QCD at the Light-Front

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

June 17, 2010

Stan Brodsky

SLAC & CP3-Origins
Local Organizing Committee

Chairpersons

- Joannis Papavassiliou (Universidad de Valencia-IFIC, Spain)
- Vicente Vento (Universidad de Valencia-IFIC, Spain)

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- Aurore Courtoy (INFN Sezione di Pavia, Italy)
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Dirac’s Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!

\[ \tau = t + \frac{z}{c} \]

\[ \sigma = ct - z \]

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Instant Form

Front Form

Plessas: Point Form
Each element of flash photograph illuminated at same Light Front time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Causal, Trivial Vacuum

Laser Physics is Light-Front Physics
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

Process Independent
Direct Link to QCD Lagrangian!

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ \sum^n_i x_i = 1 \]

\[ \sum^n_i \vec{k}_{\perp i} = \vec{0}_{\perp} \]
"Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out. " - P.A.M. Dirac (1977)
1967 SLAC Experiment:
Scatter 20 GeV/c Electrons on protons in a Hydrogen Target
Discovery of the Quark Structure of Matter

\[ e^p \rightarrow e^'X \]

Deep inelastic scattering: Experiments on the proton and the observation of scaling

Friedman, Kendall, Taylor: Nobel Prize
Deep inelastic electron-proton scattering

- Rutherford scattering using *very* high-energy electrons striking protons

Discovery of quarks!
$ep ightarrow e'X$

$\nu = 2 M_p \nu$

$Q^2 = \mathbf{q}^2 - \nu^2$

Measure rate as a function of energy loss $\nu$ and momentum transfer $Q$

$$x_{bj} = \frac{1}{\omega_{bj}} = \frac{Q^2}{2q \cdot p}$$

Discovery of Bjorken Scaling

Electron scatters on point-like quarks!
Quarks in the Proton

\[ p = (u \ u \ d) \]

- Feynman & Bjorken: "Parton" model
- Zweig: "Aces, Deuces, Treys"
- Bjorken: Scaling
- Gell Mann: "Three Quarks for Mr. Mark"

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QCD at the Light-Front

1 \text{ fm}
\[ 10^{-15} m = 10^{-13} \text{ cm} \]

Brodsky, SLAC & CP3
Fixed $\tau = t + z/c$

\[
x = \frac{k^+}{p^+} = \frac{k^0 + k^z}{p^0 + p^z} \sim x_{bj} = \frac{Q^2}{2p \cdot q}
\]
\[ x = \frac{k^+}{P^+} \]

Fixed \( \tau = t + \frac{z}{c} \)

\[ p^\mu = (p^+, p^-, p^+_{\perp}) = (P^+, \frac{M^2}{P^+}, 0_{\perp}) \]

\[ q^\mu = (q^+, q^-, q^+_{\perp}) = (0, \frac{2q \cdot p}{P^+}, q^+_{\perp}) \]

\[ p^\mu = (p^+, p^-, p^+_{\perp}) = (P^+, \frac{M^2}{P^+}, 0_{\perp}) \]

\[ q^\mu = (q^+, q^-, q^+_{\perp}) = (0, \frac{2q \cdot p}{P^+}, q^+_{\perp}) \]

\[ 2q \cdot p + M^2 = \frac{(k^+_{\perp} + q^+_{\perp})^2 + m^2}{x} + \frac{k^2_{\perp} + M_s^2}{1 - x} \]

\[ x = \frac{Q^2}{2q \cdot p} = x_{bj} \]

\[ Q^2 = q^2_{\perp} \]

\( P \)-conservation

plus mass, transverse momentum and final-state interaction corrections
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Fixed \( \tau = t + z/c \)

\[ \sum^n_i x_i = 1 \]
\[ \sum^n_i \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Structure functions and other distributions computed from the square of the LFWFs

Goal: Predict all features from first principles

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QCD at the Light-Front
Stan Brodsky, SLAC & CP3
QCD Lagrangian

Generalization of QED

Yang Mills Gauge Principle:
Color Rotation and Phase
Invariance at Every Point of
Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

QCD at the Light-Front

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QED Lagrangian

\[ \mathcal{L}_{\text{QED}} = -\frac{1}{4} Tr(F_{\mu\nu} F^{\mu\nu}) + \sum_{\ell=1}^{n_\ell} i \bar{\Psi}_\ell D_\mu \gamma^\mu \Psi_\ell + \sum_{\ell=1}^{n_\ell} m_\ell \bar{\Psi}_\ell \Psi_\ell \]

\[ i D^\mu = i \partial^\mu - e A^\mu \quad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \]

Yang Mills Gauge Principle:
Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Landau Pole
Fundamental Couplings of QED and QCD

\[ \bar{\psi} \gamma^\mu A^\mu \psi \]

\[ L_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f \]

\[ G^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu - g[A^\mu, A^\nu] \]

Gluon vertices

Gluon self couplings
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z. \]

Conserved LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

\[ l_j^z = -i\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right) \]

\( n-1 \) orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!
Calculation of Form Factors in Equal-Time Theory

**Instant Form**

\[
e_{d d d d d d d d} = \sum_{\text{terms}}\\
\text{Need vacuum-induced currents}
\]

Calculation of Form Factors in Light-Front Theory

**Front Form**

\[
e_{d d d d d d d d} = \sum_{\text{terms}}\\
\text{Complete Answer} \quad \text{Absent for } q^+ = 0 \quad \text{zero !!}
\]
\[ <p + q | j^+(0) | p > = 2p^+ F(q^2) \]

\[ q_\perp^2 = Q^2 = -q^2 \]

\[ q^+ = 0 \quad \vec{q}_\perp \]

\[ x, \vec{k}_\perp \]

\[ x, \vec{k}_\perp + \vec{q}_\perp \]

\[ \psi(x_i, \vec{k}_\perp i) \]

\[ \psi(x_i, \vec{k}'_\perp i) \]

Form Factors are Overlaps of LFWFs

\[ \gamma^* \]

Fixed \( \tau = t + z/c \)

\[ Drell & Yan, West \]

\[ p \]

\[ p + q \]

\[ \vec{q}_\perp \]

\[ \vec{k}'_\perp i = \vec{k}_\perp i + (1 - x_i)\vec{q}_\perp \]

Dressed & Yan, West

\[ \vec{k}'_\perp i = \vec{k}_\perp i - x_i\vec{q}_\perp \]
\( \frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times \)

\[
\left[ -\frac{1}{q_L} \psi^*_a(x_i, k'_{\perp i}, \lambda_i) \psi^\dagger_a(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q_R} \psi^*_a(x_i, k'_{\perp i}, \lambda_i) \psi^\dagger_a(x_i, k_{\perp i}, \lambda_i) \right]
\]

\[
k'_{\perp i} = k_{\perp i} - x_i q_\perp
\]

\[
k'_{\perp j} = k_{\perp j} + (1 - x_j) q_\perp
\]

\[
q_{R,L} = q^x \pm iq^y
\]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

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QED $g-2$ in LFPT

Roscies, Suaya, and sjb

Alternate denominator renormalization
Anomalous gravitomagnetic moment $B(0)$

Terayev, Okun, et al: $B(0)$ Must vanish because of Equivalence Theorem

Hwang, Schmidt, sjb; Holstein et al

$B(0) = 0$

Each Fock State

QCD at the Light-Front

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**J=0 Fixed Pole Contribution to DVCS**

- J=0 fixed pole -- direct test of QCD locality -- from seagull or instantaneous contribution to Feynman propagator

Real amplitude, independent of $Q^2$ at fixed $t$

Szczepaniak, Llanes-Estrada, sjb

Close, Gunion, sjb
QCD Factorization
DVCS in hard-scattering domain

\[ e p \rightarrow e' \gamma p \]

\[ T_H[\gamma^* + (uud) \rightarrow \gamma + (uud)] \]

Universal distribution amplitudes.
Renormalization Group Invariance:
Renormalization scale is unambiguous -- BLM

\[ J=0 \text{ Fixed pole from instantaneous gluon} \]
Hadron Distribution Amplitudes

\[ \phi_M(x, Q) = \int_0^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp) \]

\[ \sum_i x_i = 1 \]

Fixed \( \tau = t + z/c \)

\[ k^2_\perp < Q^2 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE

- Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

Lepage, sjb

Efremov, Radyushkin

Sachrajda, Frishman Lepage, sjb

Braun, Gardi

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QCD at the Light-Front

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Deeply Virtual Compton Scattering

\[ \gamma^* p \rightarrow \gamma p \]

\[ T(\gamma^*(q)p \rightarrow \gamma(k) + p) \sim \epsilon \cdot \epsilon' \sum_R s^\alpha_R(t) \beta_R(t) \]

\[ \alpha_R(t) \rightarrow 0 \]

\[ \beta_R(t) \sim \frac{1}{t^2} \]

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^2} \frac{1}{t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]
Regge domain

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s^\alpha_R(t) \beta_R(t) \quad s \gg -t, Q^2 \]

\[ \alpha_R(t) \rightarrow 0 \text{ at } t \rightarrow -\infty \]

\[ \beta_R(t) \sim \frac{1}{t^2} \]

\[ \frac{d\sigma}{dt}(\gamma^* p \rightarrow \gamma p) \rightarrow \frac{1}{s^2} \beta^2_R(t) \sim \frac{1}{s^2 t^4} \sim \frac{1}{s^6} \text{ at fixed } \frac{t}{s}, \frac{Q^2}{s} \]

Fundamental test of QCD
$J=0$ Fixed pole in real and virtual Compton scattering

Effective two-photon contact term

Seagull for scalar quarks

Real phase

$$M = s^0 \sum e_q^2 F_q(t)$$

Independent of $Q^2$ at fixed $t$

$<1/x>$ Moment: Related to Feynman-Hellman Theorem

Fundamental test of local gauge theory

No ambiguity in D-term

$Q^2$-independent contribution to Real DVCS amplitude

$$s^2 \frac{d\sigma}{dt} (\gamma^* p \rightarrow \gamma p) = F^2(t)$$
**Exclusive Electroproduction**

$$ep \rightarrow e' \pi^+ n$$

Hard Reggeon Domain

\[ s \gg -t, Q^2 \gg \Lambda_{QCD}^2 \]

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_R s_R^{\alpha}(t) \beta_R(t) \]

\[ \alpha_R(t) \rightarrow -1 \quad \text{Reflects elementary exchange of quarks in } t\text{-channel} \]

\[ \beta_R(t) \sim \frac{1}{t^2} \quad \frac{d\sigma}{dt} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]
Regge domain

\[ T(\gamma^* p \rightarrow \pi^+ n) \sim \epsilon \cdot p_i \sum_{R} s^\alpha_R(t) \beta_R(t) \quad s \gg -t, Q^2 \]

\[ \alpha_R(t) \rightarrow -1 \text{ at } t \rightarrow -\infty \]

\[ \beta_R(t) \sim \frac{1}{t^2} \]

\[ \frac{d\sigma}{dt}(\gamma^* p \rightarrow \pi^+ n) \rightarrow \frac{1}{s^3} \beta^2_R(t) \]

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^3} \frac{1}{t^4} \sim \frac{1}{s^7} \text{ at fixed } \frac{Q^2}{s}, \frac{t}{s} \]

Fundamental test of QCD

Reflects elementary exchange of quarks in t-channel
\[ |p, S_z \rangle = \sum_{n=3}^{\infty} \Psi_n(x_i, \vec{k}_\perp, \lambda_i) |n; \vec{k}_\perp, \lambda_i \rangle \]

**sum over states with \(n=3, 4, \ldots\) constituents**

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_\perp, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \(P^\mu\).

The light-cone momentum fraction

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]

are boost invariant.

\[ \sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp. \]

**Intrinsic heavy quarks**

\[ c(x), b(x) \text{ at high } x! \]

Mueller: gluon Fock states \(\rightarrow\) BFKL Pomeron  

\[ \bar{s}(x) \neq s(x), \quad \bar{u}(x) \neq \bar{d}(x) \]

**Fixed LF time**

**Hidden Color**
Intrinsic glue, sea, heavy quarks
QCD and the LF Hadron Wavefunctions

AdS/QCD
Light-Front Holography
LF Schrodinger Eqn

Hidden Color
sea and gluon distributions

Heavy Quark Fock States
Intrinsic Charm

color transparency

Quark & Flavor Structure

J=0 Fixed Pole
DVCS, GPDs, TMDs

Hadronization at Amplitude Level

In-hadron condensates

Counting Rules

ψ_n(x_i, k^⊥_i, λ_i)

Hard Exclusive Amplitudes
Form Factors
Counting Rules

Orbital Angular Momentum
Spin, Chiral Properties
Crewther Relation

Distribution amplitude
ERBL Evolution
φ_p(x_1, x_2, Q^2)

Baryon Decay
Measurement of Charm Structure Function


First Evidence for Intrinsic Charm

Never been checked!

DGLAP / Photon-Gluon Fusion: factor of 30 too small
Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Color-Octet Fock State!
- Probability $P_{Q\bar{Q}} \propto \frac{1}{M_Q^2}$
  $P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}}$
  $P_{cc/p} \approx 1\%$
- Large Effect at high $x$
- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)
- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin, Tung)
- Many empirical tests

M. Polyakov et al.
Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

**Data/Theory**

\[
\frac{\Delta \sigma(\bar{p}p \rightarrow \gamma cX)}{\Delta \sigma(\bar{p}p \rightarrow \gamma bX)}
\]

\[
\bar{p}p \rightarrow \gamma bX
\]

\[
\bar{p}p \rightarrow \gamma cX
\]

Ratio insensitive to gluon PDF, scales

Signal for significant IC at $x > 0.1$
- EMC data: \( c(x, Q^2) > 30 \times \text{DGLAP} \)
\( Q^2 = 75 \text{ GeV}^2, \ x = 0.42 \)

- High \( x_F \) \( pp \rightarrow J/\psi X \)
- High \( x_F \) \( pp \rightarrow J/\psi J/\psi X \)
- High \( x_F \) \( pp \rightarrow \Lambda_c X \) \( \text{ISR} \)
- High \( x_F \) \( pp \rightarrow \Lambda_b X \) \( \text{ISR} \)
- High \( x_F \) \( pp \rightarrow \Xi(c\bar{c}d)X \) \( \text{SELEX} \)
Leading Hadron Production from Intrinsic Charm

Coalescence of Comoving Charm and Valence Quarks Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$
Production of a Double-Charm Baryon

**SELEX high $x_F$**

$< x_F > \geq 0.33$
Leading Hadron Production from Intrinsic Charm

Coalescence of Comoving Charm and Valence Quarks Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$
800 GeV p-A (FNAL) \( \sigma_A = \sigma_p^A \alpha \)

**PRL 84, 3256 (2000); PRL 72, 2542 (1994)**

open charm: no A-dep at mid-rapidity

\[ \frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) \]

Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization!

Violation of factorization in charm hadroproduction.


J/ψ nuclear dependence vrs rapidity, x_{Au}, x_F

PHENIX compared to lower energy measurements

Violates PQCD factorization!

\[ \frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) \]

Hoyer, Sukhatme, Vanttinen

Kopeliovich, NP A696:669, 2001
Scattering on front-face nucleon produces color-singlet $c\bar{c}$ pair

Octet-Octet IC Fock State

Color-Opaque IC Fock state interacts on nuclear front surface

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$
\begin{align*}
\pi A \rightarrow J/\psi X \\
p A \rightarrow J/\psi X
\end{align*}

Excess beyond conventional PQCD subprocesses

\[ \frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F} \]

J. Badier et al, NA3

*A^{2/3} component*
• IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \rightarrow J/\psi X$
  (Mueller, Gunion, Tang, SJB)

• Color Octet IC Explains $A^{2/3}$ behavior at high $x_F$ (NA3, Fermilab)
  (Kopeliovitch, Schmidt, Soffer, SJB)

• IC Explains $J/\psi \rightarrow \rho\pi$ puzzle
  (Karliner, SJB)

• IC leads to new effects in $B$ decay
  (Gardner, SJB)

Higgs production at $x_F = 0.8$ !

Goldhaber, Kopeliovich, Schmidt, Soffer, sjb
Measurement of $\gamma + b + X$ and $\gamma + c + X$ Production Cross Sections in $p\bar{p}$ Collisions at $\sqrt{s} = 1.96$ TeV

**Data/Theory**

\[ \frac{\Delta \sigma(\bar{p}p \rightarrow \gamma cX)}{\Delta \sigma(\bar{p}p \rightarrow \gamma bX)} \]

**Ratio insensitive to gluon PDF, scales**

Signal for significant IC at $x > 0.1$
Intrinsic Charm Mechanism for Exclusive Diffraction Production

\[ p + p \rightarrow J/\psi + p + p \]

\[ x_{J/\psi} = x_c + x_{\bar{c}} \]

Kopeliovich, Schmidt, Soffer, sjb

Intrinsic \( c\bar{c} \) pair formed in color octet \( 8_c \) in proton wavefunction
Large Color Dipole
Collision produces color-singlet \( J/\psi \) through color exchange

RHIC Experiment
A Unified Description of Hadron Structure

\[ \psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

- Elastic form factors
- B-Decays
- Real Compton scattering at high \( q^2 \)
- GPDs
- Deeply Virtual Compton Scattering
- Deeply Virtual Meson production
- Parton momentum distributions TMDs
- Hadronization at the amplitude level
- Distribution Amplitudes
- LFWFs

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QCD at the Light-Front

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GPDs & Deeply Virtual Exclusive Processes
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)

\[ H(x, \xi, t), E(x, \xi, t), \ldots \] “Generalized Parton Distributions”

Timelike DVCS: Mukhurjee, Afanasev, Carlson, sjb

\[ \sqrt{t} - \text{Fourier conjugate to transverse impact parameter} \]

\[ \xi - \text{longitudinal momentum transfer} \]

\[ x - \text{quark momentum fraction} \]

\[ \gamma^* \rightarrow \gamma \]

\[ x+\xi \]

\[ x-\xi \]

GPDs

\[ H, \tilde{H}, E, \tilde{E} \]
Light-Front Wave Function Overlap Representation

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

Bakker & Ji
Lorce

DGLAP region

ERBL region

DGLAP region

Diehl, Hwang, sjb, NPB596, 2001
See also: Diehl, Feldmann, Jakob, Kroll

Bakker & Ji
Lorce

QCD at the Light-Front

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Example of LFWF representation of GPDs \((n+1 \Rightarrow n-1)\)

\[
\frac{1}{\sqrt{1 - \xi}} \frac{\Delta^1 - i \Delta^2}{2M} \frac{E_{(n+1 \rightarrow n-1)}(x, \xi, t)}{2M} 
= \left(\sqrt{1 - \xi}\right)^{3-n} \sum_{n, \lambda_i} \int \prod_{i=1}^{n+1} dx_i \frac{d^2 \vec{k}_{\perp i}}{16\pi^3} \frac{16\pi^3 \delta \left(1 - \sum_{j=1}^{n+1} x_j \right)}{16\pi^3} \delta^{(2)} \left(\sum_{j=1}^{n+1} \vec{k}_{\perp j} \right) \delta(x_{n+1} + x_1 - \xi) \delta^{(2)} \left(\vec{k}_{\perp n+1} + \vec{k}_{\perp 1} - \vec{\Delta}_{\perp} \right) \times \delta(x - x_1) \psi_{(n-1)}^{\uparrow *} (x_i', \vec{k}'_{\perp i}, \lambda_i) \psi_{(n+1)} \downarrow (x_i, \vec{k}_{\perp i}, \lambda_i) \delta_{\lambda_1 - \lambda_{n+1}}.
\]

where \(i = 2, \ldots, n\) label the \(n - 1\) spectator partons which appear in the final-state hadron wavefunction with

\[
x_i' = \frac{x_i}{1 - \xi}, \quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1 - \xi} \vec{\Delta}_{\perp}.
\]
**Single-spin asymmetries**

\[ i \vec{S}_p \cdot \vec{q} \times \vec{p}_q \]

**Pseudo-\( T \)-Odd**

**Leading T\( w\)ist Sivers Effect**

Hwang, Schmidt, sjb

Collins, **Burkardt**, Ji, Yuan

**QCD S- and P- Coulomb Phases**

--Wilson Line

**Leading-Twist Rescattering**

Violates pQCD Factorization!

**Light-Front Wavefunction**

\( S \) and \( P \)-Waves

**Sign reversal in DY!**
Final State Interactions Produce T-Odd (Sivers Effect)

- Bjorken Scaling!
- Arises from Interference of Final-State Coulomb Phases in S and P waves
- Relate to the quark contribution to the target proton anomalous magnetic moment
- Sum of Sivers Functions for all quarks and gluons vanishes. (Zero anomalous gavitomagnetic moment) \[ \vec{S} \cdot \vec{p}_{jet} \times \vec{q} \]
Final-State Interactions Produce Pseudo-T-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!

- Requires nonzero orbital angular momentum of quark

- Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;

- Wilson line effect -- lc gauge prescription \textit{Stefanis}

- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases

- QCD phase at soft scale!

- New window to QCD coupling and running gluon mass in the IR

- QED S and P Coulomb phases infinite -- difference of phases finite!

- Alternate: Retarded and Advanced Gauge: Augmented LFWFs \textit{Pasquini, Xiao, Yuan, sjb Mulders, Boer Qiu, Sterman}
### Static
- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and $J^z$
- DGLAP Evolution; mod. at large $x$
- No Diffractive DIS

### Dynamic
- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Sum Rules Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon Diffractive DIS

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
QCD Lagrangian

\[ \mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f \]

\[ iD^\mu = i\partial^\mu - gA^\mu \]

\[ [D^\mu, D^\nu] = igG^{\mu\nu} \]

\[ \lim_{N_C \to 0} \text{at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F \]

Analytic limit of QCD: Abelian Gauge Theory
QED ($N_c=0$): Underlies Atomic Physics, Molecular Physics, Chemistry, Electromagnetic Interactions

QCD: Underlies Hadron Physics, Nuclear Physics, Strong Interactions, Jets

**Theoretical Tools**

- Feynman diagrams and perturbation theory
- Bethe Salpeter Equation, Dyson-Schwinger Equations
- Lattice Gauge Theory, Hägler, Lepage
- Light-Front Methods: Discretized Light-Front Quantization, Transverse Lattice
- AdS/CFT
LF Quantization
Bjorken, Kogut, Soper, Susskind

LFWFs and Exclusive QCD:
Lepage and SJB, Efremov, Radyushkin

RGE and LF Hamiltonians:
Glazek & Wilson

DLCQ:
Hornbostel, Pauli, & SJB
Pinsky, Hiller

Renormalization of $H_{LF}$
Hiller, Chabysheva, Pauli, Pinsky, McCartor, Suaya, sjb

Rotation Invariance, Regularization
Karmanov, Mathiot

Zero-Modes: Standard Model
Srivastava, sjb
Light-Front formalism links dynamics to spectroscopy

\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

Heisenberg Matrix Formulation

\[ H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k^2}{x} \right]_i + H^{int}_{LF} \]

\[ H^{int}_{LF}: \text{Matrix in Fock Space} \]

\[ H^{QCD}_{LF} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h> \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions
\[ (M_{\pi}^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q} | V | q\bar{q} \rangle & \langle q\bar{q} | V | q\bar{g}g \rangle & \cdots \\ \langle q\bar{g}g | V | q\bar{q} \rangle & \langle q\bar{g}g | V | q\bar{g}g \rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{g}g/\pi} \\ \vdots \end{bmatrix} \]

\[ A^+ = 0 \]
**Light-Front QCD**

**Heisenberg Matrix Formulation**

\[
H^{QCD}_{LF} |\Psi_h> = M_h^2 |\Psi_h>
\]

**Discretized Light-Cone Quantization**

**Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions**

**DLCQ:** Frame-independent, No fermion doubling; Minkowski Space

**DLCQ:** Periodic BC in \(x^-\). Discrete \(k^+\); frame-independent truncation
Spectra for $N = 3$, baryon number $B = 0, 1$ and $2$ as a function of $g/m$; $K$ fixed.

Light Cone Quantized QCD in (1+1)-Dimensions.
Quantum chromodynamics and other field theories on the light cone

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1. Introduction
2. Hamiltonian dynamics
   2.1. Abelian gauge theory: quantum electrodynamics
   2.2. Non-abelian gauge theory: Quantum chromodynamics
   2.3. Parametrization of space–time
   2.4. Forms of Hamiltonian dynamics
   2.5. Parametrizations of the front form
   2.6. The Poincaré symmetries in the front form
   2.7. The equations of motion and the energy–momentum tensor
   2.8. The interactions as operators acting in Fock space
3. Bound states on the light cone
   3.1. The hadronic eigenvalue problem
   3.2. The use of light-cone wavefunctions
   3.3. Perturbation theory in the front form
   3.4. Example 1: The $q\bar{q}$-scattering amplitude
   3.5. Example 2: Perturbative mass renormalization in QED (KS)
   3.6. Example 3: The anomalous magnetic moment
   3.7. (1 + 1)-dimensional: Schwinger model (LB)
   3.8. (3 + 1)-dimensional: Yukawa model
4. Discretized light-cone quantization
   4.1. Why discretized momenta?
   4.2. Quantum chromodynamics in 1 + 1 dimensions (KS)
   4.3. The Hamiltonian operator in 3 + 1 dimensions (BL)
   4.4. The Hamiltonian matrix and its regularization
   4.5. Further evaluation of the Hamiltonian matrix elements
   4.6. Retrieving the continuum formulation
   4.7. Effective interactions in 3 + 1 dimensions
   4.8. Quantum electrodynamics in 3 + 1 dimensions
   4.9. The Coulomb interaction in the front form
5. The impact on hadronic physics
   5.1. Light-cone methods in QCD
   5.2. Moments of nucleons and nuclei in the light-cone formalism
   5.3. Applications to nuclear systems
   5.4. Exclusive nuclear processes
   5.5. Conclusions
6. Exclusive processes and light-cone wavefunctions
   6.1. Is PQCD factorization applicable to exclusive processes?
   6.2. Light-cone quantization and heavy particle decays
   6.3. Exclusive weak decays of heavy hadrons
   6.4. Can light-cone wavefunctions be measured?
Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable

de Teramond, Deur, Shrock, Roberts, Tandy
Goal: 
*Use AdS/QCD duality to construct a first approximation to QCD*

**Central problem for strongly-coupled gauge theories**
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_{\perp})$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond
Conformal Theories are invariant under the Poincare and conformal transformations with $M^{\mu\nu}$, $P^\mu$, $D$, $K^\mu$, the generators of $SO(4,2)$.

$SO(4,2)$ has a mathematical representation on AdS$_5$. 
\textbf{AdS/CFT:} Anti-de Sitter Space / Conformal Field Theory

\textbf{Maldacena:}

\textit{Map AdS}_5 \times \textit{S}_5 \text{ to conformal N=4 SUSY}

- **QCD is not conformal:** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- Use mathematical mapping of the conformal group $\text{SO}(4,2)$ to AdS$_5$ space
Deur, Korsch, et al.

\[ \alpha_{s,g}/\pi \text{ JLab, GDH limit, Burkert-Ioffe} \]

\[ \text{Fit, pQCD evol. eq.} \]

Cornwall, Bloch et al., Godfrey-Isgur, Fischer et al., Bhagwat et al., Maris-Tandy, DSE gluon couplings

\[ Q \text{ (GeV)} \]

Lattice QCD