AdS/QCD and Novel Heavy-Ion Phenomena

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• **Light-Front Holography**

\[ \phi(z) \]

- **Light-Front Wavefunctions:**
  Schrödinger Wavefunctions of Hadron Physics

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond

Novel Heavy-Ion Phenomena

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Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances
- Analogous to the Schrodinger Theory for Atomic Physics
  - \textit{AdS/QCD Light-Front Holography}
  - \textit{Hadronic Spectra and Light-Front Wavefunctions}
Conformal Theories are invariant under the Poincare and conformal transformations with 

$$M^{\mu\nu}, P^\mu, D, K^\mu,$$

the generators of $SO(4,2)$

$SO(4,2)$ has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (dx^\mu dx_\mu - dz^2)$$

$\rightarrow$ invariant measure

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z$$

$x^2 = x^\mu dx_\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
We will consider both holographic models.

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
AdS Schrödinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action Dilaton-Modified AdS$_5$
Effective LF Schrödinger wave equation

\[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \] \[ \phi_S(z) = \mathcal{M}^2 \phi_S(z) \]

with eigenvalues \( \mathcal{M}^2 = 2\kappa^2 (2n + 2L + S) \).

Compare with Nambu string result (rotating flux tube):

\[ M_n^2(L) = 2\pi\sigma (n + L + 1/2) \]

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \text{ GeV} \).

Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

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Hard-wall model breaks chiral symmetry!
Casher mechanism

Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ P^+, \vec{P}_\perp \]

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

\[ \sum_{i}^{n} x_i = 1 \]

\[ \sum_{i}^{n} \vec{k}_\perp i = \vec{0}_\perp \]

**Invariant under boosts! Independent of \( P^\mu \)**

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Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_{\perp})$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi > = M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

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Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements.
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! Frame Independent

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x)b^2_\perp.
\]

\[
U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)
\]

G. de Teramond, sjb

soft wall confining potential:

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Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

\[ \mathcal{M}^2 = \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} |\psi(x, \vec{k}_\perp)|^2 + \text{interactions} \]

\[ = \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left(-\vec{\nabla}^2_{\vec{b}_\perp}\right) \psi(x, \vec{b}_\perp) + \text{interactions}. \]

Change variables \((\zeta, \varphi), \zeta = \sqrt{x(1-x)} \vec{b}_\perp: \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left(\zeta \frac{d}{d\zeta}\right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}\)

\[ \mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2}\right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

\[ = \int d\zeta \phi^*(\zeta) \left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi(\zeta) \]
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb

\[ \bar{H}(\bar{p}e^+) \]

\[ b_\perp \leq \frac{1}{m_{\text{red}}\alpha} \]

\[ y_{\bar{p}} \sim y_{e^+} \]

Coalescence of off-shell co-moving positron and antiproton.

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k_\perp^2) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

“Soft Wall” model

\( \kappa = 0.375 \text{ GeV} \)

massless quarks

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Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

- \[ q^2(GeV^2) \]

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

Data Compilation
Baldini, Kloe and Volmer
de Teramond, sjb
See also: Radyushkin

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\[ |\pi^+ > = |u\bar{d} > \]
\[ m_u = 2 \text{ MeV} \]
\[ m_d = 5 \text{ MeV} \]
\[ |K^+ > = |u\bar{s} > \]
\[ m_s = 95 \text{ MeV} \]

\[ |D^+ > = |c\bar{d} > \]
\[ m_c = 1.25 \text{ GeV} \]
\[ |\eta_c > = |c\bar{c} > \]

\[ |B^+ > = |u\bar{b} > \]
\[ m_b = 4.2 \text{ GeV} \]
\[ |\eta_b > = |b\bar{b} > \]
\[ \kappa = 375 \text{ MeV} \]
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit required
- Add heavy quark masses to LF kinetic energy; linear quark mass terms
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
**AdS$_5$ Black hole simulation of temperature**

\[
ds^2 = \frac{R^2}{z^2} \left[ -f(z) dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)} \right]
\]

\[
f(z) = 1 - \frac{z^4}{z_0^4}
\]

**Hawking Temperature**

\[
T_H = \frac{r_0}{\pi R^2} = \frac{1}{\pi z_0}
\]

\[
z = \frac{R^2}{r}
\]

D. T. Son, et al
D. T. Son, et al

\[ \frac{\eta}{s} = \frac{\hbar}{4\pi} \]

Gauge/gravity duality provides unexpected tools to compute the viscosity of some strongly coupled theories.

The class of theories with gravity dual description is limited, but contains very interesting theories with infinite coupling.

The calculation of the viscosity is easy: viscosity \( \propto \) absorption cross section of low-energy gravitons by the black hole.

In this class, the ratio \( \frac{\eta}{s} \) is equal to a universal number \( \frac{\hbar}{4\pi} \), much smaller than in any other system in Nature.

The ratio \( \frac{\eta}{s} \) is the measure of perfectness of the QGP.

sjb: \textit{AdS/CFT gives a model of perfect quantum coherence.} \textit{Temperature not due to classical heating.}
Are QGP phenomena actually due to Quantum Coherence?

- Large longitudinal coherence at high energies
- Coherence: LPM effect limits energy loss; Glauber theory of nuclear shadowing in DIS
- Color transparency in Diffractive dijets (Ashery)
- Laser cascade mechanism sets up coherent system in central heavy ion collisions
- Ridge: Coherence over large longitudinal momenta
- Large $v_2$: $\Delta p_x \sim \hbar / \Delta x$
- Small $\eta / s \sim \hbar / 4\pi$
Laser Cascade: Quantum Coherent

Uncertainty principle:
Narrow overlap -- peaked transverse momenta

\[ \Delta p_x \sim \frac{\hbar}{\Delta x} \]

Additive rule for coalescing sideways transverse momenta: Flavor-independent?
coalesce quarks via LFWFs

\[ p_T(B)/3 \sim p_T(M)/2 \sim \pi/\Delta b \]
M. Kaneta  
(PHENIX)  
QM2004  

\[ v_2 = \langle \cos 2(\phi - \Psi_r) \rangle, \quad \phi = \tan^{-1}\left(\frac{p_y}{p_x}\right) \]
Probing Hot QCD Matter with Hard-Scattered Probes

leading particle

leading particle
The highly relativistic nucleus A hits the nucleus B at rest.

\[ s_{AB} = (p_A + p_B)^2 = M_A^2 + M_B^2 + 2E_{lab}^A M_B \]
\[ p_A = (P^+, \frac{M_A^2 + \ell_\perp^2}{P^+}, \vec{\ell}_\perp) \]
\[ p_B = (P^+, \frac{M_B^2 + \ell_\perp^2}{P^+}, -\vec{\ell}_\perp) \]

Both beams move along the positive z direction, and \( s = (p_A + p_B)^2 = 2M_A^2 + 2M_B^2 + 4\ell_\perp^2 \) is represented by the oppositely directed transverse momenta \( \pm \vec{\ell}_\perp \) of the colliding nuclei.

Note that the value of \( P^+ \) is irrelevant.

As \( \tau \) progresses, the constituents from A and B each interact as their coordinates \( \sigma_i \) and \( \vec{b}_\perp i \) overlap.
Light-Front Description of Heavy Ion Collisions

- Nuclear LFWFs are momentum independent
- No effects on wavefunction from boost
- Process independent
- Three-dimensional
- Small $x$ gluons and sea quarks in any frame
- Dynamical effects arise from interactions
- Wilson line give ISI and FSI
- Nuclear shadowing and antishadowing not in nuclear wavefunction -- Glauber multistep diffractive interactions
What is the dynamical mechanism which creates the QGP?

- How do the parameters of the QGP depend on the initial and final state conditions?

- A dynamical model: “Gluonic Laser”
Gluonic Laser

Gluonic bremsstrahlung from initial hard scattering backscatters on nuclear ``mirrors''

\[ gq \rightarrow \gamma q \]

QCD cascade mechanism for forming quark-gluon plasma inside overlap ellipse

Coherent

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Possible time sequence of a RHIC Ion-Ion Collision

- Nuclei collide; nucleons overlap within an ellipse

- Initial hard collision between quarks and/or gluons producing high $p_T$ trigger hadron or photon

- Induced gluon radiation radiated from initial parton collision

- Collinear radiation back-scatters on other incoming partons

- Cascading gluons creates multi-parton quark-gluon plasma within ellipse, thermalization

- Stimulated radiation contributes to energy loss of away-side jet

- Coherence creates hadronic momentum along minor axis

- Same final state for high $p_T$ direct photons and mesons

- Baryons formed in higher-twist double-scattering process at high $x_T$; double induced radiation and thus double $v_2$. 
Consequences of Gluon Laser Mechanism

Ridge created by trigger bias (Cronin effect)
Momenta of initial colored partons biased towards trigger

Soft gluon radiation from initial state partons emitted in plane of production; fills rapidity

Quantum Coherent

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