Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Conformal behavior at short distances + Confinement at large distance

Semi-Classical QCD / Wave Equations

Boost Invariant 3+1 Light-Front Wave Equations

Hadron Spectra, Wavefunctions, Dynamics

Integrable!

Holography
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k_\perp^2) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \]

"Soft Wall" model

\[ \kappa = 0.375 \text{ GeV} \]

massless quarks

Note coupling \( k_\perp^2, x \)

Connection of Confinement to TMDs
Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \, \frac{1}{g_5^2} \, G^2$$

Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} \, g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement.

YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_{0}^{\infty} \zeta \, d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

\[ \frac{\alpha_s(Q)}{\pi} = e^{-Q^2/4\kappa^2} \]

\[ \kappa = 0.54 \text{ GeV} \]

Deur, de Teramond, sjb
\[ \beta_{\text{AdS}}(Q^2) = \frac{d}{d \log Q^2} \alpha_{s}^{\text{AdS}}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2} \]
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Running Coupling from nonperturbative dynamics
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$. Spectrum is independent of $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
### Light-Front QCD

#### Heisenberg Equation

\[
H_{QCD}^{LC} |\psi_h\rangle = M_h^2 |\psi_h\rangle
\]

### Use AdS/QCD basis functions

### QCD and Light-Front Holography

---

**Zakopane**

**June 11-12, 2010**

*Stan Brodsky, SLAC & CP3*
Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- Hamiltonian light-front field theory within an AdS/QCD basis.

J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,
Key QCD Questions for Phenomenology

- Quark and Gluon Confinement
- Hadronic Spectrum
- Light Front Wavefunctions, Distribution Amplitudes
- Running Coupling at Low Scales
- Rescattering Phenomena
- Hadronization at Amplitude Level
- Heavy Quark Distributions
Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

Munger, Schmidt, sjb

\[ \bar{H}(\bar{p}e^+) \]

\[ b_\perp \leq \frac{1}{m_{red} \alpha} \]

\[ y_{\bar{p}} \simeq y_{e^+} \]

Coalescence of off-shell co-moving positron and antiproton.

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

\[ \tau = x^+ \]
Features of LF T-Matrix Formalism

“Event Amplitude Generator”

• Coalesce color-singlet cluster to hadronic state if
  \[ \mathcal{M}_n^2 = \sum_{i=1}^{n} \frac{k_{i\perp}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2 \]

• The coalescence probability amplitude is the LF wavefunction
  \[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

• No IR divergences: Maximal gluon and quark wavelength from confinement

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i} \quad \text{QCD and Light-Front Holography} \]

\[ P^+ = P^0 + P^z \]
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Baryon Production

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Higher Fock State Coalescence \[ |uuds\bar{s}\bar{s}\rangle \]

Asymmetric Hadronization! \[ D_s \rightarrow p(z) \neq D_s \rightarrow \bar{p}(z) \]

B-Q Ma, sjb

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QCD and Light-Front Holography
Stan Brodsky, SLAC & CP3
• Same principle as antihydrogen production: off-shell coalescence

• coalescence to hadron favored at equal rapidity, small transverse momenta

• leading heavy hadron production: D and B mesons produced at large $z$

• hadron helicity conservation if hadron LFWF has $L^z = 0$

• Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i \]

\[ P^+ = P^0 + P^z \]

QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
In the limit where \((1-x_F)Q^2\) is fixed as \(Q^2 \to \infty\)

**Light-Front Wavefunctions from AdS/CFT**

Entire pion \(\psi\) contributes to hard process

Virtual photon is longitudinally polarized

\[ \pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F \]

\[ x \rightarrow 0 \quad \text{and} \quad x \rightarrow 1 \]

Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer Vanttinen
Pion appears directly in subprocess at large $x_F$

All of the pion’s momentum is transferred to the lepton pair

Lepton Pair is produced longitudinally polarized
\[ \pi^- N \rightarrow \mu^+ \mu^- X \text{ at } 80 \text{ GeV/c} \]

\[ \frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \rho \sin 2\theta \cos \phi + \omega \sin^2 \theta \cos 2\phi. \]

\[ \frac{d^2\sigma}{dx_\pi d\cos \theta} \propto x_\pi \left( (1-x_\pi)^2 (1+\cos^2 \theta) + \frac{4}{9} \frac{\langle k_t^2 \rangle}{M^2} \sin^2 \theta \right) \]

\[ \langle k_t^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2 \]

\[ Q^2 = M^2 \quad x_\pi = x_\bar{q} \]

Dramatic change in angular distribution at large \( x_F \)

Example of a higher-twist direct subprocess

Chicago-Princeton Collaboration

Stan Brodsky, SLAC & CP3

QCD and Light-Front Holography

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Crucial Test of Leading-Twist QCD: Scaling at fixed $x_T$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E \frac{d\sigma}{d^3p} (pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

**Parton model:** $n_{eff} = 4$

**As fundamental as Bjorken scaling in DIS**

**Conformal scaling:** $n_{eff} = 2 n_{active} - 4$
\[ pp \rightarrow \gamma X \]

\[ E \frac{d\sigma}{d^3p}(pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4} \]

\[ gu \rightarrow \gamma u \]

\[ n_{active} = 4 \]

\[ n_{eff} = 2n_{active} - 4 \]

\[ n_{eff} = 4 \]
\[ \sqrt{s^n} E \frac{d\sigma}{d^3p} (pp \to \gamma X) \text{ at fixed } x_T \]

\[ x_T \text{-scaling of direct photon production is consistent with PQCD } \]
Leading-Twist Contribution to Hadron Production

\[ \frac{d\sigma}{d^3p/E} = \alpha_s^2 \frac{F(x_\perp, y)}{p_\perp^4} \]

Parton model and Conformal Scaling:

\[ G_{q/p}(x_1, p_\perp^2) \]

\[ G_{q/p}(x_2, p_\perp^2) \]

\[ D_{\pi/q}(z, p_\perp^2) \]

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QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

\[ \frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp^n(x_\perp)} \]

Key test of PQCD: power-law fall-off at fixed \( x_T \)

PP → \( \pi X \)

PP → \( \gamma X \)

CTEQ6.6 PDF
DSS/BFG FF
scales\( = p_\perp \)
y=0

5 < \( p_\perp \) < 20 GeV

70 GeV < \( \sqrt{s} \) < 4 TeV

Arleo, Aurenche, Hwang, Sickles, sjb

Pirner, Raufaisen, sjb

DSS (De Florian-Sassot-Stratmann)
\[ d\sigma(h_a h_b \rightarrow hX) = \sum_{abc} G_{a/h_a}(x_a) G_{b/h_b}(x_b) dx_a dx_b \frac{1}{2\hat{s}} |A_{fi}|^2 dX_f D_{h/c}(z_c) dz_c. \]

\[ E \frac{d^3\sigma(h_a h_b \rightarrow hX)}{d^3p} = \frac{F(y, x_{R})}{p_{T}^{n(y, x_{R})}}. \]

\[ n = 2n_{active} - 4, \]

\[ n_{eff}(p_T) = -\frac{d \ln E \frac{d^3\sigma(h_a h_b \rightarrow hX)}{d^3p}}{d \ln (p_T)} \]

\[ n_{eff} \sim 4.5 \]

\[ E \frac{d^3\sigma(h_a h_b \rightarrow hX)}{d^3p} = \left[ \frac{\alpha_s(p_{T}^2)}{p_{T}^2} \right]^{n_{active}-2} \frac{(1 - x_{R})^{2n_s-1+3\xi(p_T)}}{x_{R}^{\lambda(p_T)}} \alpha_s^{2n_s} \left( k_{x_R}^2 \right) f(y). \]

\[ \xi(p_T) = \frac{C_R}{\pi} \int_{k_{x_R}^2}^{p_{T}^2} dk_{\perp}^2 \alpha_s(k_{\perp}^2) = \frac{4C_R}{\beta_0} \ln \frac{\ln(p_{T}^2/\Lambda_{QCD}^2)}{\ln(k_{x_R}^2/\Lambda_{QCD}^2)}. \]
\[ E \frac{d\sigma}{d^3 p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{cm} = \pi/2)}{p_T^n} \]

Clear evidence for higher-twist contributions

\[ x_T = 2p_T/\sqrt{s} \]

\[ x_T = 2p_T/\sqrt{s} \]
• Significant increase of the hadron $n^{\text{exp}}$ with $x_\perp$
  - $n^{\text{exp}} \simeq 8$ at large $x_\perp$
• Huge contrast with photons and jets!
  - $n^{\text{exp}}$ constant and slight above 4 at all $x_\perp$
Protons produced in AuAu collisions at RHIC do not exhibit clear scaling properties in the available $p_T$ range. Shown are data for central (0 – 5%) and for peripheral (60 – 90%) collisions.

Continuous rise of $n_{eff}$ with $x_T$.

$E \frac{d\sigma}{d^3p} (pN \rightarrow pX) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}} x_T$

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June 11-12, 2010

QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
\[ E \frac{d\sigma}{d^3p} (pp \rightarrow HX) = \frac{F(x_T, \theta_{CM})}{p_T n_{eff}} \]

Trend consistent with RHIC at small \( x_T \)
\[ [\sqrt{s}]^n \frac{d\sigma}{d^3p/E} (pp \rightarrow \pi^0 X) \text{ at fixed } x_T = \frac{2p_T}{\sqrt{s}} \]
$$E \frac{d\sigma}{d^3p} (pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{eff}}}$$

$\gamma, \text{jets}$

$\pi$

$\sqrt{s}=38.8/31.6 \text{ GeV } E706$

$\sqrt{s}=62.4/22.4 \text{ GeV } \text{PHENIX/FNAL}$

$\sqrt{s}=62.8/52.7 \text{ GeV } R806$

$\sqrt{s}=52.7/30.6 \text{ GeV } R806$

$\sqrt{s}=200/62.4 \text{ GeV } \text{PHENIX}$

$\sqrt{s}=500/200 \text{ GeV } \text{UA1}$

$\sqrt{s}=900/200 \text{ GeV } \text{UA1}$

$\sqrt{s}=1800/630 \text{ GeV } \text{CDF}$

Arleo, Aurenche, Hwang, Sickles, sjb

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Leading-Twist Contribution to Hadron Production

\[ \frac{d\sigma}{d^3p/E} = \alpha_s^2 \frac{F(x_\perp, y)}{p_\perp^4} \]

Parton model and Conformal Scaling:
Higher-Twist Contribution to Hadron Production

\[ \frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x^2, y)}{p^6} \]

No Fragmentation Function -- Minimal \( x_1, x_2 \)
Direct Proton Production

Explains “Baryon anomaly” at RHIC

\[ n_{\text{active}} = 6 \]

\[ E \frac{d\sigma}{d^3p} (p \ p \rightarrow p \ X) \sim \frac{F(x_\perp, y^{\text{cm}})}{p^8_\perp} \]

Sickles, sjb
Protons less absorbed in nuclear collisions than pions because of dominant color transparent higher twist process.

Particle ratio changes with centrality!


Tannenbaum: “Baryon Anomaly”
The $p/\pi^+$ and $\bar{p}/\pi^-$ ratios as a function of $p_T$ increase dramatically to values $\sim 1$ as a function of centrality in Au + Au collisions at RHIC which was totally unexpected and is still not fully understood.

Particle ratios change with centrality!

Protons less absorbed in nuclear collisions than pions!

Central

Peripheral

“Baryon Anomaly”
Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Collision can produce 3 collinear quarks

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
Baryon made directly within hard subprocess

\[ b_\perp \approx 1 \text{ fm} \]

\[ b_\perp \approx 1/p_T \]

\[ uu \rightarrow p\bar{d} \]

\[ n_{\text{active}} = 6 \]

\[ n_{\text{eff}} = 2n_{\text{active}} - 4 \]

\[ n_{\text{eff}} = 8 \]

Formation Time proportional to Energy

Small color-singlet
Color Transparent
Minimal same-side energy

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QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
Proton trigger: $2.5 < p_T < 4.0$ GeV/c
associated: $1.8 < p_T < 2.5$ GeV/c

Proton production more dominated by color-transparent direct high-$n_{\text{eff}}$ subprocesses
Power-law exponent $n(x_T)$ for $\pi^0$ and $h$ spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV

Baryon can be made directly within hard subprocess

Collision can produce 3 collinear quarks

Coalescence within hard subprocess

\[ uu \rightarrow p\bar{d} \]

\[ \phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2 \]

Bjorken Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

Baryon anomaly

\[ qq \rightarrow B\bar{q} \]

\[ n_{active} = 6 \]

\[ n_{eff} = 2n_{active} - 4 \]

\[ n_{eff} = 8 \]

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QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
Lambda can be made directly within hard subprocess

Coalescence within hard subprocess

$ud \rightarrow \Lambda \bar{s}$

Small color-singlet
Color Transparent
Minimal same-side energy

$\bar{s}$ produced on away side!

$n_{active} = 6$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 8$
Baryon to Meson Ratios

\[ \frac{p}{\pi} \]

\[ \frac{\Lambda}{2K_0} \]

Transverse Momentum \( p_T \) (GeV/c)

\( \Delta \) Central Au+Au: PHENIX

\( \ast \) Central Au+Au: STAR

\( \bullet \) p+p NSD: STAR

\( \text{green} \) e\(^{+}\)+e\(^{-}\) → ggg: ARGUS

\( \text{diamond} \) e\(^{+}\)+e\(^{-}\) → q\(\bar{q}\): ARGUS

\( \Delta \) Central Au+Au: STAR

\( \text{green} \) 40%-60% central: STAR

\( \bullet \) 200 GeV p+p: STAR

\( \bigcirc \) 630 GeV p+p: UA1

Paul Sorensen
**Baryon Anomaly: Evidence for Direct, Higher-Twist Subprocesses**

- Explains anomalous power behavior at fixed $x_T$
- Protons more likely to come from direct higher-twist subprocess than pions
- Protons less absorbed than pions in central nuclear collisions because of color transparency
- Predicts increasing proton to pion ratio in central collisions
- Proton power $n_{\text{eff}}$ increases with centrality since leading twist contribution absorbed
- Fewer same-side hadrons for proton trigger at high centrality
- Exclusive-inclusive connection at $x_T = 1$
**RHIC/LHC predictions**

**PHENIX results**

Scaling exponents from $\sqrt{s} = 500$ GeV preliminary data

[A. Bezilevsky, APS Meeting]

- Magnitude of $\Delta$ and its $x_\perp$-dependence consistent with predictions
What is the dynamical mechanism which creates the QGP?

- How do the parameters of the QGP depend on the initial and final state conditions?
- A dynamical model: “Gluonic Laser”
Gluonic Laser

Gluonic bremsstrahlung from initial hard scattering backscatters on nuclear "mirrors"

Gluonic Laser

\[ gq \rightarrow \gamma q \]

analog of laser backscattering in QED

QCD cascade mechanism for forming quark-gluon plasma inside overlap ellipse

Coherent

QCD and Light-Front Holography

Zakopane
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Stan Brodsky, SLAC & CP3
Possible time sequence of a RHIC Ion-Ion Collision

- Nuclei collide; nucleons overlap within an ellipse
- Initial hard collision between quarks and/or gluons producing high $p_T$ trigger hadron or photon
- Induced gluon radiation radiated from initial parton collision
- Collinear radiation back-scatters on other incoming partons
- Cascading gluons creates multi-parton quark-gluon plasma within ellipse, thermalization
- Stimulated radiation contributes to energy loss of away-side jet
- Coherence creates hadronic momentum along minor axis
- Same final state for high $p_T$ direct photons and mesons
- Baryons formed in higher-twist double-scattering process at high $x_T$; double induced radiation and thus double $v_2$. 
Consequences of Gluon Laser Mechanism

Ridge created by trigger bias (Cronin effect)
Momenta of initial colored partons biased towards trigger

Soft gluon radiation from initial state partons emitted in plane of production; fills rapidity

Quantum Coherent

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QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
Chiral Symmetry Breaking in AdS/QCD

- Chiral symmetry breaking effect in AdS/QCD depends on weighted $z^2$ distribution, not constant condensate

\[ \delta M^2 = -2m_q \langle \bar{\psi}\psi \rangle \times \int dz \ \phi^2(z)z^2 \]

- $z^2$ weighting consistent with higher Fock states at periphery of hadron wavefunction

- AdS/QCD: confined condensate

- Suggests “In-Hadron” Condensates

Erlich et al.

de Teramond, Shrock, sjb
In presence of quark masses the Holographic LF wave equation is \( (\zeta = z) \)

\[
\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta),
\]

(1)

and thus

\[
\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.
\]

(2)

The parameter \( a \) is determined by the Weisberger term

\[
a = \frac{2}{\sqrt{x}}.
\]

Thus

\[
X(z) = \frac{m}{\sqrt{x}} z - \sqrt{x} \langle \bar{\psi} \psi \rangle z^3,
\]

(3)

and

\[
\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi}\psi \rangle \langle z^2 \rangle + \langle \bar{\psi}\psi \rangle^2 \langle z^4 \rangle,
\]

(4)

where we have used the sum over fractional longitudinal momentum \( \sum_i x_i = 1 \).

**Mass shift from dynamics inside hadronic boundary**
Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.\(^1\) Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.\(^2\) A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.\(^3\)
Bethe-Salpeter Analysis

\[ f_H P^\mu = Z_2 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} \left[ T_H \gamma_5 \gamma^\mu S(\frac{1}{2} P + q) \right] \Gamma_H(q; P) S(\frac{1}{2} P - q) \]

f_H Meson Decay Constant
T_H flavor projection operator,
Z_2(\Lambda), Z_4(\Lambda) renormalization constants
S(p) dressed quark propagator
\Gamma_H(q; P) = F.T.\langle H|\psi(x_a)\bar{\psi}(x_b)|0 \rangle
Bethe-Salpeter bound-state vertex amplitude.

\[ i \rho^H_\zeta \equiv \frac{-\langle q\bar{q}\rangle^H_\zeta}{f_H} = Z_4 \int^\Lambda \frac{d^4 q}{(2\pi)^4} \frac{1}{2} \left[ T_H \gamma_5 S(\frac{1}{2} P + q) \right] \Gamma_H(q; P) S(\frac{1}{2} P - q) \]

In-Hadron Condensate!

\[ f_H m^2_H = -\rho^H_\zeta \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q\in H} m_q \]

\[ m^2_\pi \propto (m_q + m_{\bar{q}})/f_\pi \quad \text{GMOR} \]
Simple physical argument for “in-hadron” condensate

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

\[ \langle B | \bar{q}q | B \rangle \text{ not } \langle 0 | \bar{q}q | 0 \rangle \]
**Higher Light-Front Fock State of Pion Simulates DCSB**

\[
f_\pi P^+ = \langle 0 | \bar{q} \gamma^5 \gamma^+ q | \pi \rangle
\]

**Instantaneous quark propagator contribution to \( \pi \) derived from higher Fock state**

\[
i \rho_\pi = \langle 0 | \bar{q} \gamma^5 q | \pi \rangle
\]

**Higher Fock state acts like mass insertion**

Roberts, Tandy, Shrock, sjb
Essence of the vacuum quark condensate

Stanley J. Brodsky,¹,² Craig D. Roberts,³,⁴ Robert Shrock,⁵ and Peter C. Tandy⁶

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⁶Center for Nuclear Research, Department of Physics, Kent State University, Kent OH 44242, USA

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

PACS numbers: 11.30.Rd; 14.40.Be; 24.85.+p; 11.15.Tk

ArXiv: 10005.4610
Quark and Gluon condensates reside within hadrons, not vacuum

- Bound-State Dyson Schwinger Equations
- AdS/QCD
- Analogous to finite size superconductor
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict

R. Shrock, sjb
PNAS
ArXiv:0905.1151

Casher and Susskind  Maris, Roberts, Tandy  Shrock and sjb
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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\( (\Omega_\Lambda)_{QCD} \sim 10^{45} \)

\( (\Omega_\Lambda)_{EW} \sim 10^{56} \)

\( \Omega_\Lambda = 0.76 (\text{expt}) \)

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb

``Condensates in Quantum Chromodynamics and the Cosmological Constant.”
Quark and Gluon condensates reside within hadrons, not LF vacuum.

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant

"Confined QCD Condensates"
Determinations of the vacuum Gluon Condensate

\[
< 0 | \frac{\alpha_s}{\pi} G^2 | 0 > \ [\text{GeV}^4]
\]

\[-0.005 \pm 0.003 \text{ from } \tau \text{ decay.} \]
\[+0.006 \pm 0.012 \text{ from } \tau \text{ decay.} \]
\[+0.009 \pm 0.007 \text{ from charmonium sum rules} \]

Davier et al.
Geshkenbein, Ioffe, Zyablyuk
Ioffe, Zyablyuk

Consistent with zero vacuum condensate
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
\[ H_{QCD}^{LF} \]

\[ (H_{LF}^0 + H_{LF}^I)|\Psi\rangle \geq M^2|\Psi\rangle \]

\[ \begin{aligned} &\left[ \frac{\overline{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \overline{k}_\perp) = M^2 \psi_{LF}(x, \overline{k}_\perp) \\ &\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta) \end{aligned} \]

\[ U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \]

QCD Meson Spectrum

Coupled Fock states

Effective two-particle equation

Azimuthal Basis \( \zeta, \phi \)

Confining AdS/QCD potential

Semiclassical first approximation to QCD
An analytic first approximation to QCD
AdS/QCD + Light-Front Holography

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable $\zeta$ conjugate to invariant mass squared
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales: Essential for Gauge Link phenomena
- Hadron Spectroscopy and Dynamics from one parameter $\kappa$
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates: Zero cosmological constant!
- Systematically improvable with DLCQ Methods
Deeply Virtual Compton Scattering (DVCS)

\[ H(x,\xi,t), E(x,\xi,t), \ldots \]

"Generalized Parton Distributions"
Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

QCD and Light-Front Holography
Stan Brodsky, SLAC & CP3

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Example of LFWF representation of GPDs \((n+1 \Rightarrow n-1)\)

\[
\frac{1}{\sqrt{1-\zeta}} \frac{\Delta_1 - i \Delta_2}{2M} E_{(n+1 \rightarrow n-1)}(x, \zeta, t)
\]

\[
= (\sqrt{1-\zeta})^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{dx_i \, d^2 \vec{k}_{\perp i}}{16\pi^3} \left( 1 - \sum_{j=1}^{n+1} x_j \right) \delta^{(2)} \left( \sum_{j=1}^{n+1} \vec{k}_{\perp j} \right)
\]

\[
\times 16\pi^3 \delta(x_{n+1} + x_1 - \zeta) \delta^{(2)}(\vec{k}_{\perp n+1} + \vec{k}_{\perp 1} - \vec{\Delta}_{\perp})
\]

\[
\times \delta(x - x_1) \psi_{(n-1)}^{(n-1)}(x_i', \vec{k}_{\perp i}', \lambda_i) \psi_{(n+1)}^{(n+1)}(x_i, \vec{k}_{\perp i}, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}},
\]

where \(i = 2, \ldots, n\) label the \(n - 1\) spectator partons which appear in the final-state hadron wavefunction with

\[
x_i' = \frac{x_i}{1 - \zeta}, \quad \vec{k}_{\perp i}' = \vec{k}_{\perp i} + \frac{x_i}{1 - \zeta} \vec{\Delta}_{\perp}.
\]
**J=0 Fixed Pole Contribution to DVCS**

- J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator

Real amplitude, independent of $Q^2$ at fixed $t$

Szczepaniak, Llanes-Estrada, sjb
Close, Gunion, sjb
Deeply Virtual Compton Scattering

$\gamma^* p \rightarrow \gamma p$

Hard Reggeon Domain

$s >> -t, Q^2 >> \Lambda^2_{QCD}$

$T(\gamma^*(q)p \rightarrow \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s^\alpha_R(t) \beta_R(t)$

$\alpha_R(t) \rightarrow 0$

Reflects elementary coupling of two photons to quarks

$\beta_R(t) \sim \frac{1}{t^2}$

$\frac{d\sigma}{dt} \sim \frac{1}{s^2} \frac{1}{t^4} \sim \frac{1}{s^6}$ at fixed $\frac{Q^2}{s}, \frac{t}{s}$
Deeply Virtual Compton Scattering

\[ \gamma^* p \rightarrow \gamma p \]

\[ T(\gamma^*(q)p \rightarrow \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s_R^\alpha(t) \beta_R(t) \]

\[ \alpha_R(t) \rightarrow 0 \]

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\[ \frac{d\sigma}{dt} \sim \frac{1}{s^2} \frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]

Seagull interaction
(instantaneous quark exchange or Z-graph)

Hard Reggeon Domain

\[ s >> -t, Q^2 >> \Lambda^2_{QCD} \]

Reflects elementary coupling of two photons to quarks
Regge domain

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_{R} s_{R}^\alpha(t) \beta_{R}(t) \quad s \gg -t, Q^2 \]

Fundamental test of QCD

\[ \frac{d\sigma}{dt}(\gamma^* p \rightarrow \gamma p) \rightarrow \frac{1}{s^2} \beta_{R}^2(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s} \]
Exclusive Electroproduction

\[ ep \rightarrow e' \pi^+ n \]

**Hard Reggeon Domain**

\[ s >> -t, Q^2 >> \Lambda_{QCD}^2 \]

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s^\alpha_R(t) \beta_R(t) \]

\[ \alpha_R(t) \rightarrow -1 \]

\[ \beta_R(t) \sim \frac{1}{t^2} \]

Reflects elementary exchange of quarks in t-channel

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]
Regge domain

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^\alpha(t) \beta_R(t) \quad s >> -t, Q^2 \]

\[ \frac{d\sigma}{dt}(\gamma^* p \rightarrow \pi^+ n) \rightarrow \frac{1}{s^3} \beta^2_R(t) \]

Fundamental test of QCD

Reflects elementary exchange of quarks in t-channel

\[ \alpha_R(t) \rightarrow -1 \text{ at } t \rightarrow -\infty \]

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^3} \frac{1}{t^4} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]
Effective two-photon contact term

Seagull for scalar quarks

Real phase

\[ M = s^0 \sum e_q^2 F_q(t) \]

Independent of \( Q^2 \) at fixed \( t \)

\( <1/x> \) Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

No ambiguity in D-term

\( Q^2 \)-independent contribution to Real DVCS amplitude

\[ s^2 \frac{d\sigma}{dt}(\gamma^*p \rightarrow \gamma p) = F^2(t) \]
Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

QCD S- and P- Coulomb Phases --Wilson Line

Leading-Twist Rescattering Violates pQCD Factorization!

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QCD and Light-Front Holography

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Stan Brodsky, SLAC & CP3
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale!
- New window to QCD coupling and running gluon mass in the IR
- QED S and P Coulomb phases infinite -- difference of phases finite!
- Alternate: Retarded and Advanced Gauge: Augmented LFWFs

Pasquini, Xiao, Yuan, sjb Mulders, Boer Qiu, Sterman

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QCD and Light-Front Holography Stan Brodsky, SLAC & CP3
Final State Interactions Produce T-Odd (Sivers Effect)

- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment
- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero anomalous gavitomagnetic moment)

\[ \vec{S} \cdot \vec{p}_{jet} \times \vec{q} \]

Hwang, Schmidt. sjb; Burkardt
Measure single-spin asymmetry $A_N$ in Drell-Yan reactions

Leading-twist Bjorken-scaling $A_N$ from $S, P$-wave initial-state gluonic interactions

Predict: $A_N(DY) = -A_N(DIS)$ Opposite in sign!

$pp \uparrow \rightarrow \ell^+ \ell^- X$

$\vec{S} \cdot \vec{q} \times \vec{p}$ correlation
Drell-Yan angular distribution

Unpolarized DY

- Experimentally, a violation of the Lam-Tung sum rule is observed by sizeable $\cos^2\Phi$ moments
- Several model explanations
  - higher twist
  - spin correlation due to non-trivial QCD vacuum
  - Non-zero Boer Mulders function

\[
\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2\theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2\theta \cos 2\phi \right)
\]

Experiment: $\nu \approx 0.6$

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QCD and Light-Front Holography

Stan Brodsky, SLAC & CP3
Measurement of Angular Distributions of Drell-Yan Dimuons in $p + d$ Interaction at 800 GeV/c

(FNAL E866/NuSea Collaboration)

Parameter $\nu$ vs. $p_T$ in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4 \text{ GeV}/c^2$ are also shown.

Huge Effect in $\pi W \rightarrow \mu^+\mu^- X$
Negligible Effect $pd \rightarrow \mu^+\mu^- X$
DY $\cos 2\phi$ correlation at leading twist from double ISI

Boer, Hwang, sjb
DY $\cos 2\phi$ correlation at leading twist from double ISI

Product of Boer-Mulders Functions

$$h_1^\perp(x_1, p_\perp^2) \times \bar{h}_1^\perp(x_2, k_\perp^2)$$
T-Odd: Require ISI or FSI

Unpolarized Distribution

Boer-Mulders Function

Bj Sum Rule

Transversity

Sivers Function
Double Initial-State Interactions

generate anomalous $\cos 2\phi$

Boer, Hwang, sjb

Drell-Yan planar correlations

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$

PQCD Factorization (Lam Tung): $1 - \lambda - 2\nu = 0$

Violates Lam-Tung relation!
\( \text{DY} \cos 2\phi \) correlation at leading twist from double ISI

\[
F = \mathcal{F} \left[ \left( 2 \hat{h} \cdot p_\perp \hat{h} \cdot k_\perp - p_\perp \cdot k_\perp \right) h_1^+ \bar{h}_1^+ \right]
\]

\[
= \int d^2p_\perp d^2k_\perp \delta^2 (p_\perp + k_\perp - q_\perp) \left( 2 \hat{h} \cdot p_\perp \hat{h} \cdot k_\perp - p_\perp \cdot k_\perp \right) \times h_1^+(\Delta, p_\perp^2) \bar{h}_1^+(\bar{\Delta}, k_\perp^2),
\]

(40)

\[
G = \mathcal{F} [f_1 \bar{f}_1]
\]

\[
= \int d^2p_\perp d^2k_\perp \delta^2 (p_\perp + k_\perp - q_\perp) f_1(\Delta, p_\perp^2) \bar{f}_1(\bar{\Delta}, k_\perp^2),
\]

Product of Boer-Mulders Functions

\[
h_1^+(x_1, p_\perp^2) \times \bar{h}_1^+(x_2, k_\perp^2)
\]
### Static
- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and $J^z$
- DGLAP Evolution; mod. at large $x$
- No Diffractive DIS

### Dynamic
- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS

\[ \psi_n(x_i, \vec{k}_\perp, \lambda_i) \]
Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer

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QCD and Light-Front Holography
Stan Brodsky, SLAC & CP3
The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.
Problem for factorization when both ISI and FSI occur
$\cos 2\phi$ correlation for quarkonium production at leading twist from double ISI
Enhanced by gluon color charge
Anti-Shadowing

\[ Q^2 = 5 \text{ GeV}^2 \]

Nuclear Shadowing in QCD

Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF!

Dynamical effect due to virtual photon interacting in nucleus
The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken $x_B$:

$$\frac{1}{Mx_B} = \frac{2\nu}{Q^2} \geq L_A.$$ 

If the scattering on nucleon $N_1$ is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $q$ flux reaching $N_2$.

→ Shadowing of the DIS nuclear structure functions.

**Observed HERA DDIS produces nuclear shadowing**
Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate $T$-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Antishadowing (Reggeon exchange) is not universal!

Schmidt, Yang, sjb
$Q^2 = 5 \text{ GeV}^2$

Extrapolations from NuTeV

SLAC/NMC data

Scheinbein, Yu, Keppel, Morfin, Olness, Owens
Antiquark interacts with target nucleus at energy $\tilde{s} \propto \frac{1}{x_{bj}}$

Regge contribution: $\sigma_{\bar{q}N} \sim \tilde{s}^{\alpha_R-1}$

Nonsinglet Kuti-Weisskoff $F_{2p} - F_{2n} \propto \sqrt{x_{bj}}$ at small $x_{bj}$.

Shadowing of $\sigma_{\bar{q}M}$ produces shadowing of nuclear structure function.
Phase of two-step amplitude relative to one step:

$$\frac{1}{\sqrt{2}}(1 - i) \times i = \frac{1}{\sqrt{2}}(i + 1)$$

Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

Critical test: Tagged Drell-Yan
Shadowing and Antishadowing in Lepton-Nucleus Scattering

- Shadowing: **Destructive Interference** of Two-Step and One-Step Processes
  *Pomeron Exchange*

- Antishadowing: **Constructive Interference** of Two-Step and One-Step Processes!
  *Reggeon and Odderon Exchange*

- Antishadowing is Not Universal!
  Electromagnetic and weak currents: different nuclear effects!
  Potentially significant for NuTeV Anomaly}
Nuclear Antishadowing not universal!

Schmidt, Yang; sjb

Modifies NuTeV extraction of \( \sin^2 \theta_W \)
• Color Confinement: Maximum Wavelength of Quark and Gluons

• Conformal symmetry of QCD coupling in IR

• Conformal Template (BLM, CSR, BFKL scale)

• Motivation for AdS/QCD

• QCD Condensates inside of hadronic LFWFs

• Technicolor: confined condensates inside of technihadrons -- alternative to Higgs

• Simple physical solution to cosmological constant conflict with Standard Model
An analytic first approximation to QCD
AdS/QCD + Light-Front Holography

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable $\zeta$ conjugate to invariant mass squared
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- QCD Coupling at all scales: Essential for Gauge Link phenomena
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