_\nu(0))|0\rangle = \frac{i\delta^{ab}}{(2\pi)^4}\int d^4k\, e^{-ik\cdot x}\,\frac{D_{\mu\nu}(k)}{k^2+i\epsilon}$$

$\sum_{\lambda=1,2} \epsilon^{(\lambda)}_\mu \epsilon^{(\lambda)}_\nu$

$$D_{\mu\nu}(k) = -g_{\mu\nu} + \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} - \frac{k^2}{(n\cdot k)^2}\, n_\mu n_\nu$$

$$n_\mu = 2\delta_\mu^{+}$$

"Doubly-Transverse"

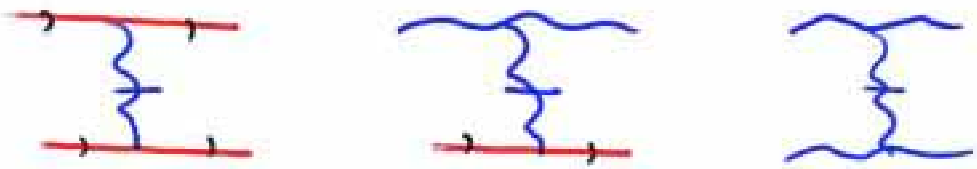
$$n^\mu D_{\mu\nu}(k) = 0 \quad : \quad A^+ = 0$$

$$k^\mu D_{\mu\nu}(k) = 0 \quad : \quad \partial^\mu A_\mu = 0$$

- Lorentz condition automatically satisfied
- Covariant Feynman rules $A^+=0$ gauge
- Ghost-Free.
- Instantaneous Interactions
- Mandelstam-Liebbrandt prescription for $n\cdot k$

$k \cdot n = k^+$

"Instantaneous" L.F. Interactions

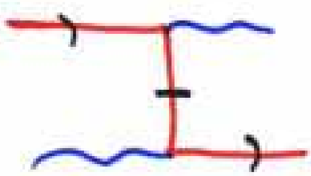


$$e^2 \frac{J \cdot n \; J \cdot n}{(k \cdot n)^2}$$ from elimination of A^-

In tree graphs:

$$\left\{ \begin{matrix} K^{\mu\nu} \\ D^{\mu\nu} \end{matrix} \right. \rightarrow -g^{\mu\nu} + \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n}$$

"text book l.c.g."



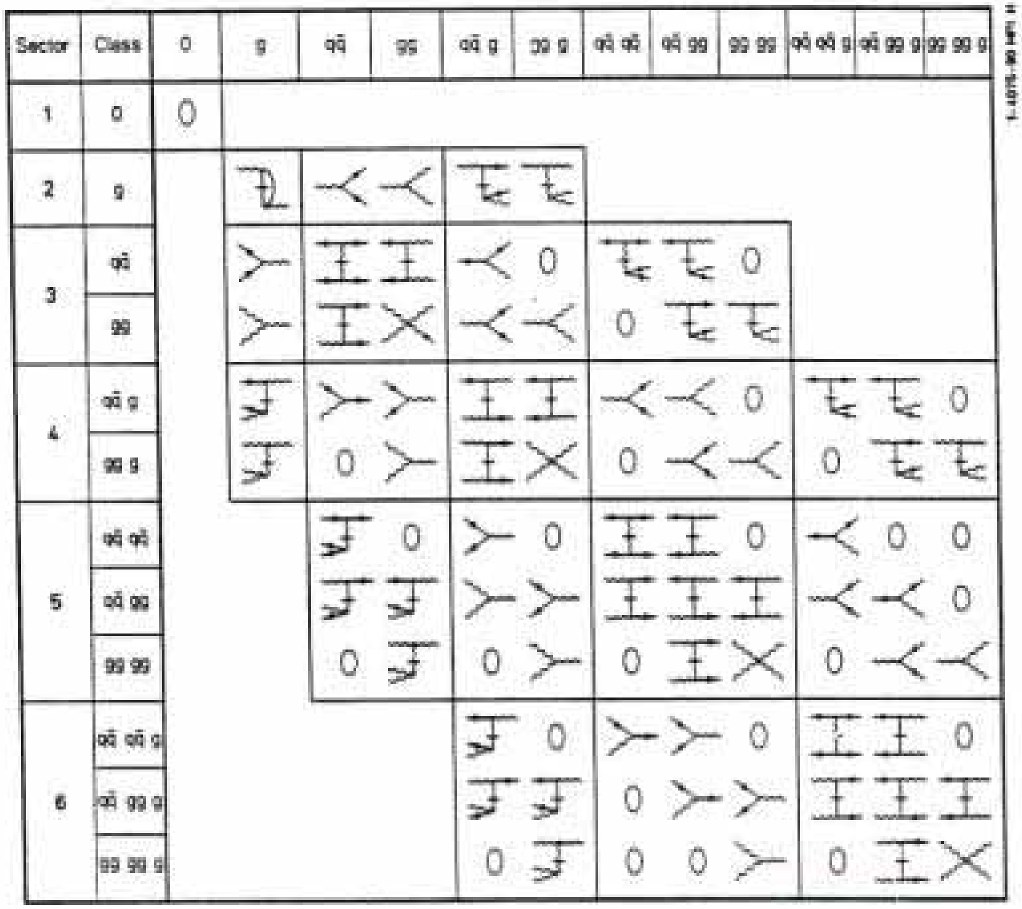
$$\bar{u} \gamma \cdot \epsilon^* \frac{\gamma \cdot n}{k \cdot n} \gamma \cdot \epsilon \, u$$

from elimination of $\psi^- = \Lambda^- \psi$

In tree graphs, restores Feynman propagator

$$\not{k}_{on} + m \Rightarrow \not{k}_{Feynman} + m$$

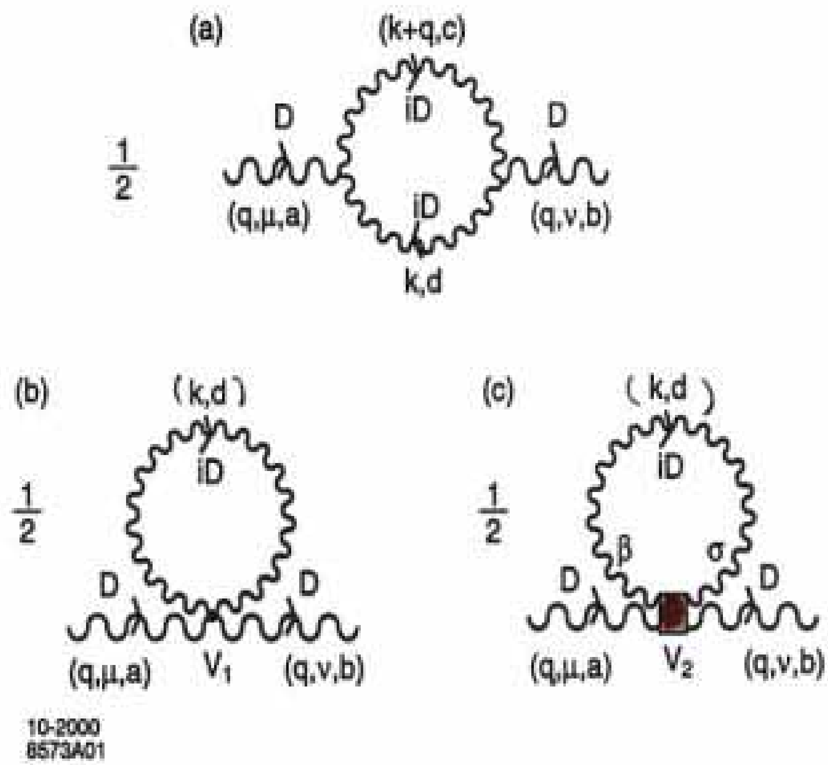
QCD (3+1)



DLCQ: $\langle n | H_{LC}^{I} | m \rangle$

$$\left(M^2 - \sum_n \frac{k_\perp^2 + m^2}{x}\right)\psi_n = \sum_m \langle n | H_{LC}^{I} | m \rangle \psi_m$$

PBC: $k^+ = \frac{2\pi}{L} n \quad , \quad \vec{k}_\perp = \frac{2\pi}{L_\perp} \vec{n}_\perp$

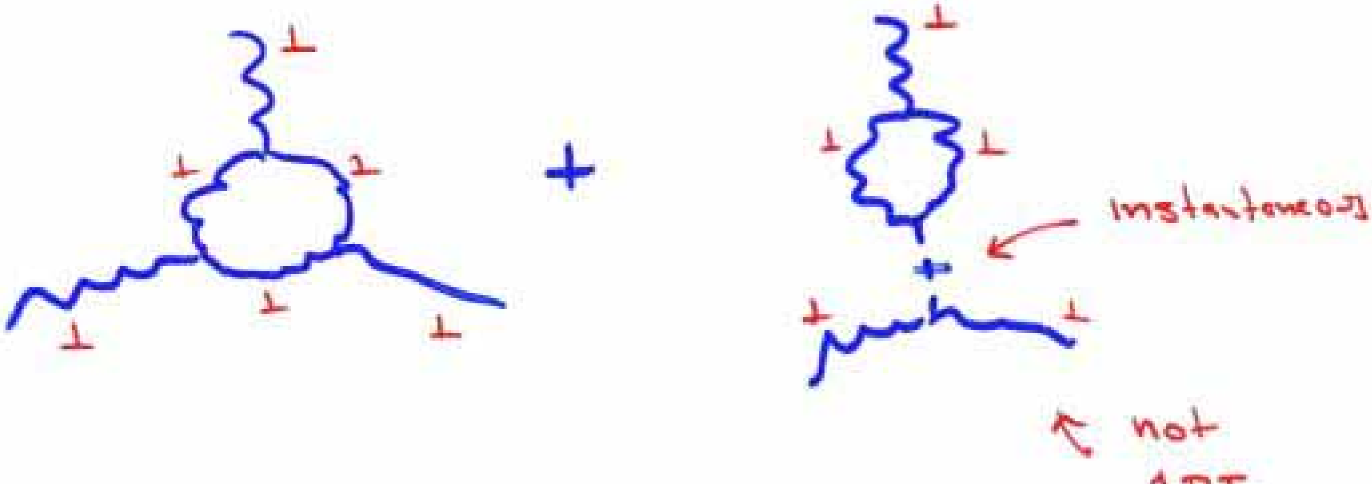


gluon self-energy correction at one-loop in l.c.g.

P. Srivastava
+
SJB

Key point

→ Renormalization of QCD in l.c.g.



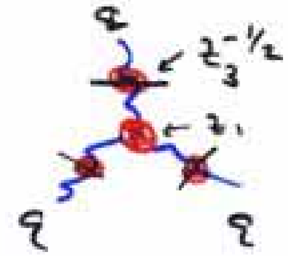
* Both contribute to renormalization of proper three-gluon vertex.

* Avoid conflict with Källen-Lehman repn.

P.P.S. + S.J.B.
C. Thorn

Calculation of one-loop renormalization constants

Light-front QCD ; l.c.g (M-L presc.) ; dim. reg.



Coupling renormalization

$$Z_g = \frac{Z_1}{(Z_3)^{3/2}}$$

$$Z_3 = 1 + \frac{g^2}{16\pi^2} C_A \frac{11}{3} \left[\frac{2}{\epsilon} - \ln \frac{Q^2}{4\pi\mu^2} \right]$$

$$Z_1 = 1 + \frac{g^2}{16\pi^2} C_A \frac{11}{3} \left[\frac{2}{\epsilon} - \ln \frac{Q^2}{4\pi\mu^2} \right]$$

$$Z_g = \frac{Z_1}{Z_3^{3/2}} = 1 - \frac{g^2}{16\pi^2} C_A \left(\frac{11}{6}\right) \left[\frac{2}{\epsilon} - \ln \frac{Q^2}{4\pi\mu^2} \right]$$

$\therefore \quad \beta_0 = \frac{11}{3} C_A$ asymptotic freedom

$$\alpha_S(Q^2) - \alpha_S(Q_0^2) = -\frac{\alpha_S^2}{4\pi} \beta_0 \ln Q^2/Q_0^2$$

$Z_1 = Z_3$ no dep. on n^μ

Thus l.c.g gives consistent

covariant Feynman rules

* Unitary, ghost free
* Ward-identities satisfied
* Lorentz condition
* Renorm. consts. scalars (indep of $n.A=0$)

  - at least to one-loop

Well-matched to "pinch technique" scheme

Advances in L-F Quantization

- Progress in DLCQ

  3+1 Yukawa Theory with Pauli-Villars reg.
  — numerical results

  "Exact" soln. to model theory

  Hiller, McCartor, & B

- U.V. Regularization of DLCQ in gauge theory

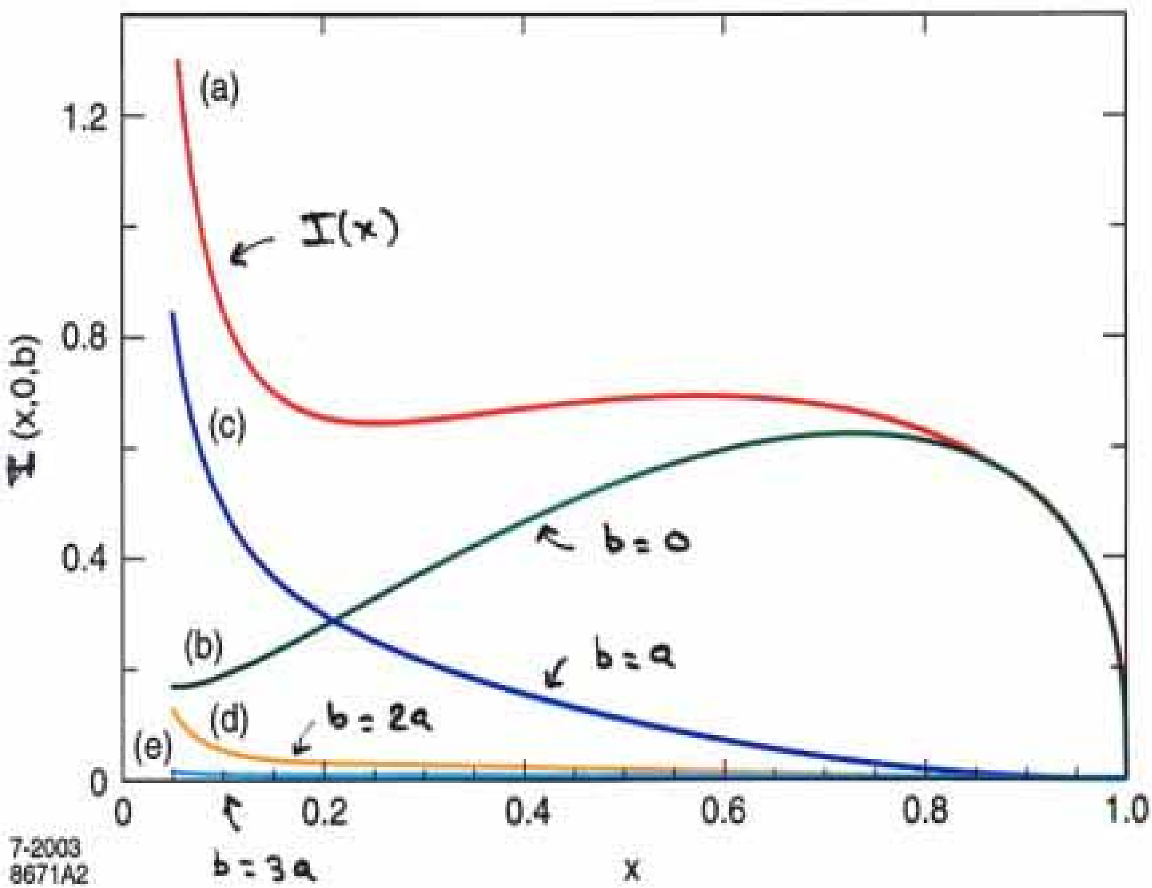
  Prokhvatilov, Franke, Paston

- Supersymmetric DLCQ !

  Hiller, Pinsky, Trittman

- Transverse Lattice Bardeen, Rabinovici, Dalley, Burkardt, Seal

  $\phi_\pi(x,Q)$, $B \to D \ell \nu$



$$I(x, \vec{b}_\perp) = \sum_n \int |\psi^n_\pi(x, \vec{b}_\perp)|^2 e^{i k_\perp \cdot b_\perp}$$

Transverse Lattice
plus DLCQ

S. Dalley

P. Srivastava
S.J.B.

Light-Front Quantization of Standard Model $SU(2) \otimes U(1)$

* $n \cdot A = A^+ = 0$ gauge

  $\Rightarrow$ physical gauge quanta
  unitary
  renormalizable

* SSB from zero mode of scalar field

$$\phi^{(0)} = \frac{v}{\sqrt{2}}$$

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x) + i\eta(x))$$

$$\partial \cdot A = M\eta \quad , \quad M = ev$$

* Nambu-Goldstone field $\eta(x)$ restores ϵ_L :

$$\epsilon^{(L)}_{\mu} = \frac{n_\mu M}{n \cdot k} - \frac{k_\mu M}{k^2} \qquad (\epsilon \cdot k = 0)$$

* LC vacuum remains trivial

* Amplitude Event Generator

  $\Rightarrow$ Renormalized Amplitudes from LF T-O PTh
  Ghost-Free, $\Pi d^2k_\perp dx$, DLCQ discret.

Quantize Theory:

$$A^\mu(x) = \frac{1}{\sqrt{(2\pi)^3}} \int d^2k_\perp \, dk^+ \, \frac{\theta(k^+)}{\sqrt{2k^+}}$$

$$\sum_{\alpha=1,2,3} E_{(\alpha)}^{\mu}(k) \left[a_\alpha(k^+, k^\perp) e^{-ik\cdot x} + a^\dagger_\alpha(k^+, k^\perp) e^{ik\cdot x} \right]$$

$$\left[a_\alpha(k), a^\dagger_\beta(l) \right] = \delta_{\alpha\beta} \, \delta^2(k_\perp - l_\perp) \, \delta(k^+ - l^+)$$

$$E_{(\alpha)}^{\mu=+} = 0 \quad , \quad E^\mu_{1,2} = \left(0, \frac{2\,\vec{\epsilon}_\perp \cdot \vec{k}_\perp}{k^+}, \vec{\epsilon}_\perp\right)$$

$$E_3^\mu = -\frac{M}{k^+} n^\mu = -\frac{M}{k^+}(0, 2, 0_\perp)$$

← null norm

$$\langle 0 | T(A_\mu(x), A_\nu(y)) | 0 \rangle$$

$$= \frac{i}{(2\pi)^4} \int d^4k \, \frac{K_{\mu\nu}(k)}{(k^2 - M^2 + i\epsilon)} \, e^{-ik\cdot(x-y)}$$

$$K^{\mu\nu} = -g^{\mu\nu} + \frac{n^\mu k^\nu + n^\nu k^\mu}{n\cdot k} - \frac{(k^2 - M^2)}{(n\cdot k)^2} n^\mu n^\nu$$

Good high-energy behavior unlike

Proca : $-g^{\mu\nu} + \frac{k^\mu k^\nu}{M^2}$

Simple Example of SSB on LF : U(1) Theory

$$\mathcal{L} = |\partial^\mu \phi|^2 - V(\phi^+\phi)$$

$$= \partial_+ \phi^+ \partial_- \phi + \partial_- \phi^+ \partial_+ \phi - \partial_\perp \phi^+ \partial_\perp \phi - V(\phi^+\phi)$$

$$V(\phi^+\phi) = \mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2 \qquad [\lambda > 0,\ \mu^2 < 0]$$

Make separation:

$$\phi(\tau, x^-, x^\perp) = \omega(\tau, x^\perp) + \varphi(\tau, x^-, x^\perp)$$

↑ zero long. mom mode, dynamical, c-no.

↑ quantum fluctuations $\langle 0|\varphi|0\rangle = 0$

Constraint Eq. (from Euler-Lagrange)

$$\int dx_\perp\, dx^- \left[\partial_\perp \partial_\perp \phi - \frac{\delta V}{\delta \phi^+}\right] = 0 \qquad (\text{and } \phi \leftrightarrow \phi^+)$$

See also: Heinzl + Zamolodi...

For simplicity, assume $\partial_\perp \omega = 0$. Then

$$\frac{\delta V}{\delta \phi^+}\bigg|_{\phi=\omega},\quad \frac{\delta V}{\delta \phi}\bigg|_{\phi=\omega^+} = 0$$

$$\Rightarrow \quad \omega = 0 \quad \text{or} \quad \omega^+\omega = -\mu^2/2\lambda$$

Broken phase

U(1) invariance:

$$\omega = \frac{v}{\sqrt{2}}, \quad v = +\sqrt{\frac{-\mu^2}{\lambda}}$$

Simple example of SSB on LF

Abelian Higgs Model in LC Gauge

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \phi|^2 - V(\phi^+\phi)$$

$$D_\mu = \partial_\mu + ieA_\mu$$

$$V(\phi) = \mu^2 \phi^+\phi + \lambda(\phi^+\phi)^2$$

$\mu^2 < 0, \quad \lambda > 0$

$$\phi(x) = \frac{1}{\sqrt{2}} v + \varphi = \frac{1}{\sqrt{2}}[v + h(x) + i\eta(x)]$$

$$v^2 = -\mu^2/\lambda$$

↑ zero mode ↑ higgs ↑ Nambu-Goldstone

Find $\partial \cdot A - M\eta = 0$ ('t Hooft cond.)

$$H_0^{LF} = \frac{1}{2}(\partial_\perp A_\perp)(\partial_\perp A_\perp) + \frac{1}{2} M^2 A_\perp A_\perp$$

$$+ \frac{1}{2}(\partial_\perp \eta)(\partial_\perp \eta) + \frac{1}{2} M^2 \eta^2$$

$$+ \frac{1}{2}(\partial_\perp h)(\partial_\perp h) + \frac{1}{2} m_h^2 h^2$$

$$M = ev, \quad m_h^2 = 2\lambda v^2 = -2\mu^2$$

More interaction [Abelian Higgs Theory]

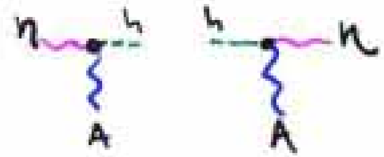
also fermion coupl
to h, $\eta = G$

$$-\mathcal{H}_{int} = \mathcal{L}_{int} =$$

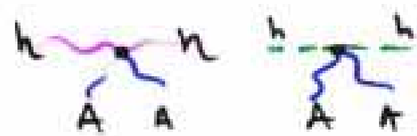
$$e M A_\mu A^\mu h$$

$$-e^2 \frac{m_h^2}{2M}(\eta^2+h^2)h$$

$$+e(h\partial_\mu \eta - \eta \partial_\mu h)A^\mu$$

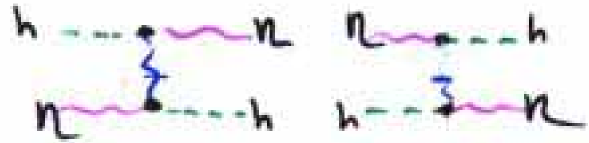


$$+\frac{e^2}{2}(h^2+\eta^2)A^\mu A_\mu$$



$$-\frac{\lambda}{4}(\eta^2+h^2)^2$$

$$-\frac{e^2}{2}\left(\frac{1}{\partial_-}J^+\right)\left(\frac{1}{\partial_-}J^+\right)$$



$$J^+ = h\partial^+\eta - \eta\partial^+ h$$

In tree graphs $\quad E^\mu_{(3)eff} = E^\mu_{(3)} - \frac{k^\mu k\cdot E_{(3)}}{k^2}$

$$k\cdot E_{(3)eff} = 0, \quad E^2_{(3)eff} = -1$$

Nambu Goldstone provides part of long. mode

Noether U(1) Symmetry current:

$$J_\mu = i\left[\phi^+ \partial_\mu \phi - \phi\, \partial_\mu \phi^+\right]$$

$$\partial_\mu J^\mu = 0.$$

- However the U(1) generator current

$$j_\mu = i\left[\varphi^+ \partial_\mu \varphi - \varphi\, \partial_\mu \varphi^+\right]$$

is not conserved! $\quad [\phi = v/\sqrt{2} + \varphi]$

$$j_\mu = J_\mu - \frac{iv}{\sqrt{2}}\, \partial_\mu(\varphi - \varphi^+)$$

$$\partial^\mu j_\mu = \frac{iv}{\sqrt{2}}\, \Box(\varphi - \varphi^+) \neq 0!$$

- The U(1) generator is $\quad G(x^+) \equiv \int d^2x^\perp dx^-\, j_-$

$$[\varphi(x), G] = \varphi, \quad [\varphi^+(x), G] = \varphi^+$$

$$G = \int d^2k_\perp dk^+\, \theta(k^+)\left[a^+(k)\, a(k) - b^+(k)\, b(k)\right]$$

$$\boxed{G\,|0\rangle_{LF} = 0}$$

, $|0\rangle_{LF}$ remains invariant indep. of SSB!

although j_μ not conserved.

PPS
PPS + SSB

Light-Front Quantization of Standard Model

⇒ New insight into Higgs mechanism

$$\phi_i(\tau, x^-, \vec{x}_\perp) = \omega_i(\tau, x_\perp) + \varphi_i(\tau, x^-, \vec{x}_\perp)$$

↑ isospin multiplet — ↑ zero mode — ↑ quantized field

Condition for SSB: (tree level)

$$V_i'(\omega) - \partial_\perp \partial_\perp \omega_i = 0$$

Construct generators:

$$G_a = -i \int dx^\perp dx^- (\partial_- \varphi_i)(t_a)_{ij} \varphi_j$$

SSB → Current not conserved; $[H_{LF}, G_a] \neq 0$

However $G_a |0\rangle_{LF} = 0$!

* LF Vacuum retains symmetry.

⇒ Speculation: Interpret $\omega_i \neq 0$ as "external field" remnant of Higgs field from early cosmology

Light-Front Quantized QCD and Zero-Modes

$k^+ = 0$; structure possible in $\vec{k}_\perp$

$|0\rangle_{LF}$ remains trivial

→ Use DLFQ to separate, constrain zero modes

T. Maskawa + K. Yamawaki

- θ-vacuum in QED(1+1)

G. McCartor

- Phases of $\phi^4(1+1)$

P. Srivastava

- Chiral Symmetry-Breaking, Nambu-Goldstone Phase QCD(3+1)

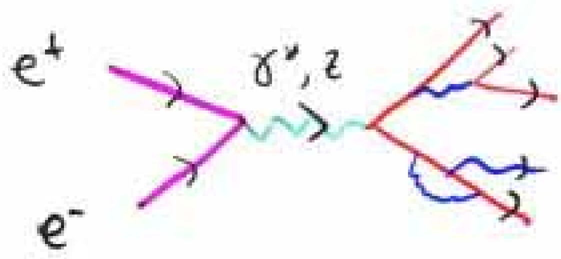
Y. Kim, S. Tsujimaru, K. Yamawaki

Need interpolating π, σ fields $m_\pi^2 \neq 0$

$Q|0\rangle_{LF} = 0$, $\partial^\mu J_\mu \neq 0$ {dynamical fields

- No deriv. of SSB from $\mathcal{L}_{QCD}$ directly

Event Amplitude Generator



High accuracy needed for QCD backgrounds to Higgs, SUSY, ..

Conventional method:

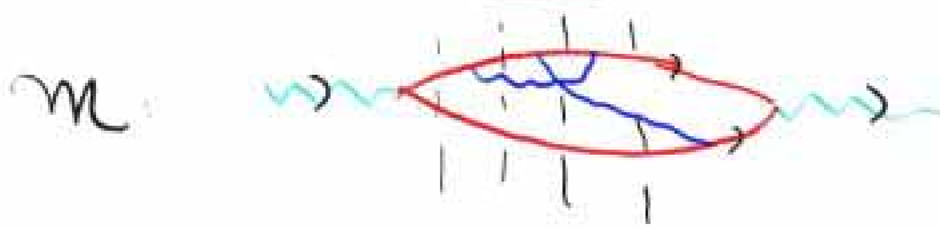
- generate probabilities
- physical phase space — physical polarization

but — virtual contributions — Feynman gauge, dimensional regulariz.
d^4k

Light-cone method:

disc. with J. Hiller, G. McCartor, D. S. Hwang

See also Soper, Sterman

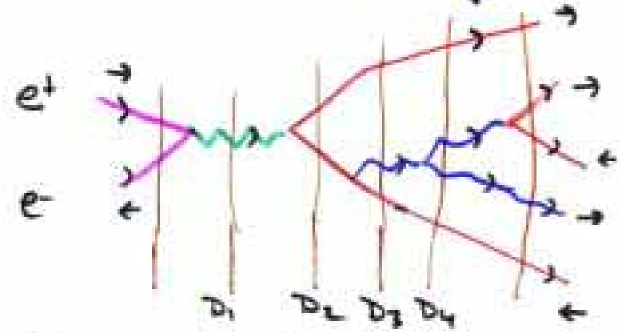


* generate amplitudes, specific l.c. spin
* Physical phase-space, pol: real + virtual
* Mren from alternating denom. method.

"Event Amplitude Generator"

General amplitude from LF TOPTh { Tree + Virtual

$$\mathcal{M} = \sum_{\text{time-orderings}} \mathcal{M}_\alpha \quad \text{(specific spins } S_z\text{)}$$



$$\mathcal{M}_\alpha = H_I \frac{1}{D_1} H_I \frac{1}{D_2} \cdots$$

Σk^+, $\Sigma k_\perp$, ΣJ_z conserved

$k^+ > 0$: Few surviving LF time-orderings

Physical polarization sums: $\sum_{(i)} \epsilon_\mu^{(i)} \epsilon_\nu^{(i)}$

$(i) = 1, 2, 3$

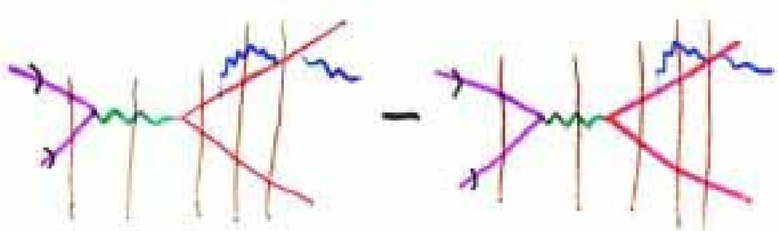
↑ Standard Model W

Compute renormalized amplitude

– "alternating denominator" method

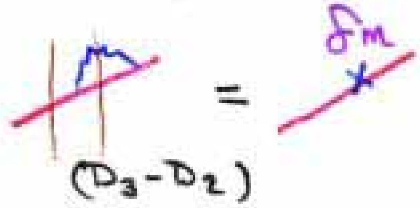
Roskies Suaya SJB

Example:



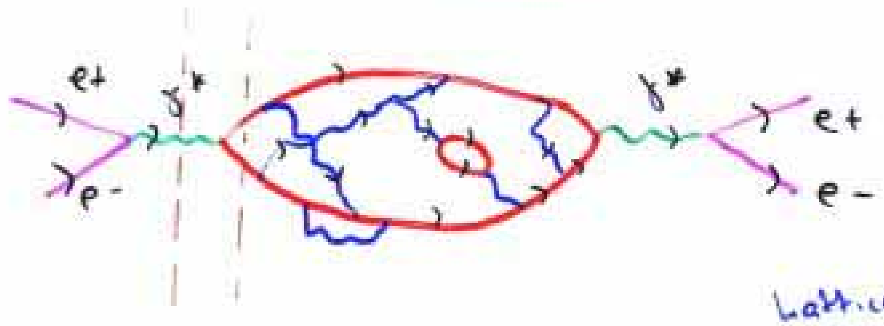
$D_1 \; D_2 \; D_3 \; D_4$ − $D_1 \; D_2 (D_3 - D_2) \; D_4$

equivalent to subtracting mass counterterm!



$\pi \, d^2k_\perp dx$, unitary, no ghosts.

Calculate $e^+e^- \to X$ in LF PTh



Lattice: Blue

Im $\langle e^+e^- | T | e^+e^- \rangle$:

$$T = H_I + H_I \frac{1}{D} H_I + \cdots$$

- $k^+_i > 0$, $D = s - \sum_{i=1}^{n} \left(\frac{m^2 + k_\perp^2}{x}\right)_i$; Few T.O.'s on light-front
- "Alternating" denominators for renormalization
- Input $m(Q_0^2)$, $\alpha_s(Q_0^2)$; no condensates! (as in S-D)
- Non-perturbative : LF Lippman-Schwinger

$$T = H_I + H_I \frac{1}{D} T$$

- Poles : QCD Spectrum $J^{PC} = 1^{--}$
- Practical : DLCQ PBC

? Instanton Effects ; 't Hooft Interactions ?

DLCQ : Discretised Light-Cone Quantization

H.C. Pauli
SJB

Assume PBC for $-L < x^- < L$

$$P^+ = \frac{2\pi}{L} K$$

$$k^\pm = k^0 \pm k^3$$

$$k_i^+ = \frac{2\pi}{L} n_i$$

$$\sum_i n_i = K$$

↑ positive integers

∴ Finite Fock Basis ! $\sum_i \frac{k_\perp^2 + m^2}{x} < \Lambda^2$

DLCQ: Diagonalize Finite Hermitian Matrix

$$\langle n | H_{LF} | m \rangle$$

→ Spectrum , LFWFs

Continuum limit $K \to \infty$

A new program for LF QCD:

Construct correlators: J^2, J_z proj.

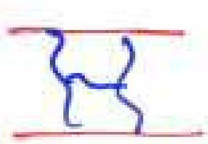
e.g. $q\bar{q} \cdots q\bar{q}$

$$T = H_I^{LF} + H_I^{LF} \frac{1}{M^2 - H_0^{LF}} T$$

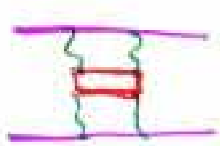
- Lippmann-Schwinger resummation
- DLCQ discrete momentum grid.
- Locate poles: QCD spectrum, $\psi_{LF}^{(n)}(x, k_\perp)$
- Mimic LGTh
- Accurate in QED! Brezin, Itzykson Zinn-Justin.

* Question: where is t'Hooft interaction? topology?

Note: V_{eff}^{QCD}:



not an eff. charge



QED: $(Z_1\alpha)^n (Z_2\alpha)^m$