$\boxed{\text{AdS/CFT Correspondence}}$

Anti de Sitter — conformal field theory

Maldacena (1998)

Remarkable duality between

Supergravity string theory in 10 dimensions

and

Supersymmetric Yang-Mills field theory in 3+1 Minkowski space-time

$$\boxed{AdS_5 \otimes S^5 \Leftrightarrow SO(4,2) \otimes (\mathcal{N}=4)}$$

5 dim. Sphere

symmetries of Conformal Transformations + Poincare Invariance of Minkowski space

$\mathcal{N}=4$ SUSY

References:

* J.M. Maldacena hep-th/9711200 /9803002

* J. Polchinski + M.J. Strassler
hep-th/0109174, 0205211

* R.C. Brower + C.I. Tan
hep-th/0207071

* S.J. Rey + J.T. Lee
hep-th/9803001

* S.J. Brodsky + G.F. de Teramond
hep-th/0310227

* O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri
+ Y. Oz hep-th/9905111

Theories with Conformal Symmetry

invariant under Poincare transformations $M^{\mu\nu}, P^\nu$
+ Conformal transformations D, K^ν

generators form group

$$\boxed{SO(4,2)} \qquad (d=4)$$

$SO(4,2)$ has representations on both

Minkowski space $R^{(3,1)}$

and AdS_5

Minkowski metric

$$ds^2 = dt^2 - d\vec{x}^2$$

AdS_5 metric

fifth dim.

$$ds^2 = \frac{r^2}{R^2}\left(dt^2 - d\vec{x}^2\right) - \frac{R^2}{r^2}dr^2$$

Dilatations

$$x^\mu \to \lambda x^\mu \quad ; \quad (x^\mu, r) \to \left(\lambda x^\mu, \frac{r}{\lambda}\right)$$

Theories with conformal symmetry

Maxwell's Equations with $J^\mu = 0$

Spin-J equations for $m = 0$

In general, conformal symmetry broken

quantum corrections 

mass terms

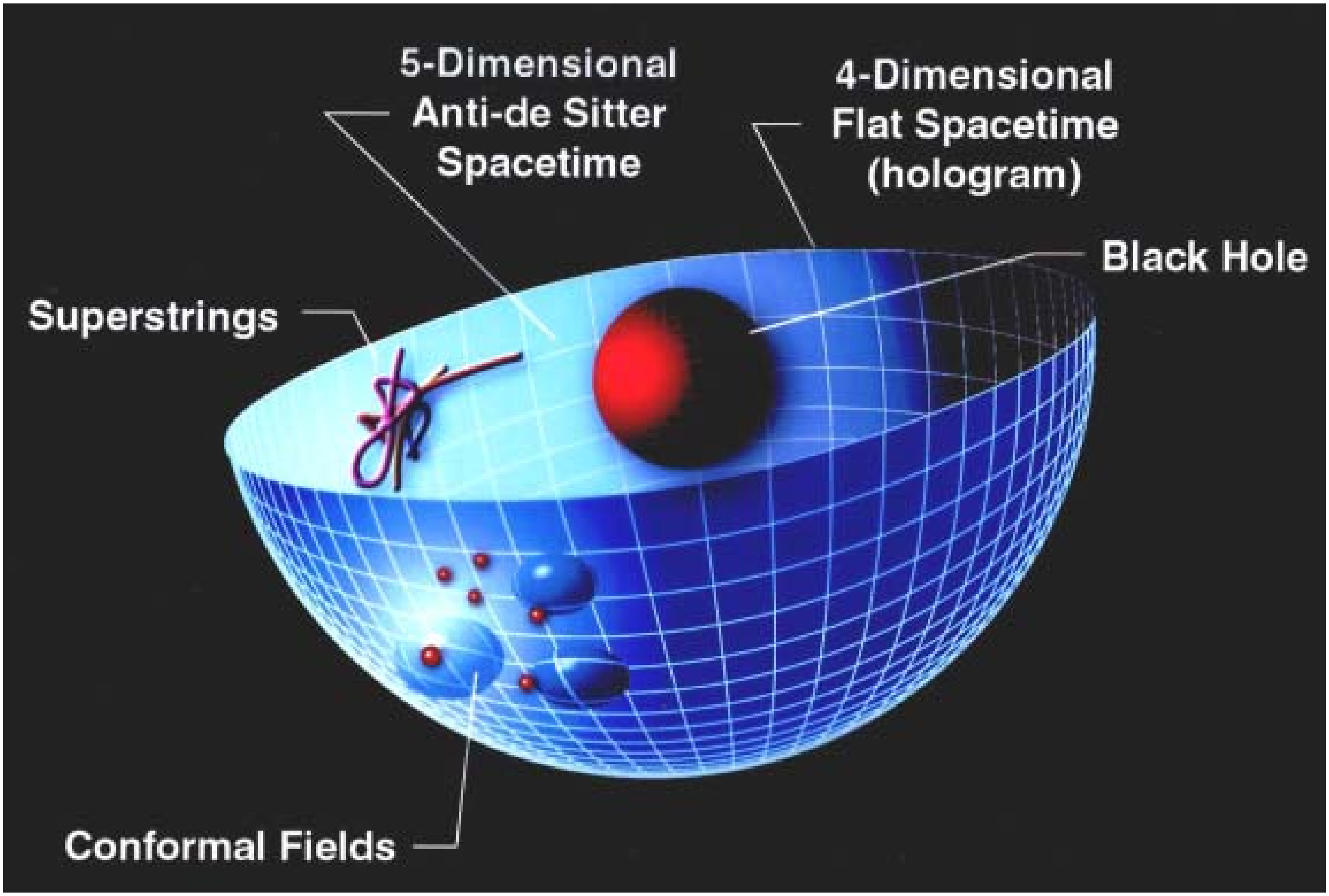
Exception: $\mathcal{N} = 4$ super-Yang Mills
$m = 0$

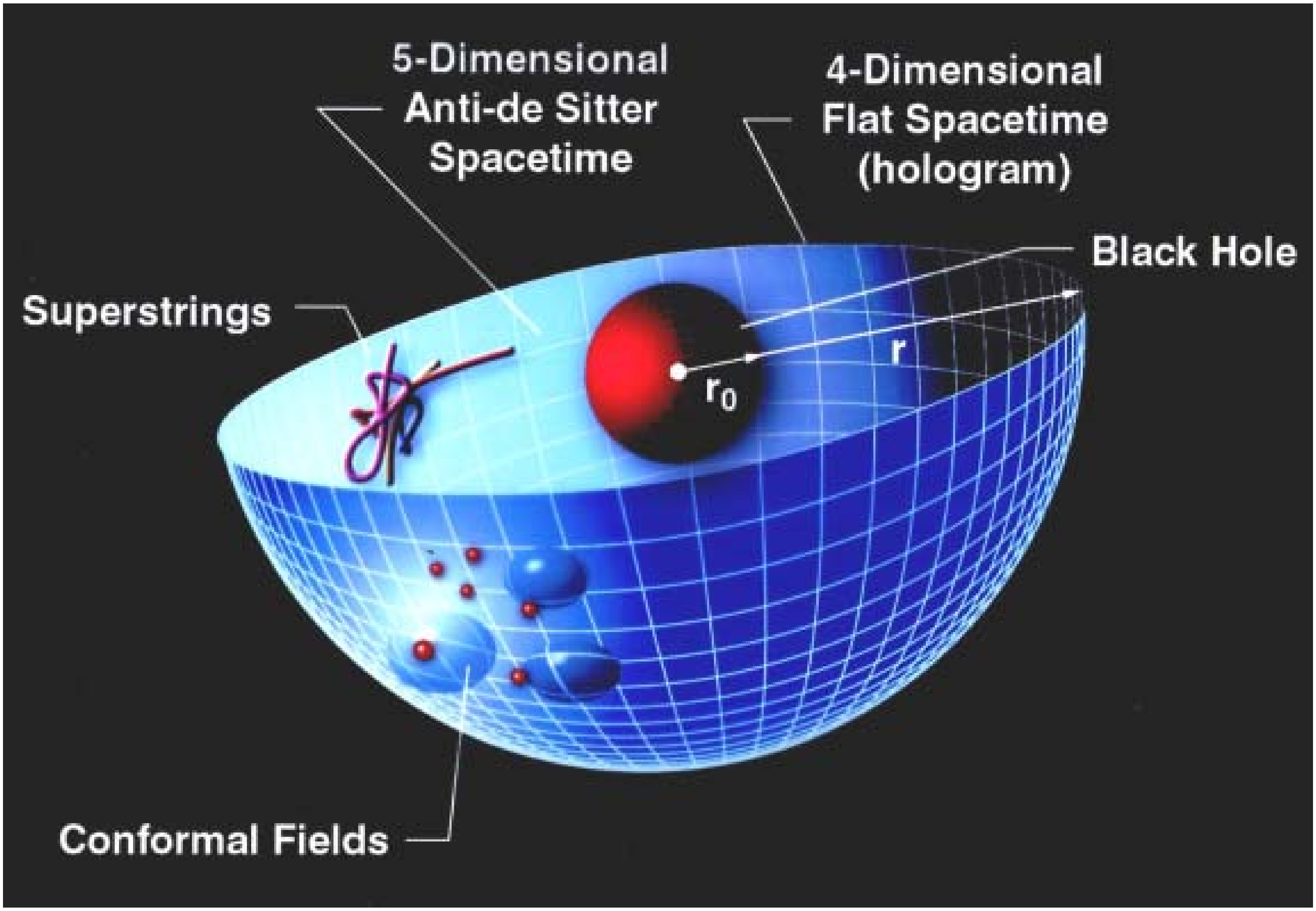
$\mathcal{N} = 4$ isomorphic to symmetries on S^5

$\Rightarrow$ $\therefore$ Study $AdS_5 \otimes S^5$ 10 dim

to learn about supersymmetric Yang-Mills theory in 3+1 dim

" AdS / CFT Correspondence "


5-Dimensional
Anti-de Sitter
Spacetime
4-Dimensional
Flat Spacetime
(hologram)
Black Hole
Superstrings
Conformal Fields


5-Dimensional
Anti-de Sitter
Spacetime
4-Dimensional
Flat Spacetime
(hologram)
Black Hole
Superstrings
r
r_0
Conformal Fields

Maldacena: $$R = (4\pi g_s N_c)^{1/4} (\alpha')^{1/2}$$

$g_s = g_{YM}^2$ String scale

Thus radius R of S^5 and metric of AdS_5 related to YM coupling

AdS_5 : $$ds^2 = \frac{r^2}{R^2}(dt^2 - d\vec{x}^2) - \frac{R^2}{r^2} dr^2$$

3+1 Gauge theory defined on boundary $r \to \infty$

"holographic principle"

Solve theory in "bulk" $AdS_5 \otimes S^5$ and project results on boundary

<u>Hard scattering</u> : $(\Delta x_\mu)^2 \sim O\left(\frac{1}{Q^2}\right)$

(ds^2 invariant): is also $(\Delta r)^2 = O(Q^2 R^4)$ in bulk

Polchinski + Strassler

$$r \sim Q R^2$$

short distance is large r

G. dT + S.J.B

Example of conformal string theory calculation

$AdS_5 \otimes S^5$: $\qquad z = R^2/r$

$$ds^2 = \frac{r^2}{R^2}\left(dt^2 - d\vec{x}^2\right) - \frac{R^2}{r^2}dr^2 - R^2 d\Omega_5^2(y)$$

$$= \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right) - R^2 d\Omega_5^2$$

10-dimensional string amplitude $\Phi(x, z, y)$

Laplace Eqn $S=0$

$$\frac{1}{\sqrt{g}}\frac{\partial}{\partial x^A}\left(\sqrt{g}\, g^{AB}\frac{\partial}{\partial x^B}\Phi\right) = 0$$

$$\sqrt{g} = \left(R^2/z\right)^{d+1}\sqrt{\Omega(y)} \qquad d=4$$

$$\Phi(x, z, y) = \sum_\ell \Psi_\ell(x, z)\,\phi_\ell(y)$$

spherical harmonics

hadronic plane wave in 3+1: $\Psi_\ell(x, z) = e^{-iP\cdot x} f(z)$

"dilaton" amplitude

$$* \quad \left[z^2\frac{d}{dz^2} - (d-1)z\frac{\partial}{\partial z} - (\lambda R)^2 + z^2 M^2\right]F(z) = 0$$

$$* \quad (\lambda R)^2 = \ell(\ell+d)$$

Solution: Cylindrical Bessel Functions

$$\Psi(x,r) = C_1 e^{-iP\cdot x}\, r^{-d/2} \begin{cases} J_\alpha\left(\frac{mR^2}{r}\right) \\ N_\alpha\left(\frac{mR^2}{r}\right) \end{cases}$$ ← ok

where

$$\alpha^2 = \left(\frac{d}{2}\right)^2 + (\lambda R)^2$$

$d=4$

$$(\lambda R)^2 = l(l+d)$$

eigenvalues on S^{d+1}

At large r:

$$\psi(r) \sim r^{-\Delta}$$

$$\Delta = \frac{1}{2}\left(d + \sqrt{d^2 + 4\lambda^2 R^2}\right)$$

$$= d + l$$

Thus for hard scattering $r \sim QR^2$

$$\psi(Q) \sim Q^{-\Delta}$$

Can we relate hadron wavefunctions in 3+1 to the $AdS_5 \otimes S^5$ solution?

Use light-front Fock expansion at fixed $\tau = t + z/c$ $\quad H_{LF} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle$

$x_i = k_i^+ / P^+$

n-1 integrations

$$|\Psi_h(P^+, \vec{P}_\perp\rangle = \sum_{n, \lambda_i} \int [dx_i \, d^2 k_{\perp i}] \, \psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$|n : x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\rangle$$

$$\sum_{i=1}^{n} x_i = 1 \quad , \quad \sum_i \vec{k}_{\perp i} = 0 \quad , \quad \lambda_i = S_z^i$$

Introduce UV regulator: $\quad k_{\perp i}^2 < \Lambda^2 = Q^2$

$$|\Psi_h(Q)\rangle \quad , \quad |\Psi_{n/h}(Q)\rangle$$

$$\Psi_{n/h}(Q) \sim \int^{Q} [d^2 k_\perp]^{n-1} [a^+(k_\perp)]^n \, \psi_n(\vec{k}_\perp)$$

Identify $\quad Q$-dep. of $\Psi_{n/h}(Q)$

with $\Psi(Q) \sim Q^{-\Delta_n}$

from AdS_5 with $m^2 \Rightarrow M_n^2 = \sum_{i=1}^{n} \frac{k_\perp^2 + m^2}{x}$

The large $k_\perp$ behavior of LFWFs

is predicted from OPE, conformal QCD

Belitsky
Ji, Yuan

$$\psi_{n/h}(\vec{k}_\perp) \sim (k_\perp)^{\ell} \left[\frac{1}{k_\perp^2}\right]^{n+\ell-1}$$

ie: $\Delta_{n,\ell} = n + \ell$

Compare with result from $AdS_5 \times S^5$:

$$\Delta = d + \ell(S^5)$$

These match for meson $(n = 2)$

if $\boxed{\ell = \ell(S^5) + 2}$

Orbital angular momentum from $S5$

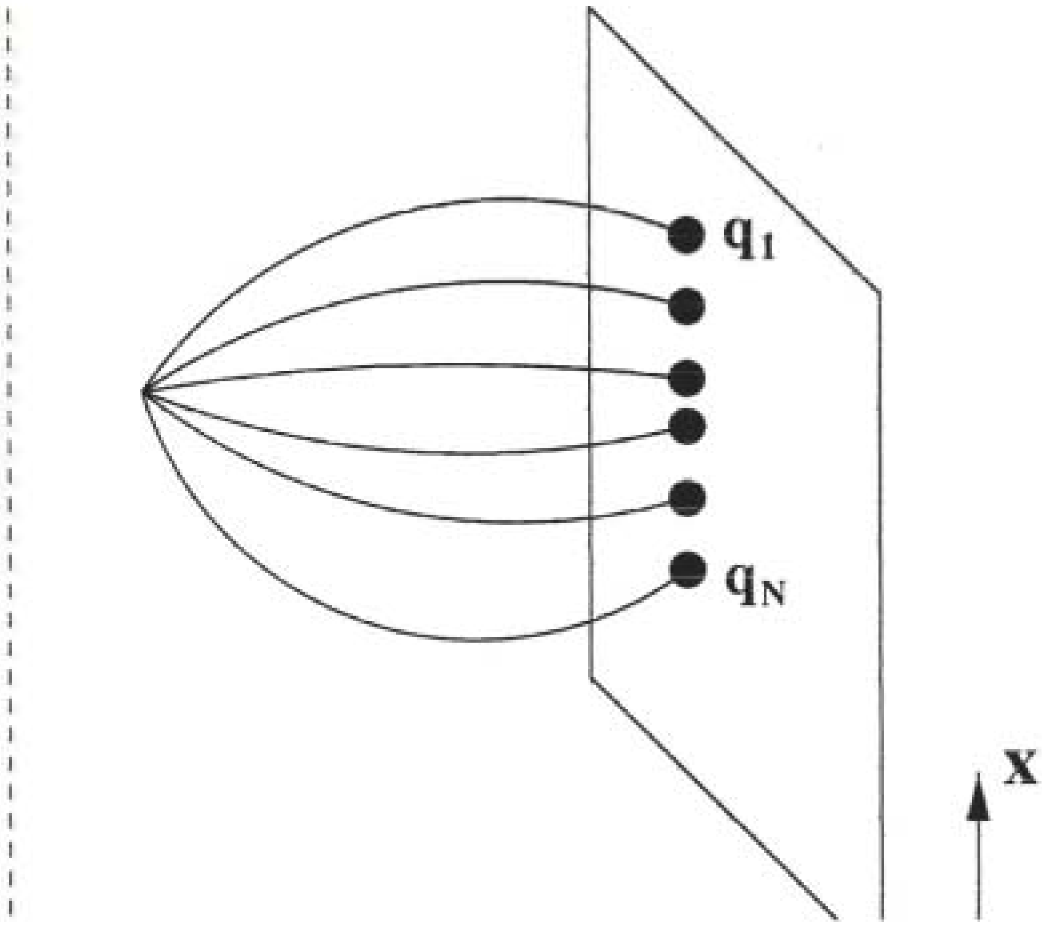
Minimum: $\ell = 0 \Rightarrow$

$$\ell(S^5) = -2$$

and $(\lambda R)^2 = -4$.

Allowed!

See Maldacena, Tesi 2003



q_1
q_N
X

Form of ψ_{LF} from QCD + Conf. Sym.

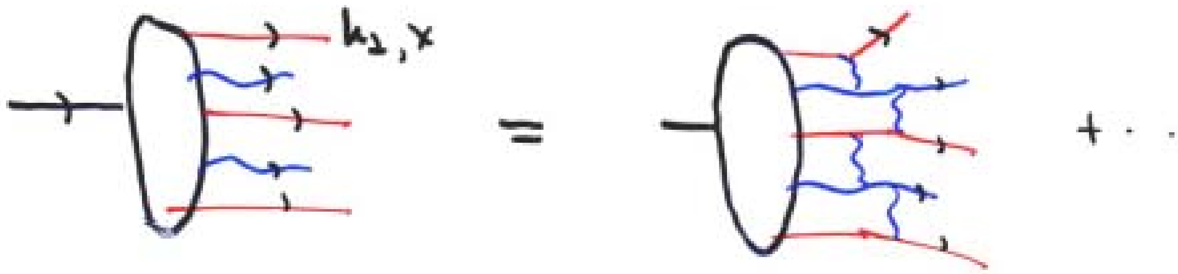
(anom dim ignored)

$$\psi_{n/h}(x_i, \vec{k}_{\perp i}, \lambda_i, l_{z i})$$

GdT
S13

$$\sim \frac{(g_s N_c)^{\frac{1}{2}(n-1)}}{\sqrt{N_c}} \prod_{i=1}^{n-1} (k_{\perp i}^{\pm})^{|l_{z i}|}$$

$$\times \left[\frac{\Lambda_0}{\mathcal{M}^2 - \sum_{i=1}^{n} \frac{(k_\perp^2 + m^2)_i}{x_i} + \Lambda_0^2} \right]^{n+|l_z|-1}$$



AdS:

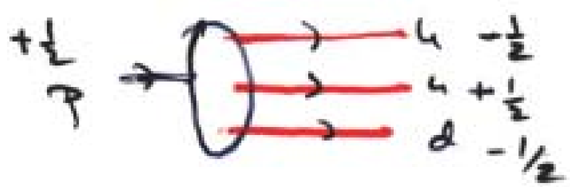
$$(g_s N_c)^{\frac{1}{2}(n-1)} \quad \text{instead of} \quad (g_{YM}^2 N_c)^{n-1}$$

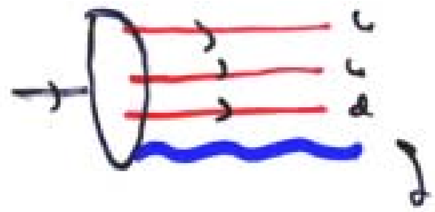
S. J. Rey, J. T. Lee (2001)
J. Maldacena (1998)

ADS / CFT $\Rightarrow$

* Orbital Angular Momentum

* Higher Fock States





$L_z = 1$

$$\psi^{L_z=1}_{qqq} \sim \frac{f_B \left(\frac{1}{\alpha'_{QCD}}\right)^{1/2} \vec{S}\cdot \vec{n}\times \vec{k}_\perp}{\left[k_\perp^2 + \frac{1}{\alpha'_{QCD}}\right]^3}$$

$$\psi^{L_z=0}_{qqqg} \sim \frac{f_B \left(\frac{1}{\alpha'_{QCD}}\right)^{1/2} \vec{S}\cdot \vec{n}\times \vec{\epsilon}_\perp}{\left[k_\perp^2 + \frac{1}{\alpha'_{QCD}}\right]^3}$$

$$\psi^{L_z=1}_{qqqg} \sim \frac{f_B \left[\frac{1}{\alpha'_{QCD}}\right] \vec{k}_\perp\cdot(\vec{n}\times \vec{\epsilon}_\perp)}{\left[k_\perp^2 + \frac{1}{\alpha'_{QCD}}\right]^3}$$

$$\Rightarrow F_2(Q^2) = \frac{1}{Q}\int \psi^{L_z=0}_{qqq}\,\psi^{L_z=1}_{qqq}\, d^2k_\perp dx \sim \frac{1}{Q^6}\frac{1}{\alpha'^2_{QCD}}$$

$\times$ logs

Predictions from conformal QCD:
+ $AdS_5 \times S^5$ + large N_c

large Q^2 Form Factors

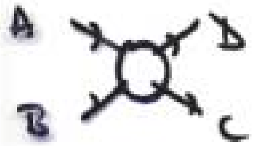
$$F(Q^2) \sim (g_{YM}^2 N_c)^{\frac{(n-1)}{2}} \left(\frac{\Lambda_0}{Q}\right)^{2n+|\ell_z|-2}$$

$$F_2(Q^2)/F_1(Q^2) \sim \left(\frac{M\Lambda_0}{Q^2}\right)$$

mod logs, anom. dim.

$$\mathcal{M}(Q^2)_{AB\to CD}$$

Polchinski-Strassler



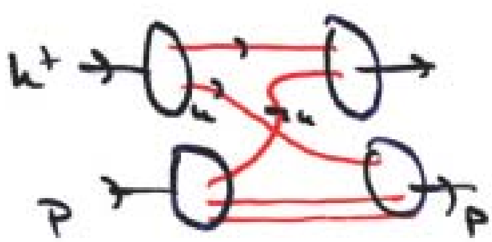
$$\sim \frac{(g_{YM}^2 N_c)^{\frac{1}{4}(n_T-2)}}{N_c} \left[\frac{\Lambda_0}{Q}\right]^{n+|\ell_z|-4}$$

← QIM

$$n_T = n_A + n_B + n_C + n_D$$

BF, MMT

- Non-perturbative derivation
- large N_c: QIM dominant



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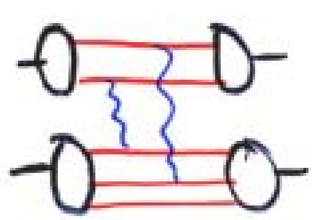


suppressed at large N_c

General Features of Exclusive Reactions at Large Momentum Transfer

- Power Law Scaling–Quark Counting Rules

F: ($\lambda = \lambda' = \pm 1/2$)
B: ($\lambda = \lambda' = 0$)

$$F_H(Q^2) \sim \{\frac{1}{Q^2}\}^{n_H - 1}$$

SJB + Farrar
Muradyan, ...

$$\frac{d\sigma}{dt}(AB \to CD) \sim \frac{1}{s^{N-2}} f(\theta_{cm})$$

$$(N = n_A + n_B + n_C + n_D)$$

Sudakov Suppression of Landshoff pinch contributions

- Hadron Helicity Conservation

$$\sum_{initial} \lambda_{had} = \sum_{final} \lambda_{had}$$

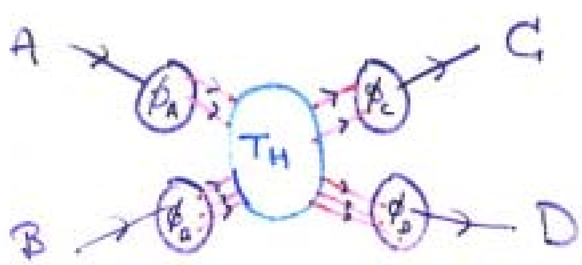
- Factorization Theorem

$$\mathcal{M}_{AB \to CD} = \int \phi_A \phi_B T_H \phi_C \phi_D \ \prod dx_i$$

T_H : Hard q-g Scattering Amplitude

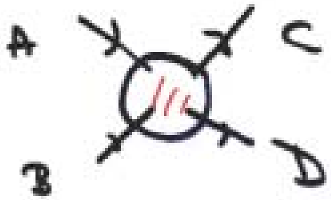
$\phi_A(x_i, Q)$: Hadron Distribution Amplitude

Lepage SJB
Efremov Radyushkin
Chernyak Zhitnitsky
Farrar Jackson
Mueller Duncan



Conformal Invariance and QCD

BF, MMT



$$\frac{d\sigma}{dt}(AB \to CD) \Rightarrow \frac{1}{s^{n_{TOT}-2}} F(t/s)$$

$$-t/s = \tfrac{1}{2}(1-\cos\theta_{cm})$$

high momentum transfer

Examples	n_{TOT}	$d\sigma/dt$
$eq \to eq$, $qq \to qq$, $\gamma q \to gq$	$2+2+2+2$	$\frac{1}{s^2}$
$\gamma p \to \pi^+ n$	$1+3+2+3$	$\frac{1}{s^7}$
$pp \to pp$	$3+3+3+3$	$\frac{1}{s^{10}}$
$ep \to ep$, $\gamma p \to \gamma p$	$1+3 \to 1+3$	$\frac{1}{s^6}$

Form Factors

$$F_H^2(t) = \frac{\frac{d\sigma}{dt}(eH \to eH)}{\frac{d\sigma}{dt}(eq \to eq)} \Rightarrow F_H(t) \approx \frac{1}{t^{n-1}}$$

New derivation (without perturbation theory)

AdS/CFT

Maldacena
Polchinski + Strassler

Test of QCD fixed angle scaling

in $pp \to pp$

$\frac{d\sigma}{dt}(pp \to pp)$ $\qquad$ $\mathcal{M}_{pp \to pp} \simeq \frac{1}{s^4} \tilde{\mathcal{M}}(\theta_{cm})$

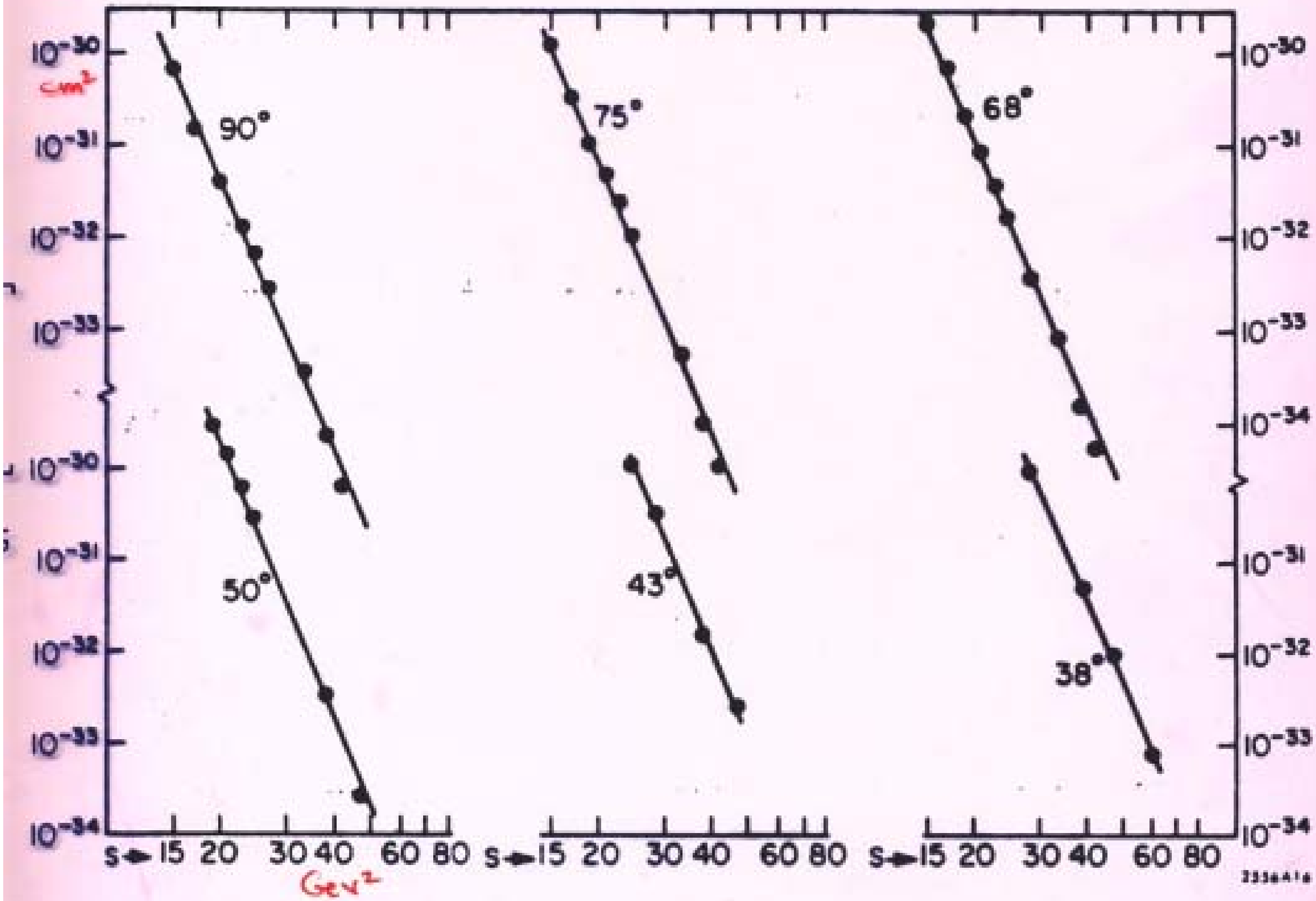
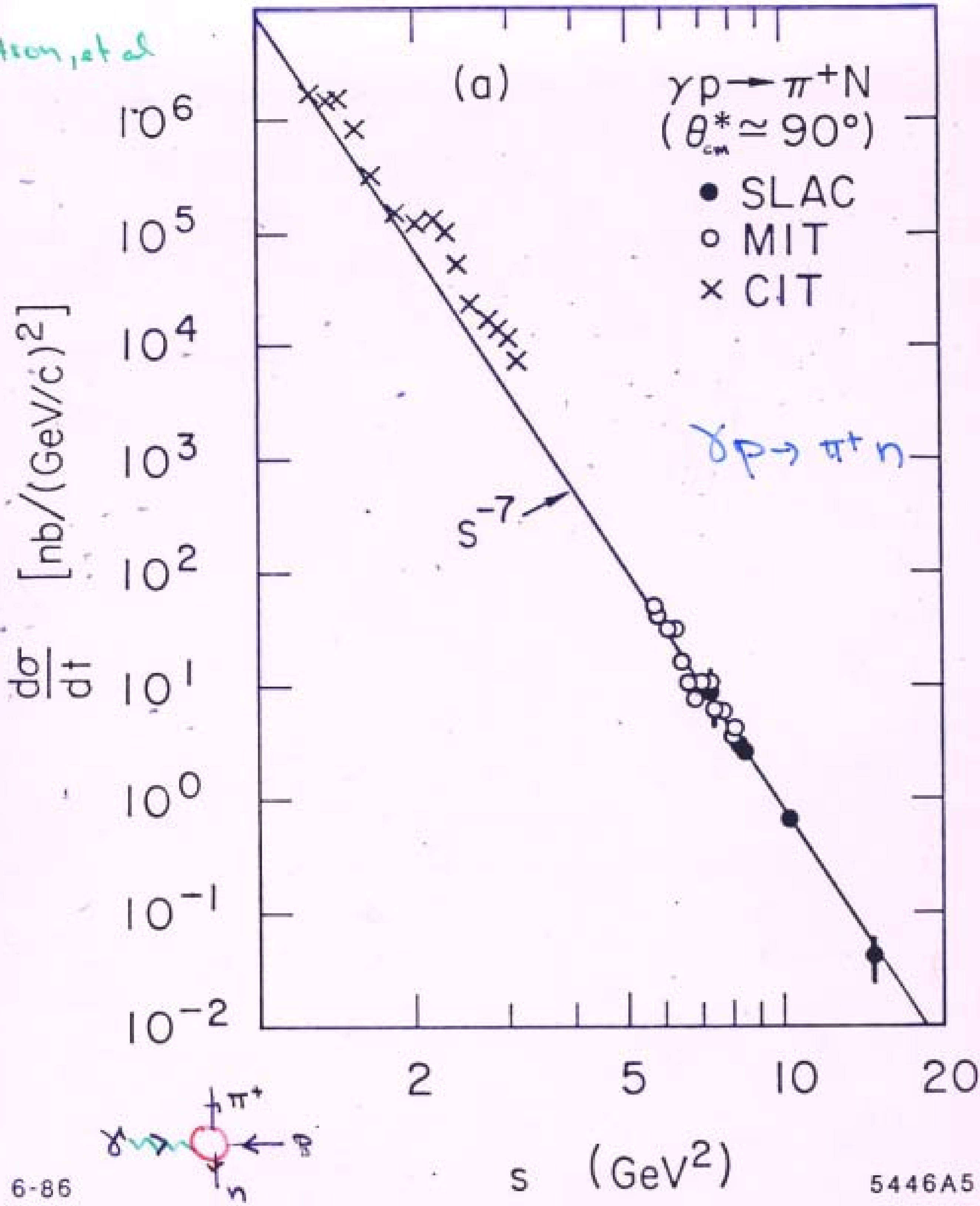


Figure 22. Test of fixed θ_{CM} scaling for elastic pp scattering. The data compilation is Landshoff and Polkinghorne.

$$\frac{d\sigma}{dt} = \frac{1}{s^N} f(\theta_{cm}) \quad \begin{cases} N = 9.7 \pm 0.5 & (\text{Expt}) \\ N_{QCD} = 10 - \epsilon & \text{Landshoff} \\ \quad = 9.66 & \text{Mueller, Botts + Sterman} \end{cases}$$

Dim. Counting: $N = 12 - 2 = 10$

Ritson, et al



6-86

5446A5

SJB Farrar et al

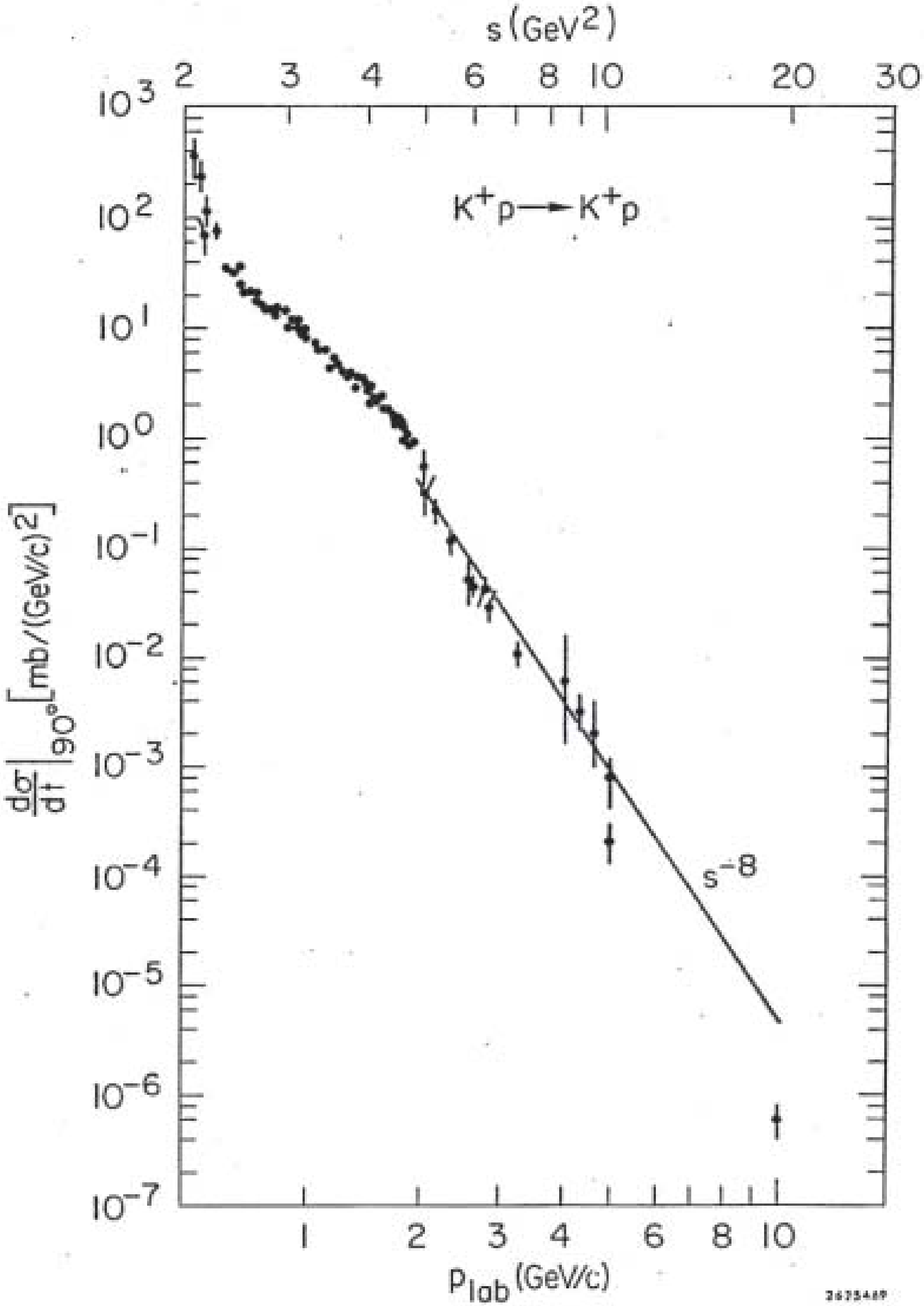
no x~1 endpoint contribution

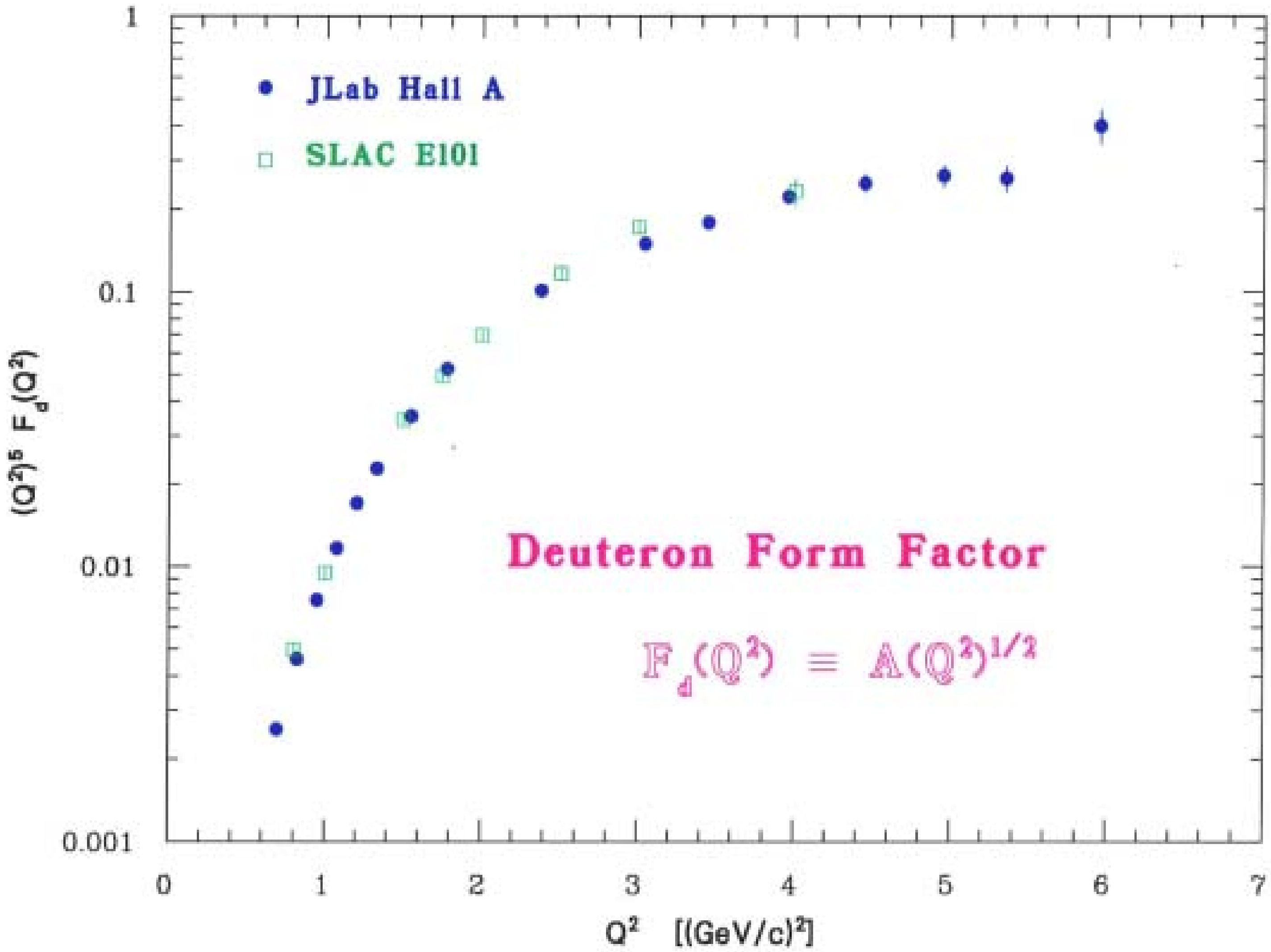
Dimensional counting

$(3+1+3+2)-2=7$

QCD:

$$\frac{d\sigma}{dt}(\gamma p \to \pi^+ n) = \frac{1}{s^7} f(\theta_{cm})$$

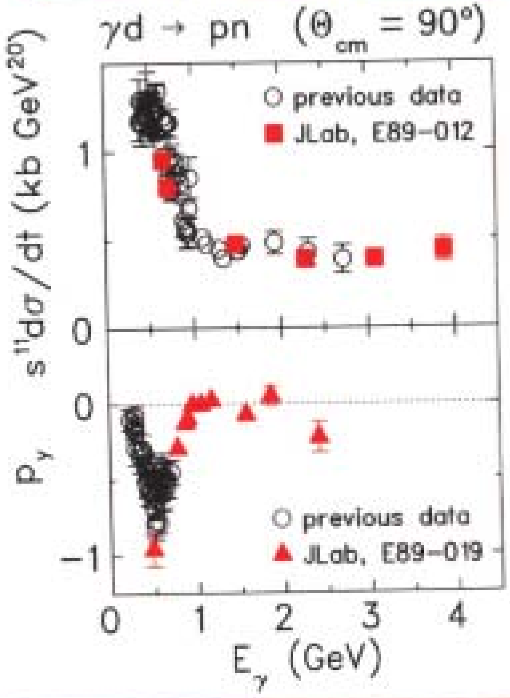

s (GeV^2)
2
3
4
6
8
10
20
30
10^3
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
$K^+p \to K^+p$
s^{-8}
$\frac{d\sigma}{dt}\big|_{90°}$ [mb/(GeV/c)^2]
1
2
3
4
6
8
10
p_{lab} (GeV/c)

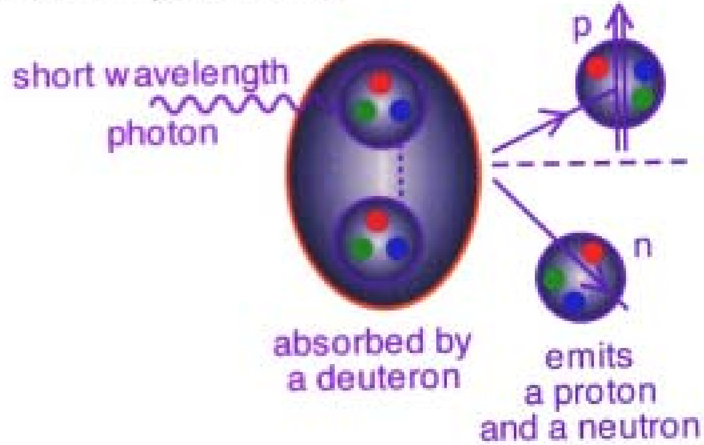

JLab Hall A
SLAC E101
Deuteron Form Factor
$F_d(Q^2) \equiv A(Q^2)^{1/2}$
$(Q^2)^5\ F_d(Q^2)$
Q^2 [(GeV/c)^2]
1
0.1
0.01
0.001
0
1
2
3
4
5
6
7

SHORT-DISTANCE STRUCTURE of the DEUTERON

Jefferson Lab (E89 - 012, E89 - 019)

Do we see the effects of quarks and gluons in a nuclear reaction ?



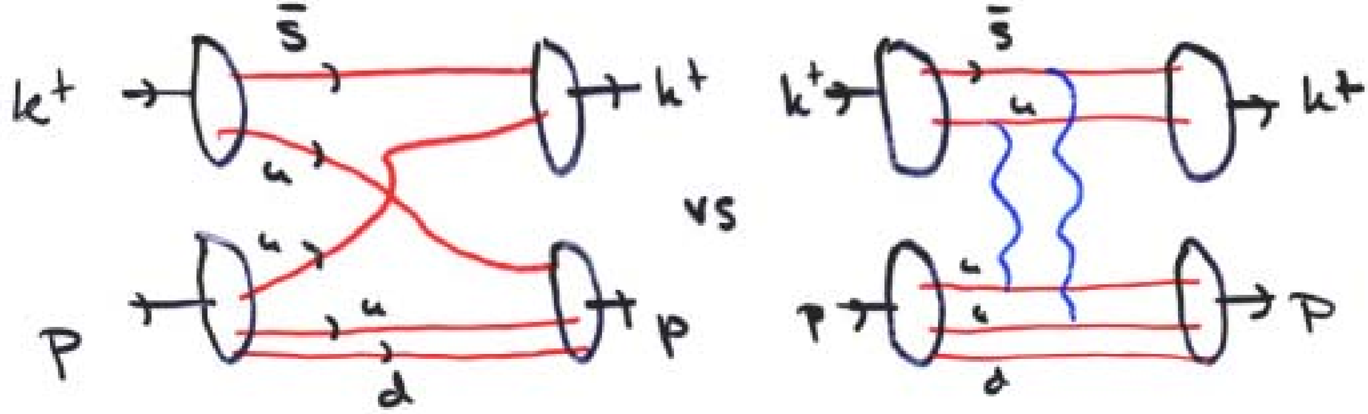


- **Reaction probability is consistent with quark counting rules at high photon energy.**
- **Polarization vanishes at same photon energy that reaction probability begins scaling.**
- **First glimpse of the transition region.**

Angular Distribution $\qquad -t/s = \frac{1}{2}(1-\cos\theta_{cm})$

$$\frac{d\sigma}{dt} = \frac{1}{s^{n_{TOT}-2}} F(t/s)$$

determined by scattering mechanism



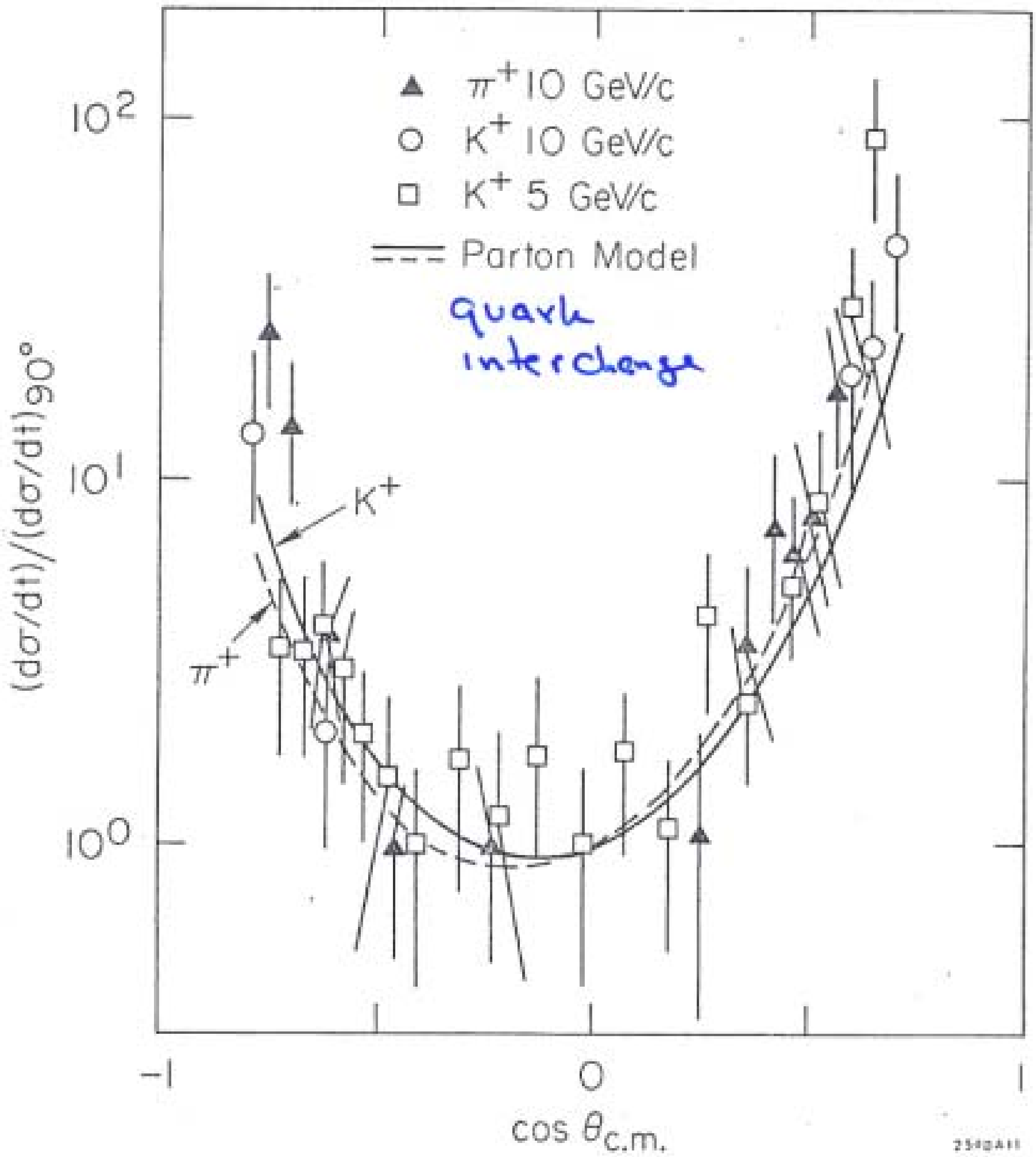
Quark Interchange $\qquad\qquad$ gluon exchange

analogous to spin exchange in atom - atom scattering $\qquad\qquad$ Van der Waals

Large N_c : Quark Interchange Dominant

$$\mathcal{M} \sim \frac{1}{u}\frac{1}{t^2}$$

't Hooft limit , AdS/CFT



VC.3 Comparison with interchange model predictions for $K^+p \to K^+p$ and $\pi^+p \to \pi^+p$ elastic scattering. From Lundby (1973).

Compare String Theory / Dim. Counting

with PQCD Scaling

$2 \to 2$ $\quad \frac{1}{N^2} \to \frac{1}{N}$ for quark composites

* Higher order corrections in α_s ?.

  Exact in string theory!

* Anom. Dimensions from Evolution of $\phi_H(x, Q^2)$

$$e^{\gamma_n \zeta(Q^2, Q_0^2)} = \begin{cases} \left(\ln Q^2/\Lambda^2\right)^{\gamma_n C_F/\beta} & \text{ASF} \\ \left(Q^2/\Lambda^2\right)^{\frac{\alpha_s C_F}{4\pi}\gamma_n} & \text{conf} \end{cases}$$

$$\zeta(Q^2, Q_0^2) = \frac{C_F}{4\pi} \int_{Q_0^2}^{Q^2} \frac{d\ell^2}{\ell^2} \alpha_s(\ell^2)$$

* Pinch contributions: suppressed by $(Q^2)^{-C \ln Q^2}$ in conf. theory

  Large $N_C \Rightarrow$ suppressed.

  Farrar + SJB
  Mueller Dun

* What is magnitude of α_s in conf theory?

Compare String Theory / Dim. Counting with PQCD Scaling

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$$\xi(Q^2, Q_0^2) = \frac{C_F}{4\pi}\int_{Q_0^2}^{Q^2} \frac{d\ell^2}{\ell^2}\, \alpha_s(\ell^2)$$

* Pinch contributions: suppressed by $(Q^2)^{-C \ln Q^2}$ in conf. theory

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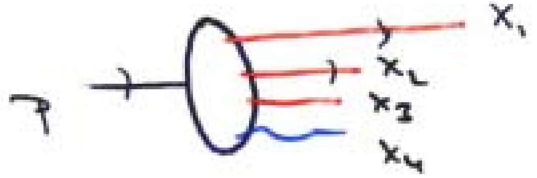
* What is magnitude of α_s in conf theory?

Structure Functions at $x \to 1$



$$M_n^2 = \sum_{i=1}^{n} \frac{k_\perp^2 + m^2}{x_i}$$

$\to \infty$ for $x_1 \to 1$

$x_{i \neq 1} \to 0$

$$\sum x_i = 1$$

SJB, Gunion, Scadron

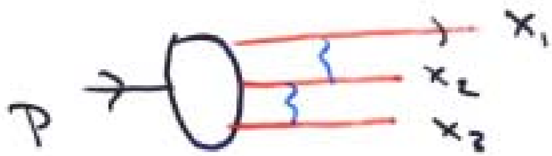
Farrar + Jackson

SJB + GPL

Burkardt, Schmidt, SJB

Leading order

Iterate kernel:



$$q(x) \sim (1-x)^{2n_s - 1 + 2\Delta h}$$

$u/d = 5:1$

$$= \begin{cases} (1-x)^3 & q\uparrow \\ (1-x)^5 & q\downarrow \end{cases}$$

$$g(x) \sim \begin{cases} (1-x)^4 & g\uparrow \\ (1-x)^6 & g\downarrow \end{cases}$$

note: minimal evolution at $x \to 1$ (off shell effect)

Dirac Equation in 10-dimension

$$\chi(x,r,y) = \sum_l \Psi_l(x,r)\,\eta_l(y)$$

$$\Psi(x,r) = C\, e^{-iP\cdot x}\, r^{-\frac{d+1}{2}}$$

$$\times\Big[J_\alpha\left(\frac{mR^2}{r}\right) u_+(P)$$

$$+ J_{\alpha+1}\left(\frac{mR^2}{r}\right) u_-(P)\Big]$$

$$\alpha = \lambda R - \tfrac{1}{2} \qquad m^2 = P^2$$

$$\hat{\Gamma}\, u_\pm = \pm u_\pm$$

$$\hat{\Gamma} = i\,\Gamma_1 \cdots \Gamma_d \qquad \text{chirality } d \text{ even}$$

$$\lambda R = \pm\left(l + \tfrac{d}{2} + \tfrac{1}{2}\right)$$

$$\Psi \sim r^{-\Delta}, \qquad \Delta = \tfrac{d}{2} + |\lambda R|$$

Towards realistic QCD

break conformal symmetry : $h(r) \neq 1$

introduce $\Lambda_0 = \Lambda_{QCD}$

physics at $r \sim r_0 = \Lambda_0 R^2$

Modify AdS_5 metric : $z = R^2/r$

$$ds^2 = \frac{r^2}{R^2} h(r)\, dx_\mu^2 - \frac{R^2}{r^2} h^{-1}(r)\, dr^2$$

$$- R^2 h^{-1}(r)\, d\Omega_5^2(y)$$

$$\Phi(x, z, y) = \sum_\ell \psi_\ell(x, z)\, \phi_\ell(y)$$

* $$\left[h^2(z) \left(z^2 \frac{d^2}{dz^2} - (d-1) z\, \partial_z - (\lambda R)^2 \right) + z^2 M^2 \right] f(z) = 0$$

$$\psi_\ell(x, z) = C_1 e^{-iP \cdot x} f(z)$$

GDT
SDB

$$(\lambda R)^2 = \ell(\ell + d) \qquad d = 4$$

$h(z)$ sub.t.

$d=4$

Write $F(z) = z^{d/2} Z_\alpha(z)$

$$\zeta = z\mathcal{M}/h(z)$$

$$\left[\frac{\partial^2}{\partial \zeta^2} + \frac{1}{\zeta}\frac{\partial}{\partial \zeta} + \left(1 - \frac{\alpha^2}{\zeta^2}\right)\right] Z_\alpha(\zeta) = 0$$

$$\alpha^2 = (d/2)^2 + (\lambda R)^2$$

$$(\lambda R)^2 = l(l+d)$$

regular soln

Solution:

$$\Psi(x,z) = C\, e^{-iP.x}\, z^{d/2} \left\{ \begin{array}{l} J_\alpha\left(\frac{z\mathcal{M}}{h(z)}\right) \\ N_\alpha\left(\frac{z\mathcal{M}}{h(z)}\right) \end{array} \right.$$

- independent of $h(z)$!

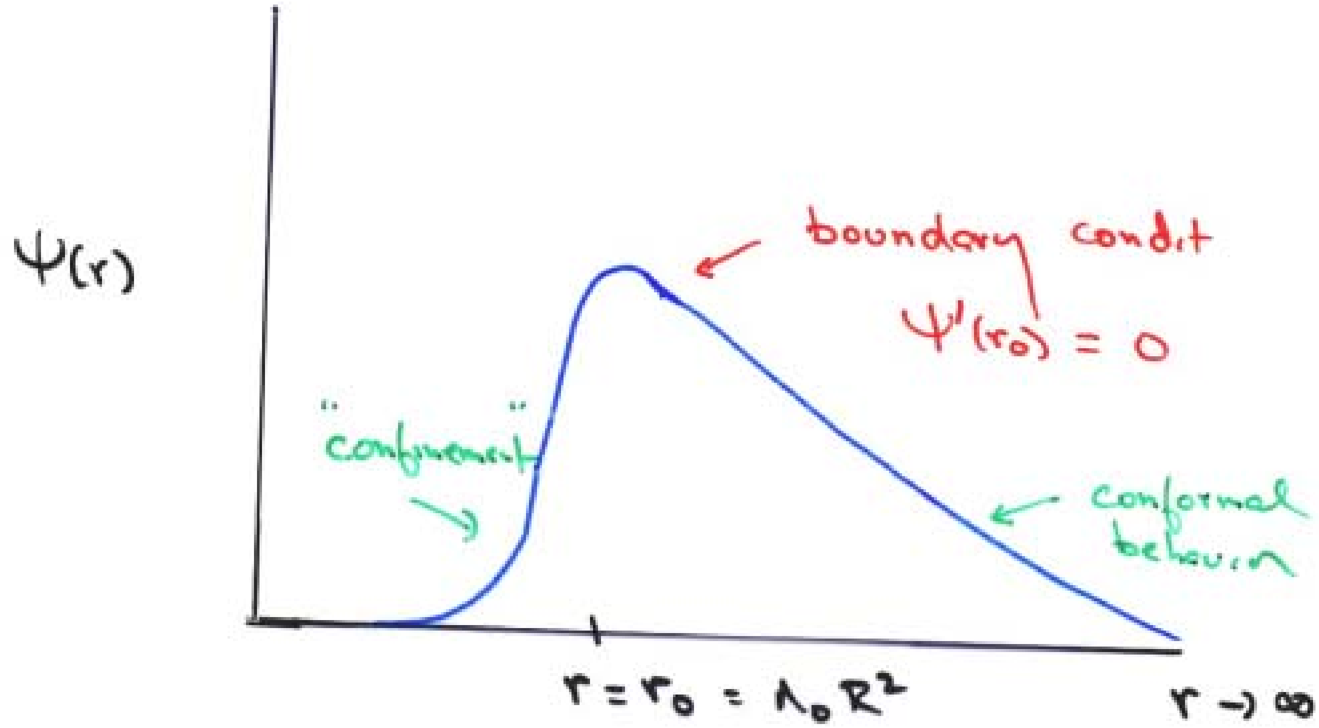
$$z = R^2/r$$

Model form

$$h(r) = 1 + e^{-(r-r_0)/\epsilon r_0}$$

Conformal at $r \gg r_0$ $\quad h(r) \simeq 1$

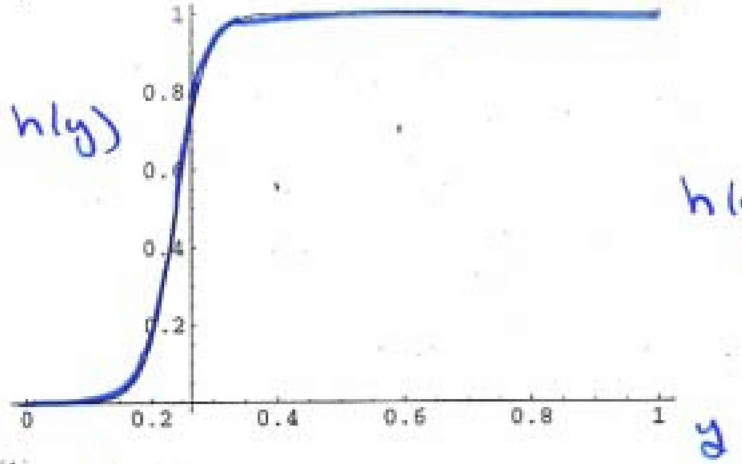
Barrier at $r < r_0$



Produces eigenvalues $\mathcal{M}^2 = \mathcal{M}^2(J)$

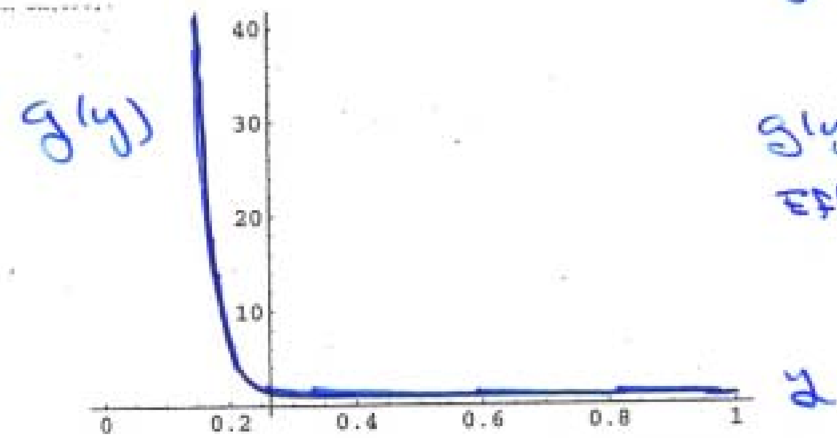
Similar to Chew-Frautschi $\mathcal{M}^2 \propto J$

One parameter Λ_0



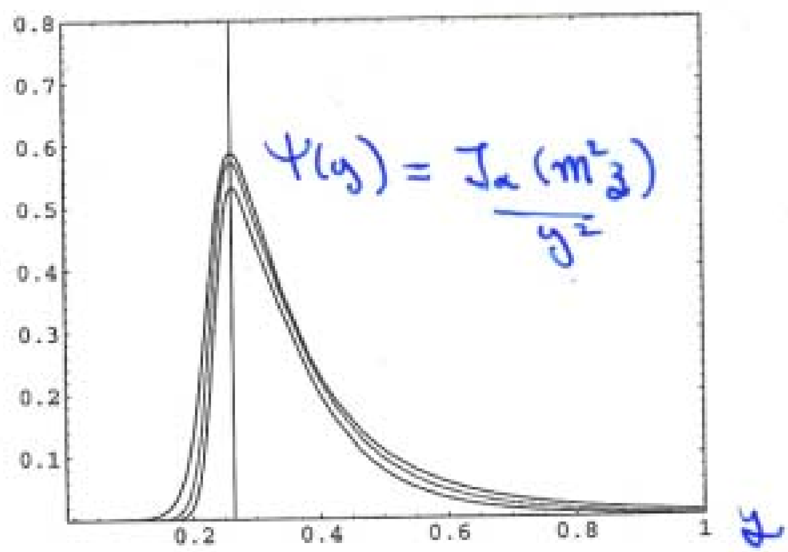
$\tau = 0.1$

$h(y) = 1 + e^{-(y-y_0)/\tau y_0}$



$g(y) = 1/h(y)$

Effective coupling

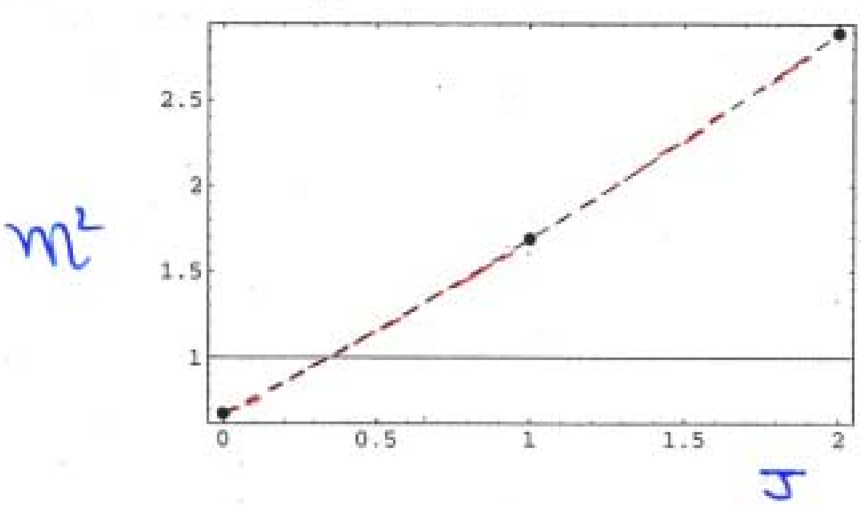


$z = \frac{1}{y\,h(y)}$

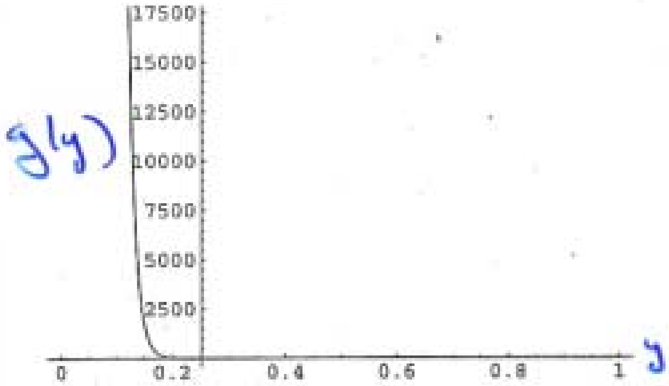
$m = 0.82$, $J=0$

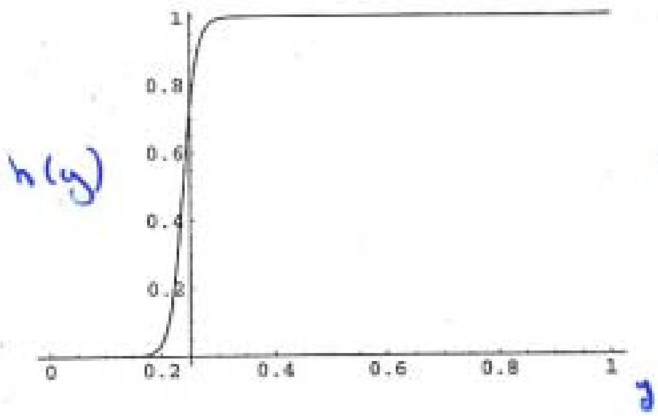
1.3 $J=1$

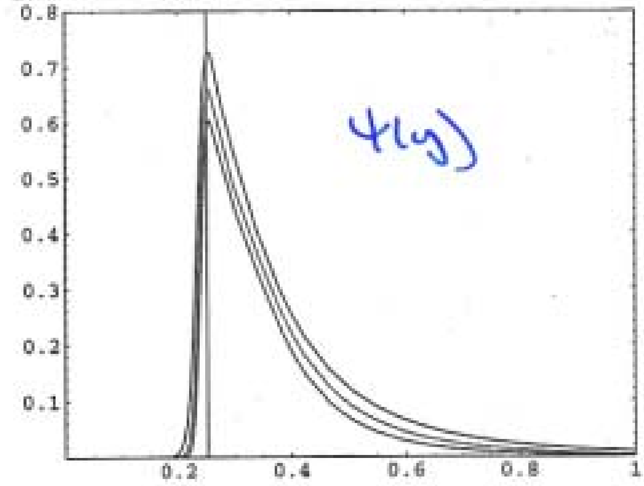
1.7 $J=2$

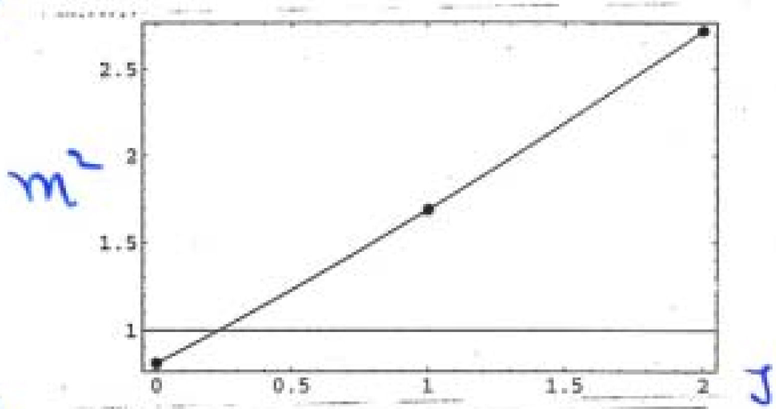


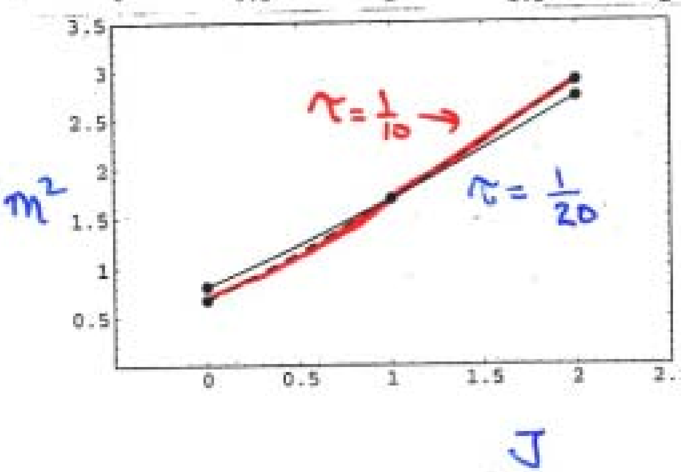
Regge plot





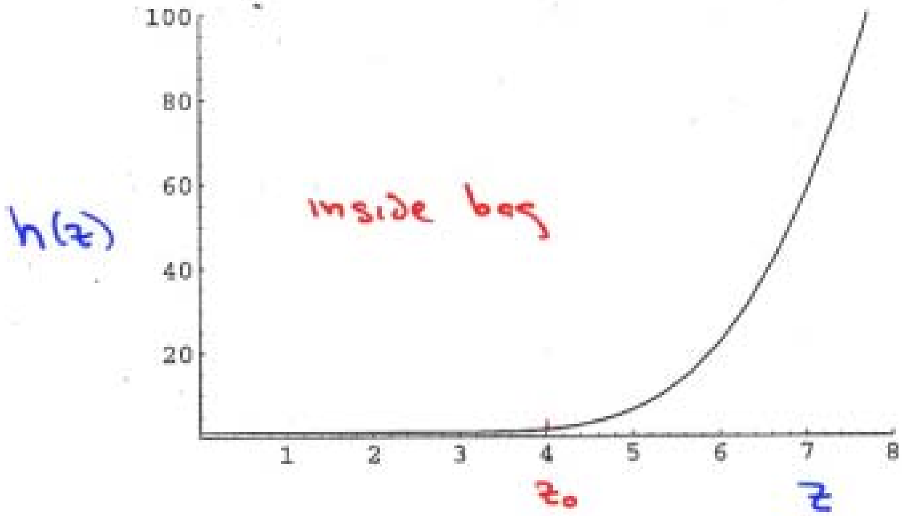






$$h(y) = 1 + e^{-(y-y_0)/\tau y_0}$$

$$\tau = 0.05$$



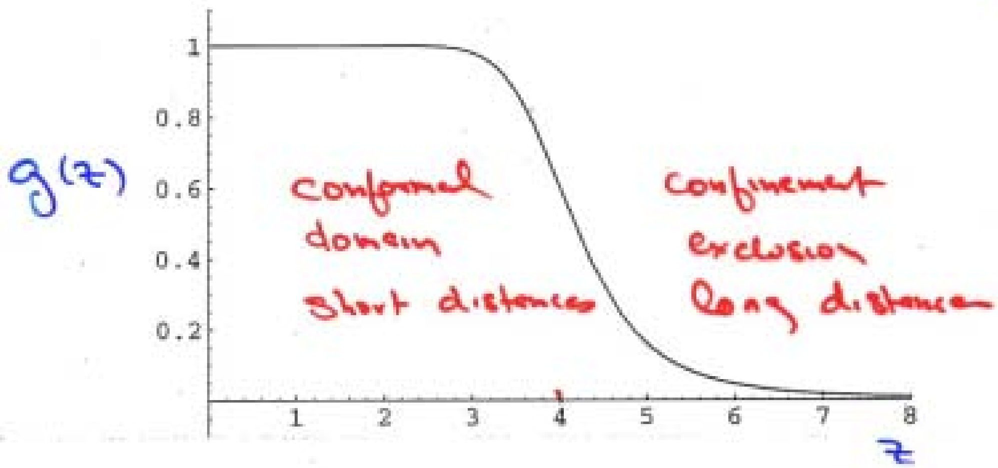
$$z = 1/y$$

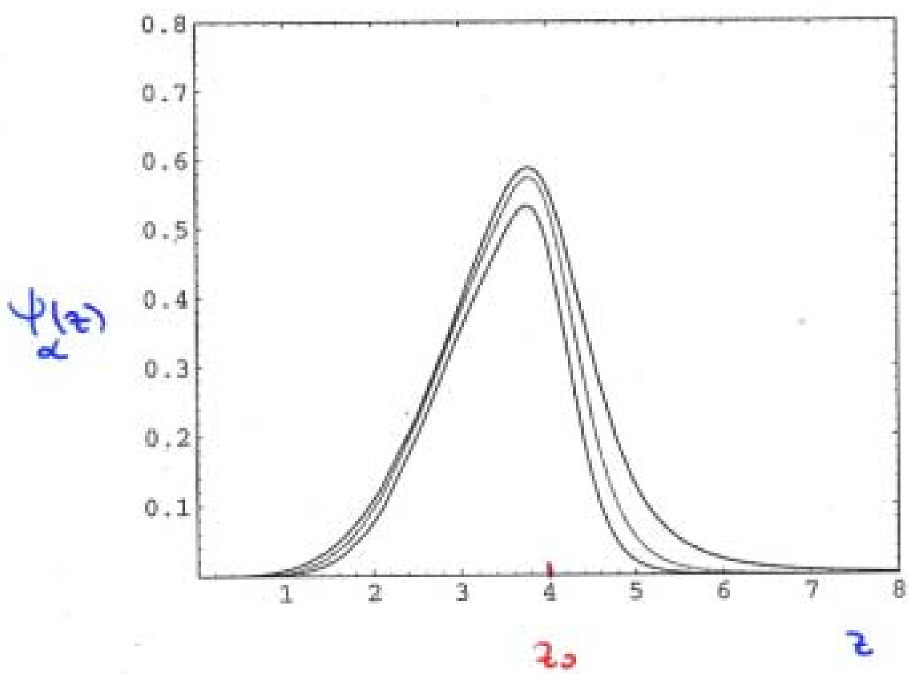
$$z^0 = 1/y_0 = 1/.24$$

$$h(y) = 1 + e^{-(y-y_0)/\tau y_0}$$

$$\tau = 1/10$$

$$g(y) = \frac{1}{h(y)}$$





Conformal Symmetry

$\mathcal{L}_{\text{classical}}$ renormalizable theory

* Poincaré plus
* dilation $[x^\mu \to \lambda x^\mu]$
* conformal transformation $\left[\text{inversion}\left(x^\mu \to -\frac{x^\mu}{x^2}\right) \otimes \text{translation} \otimes \text{inversion}\right]$

broken by

masses

quantum loops : running coupling, ...

For $\beta = \frac{d\alpha_s(Q^2)}{d \ln Q^2} = 0$ (e.g. fixed point theory)

* PQCD $\Rightarrow$ conformal theory

all orders proof : G. Parisi, PL 39B, 643 (1972)

cf: Y. Frishman : Proc. Moriond (1985)

$\boxed{\text{AdS/CFT} \Rightarrow \text{Near - Conformal QCD}}$

* All orders, strong coupling derivation of dimensional counting rules; counting rules for LFWFs
* $\sum_n (C_n \alpha_s^n)^* \Rightarrow \sqrt{\alpha_s}$
* $\alpha_\tau, \alpha_R \rightarrow$ I.R. Fixed point
* $\alpha_\tau \cong 0.9$

$$\Rightarrow \alpha_{excl}(Q^2) = \frac{1}{4\pi} \frac{Q^2 F_\pi(Q^2)}{|Q^2 F_{\pi\gamma}(Q^2)|^2} \cong 0.8$$

* Template for QCD
* Commensurate Scale Relations
* Systematically correct for $\beta \neq 0$, $m_q \neq 0$

Summary $\boxed{\text{Near-Conformal QCD}}$

* AdS/CFT ⇒ New insights into
  3+1 Conformal Gauge Theory
  non-perturbative

- Hard scattering: $r \sim Q R^2$ in AdS_5

  ⇒ QCD conformal predictions
  LFWFs, counting rules

+ Non-conformal ansatz $h(r) \neq 1$
  ⇒ QCD (3+1) Phenomena

* Conformal symmetry Template for QCD

  - Commensurate Scale Relations
  - Effective Charges
  - IR Fixed Point Behavior
  - Analytic Thresholds, Unification

* No scale or scheme ambiguity

G. de Teramond
SJB

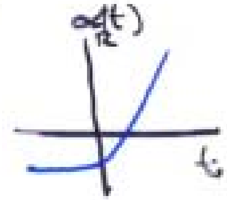
Consequences of Conformal Scaling of $\psi_{n/h}(x, k_\perp, \lambda)$

* $F_{\Delta S_z=0}(q^2) \sim \frac{1}{(q^2)^{n-1}}$ mod logs/anom dim.

* $q^2 F_2(q^2)/F_1(q^2) \sim$ const mod logs/anom dim.

* N_c large: suppression of pinch contrib.

* dominance of quark interchange



* $\alpha_R(t) \to$ neg. integer

* $\frac{d\sigma}{dt}(AB \to CD) \sim \frac{1}{s^{\sum n_i - 2}} F(\theta_c)$

* Non pert. Normalization: $\sum_n C_n \alpha_s^n \Rightarrow \sqrt{\alpha_s}$

* Orbital dependence of LFWF — Smirnov Karmanov, Hiller Hwang Karmanov, SJB

* x-dep of distribution amplitudes — D. Müller, V. Braun, P. Ball, Korchemsky

RESEARCH NEWS

A Theory of Everything Takes Shape

String theorists have broken an impasse and may be on their way to converting this mathematical structure—physicists' best hope for unifying gravity and quantum theory—into a single, coherent theory

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FRANK & ERNEST *by BOB THAVES*