

Higher Curvature Corrections to TeV-Scale Gravity at Colliders



- JHEP01(2005)028
- hep-ph/0503163

• Higher Curvature Terms...

What are they + why are they important??

• Where do we see them??

→ Signals in Randall-Sundrum...

→ Signals in ADD...

• Summary + Conclusions

T. Rizzo
ANL
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$$S = \int d^{4+n}x \sqrt{-g} \left\{ \frac{M_*^{n+2}}{2} R - \Lambda + ?? \right\}$$

Einstein-Hilbert (EH)

Action

- EH is at best an effective theory below M_*

basis for \rightarrow $\left\{ \begin{array}{l} \text{GR in 4d...} \\ \text{RS/ADD in extra dims} \end{array} \right.$

"??" terms from UV completion (strings?) may be important as we approach M_* ... what can they be?

?? R^2 , $(\partial R)^2$, $\frac{1}{R^2}$, $R_{AB} R^{BC} R_C^A$,??

Many, many possibilities!

• We need some guidance ...

• Unitary / ghost-free theory

\rightarrow no derivatives of metric > 2

[Boulware + Deser]

∴ 'Benign' modifications to Einstein's E_g s

$$R_{AB} - \frac{1}{2} g_{AB} R + \underline{\underline{L_{AB}}} = \frac{1}{M_*^{4+2}} T_{AB}$$

... is symmetric, zero covar. derivative, no derivatives of metric higher than 2nd

• String 'motivation' (Zweibach)

Lowest-order corrections to GR in string

Expansion... [⇒ Gauss-Bonnet]

⇒ Unique solution: Lovelock invariants!

{ Lanczos '32, '38
Lovelock '71

• The effective low energy gravity theory is a sum of Lovelock invariants...

$$\mathcal{L}_m \sim \underbrace{\delta_{C_1 D_1 \dots C_m D_m}^{A_1 B_1 \dots A_m B_m}} R_{A_1 B_1}{}^{C_1 D_1} \dots R_{A_m B_m}{}^{C_m D_m}$$

Totally antisymmetric Kronecker

.. in $D = 4+n$ MOST \mathcal{L}_m are zero!

- Only \mathcal{L}_m with $D \geq 2m+1$ are 'dynamical', i.e., contribute to field equations

$$\begin{array}{l}
 \left. \begin{array}{l} D=9,10 \\ D=7,8 \end{array} \right\} \\
 \left. \begin{array}{l} D=5,6 \\ D=4 \end{array} \right\} \left. \begin{array}{l} \mathcal{L}_0 = \text{a constant } (\Lambda) \\ \mathcal{L}_1 = R \end{array} \right\} \underline{\text{EH}} \\
 \\
 \mathcal{L}_2 = R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \\
 \text{(Gauss-Bonnet)} \\
 \\
 \mathcal{L}_3 = 8 \text{ terms} \dots \\
 \\
 \mathcal{L}_4 = 25 \text{ terms} \dots
 \end{array}$$

- EH is just the First Two Terms of a general expansion...

.. it is THE UNIQUE LoveLack action in 4d !!

... in 5d $R + \frac{\alpha}{M_*^2} \mathcal{L}_2 \rightarrow \Lambda$ is unique, etc (RS)

- $\sum \alpha_m \mathcal{L}_m$: often used in literature as toy models to probe quantum/stringy corrections to EH...

- What happens in ADD / RS ??

The Lovelock invariants rapidly grow in complexity ...

$$EH \left\{ \begin{aligned} L_{(0)} &= 1, \\ L_{(1)} &= -R, \\ L_{(2)} &= R^2 - 4 R_b^a R_a^b + R_{cd}^{ab} R_{ab}^{cd}, \end{aligned} \right. \quad (\text{Gauss-Bonnet})$$

and

$$L_{(3)} = -R^3 + 12 R R_b^a R_a^b - 3 R R_{cd}^{ab} R_{ab}^{cd} - 16 R_b^a R_c^b R_a^c + 24 R_c^a R_d^b R_{ab}^{cd} + 24 R_b^a R_{de}^{bc} R_{ac}^{de} + 2 R_{cd}^{ab} R_{ef}^{cd} R_{ab}^{ef} - 8 R_{ce}^{ab} R_{of}^{cd} R_{bd}^{ef}.$$

$$L_{(4)} = R^4 - 24 R^2 R_b^a R_a^b + 6 R^2 R_{cd}^{ab} R_{ab}^{cd} + 64 R R_b^a R_c^b R_a^c - 96 R R_c^a R_d^b R_{ab}^{cd} - 96 R R_b^a R_{de}^{bc} R_{ac}^{de} - 8 R R_{cd}^{ab} R_{ef}^{cd} R_{ab}^{ef} + 32 R R_{ce}^{ab} R_{of}^{cd} R_{bd}^{ef} + 48 R_b^a R_a^b R_d^c R_c^d - 96 R_b^a R_c^b R_d^c R_a^d + 384 R_b^a R_d^b R_c^c R_{ac}^{de} - 24 R_b^a R_a^b R_{cd}^{ef} R_{cd}^{ef} + 192 R_b^a R_c^b R_{ef}^{cd} R_{ad}^{ef} + 96 R_c^a R_d^b R_{ef}^{cd} R_{ab}^{ef} - 192 R_c^a R_e^b R_{of}^{cd} R_{bd}^{ef} + 192 R_c^a R_e^b R_{bf}^{cd} R_{ad}^{ef} - 192 R_b^a R_{ad}^{bc} R_{fg}^{de} R_{ce}^{fg} + 96 R_b^a R_{de}^{bc} R_{fg}^{de} R_{ac}^{fg} - 384 R_b^a R_{df}^{bc} R_{ag}^{de} R_{ce}^{fg} + 3 R_{cd}^{ab} R_{ab}^{cd} R_{gh}^{ef} R_{ef}^{gh} - 48 R_{cd}^{ab} R_{ae}^{cd} R_{gh}^{ef} R_{bf}^{gh} + 6 R_{cd}^{ab} R_{ef}^{cd} R_{gh}^{ef} R_{ab}^{gh} - 96 R_{cd}^{ab} R_{eg}^{cd} R_{ah}^{ef} R_{bf}^{gh} + 48 R_{ce}^{ab} R_{ag}^{cd} R_{bh}^{ef} R_{df}^{gh} - 96 R_{ce}^{ab} R_{ag}^{cd} R_{dh}^{ef} R_{bf}^{gh}.$$

$$L_{(5)} = -R^5 + 40 R^3 R_b^a R_a^b - 10 R^3 R_{cd}^{ab} R_{ab}^{cd} - 160 R^2 R_b^a R_c^b R_a^c + 240 R^2 R_c^a R_d^b R_{ab}^{cd} + 240 R^2 R_b^a R_{de}^{bc} R_{ac}^{de} + 20 R^2 R_{cd}^{ab} R_{ef}^{cd} R_{ab}^{ef} - 80 R^2 R_{ce}^{ab} R_{of}^{cd} R_{bd}^{ef} - 240 R R_b^a R_a^b R_d^c R_c^d + 480 R R_b^a R_c^b R_d^c R_a^d - 1920 R R_b^a R_d^b R_c^c R_{ac}^{de} + 120 R R_b^a R_a^b R_{cd}^{ef} R_{cd}^{ef} - 960 R R_b^a R_c^b R_{ef}^{cd} R_{ad}^{ef} - 480 R R_c^a R_d^b R_{ef}^{cd} R_{ab}^{ef} + 960 R R_c^a R_e^b R_{of}^{cd} R_{bd}^{ef} - 960 R R_c^a R_e^b R_{bf}^{cd} R_{ad}^{ef} + 960 R R_b^a R_{ad}^{bc} R_{fg}^{de} R_{ce}^{fg} - 480 R R_b^a R_{de}^{bc} R_{fg}^{de} R_{ac}^{fg} + 1920 R R_b^a R_{df}^{bc} R_{ag}^{de} R_{ce}^{fg} - 15 R R_{cd}^{ab} R_{ab}^{cd} R_{gh}^{ef} R_{ef}^{gh} + 240 R R_{cd}^{ab} R_{ae}^{cd} R_{gh}^{ef} R_{bf}^{gh} - 30 R R_{cd}^{ab} R_{ef}^{cd} R_{gh}^{ef} R_{ab}^{gh} + 480 R R_{cd}^{ab} R_{eg}^{cd} R_{ah}^{ef} R_{bf}^{gh} - 240 R R_{ce}^{ab} R_{ag}^{cd} R_{bh}^{ef} R_{df}^{gh} + 480 R R_{ce}^{ab} R_{ag}^{cd} R_{dh}^{ef} R_{bf}^{gh} + 640 R_b^a R_a^b R_d^c R_e^d R_c^e - 768 R_b^a R_c^b R_d^c R_e^d R_a^e - 960 R_b^a R_a^b R_c^c R_d^d R_{ad}^{ef} + 3840 R_b^a R_c^b R_e^c R_d^d R_{ad}^{ef} + 1920 R_b^a R_e^b R_d^c R_f^d R_{ac}^{ef} - 960 R_b^a R_a^b R_d^c R_{de}^{fg} R_{ce}^{fg} - 160 R_b^a R_c^b R_a^c R_{fg}^{de} R_{de}^{fg} + 1920 R_b^a R_c^b R_d^c R_{fg}^{de} R_{ae}^{fg} + 1920 R_b^a R_d^b R_c^c R_{fg}^{de} R_{ac}^{fg} - 3840 R_b^a R_d^b R_f^c R_{ag}^{de} R_{ce}^{fg} + 3840 R_b^a R_d^b R_f^c R_{eg}^{de} R_{ae}^{fg} + 1920 R_d^a R_f^b R_c^c R_{bg}^{de} R_{ac}^{fg} + 3840 R_d^a R_f^b R_g^c R_{ab}^{de} R_{ce}^{fg} - 80 R_b^a R_a^b R_{ef}^{cd} R_{gh}^{ef} R_{cd}^{gh} + 320 R_b^a R_a^b R_{eg}^{cd} R_{ch}^{ef} R_{df}^{gh} - 1920 R_b^a R_c^b R_{ae}^{cd} R_{gh}^{ef} R_{df}^{gh} + 960 R_b^a R_c^b R_{ef}^{cd} R_{gh}^{ef} R_{ad}^{gh} - 3840 R_b^a R_c^b R_{eg}^{cd} R_{ah}^{ef} R_{df}^{gh} + 240 R_c^a R_d^b R_{ab}^{cd} R_{gh}^{ef} R_{ef}^{gh} - 1920 R_c^a R_d^b R_{ae}^{cd} R_{gh}^{ef} R_{bf}^{gh} + 480 R_c^a R_d^b R_{ef}^{cd} R_{gh}^{ef} R_{ab}^{gh} - 1920 R_c^a R_d^b R_{eg}^{cd} R_{ah}^{ef} R_{bf}^{gh} - 1920 R_c^a R_e^b R_{ab}^{cd} R_{gh}^{ef} R_{df}^{gh} - 1920 R_c^a R_e^b R_{of}^{cd} R_{gh}^{ef} R_{bd}^{gh} - 3840 R_c^a R_g^b R_{ae}^{cd} R_{bh}^{ef} R_{df}^{gh} + 1920 R_c^a R_g^b R_{ae}^{cd} R_{dh}^{ef} R_{bf}^{gh} + 3840 R_c^a R_g^b R_{bc}^{de} R_{ah}^{ef} R_{df}^{gh} - 1920 R_c^a R_g^b R_{be}^{cd} R_{dh}^{ef} R_{af}^{gh} + 1920 R_c^a R_g^b R_{ef}^{cd} R_{bh}^{ef} R_{ad}^{gh} + 1920 R_c^a R_g^b R_{ch}^{cd} R_{ab}^{ef} R_{df}^{gh} + 1920 R_b^a R_{ad}^{bc} R_{ef}^{de} R_{fg}^{hi} R_{eg}^{hi} - 960 R_b^a R_{ad}^{bc} R_{fg}^{de} R_{hi}^{fg} R_{ce}^{hi} + 3840 R_b^a R_{ad}^{bc} R_{fh}^{de} R_{ci}^{fg} R_{eg}^{hi} + 240 R_b^a R_{de}^{bc} R_{ac}^{de} R_{hi}^{fg} R_{fg}^{hi} - 960 R_b^a R_{de}^{bc} R_{af}^{de} R_{hi}^{fg} R_{cg}^{hi} + 960 R_b^a R_{de}^{bc} R_{cf}^{de} R_{hi}^{fg} R_{ag}^{hi} + 480 R_b^a R_{de}^{bc} R_{fg}^{de} R_{hi}^{fg} R_{ac}^{hi} - 1920 R_b^a R_{de}^{bc} R_{fh}^{de} R_{ai}^{fg} R_{eg}^{hi} - 1920 R_b^a R_{df}^{bc} R_{ac}^{de} R_{hi}^{fg} R_{eg}^{hi} - 1920 R_b^a R_{df}^{bc} R_{ag}^{de} R_{hi}^{fg} R_{ce}^{hi} + 3840 R_b^a R_{df}^{bc} R_{ah}^{de} R_{ci}^{fg} R_{eg}^{hi} - 3840 R_b^a R_{df}^{bc} R_{ah}^{de} R_{fg}^{hi} + 1920 R_b^a R_{df}^{bc} R_{cg}^{de} R_{hi}^{fg} R_{ae}^{hi} + 3840 R_b^a R_{df}^{bc} R_{ch}^{de} R_{ei}^{fg} R_{ag}^{hi} - 1920 R_b^a R_{df}^{bc} R_{gh}^{de} R_{ei}^{fg} R_{ac}^{hi} + 20 R_{cd}^{ab} R_{ab}^{cd} R_{gh}^{ef} R_{ij}^{gh} R_{ef}^{ij} - 80 R_{cd}^{ab} R_{ab}^{cd} R_{gi}^{ef} R_{ej}^{gh} R_{fh}^{ij} + 480 R_{cd}^{ab} R_{ae}^{cd} R_{bg}^{ef} R_{ij}^{gh} R_{fh}^{ij} - 480 R_{cd}^{ab} R_{ae}^{cd} R_{gh}^{ef} R_{ij}^{gh} R_{bf}^{ij} + 1920 R_{cd}^{ab} R_{ae}^{cd} R_{gi}^{ef} R_{bj}^{gh} R_{fh}^{ij} + 24 R_{cd}^{ab} R_{ef}^{cd} R_{gh}^{ef} R_{ij}^{gh} R_{ab}^{ij} - 480 R_{cd}^{ab} R_{ef}^{cd} R_{gi}^{ef} R_{oj}^{gh} R_{bh}^{ij} - 480 R_{cd}^{ab} R_{eg}^{cd} R_{ah}^{ef} R_{ij}^{gh} R_{bf}^{ij} + 960 R_{cd}^{ab} R_{eg}^{cd} R_{ai}^{ef} R_{bj}^{gh} R_{fh}^{ij} - 1920 R_{cd}^{ab} R_{eg}^{cd} R_{ai}^{ef} R_{ff}^{gh} R_{bh}^{ij} + 1920 R_{ce}^{ab} R_{of}^{cd} R_{gi}^{ef} R_{bj}^{gh} R_{dh}^{ij} - 384 R_{ce}^{ab} R_{ag}^{cd} R_{bi}^{ef} R_{dj}^{gh} R_{fh}^{ij} + 1920 R_{ce}^{ab} R_{ag}^{cd} R_{bi}^{ef} R_{ff}^{gh} R_{dh}^{ij} - 1920 R_{ce}^{ab} R_{ag}^{cd} R_{di}^{ef} R_{ff}^{gh} R_{bh}^{ij} - 768 R_{ce}^{ab} R_{fg}^{cd} R_{hi}^{ef} R_{aj}^{gh} R_{bd}^{ij}.$$

$$S = \int d^{4+n}x \sqrt{-g} \left\{ \frac{M_*^{n+2}}{2} \left[R + \frac{\alpha}{M_*^2} \mathcal{L}_2 + \frac{\beta}{M_*^4} \mathcal{L}_3 + \frac{\gamma}{M_*^6} \mathcal{L}_4 \right] \right\}$$

all Levelock Invariants

• $\alpha, \beta, \gamma \equiv$ dimensionless constants (not generally $O(1)$)

When are $\mathcal{L}_{m \geq 2}$ important ???

\Rightarrow when $\sim \frac{R^{\text{background}}}{M_*^2}$ is big.....

• in RS, $R^{\text{background}} = \underline{-20 k^2}$

Recall: $ds^2 = \frac{-2k|y|}{e} (\eta_{\mu\nu} dx^\mu dx^\nu) - dy^2$ (Warp factor \rightarrow Curvature)

\therefore corrections in RS $\sim \underline{\alpha k^2 / M_*^2}$!

• recall, we usually demand $20k^2 / M_*^2 < 1$
in RS to AVOID large curvature !! [Drop?]

• in ADD? space + 'brane' are flat [T^n]

$\rightarrow R^{\text{background}} = 0$ (usually)

So...

⇒

- ADD relation unaltered: $\tilde{M}_{Pl}^2 = V_n M_*^{n+2}$
- Matter-graviton coupling unaltered:

$$S_{SM} = \int d^{4+n}x \sqrt{-g} \cdot \frac{1}{M_*^{1+\frac{n}{2}}} h_{\mu\nu}^{(n)} T^{\mu\nu}$$

- graviton mass spectrum unaltered

∴ ~~≠~~ + contact int' signatures as usual
insensitive to \mathcal{L}_m ! ... but

⇒ Black Holes do have big $\frac{R_{\text{black}}}{M_*^2}$!
(Return to this later....)

RS Model: can add on \mathcal{L}_2 in $D=5$

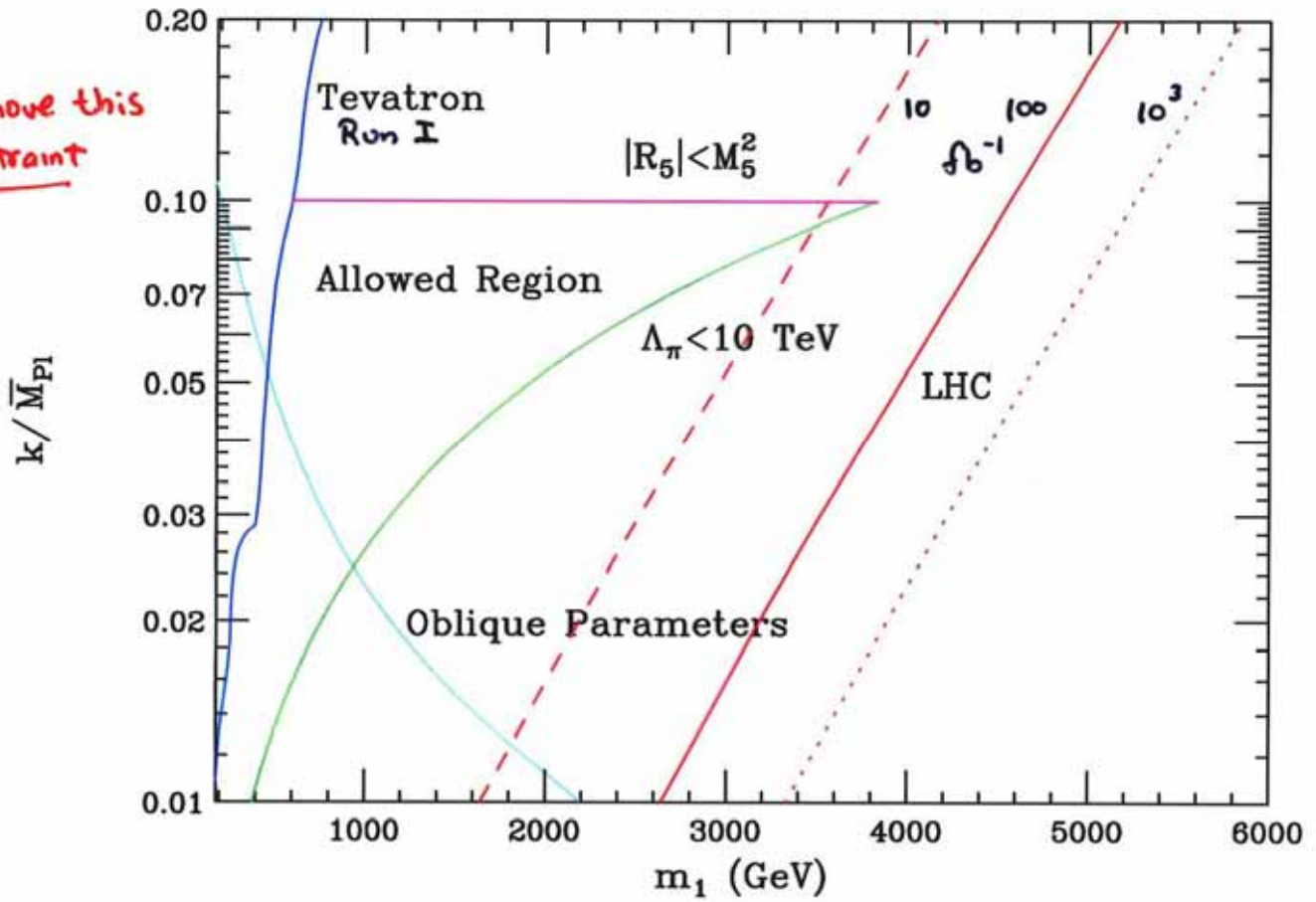
→ general sol'n structure remains
but details change [Kim, Kyae + Lee]

Still $\psi_n(y) = \frac{e^{2\sigma}}{N_n} J_2\left(\frac{m_n}{k} e^\sigma\right) \quad (\sigma = k|y|)$

↑
 $J_2 + Y_2$ Bessel functions

The LHC has good RS Coverage

Remove this constraint



first graviton KK mass

Some differences ... similar to graviton brane kinetic terms

$$\bar{M}_{\text{pl}}^2 = \frac{M_*^3}{k} \left(1 + 4\alpha \frac{k^2}{M_*^2} \right)$$

$$\int_1(x_n) + \frac{4\alpha k^2 / M_*^2}{1 - 4\alpha k^2 / M_*^2} \int_2(x_n) = 0$$

$$\underbrace{\hspace{10em}}_{\Omega}$$

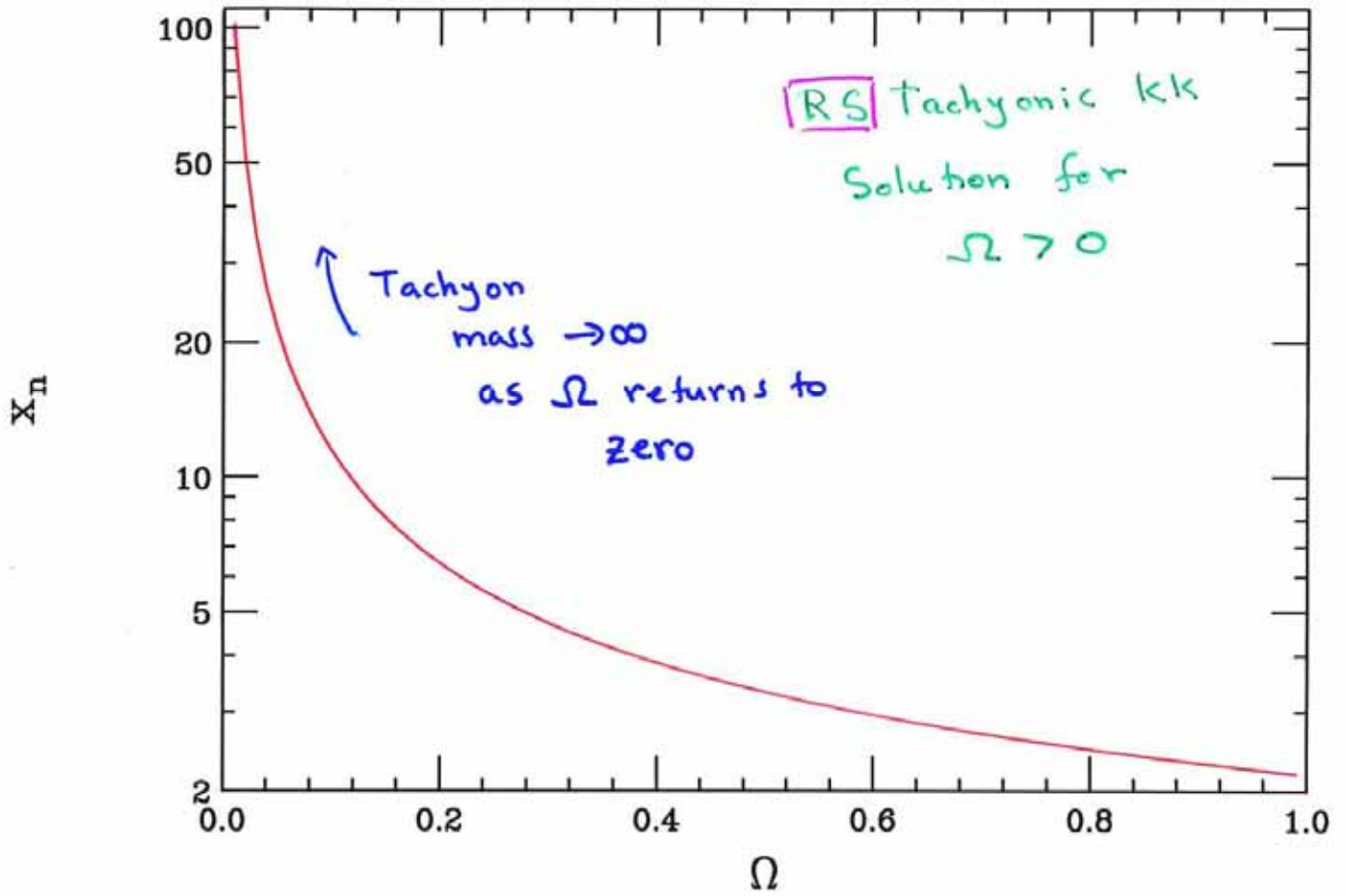
KK mass shifts

$$\mathcal{L} = \frac{1}{\Lambda_\pi} \sum \left[\frac{1 + 2\Omega}{1 + 2\Omega + \Omega^2 x_n^2} \right]^{1/2} h_{\mu\nu}^{(n)} T^{\mu\nu}$$

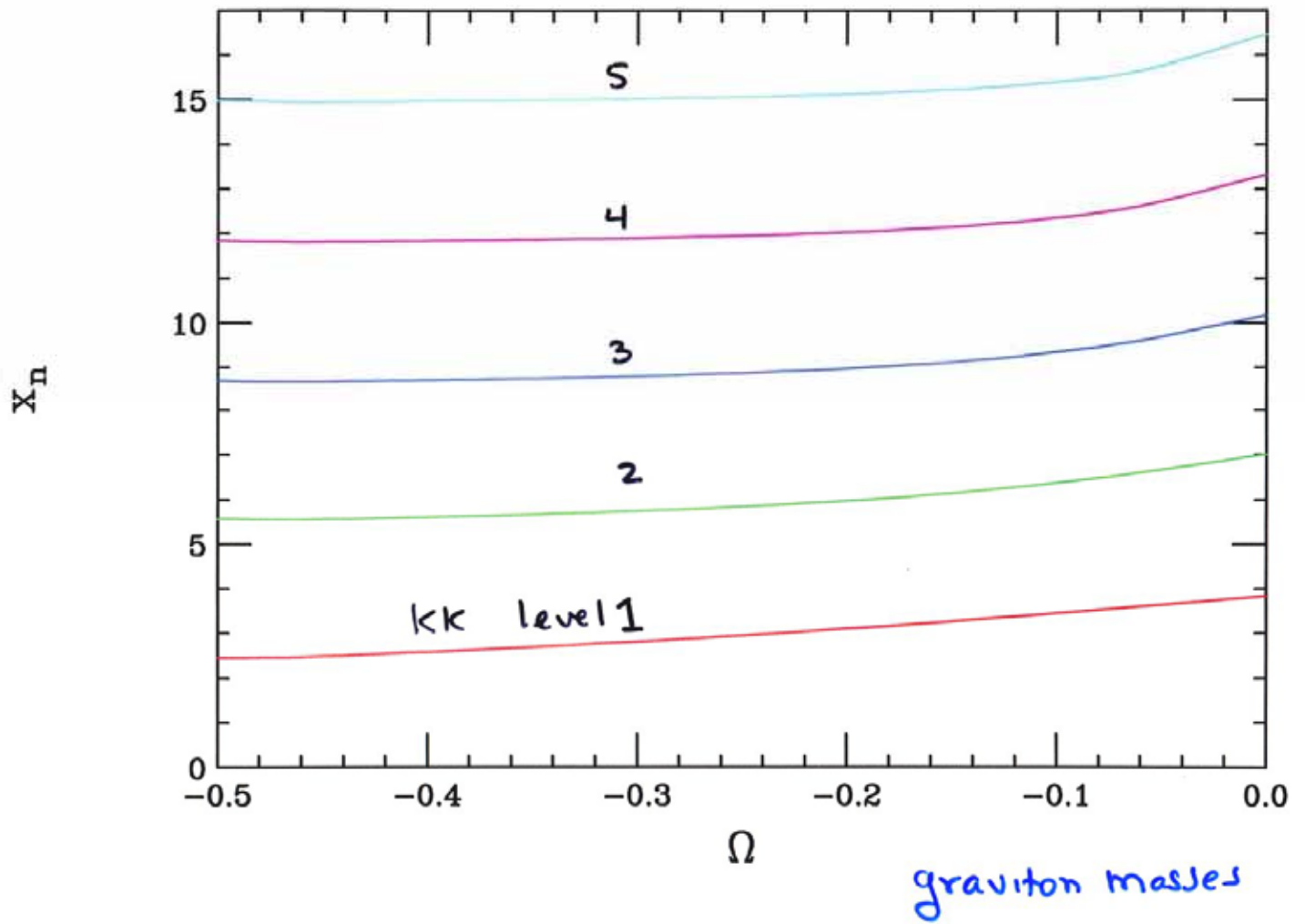
- KK coupling shifts

- $\alpha < 0$ [no tachyons] $\left\{ \begin{array}{l} \text{Charmousis + Dufaux} \\ \text{Brax, Chatillon + Steer} \end{array} \right.$
- $-\frac{1}{2} \leq \Omega \leq 0$, $\Omega = 0 \equiv$ USUAL RS model
- couplings are KK level dependent and vanishing for $\Omega = -\frac{1}{2}$!
- level shifts (weak)
- Can we measure / constrain Ω ??
 $m_2/m_1 \sim \Gamma_2/\Gamma_1$ ratios Ω to ± 0.01 ?

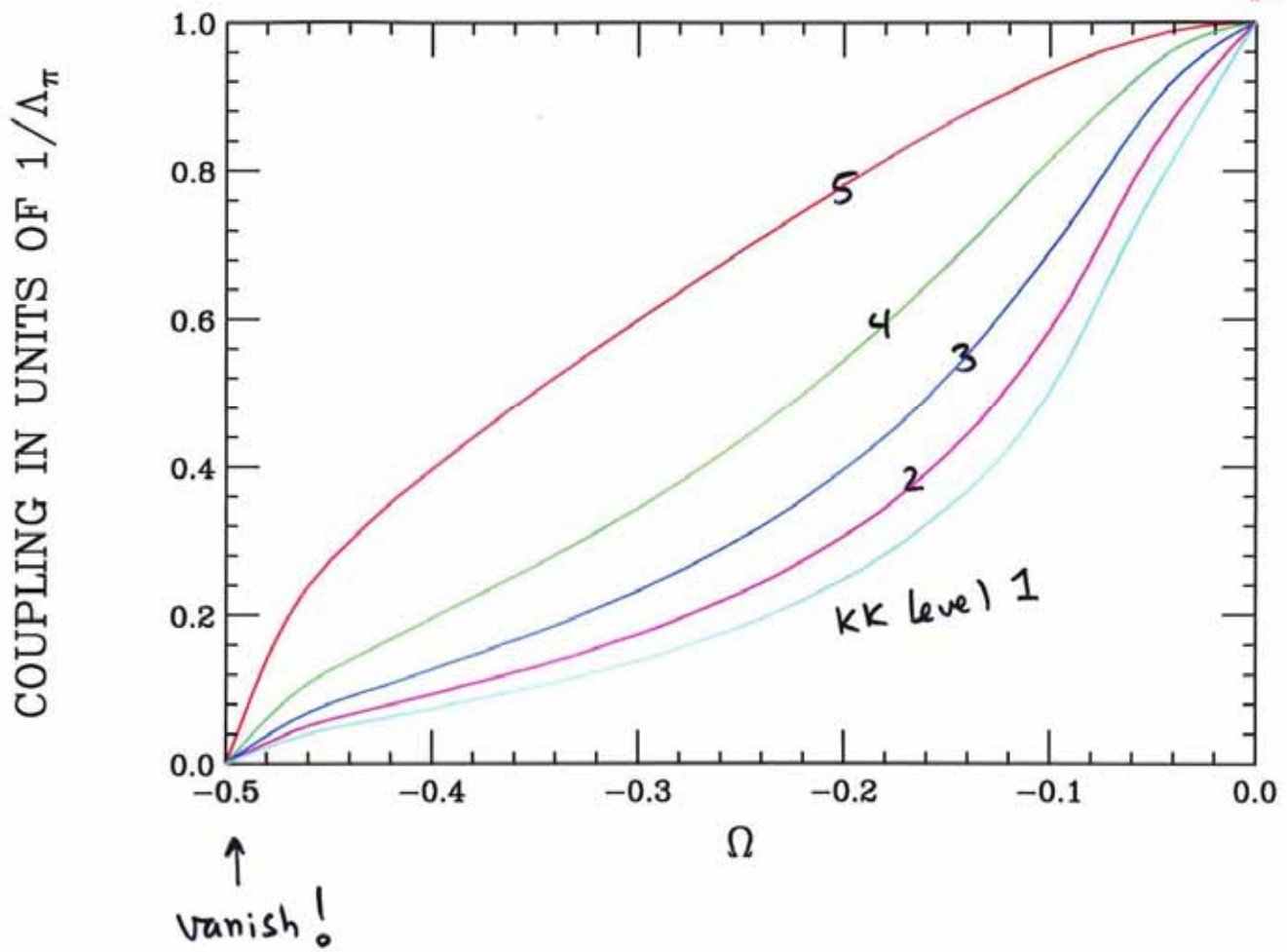
$$m_{\text{Tachyon}}^2 = -X_H^2 k^2 e^{-2\pi k r_c} < 0$$



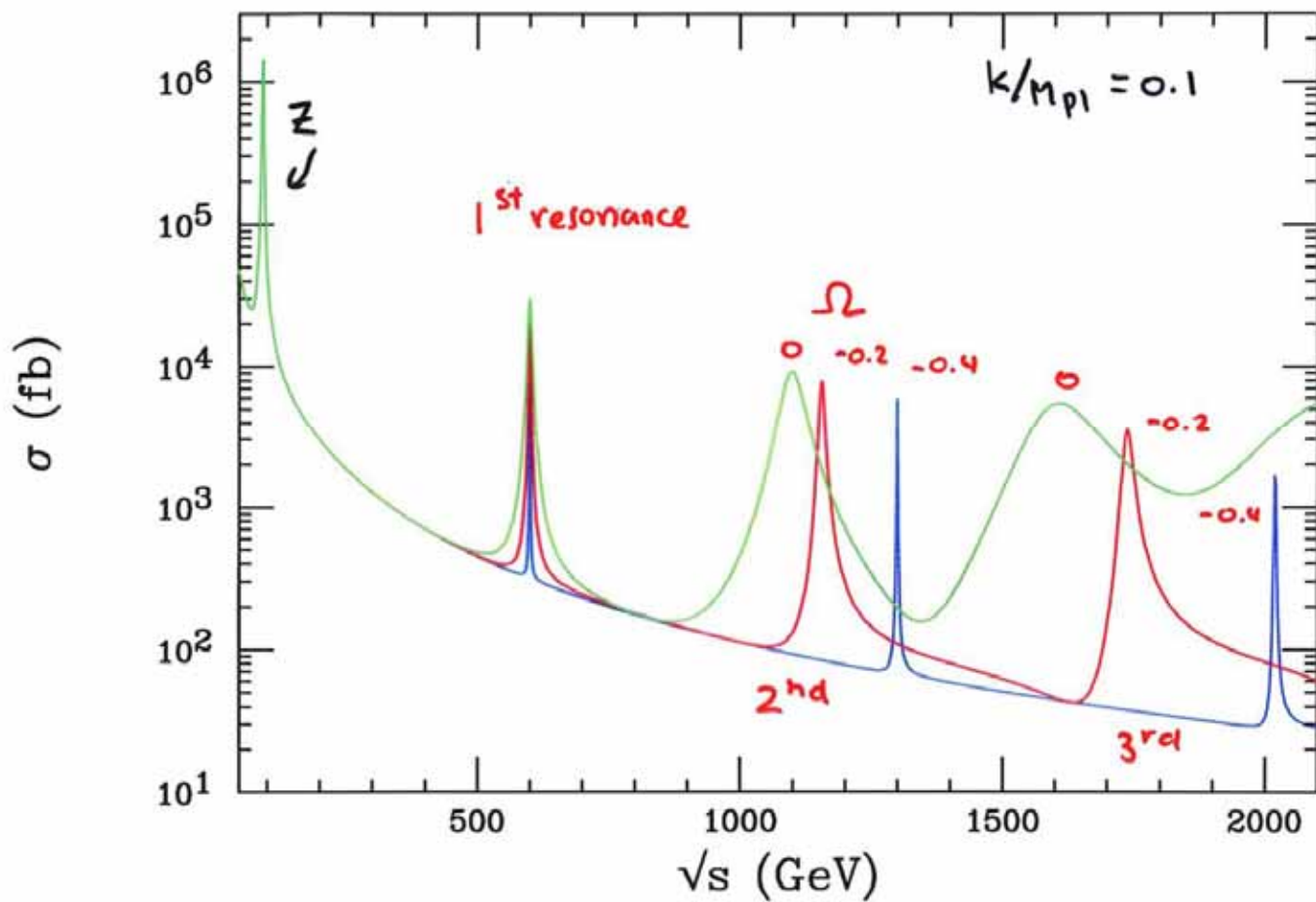
Root shifts due to \mathcal{L}_2 in RS



Graviton Coupling Strengths

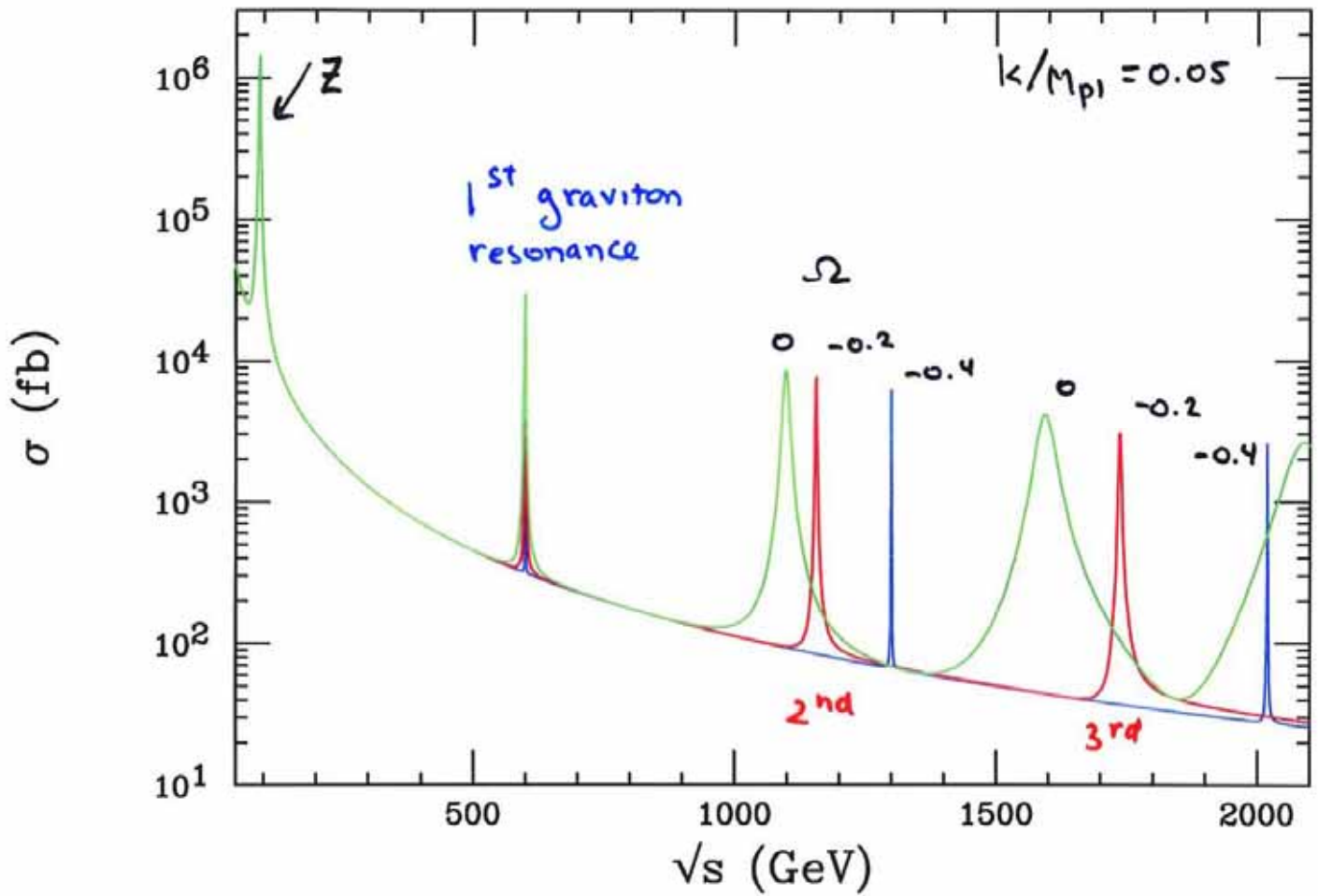


$$e^+e^- \rightarrow \mu^+\mu^-$$

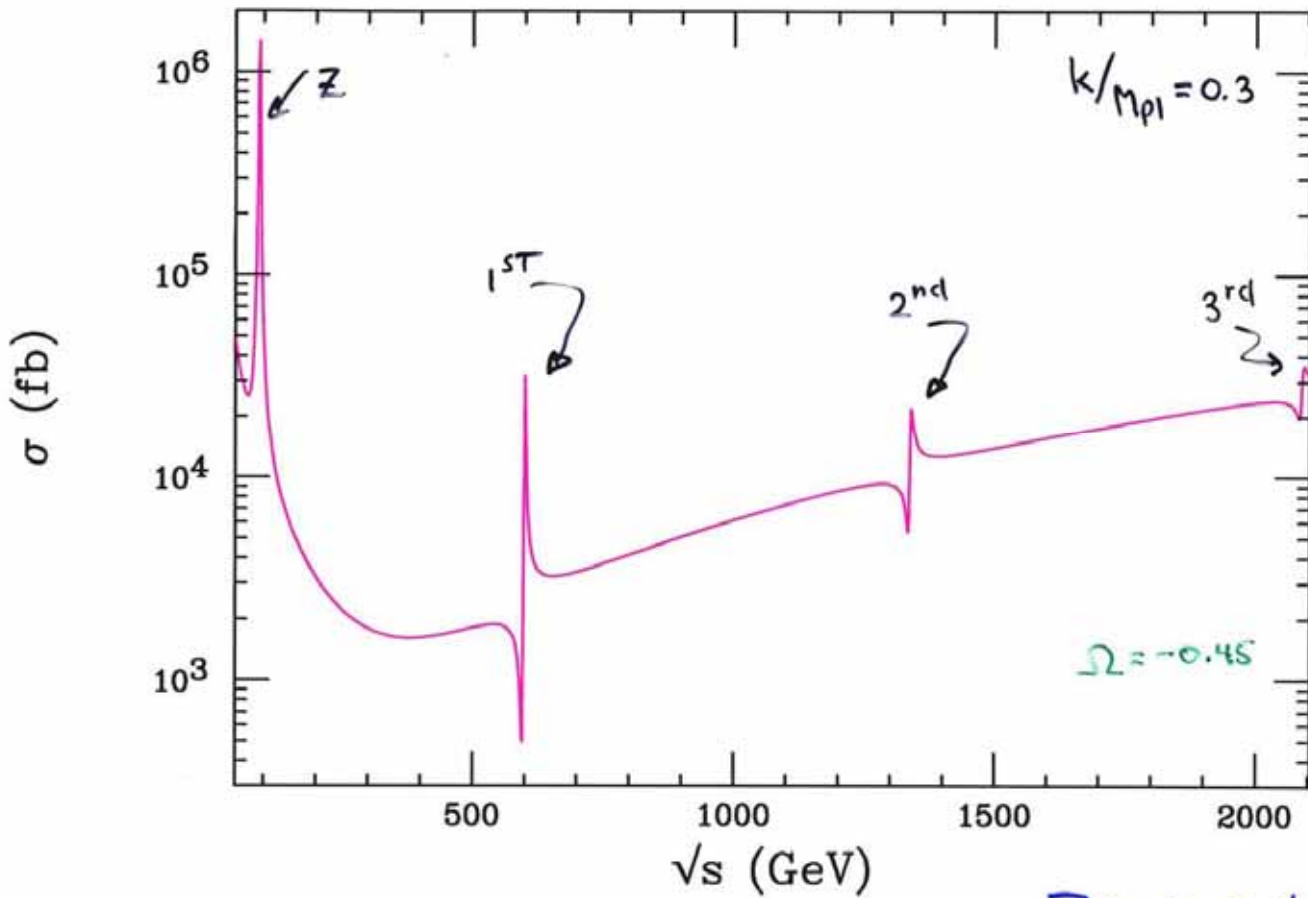


mass shifted + narrowed ..

$$e^+e^- \rightarrow \mu^+\mu^-$$

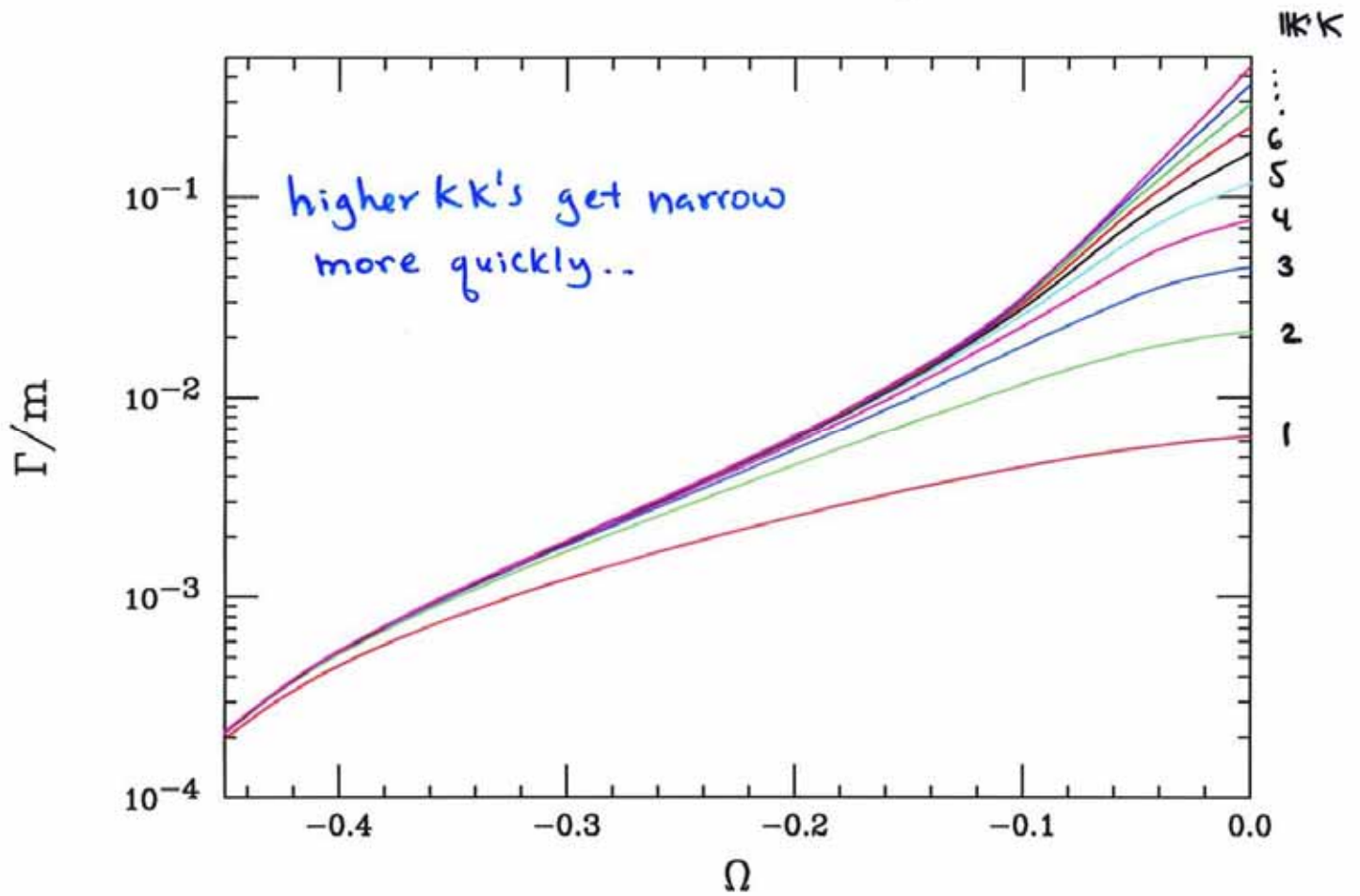


k, k spectrum shifted + resonances narrowed



... This is a traditionally excluded region of parameter space

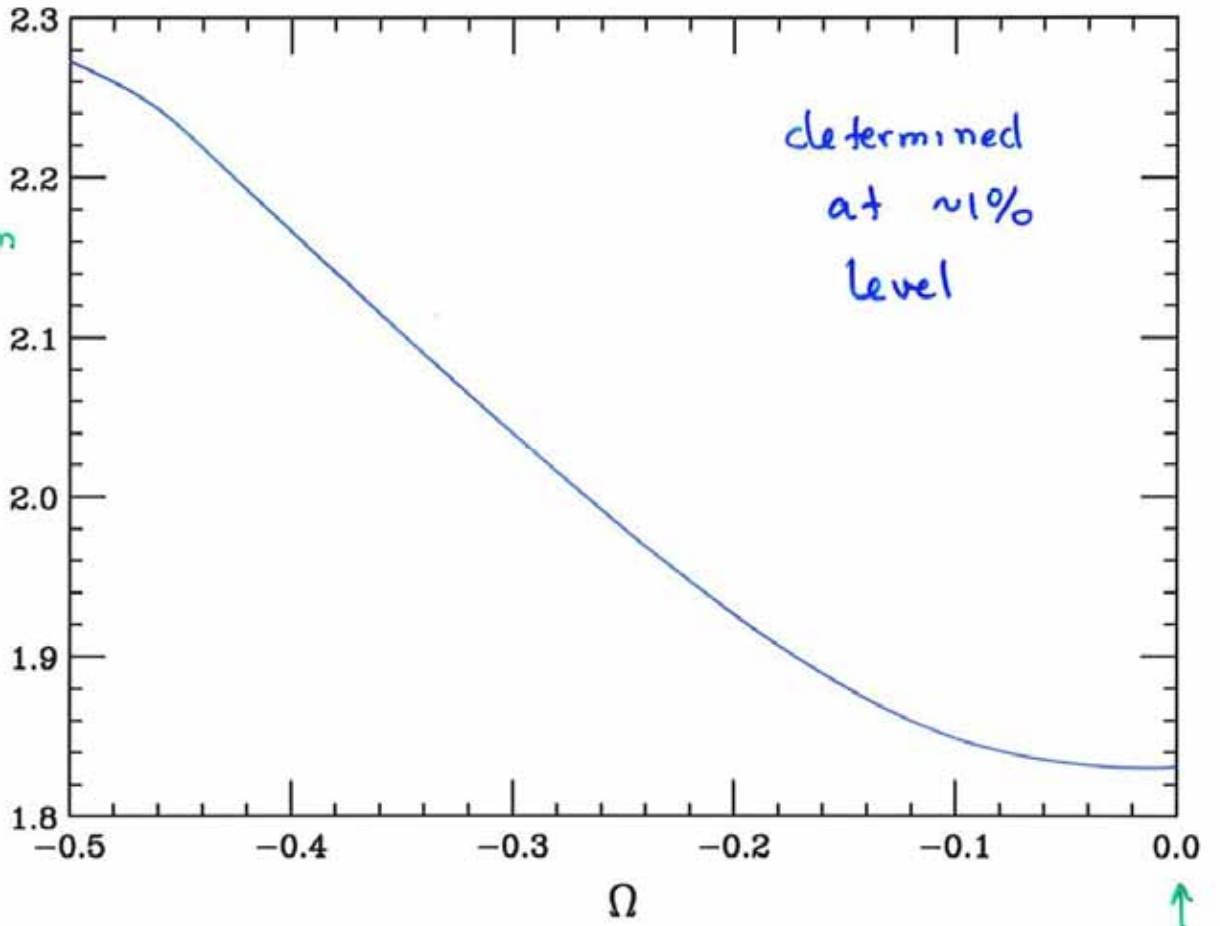
KK's are getting narrow quite quickly



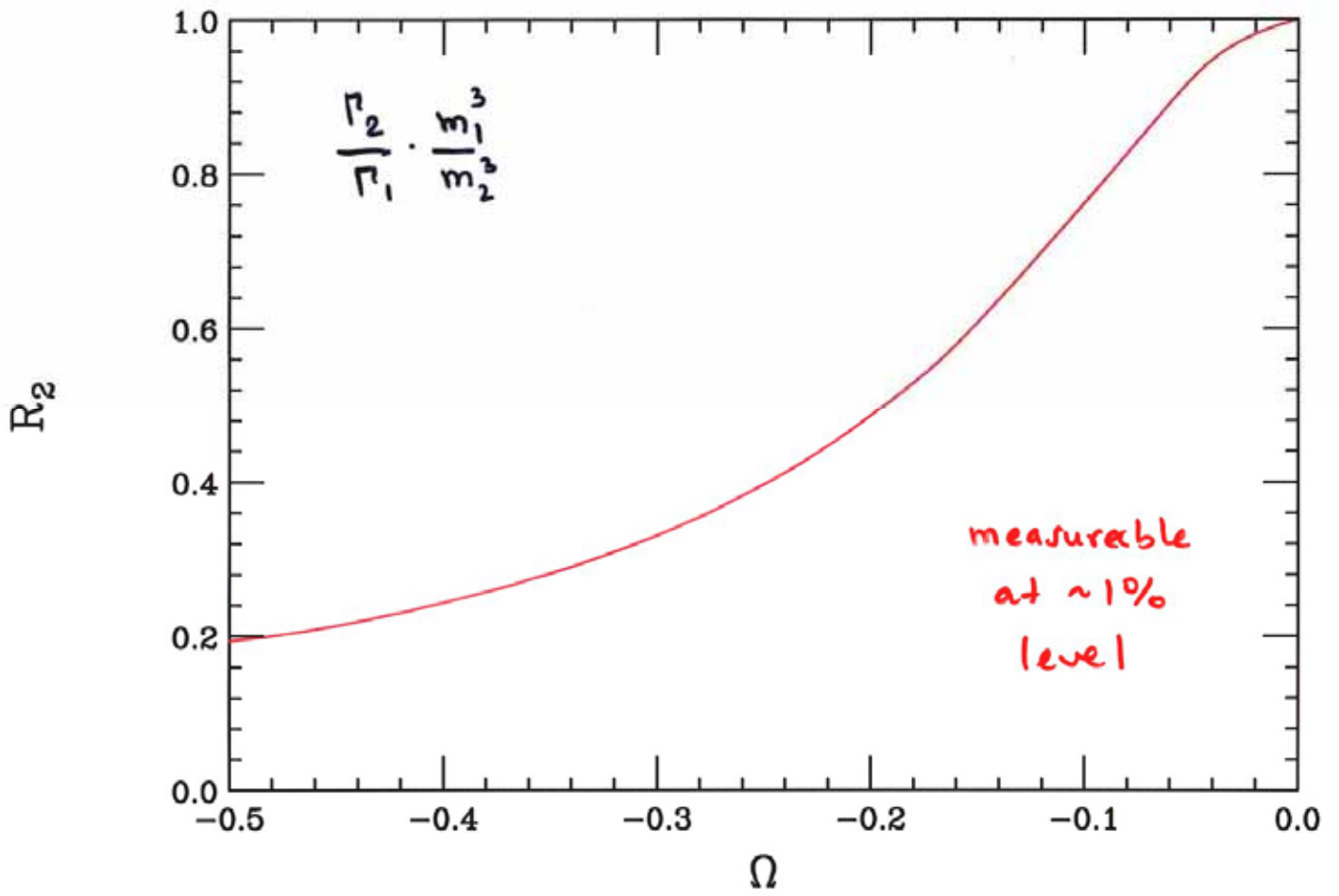
KK mass ratios

$\frac{m_1}{m_2}$ graviton

R_1



Scaled width ratios



ADD

- modification of BH properties

$$\rightarrow \text{BH} \leftarrow \hat{\sigma} \approx \pi R_s^2 \delta(\hat{S} - M_*^2)$$

↳ decays by 'Hawking radiation'
very rapidly into sm particles

$$\langle S \rangle \sim 10$$

- $R_s(m_{\text{BH}}/M_*, n) [\alpha, \beta, \gamma]$

- $T(m_{\text{BH}}/M_*, n) [\alpha, \beta, \gamma]$

∴ etc

Quantitative + Qualitative Changes

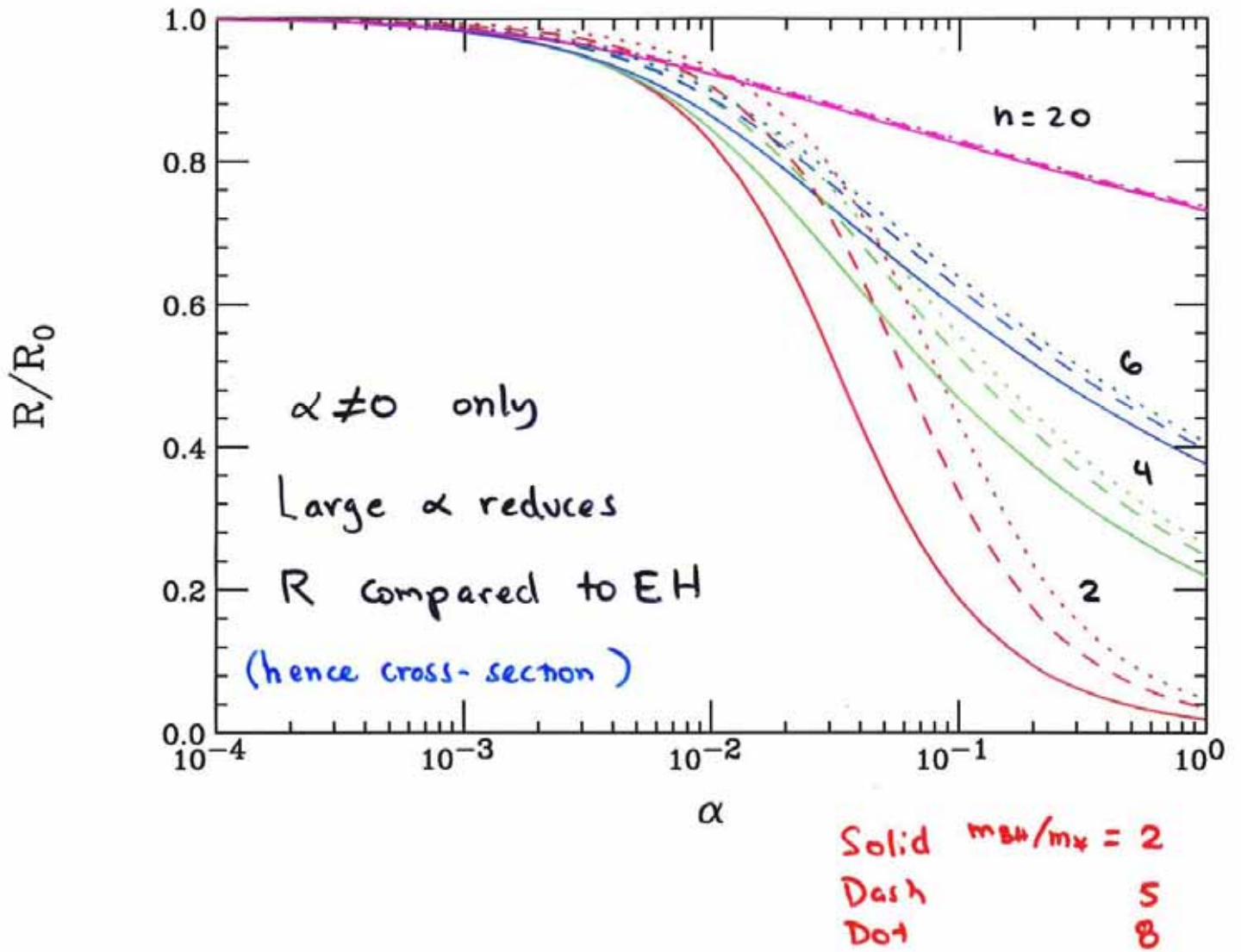
- R, T vary w/ α, β, γ for fixed $n, \frac{m_{\text{BH}}}{M_*}$

- for $n=3, \beta \neq 0$
 $= 5, \gamma \neq 0$ } No BH can form below
a critical minimum mass

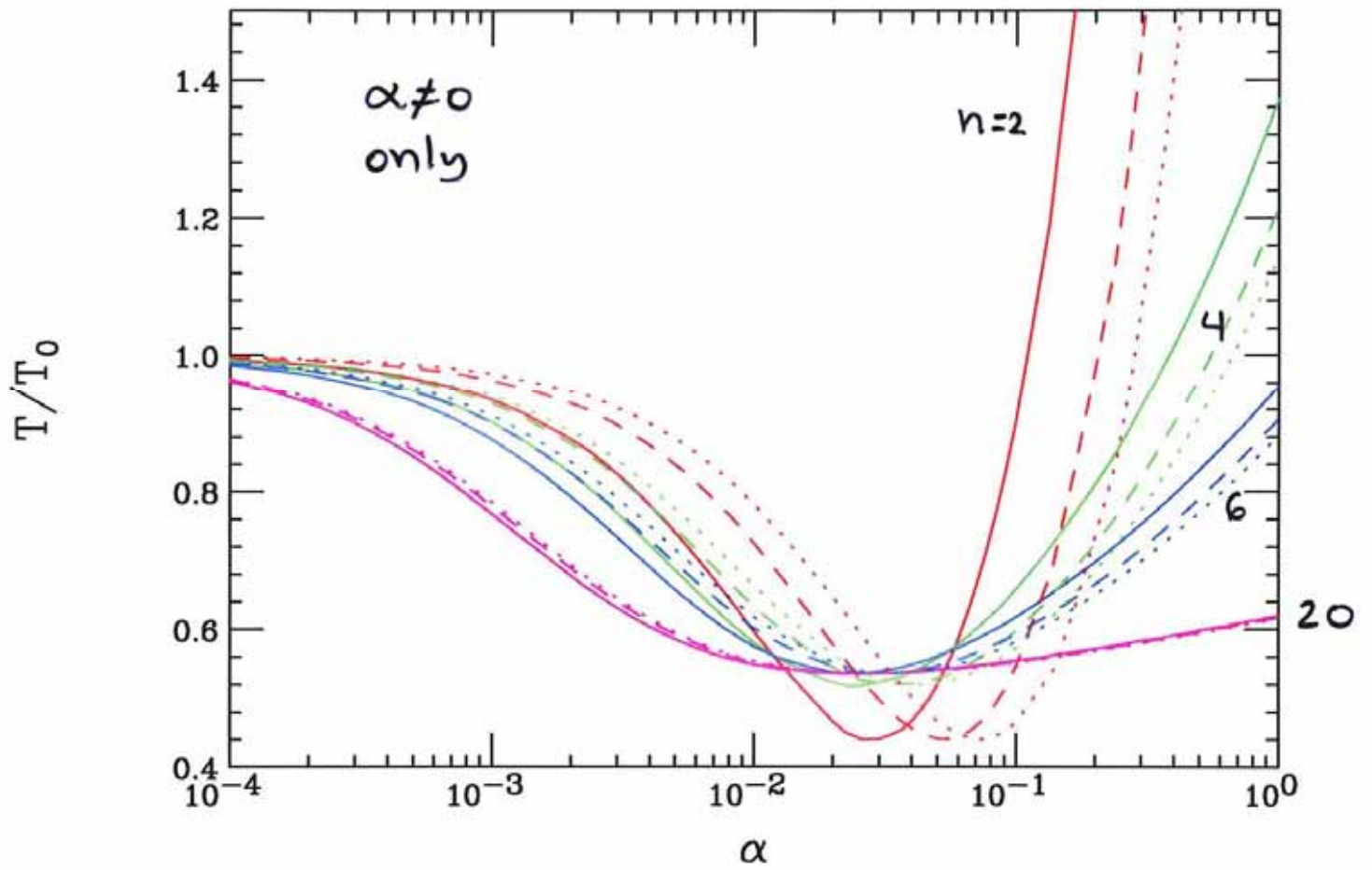
Furthermore, for $M_{\text{BH}} \approx M_{\text{crit}}$ BH are STABLE

↳ these cases

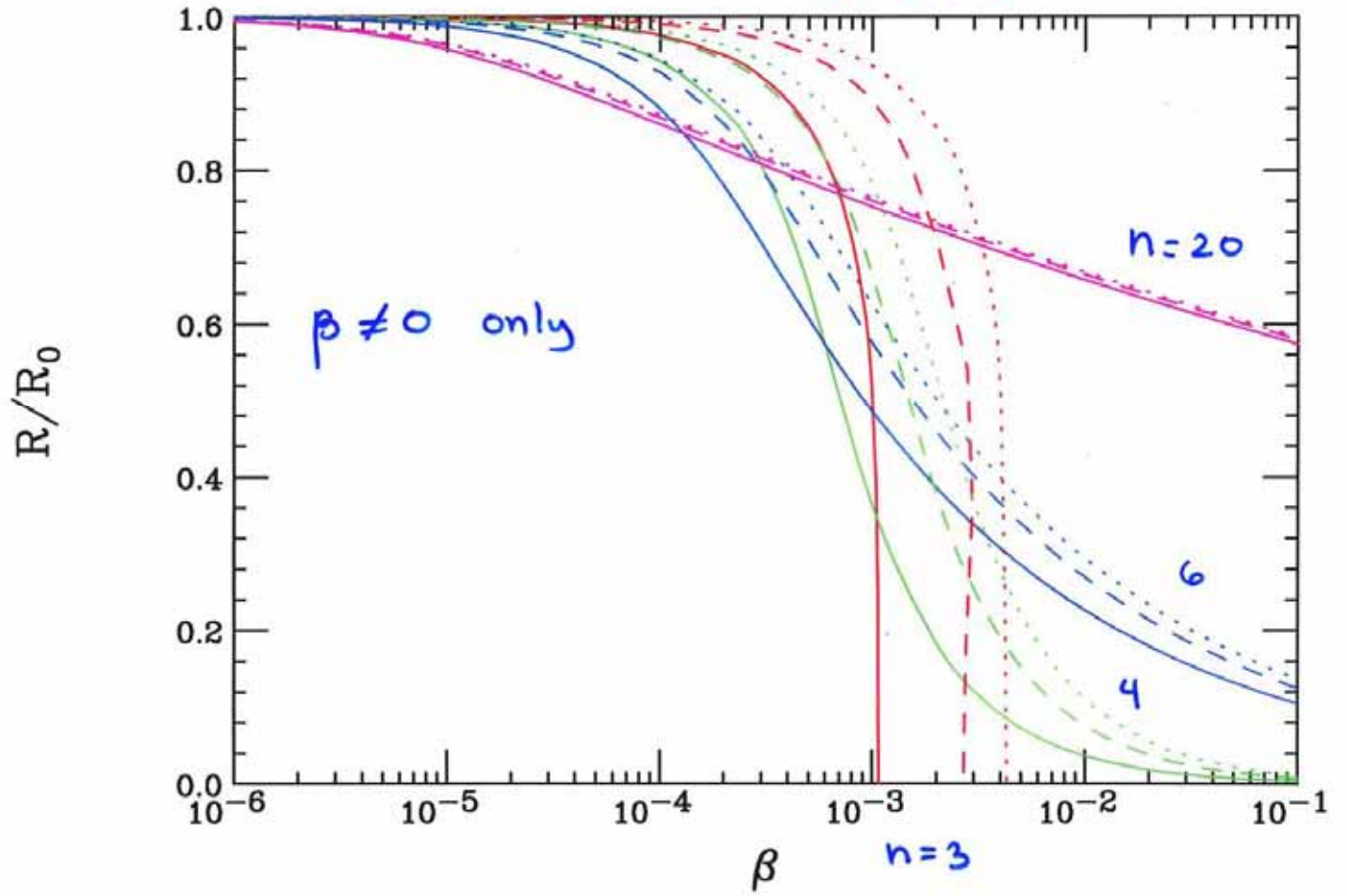
Shift in Schwarzschild radius

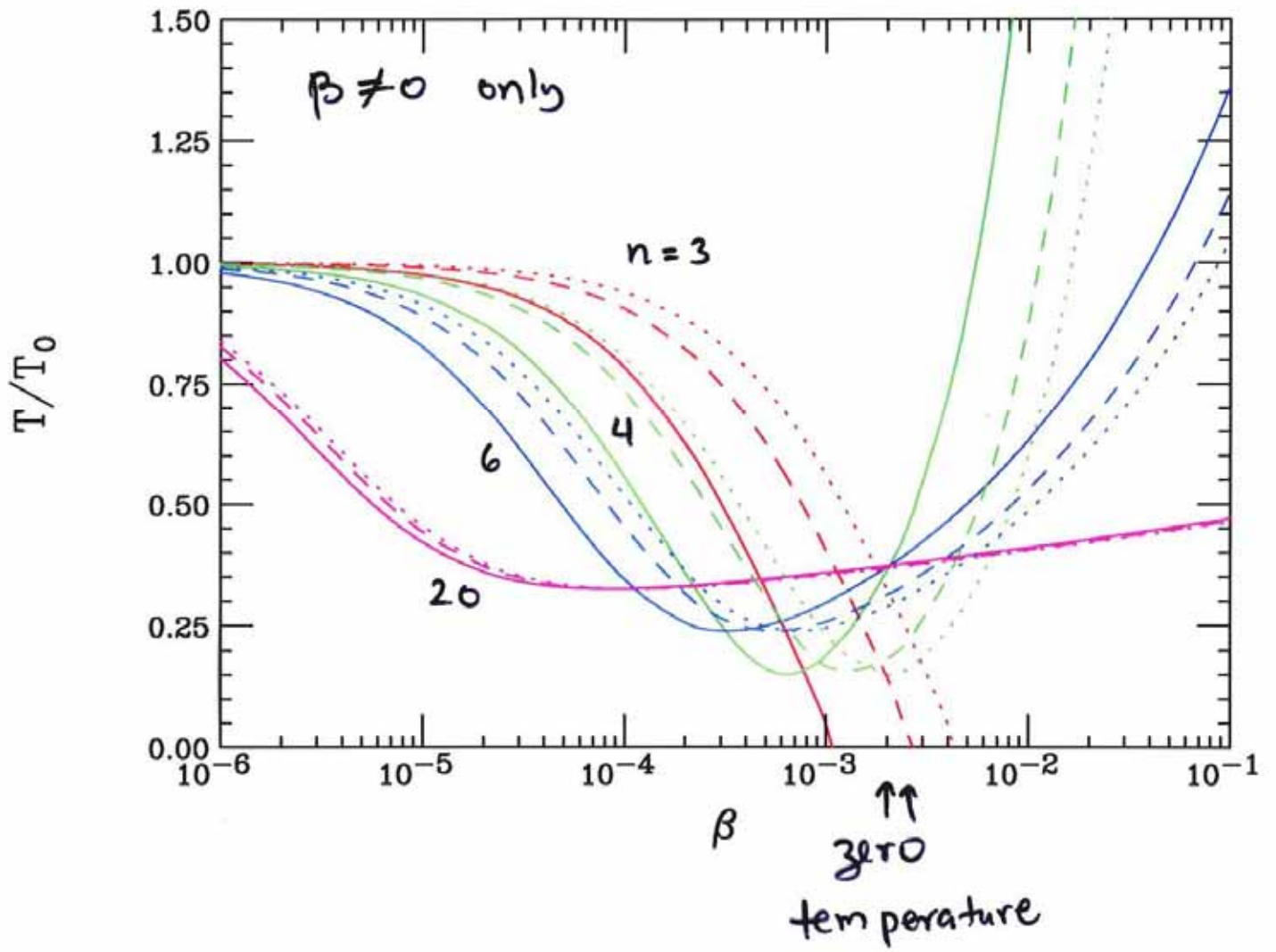


O(1) Temperature changes



Zero Radius





For $n=3$, $\beta \neq 0$

$$R_S M_* = \left[\frac{M_{BH}/M_*}{5\pi^2/2} - 24\beta \right]^{1/4}$$

\therefore unless $M_{BH} > 60\pi^3 \beta M_*$ No BH forms...

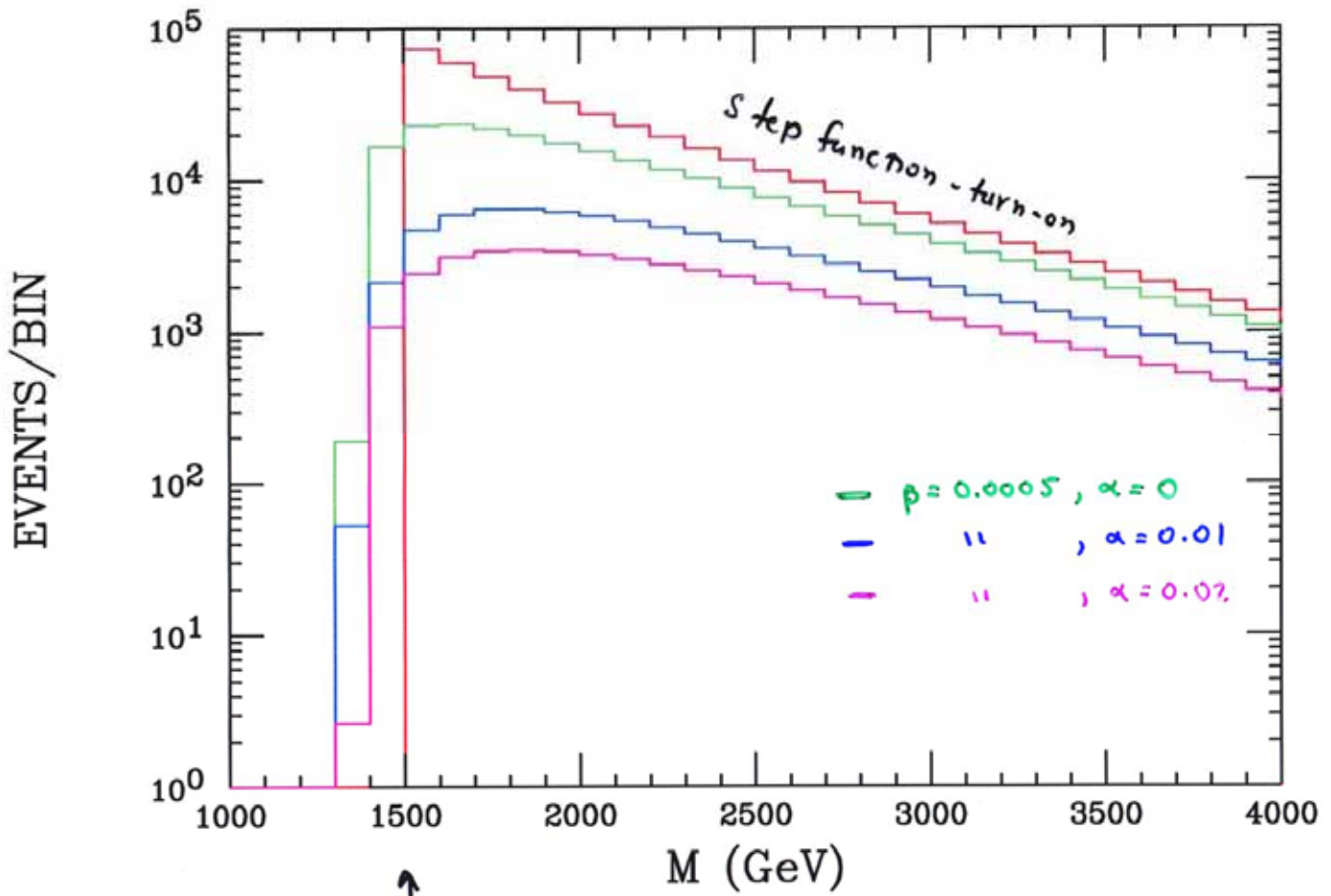
Lifetime : $\frac{dM_{BH}}{dt} \sim (\text{Area})(\text{Temp})^4$ (BB-radiation)

$$\sim \frac{(M_{BH} - 60\pi^3 \beta M_*)^{3/2}}{(M_{BH} + 120\pi^3 \beta M_*)^4}$$

- For any $M_{BH} > m_{crit} = 60\pi^3 \beta M_*$, the lifetime is ∞ long.
- These BH cool as they lose mass, so loss is reduced
- Make a BH with $M_{BH} > m_{crit}$, it will radiate until $M_{BH} \sim m_{crit}$ is reached
- σ_{BH} now have THRESHOLDS \therefore the turn-on at M_* is no longer a STEP FUNCTION!

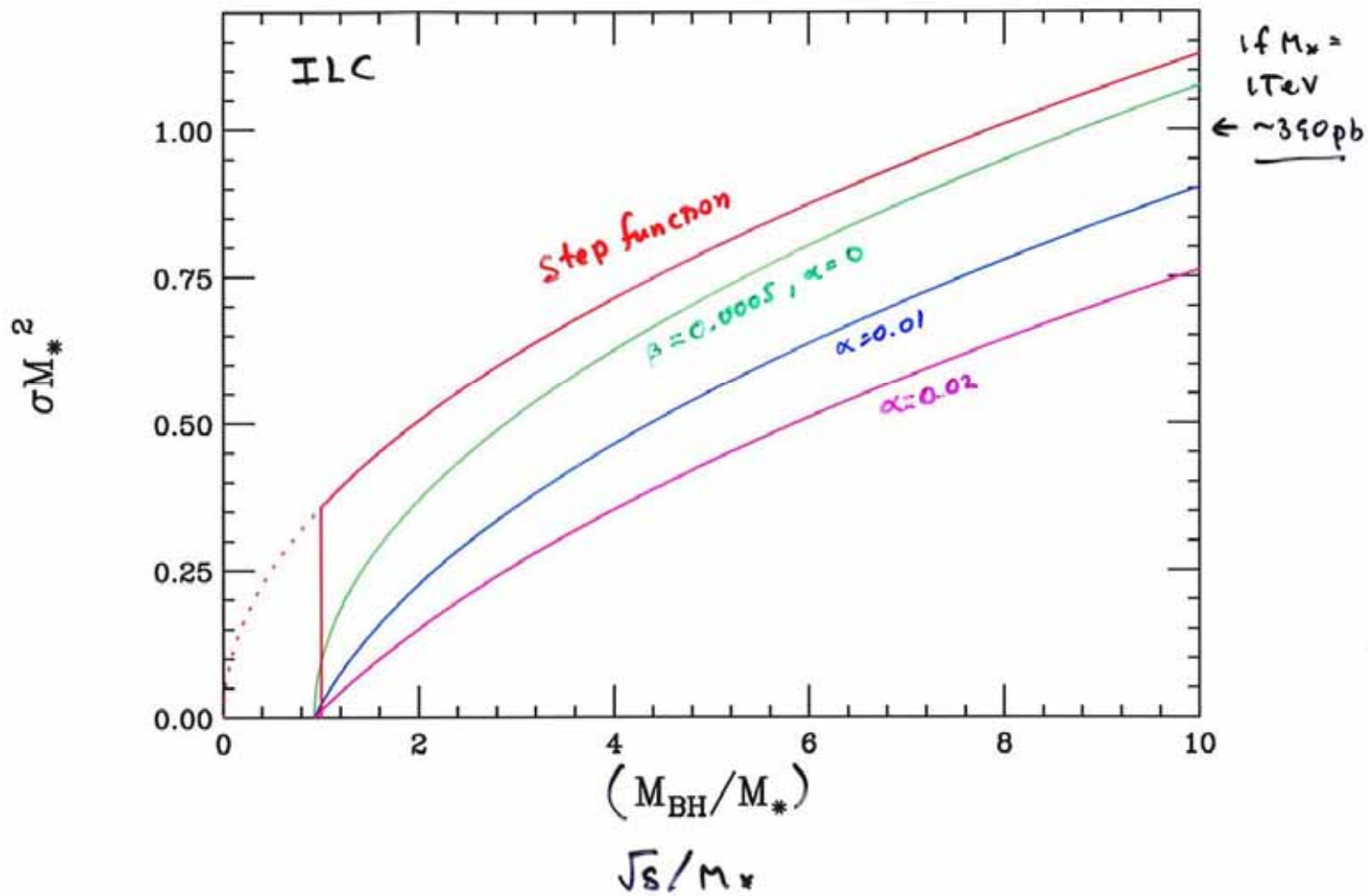
BH at LHC

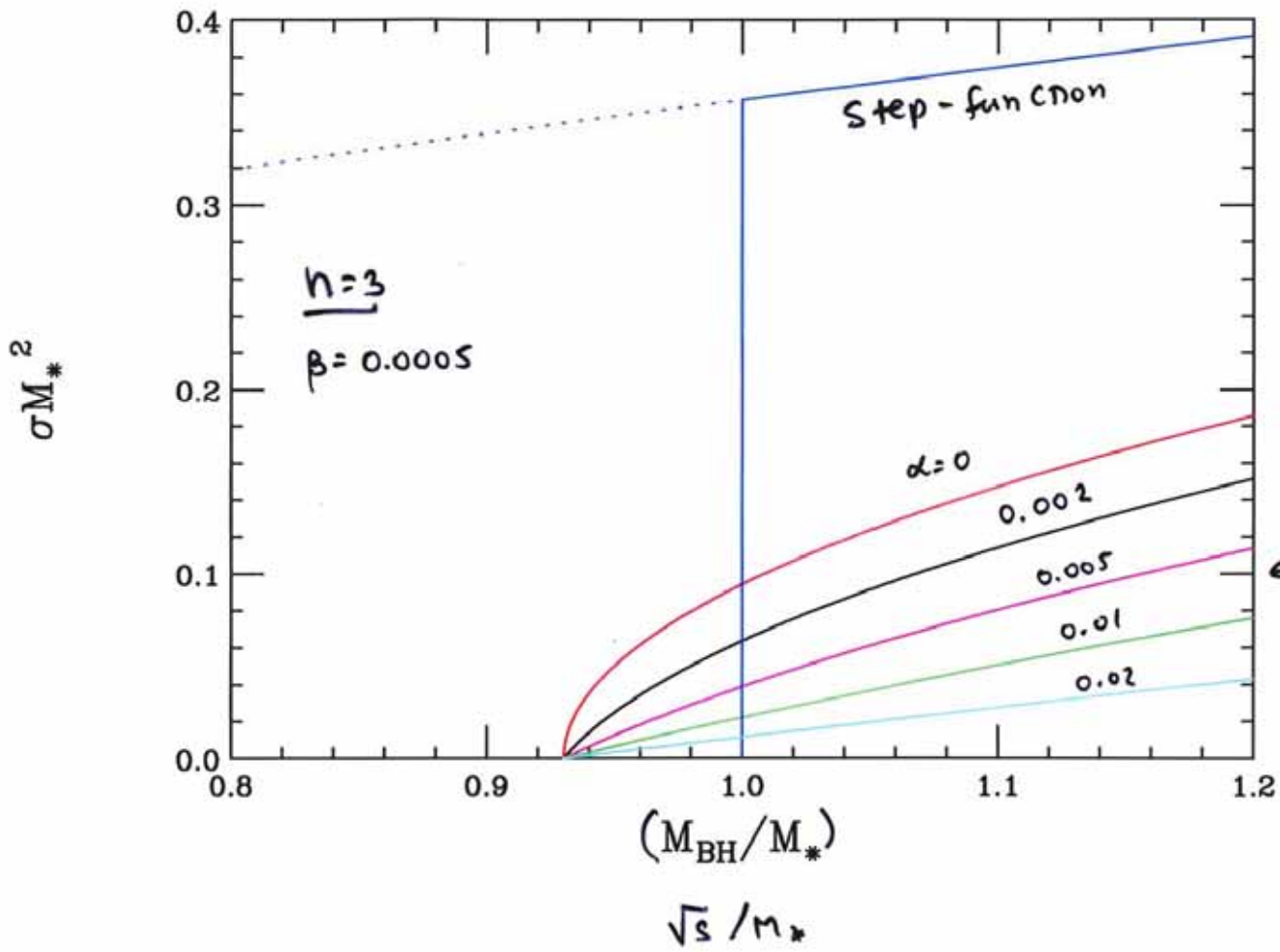
100 fb⁻¹



↑
 $M_* = 1.5 \text{ TeV}$

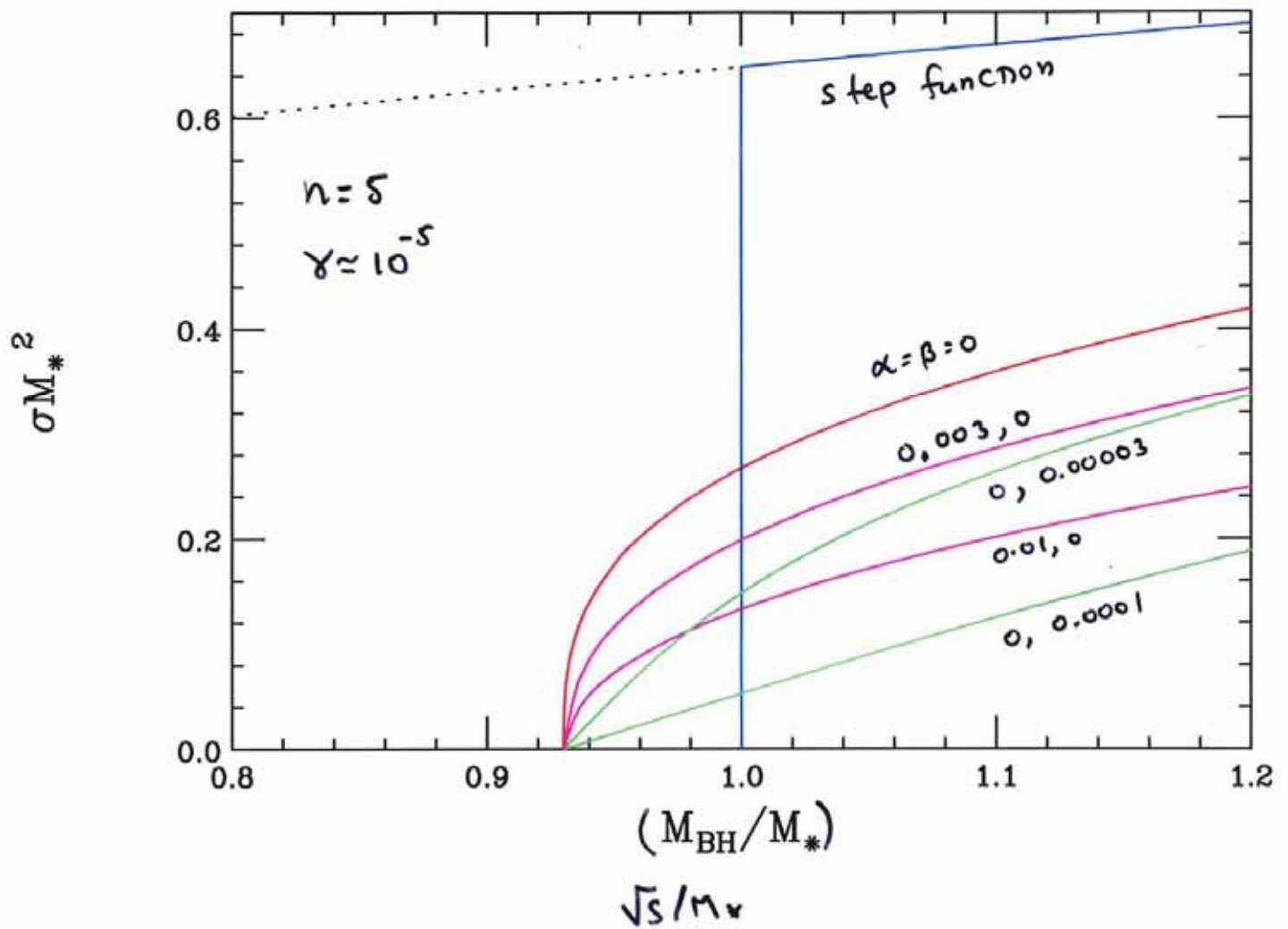
... need precision to probe parameters



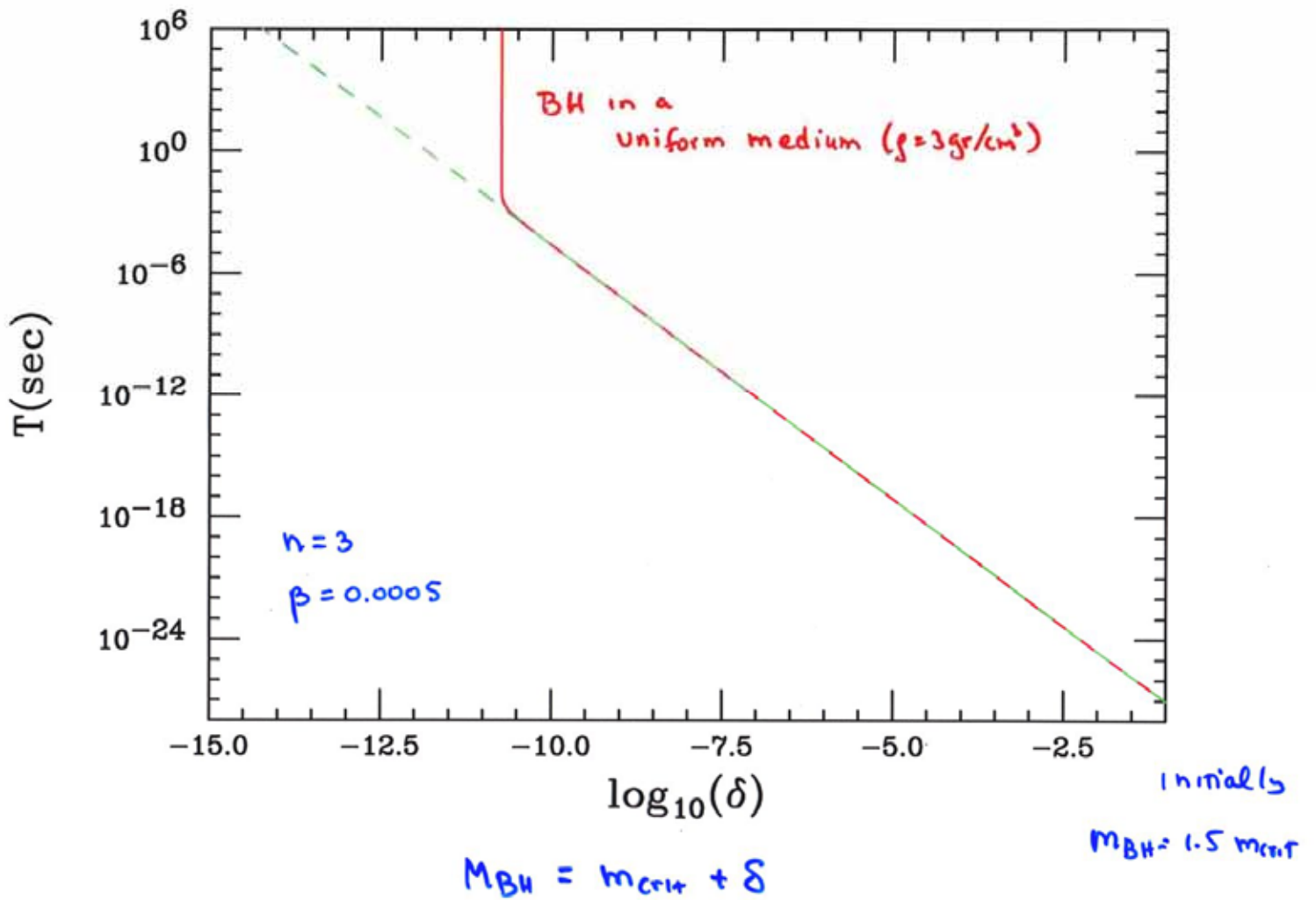


Note
 If $M_* = 1 \text{ TeV}$
 $\leftarrow 39 \text{ pb!}$

Threshold shapes will tell us $(\alpha, \beta, \delta) \dots$



Crude Estimate of BH lifetime ...



Summary

- The presence of higher curvature terms in the action for gravity can lead to visible modifications in our favorite Extra Dim theories..
 - KK spectrum + coupling shifts in RS
 - New features in BH production / properties in ADD (thresholds, stability)
- More work needs to be done to elucidate these exciting possibilities... and to find other potential signatures