LFWFS give a fundamental description of hadron observables

- LFWFS underly structure functions and generalized parton distributions.
- Parton number not conserved: n=n' & n=n'+2 at nonzero skewness
- GPDs are not densities or probability distributions
- Nonperturbative QCD: Lattice, DLCQ, Bethe-Salpeter, AdS/CFT

LFWFS give a fundamental description of hadron observables

- LFWFS underly structure functions and generalized parton distributions.
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- Nonperturbative QCD: Lattice, DLCQ, Bethe-Salpeter, AdS/CFT

Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- General relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

Remarkable Features of LFWFs

- Lorentz Boost Invariant (kinematical Lorentz transformations of front form)

- Essential check: Vanishing of "anomalous gravitomagnetic moment": $B(0) = 0$

- Exact property of LFWFS, Fock-state by Fock-state

Hwang, Schmidt, sjb

The form factors of the energy–momentum tensor for a spin-$\frac{1}{2}$ composite

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \bar{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P),$$

where $q^\mu = (P' - P)^\mu$, $\overline{P}^\mu = \frac{1}{2}(P' + P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$.

$$\left\langle P+q, \uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P, \uparrow \right\rangle = A(q^2),$$

$$\left\langle P+q, \uparrow \left| \frac{T^{++}(0)}{2(P^+)^2} \right| P, \downarrow \right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M}.$$

The angular momentum projection of a state is given by

$$\langle J^i\rangle = \frac{1}{2}\epsilon^{ijk}\int \mathrm{d}^3x \left\langle T^{0k}x^j - T^{0j}x^k \right\rangle = A(0)\langle L^i\rangle + \big[A(0) + B(0)\big]\bar{u}(P)\frac{1}{2}\sigma^i u(P).$$

$$\langle J^z\rangle = \left\langle \frac{1}{2}\sigma^z \right\rangle \big[A(0) + B(0)\big].$$

$$
\begin{aligned}
-\frac{B(0)}{2M} &= \lim_{q_\perp^1\to 0}\frac{\partial}{\partial q_\perp^1}\langle P+q,\uparrow|\frac{T^{++}(0)}{2(P^+)^2}|P,\downarrow\rangle \\
&= \lim_{q_\perp^1\to 0}\frac{\partial}{\partial q_\perp^1}\langle\Psi^\uparrow(P^+=1,\vec{P}_\perp=\vec{q}_\perp)|\frac{T^{++}(0)}{2(P^+)^2}|\Psi^\downarrow(P^+=1,\vec{P}_\perp=\vec{0}_\perp)\rangle \\
&= \lim_{q_\perp^1\to 0}\frac{\partial}{\partial q_\perp^1}\sum_a\int\prod_{k=1}^{n-1}\frac{\mathrm{d}^2\vec{k}_{\perp k}\,\mathrm{d}x_k}{16\pi^3} \\
&\quad\times\psi_a^{\uparrow *}(x_1,x_2,\ldots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad\vec{k}'_{\perp 1},\vec{k}'_{\perp 2},\ldots,\vec{k}'_{\perp n-1},(-\vec{k}'_{\perp 1}-\vec{k}'_{\perp 2}-\cdots-\vec{k}'_{\perp n-1})) \\
&\quad\times\left[\sum_{i=1}^{n-1}x_i+(1-x_1-x_2-\cdots-x_{n-1})\right] \\
&\quad\times\psi_a^{\downarrow}(x_1,x_2,\ldots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad\vec{k}_{\perp 1},\vec{k}_{\perp 2},\ldots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})).
\end{aligned}
$$

$$
\begin{aligned}
&=\int\prod_{k=1}^{n-1}\frac{\mathrm{d}^2\vec{k}_{\perp k}\,\mathrm{d}x_k}{16\pi^3}\psi_a^{\uparrow *}(x_1,x_2,\ldots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad\vec{k}_{\perp 1},\vec{k}_{\perp 2},\ldots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})) \\
&\quad\times\left[\sum_{i=1}^{n-1}\left(-1+\sum_{j=1}^{n-1}x_j+(1-x_1-x_2-\cdots-x_{n-1})\right)x_i\frac{\partial}{\partial k_{\perp i}^1}\right] \\
&\quad\times\psi_a^{\downarrow}(x_1,x_2,\ldots,x_{n-1},(1-x_1-x_2-\cdots-x_{n-1}), \\
&\qquad\vec{k}_{\perp 1},\vec{k}_{\perp 2},\ldots,\vec{k}_{\perp n-1},(-\vec{k}_{\perp 1}-\vec{k}_{\perp 2}-\cdots-\vec{k}_{\perp n-1})) \\
&=0.
\end{aligned}
$$

B(0) = 0

Fock state by Fock state

B(q²) not zero : QED: 2 photon cut

Vanishing Anomalous Gravitomagnetic Moment

Hwang, Ma, Schmidt, sjb

Equivalence Theorem

Kobsarev, Okun
Taryaev

LFWFs of Electron (n=2)

$J_z = +\frac{1}{2}$

Gives Schwinger Anomalous Moment $\frac{\alpha}{2\pi}$

$$\left\{\begin{array}{l}
\psi^{\uparrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(-k^1+ik^2)}{x(1-x)}\,\varphi\ , \qquad L_z = -1 \\
\psi^{\uparrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(+k^1+ik^2)}{1-x}\,\varphi\ , \qquad L_z = 1 \\
\psi^{\uparrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}(M-\frac{m}{x})\,\varphi\ , \qquad L_z = 0 \\
\psi^{\uparrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_\perp) = 0\ ,
\end{array}\right.$$

where

$$\varphi = \varphi(x,\vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}_\perp^2 + m^2)/x - (\vec{k}_\perp^2 + \lambda^2)/(1-x)}\ .$$

$M \to m + \lambda$:

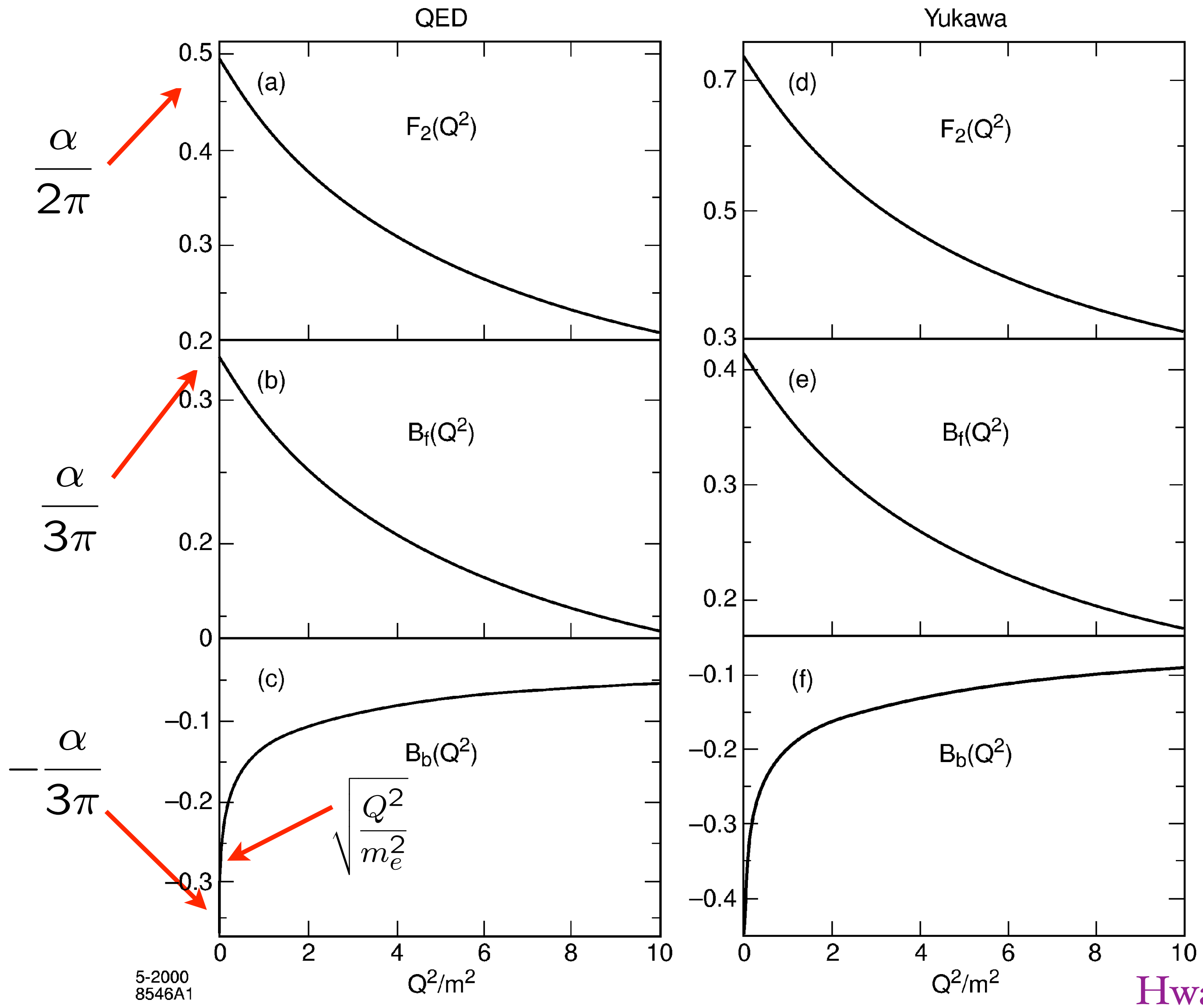
$$\left\{\begin{array}{l}
\psi^{\downarrow}_{+\frac{1}{2}\,+1}(x,\vec{k}_\perp) = 0\ , \\
\psi^{\downarrow}_{+\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}(M-\frac{m}{x})\,\varphi\ , \\
\psi^{\downarrow}_{-\frac{1}{2}\,+1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(-k^1+ik^2)}{1-x}\,\varphi\ , \\
\psi^{\downarrow}_{-\frac{1}{2}\,-1}(x,\vec{k}_\perp) = -\sqrt{2}\frac{(+k^1+ik^2)}{x(1-x)}\,\varphi\ .
\end{array}\right.$$

Spin-1 mass λ

Spin-1/2 mass m

Drell, sjb

Hwang, Schmidt, sjb



Hwang, Ma, Schmidt, sjb

Helicity-flip electromagnetic and gravitational form factors for spacelike $q^2 = -Q^2 < 0$ from the quantum fluctuations of a fermion at one-loop order in units of α/π for QED and $g^2/4\pi^2$ for the Yukawa theory. The fermion constituent mass is taken as $m_f = M$. The boson constituent is massless.

electron LFWFs provide quark + spin-one diquark model of nucleon

$$q(x,\Lambda^2)_{\text{spin}-1\ \text{diquark}}$$

$$= \int \frac{d^2\vec{k}_\perp dx}{16\pi^3}\theta(\Lambda^2-\mathcal{M}^2)\ 2\left[\frac{\vec{k}_\perp^2}{x^2(1-x)^2}+\frac{\vec{k}_\perp^2}{(1-x)^2}+(M-\frac{m}{x})^2\right]|\varphi|^2$$

$$\Delta q(x,\Lambda^2)_{\text{spin}-1\ \text{diquark}}$$

$$= \int \frac{d^2\vec{k}_\perp dx}{16\pi^3}\theta(\Lambda^2-\mathcal{M}^2)\ 2\left[\frac{\vec{k}_\perp^2}{x^2(1-x)^2}+\frac{\vec{k}_\perp^2}{(1-x)^2}-(M-\frac{m}{x})^2\right]|\varphi|^2$$

$$\delta q(x,\Lambda^2)_{\text{spin}-1\ \text{diquark}}$$

$$= \int \frac{d^2\vec{k}_\perp dx}{16\pi^3}\theta(\Lambda^2-\mathcal{M}^2)\ 4\left[\frac{\vec{k}_\perp^2}{x(1-x)^2}\right]|\varphi|^2\ .$$

Electron Transversity

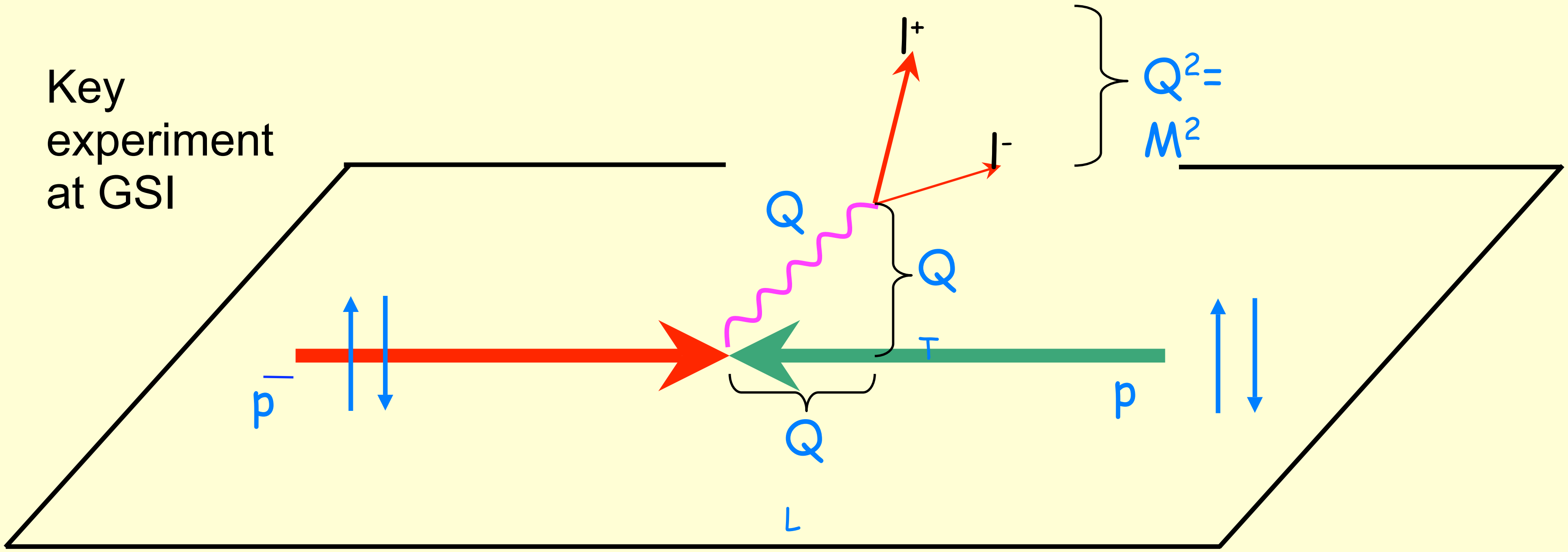
Soffer bounds automatically satisfied in LF formalism

$$\varphi=\varphi(x,\vec{k}_\perp)=\frac{e/\sqrt{1-x}}{M^2-(\vec{k}_\perp^2+m^2)/x-(\vec{k}_\perp^2+\lambda^2)/(1-x)}$$

Transversity in Drell-Yan Processes

Polarized Antiproton Beam → Polarized Proton Target
(both transversely polarized)

Key experiment at GSI



$$A_{TT} \equiv \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}} = \hat{a}_{TT} \frac{\sum_q e_q^2 h_1^q(x_1, M^2) h_1^{\bar{q}}(x_2, M^2)}{\sum_q e_q^2 q(x_1, M^2) \bar{q}(x_2, M^2)}$$

$q = u, \bar{u}, d, \bar{d}, \ldots$

M invariant Mass of lepton pair

F. Rathsman

LFWFS give a fundamental description of hadron observables

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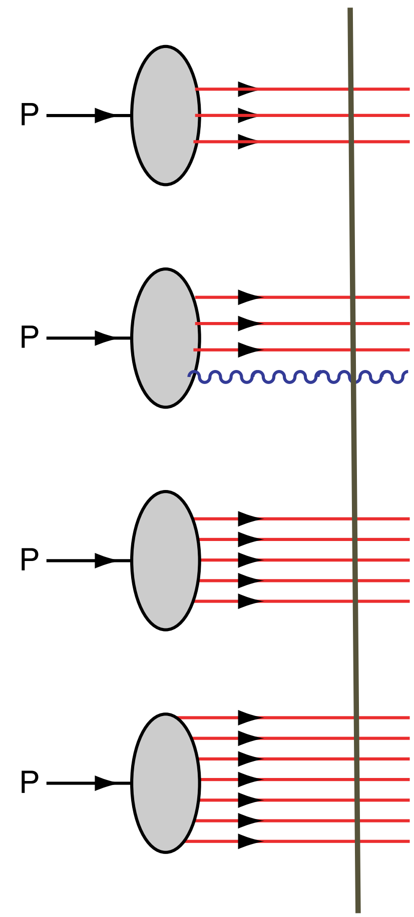
Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

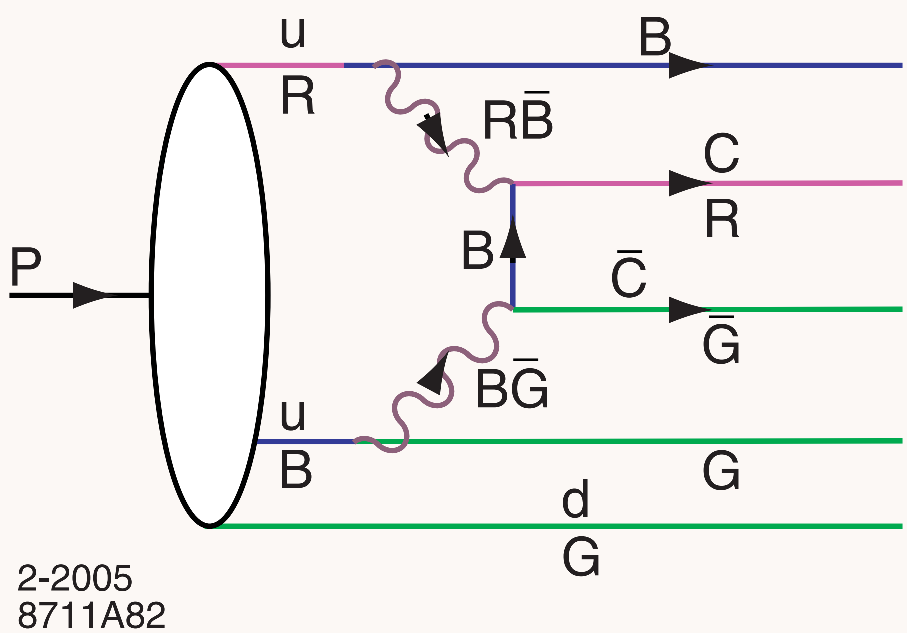
$$\psi_n(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$



$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Invariant under boosts. Independent of P^μ



$|uud c\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positr

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

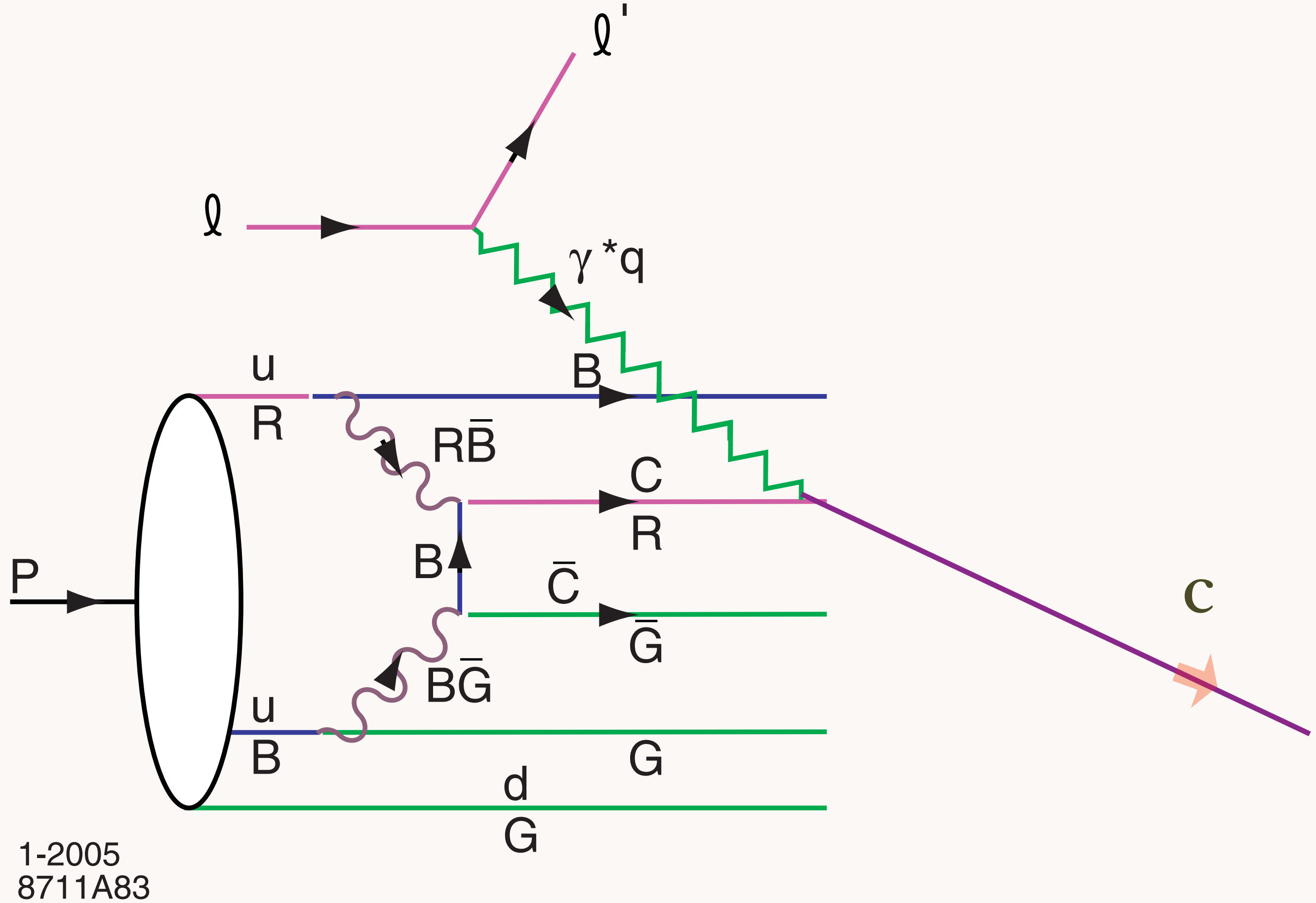
$c\bar{c}$ in Color Octet

High x charm

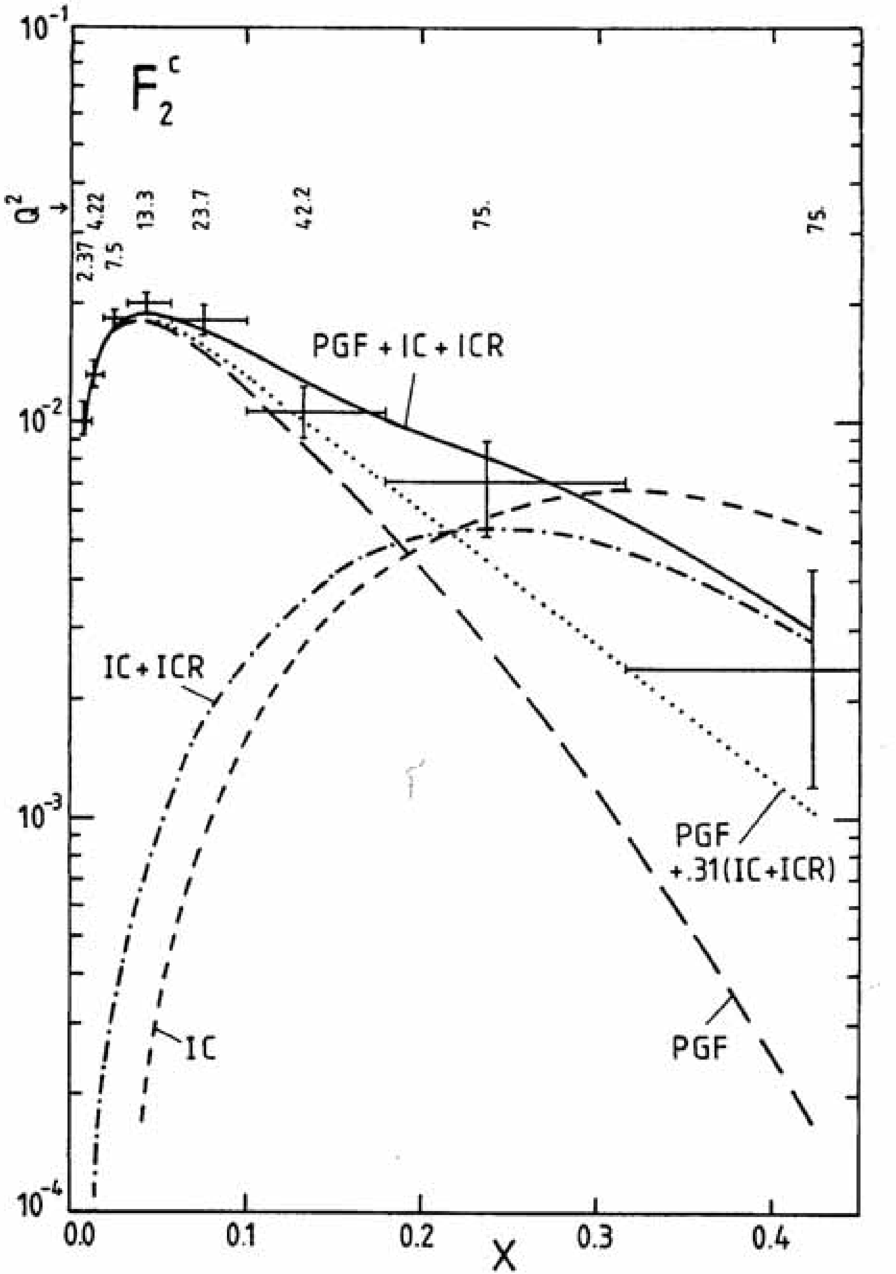
Distribution peaks at equal rapidity (velocity)

Therefore heavy particles carry the largest momentum fractions

Measure c(x) in Deep Inelastic Lepton-Proton Scattering



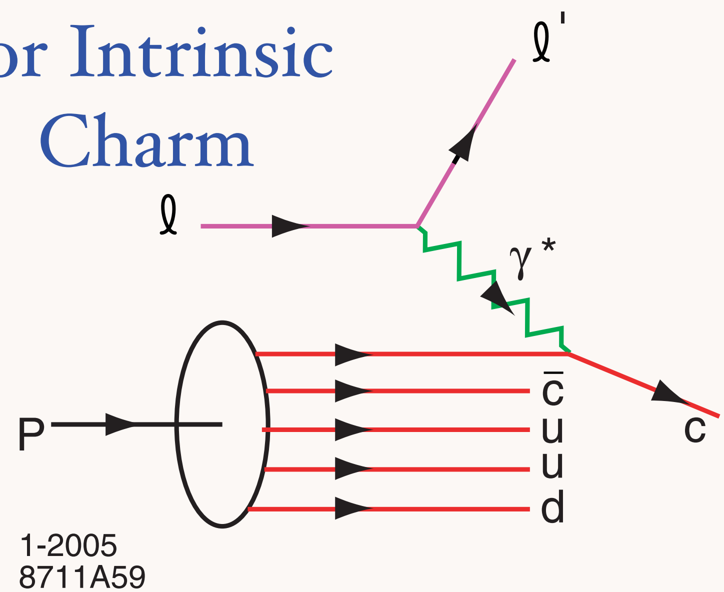
Hoyer, Peterson, SJB



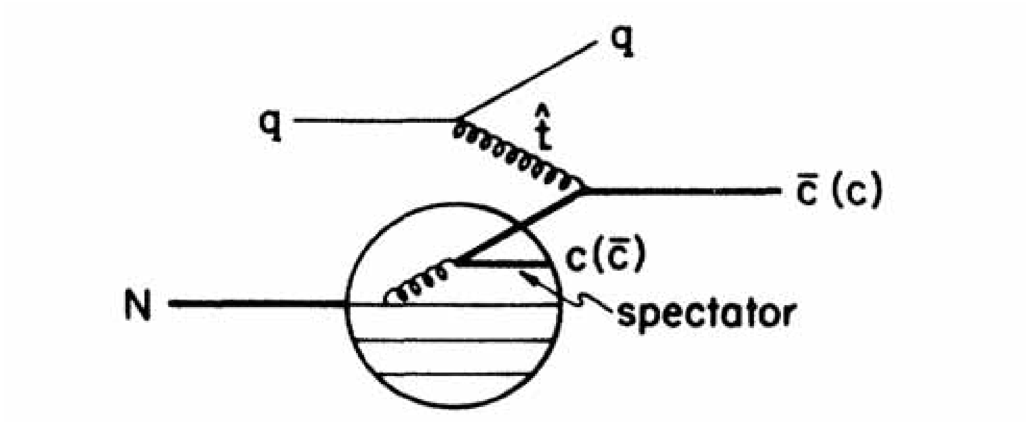
Measurement of Charm Structure Function

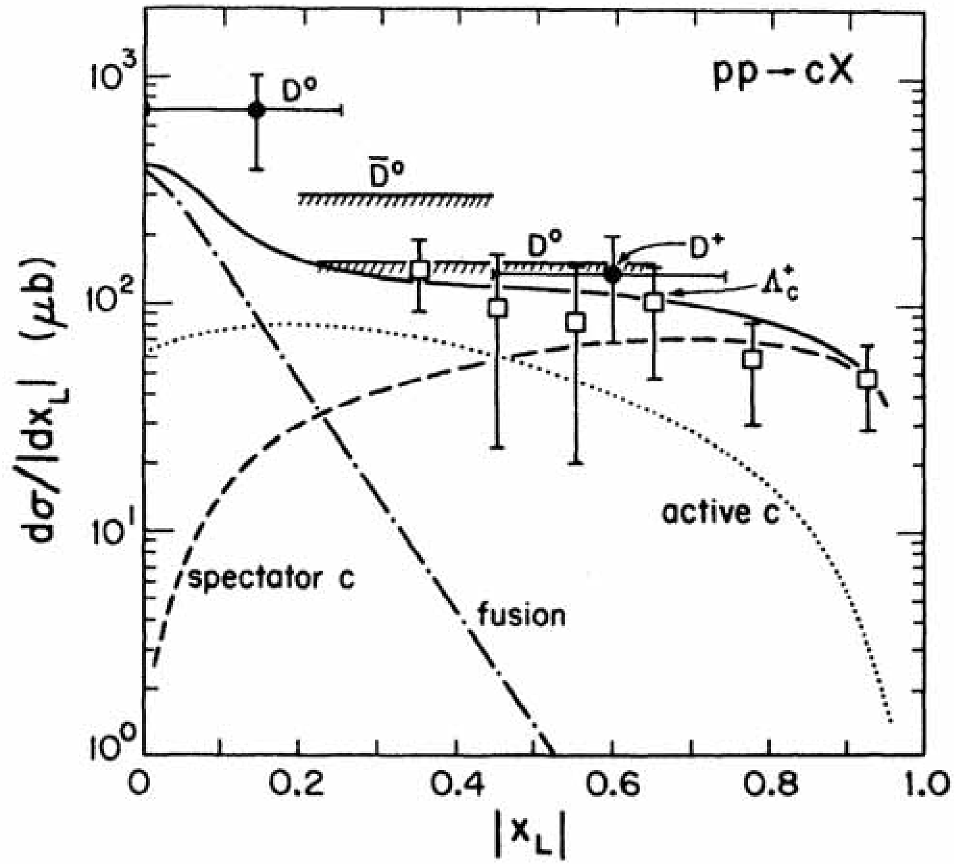
J. J. Aubert et al. [European Muon Collaboration], “P duction Of Charmed Particles In 250-Gev Mu+ - Iron teractions,” Nucl. Phys. B 213, 31 (1983).

Evidence for Intrinsic Charm



DGLAP / Photon-Gluon Fusion Factor of 30 too small





Predictions for Inclusive Charm ProductionDistributions at the ISR. Assumes active and spectator charm distribution in proton patterned on IC, plus coalescence of valence and charm quarks.

V. D. Barger, F. Halzen and W. Y. Keung,
"The Central And Diffractive Components Of Charm Production,"
Phys. Rev. D 25, 112 (1982).

- EMC data: $c(x, Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV^2, $x = 0.42$

- High x_F $pp \to J/\psi X$

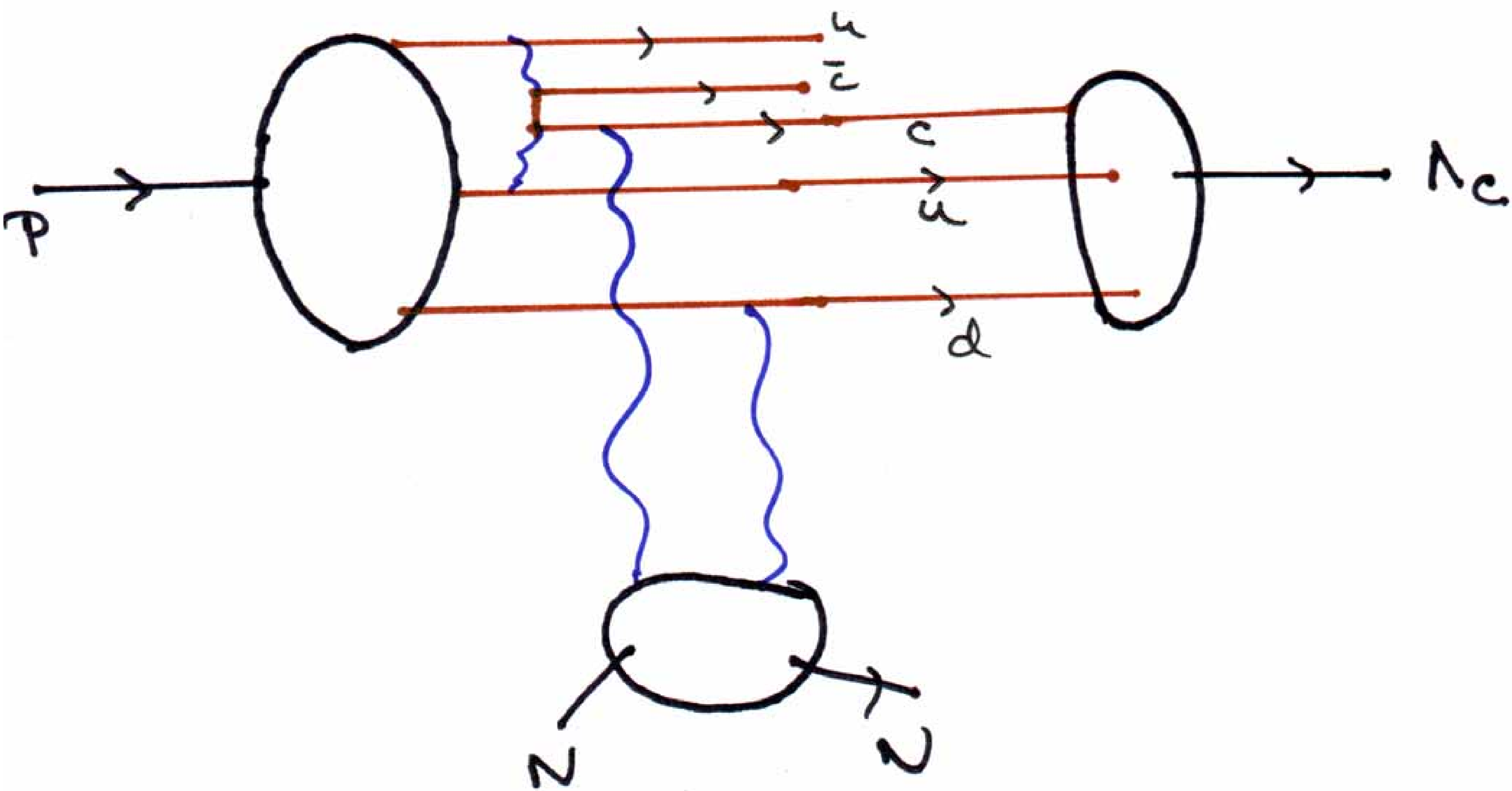
- High x_F $pp \to J/\psi J/\psi X$

- High x_F $pp \to \Lambda_c X$

- High x_F $pp \to \Lambda_b X$

- High x_F $pp \to \Xi(ccd) X$ (SELEX)

$p\ p \rightarrow p\Lambda_c X$
Diffractive Dissociation of Intrinsic Charm



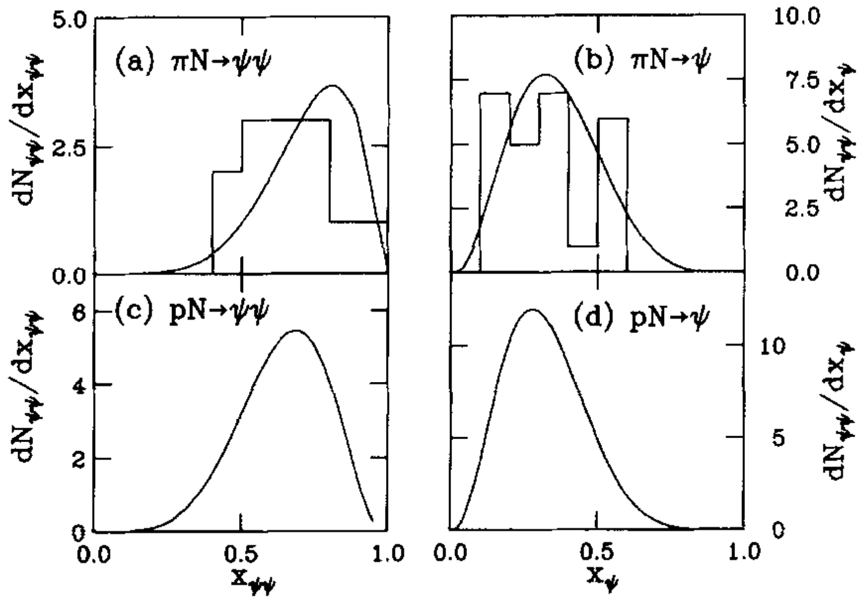


Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of J/ψ's from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/c [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single J/ψ's is twice the number of pairs.

NA3 Data

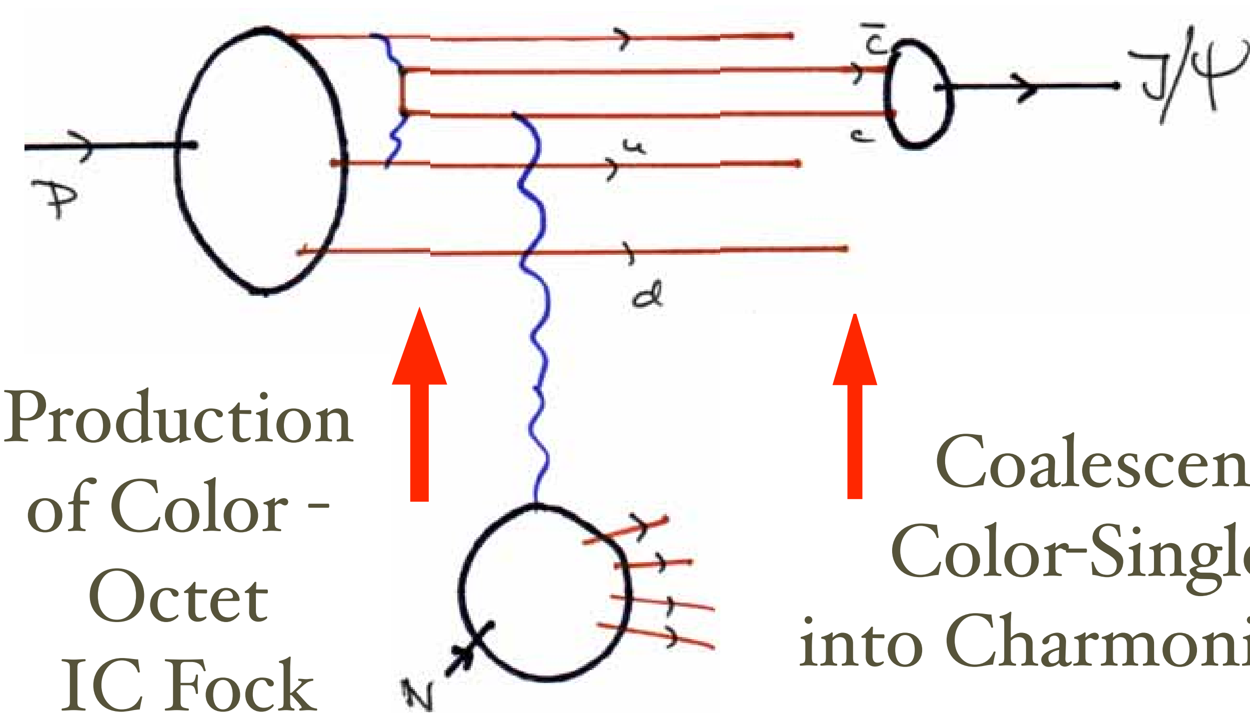
$\pi A \to J/\psi J/\psi X$

Intrinsic charm contribution to double quarkonium hadroproduction ⋆

R. Vogt [a], S.J. Brodsky [b]

The probability distribution for a general n-parti intrinsic $c\bar{c}$ Fock state as a function of x and k_T written as

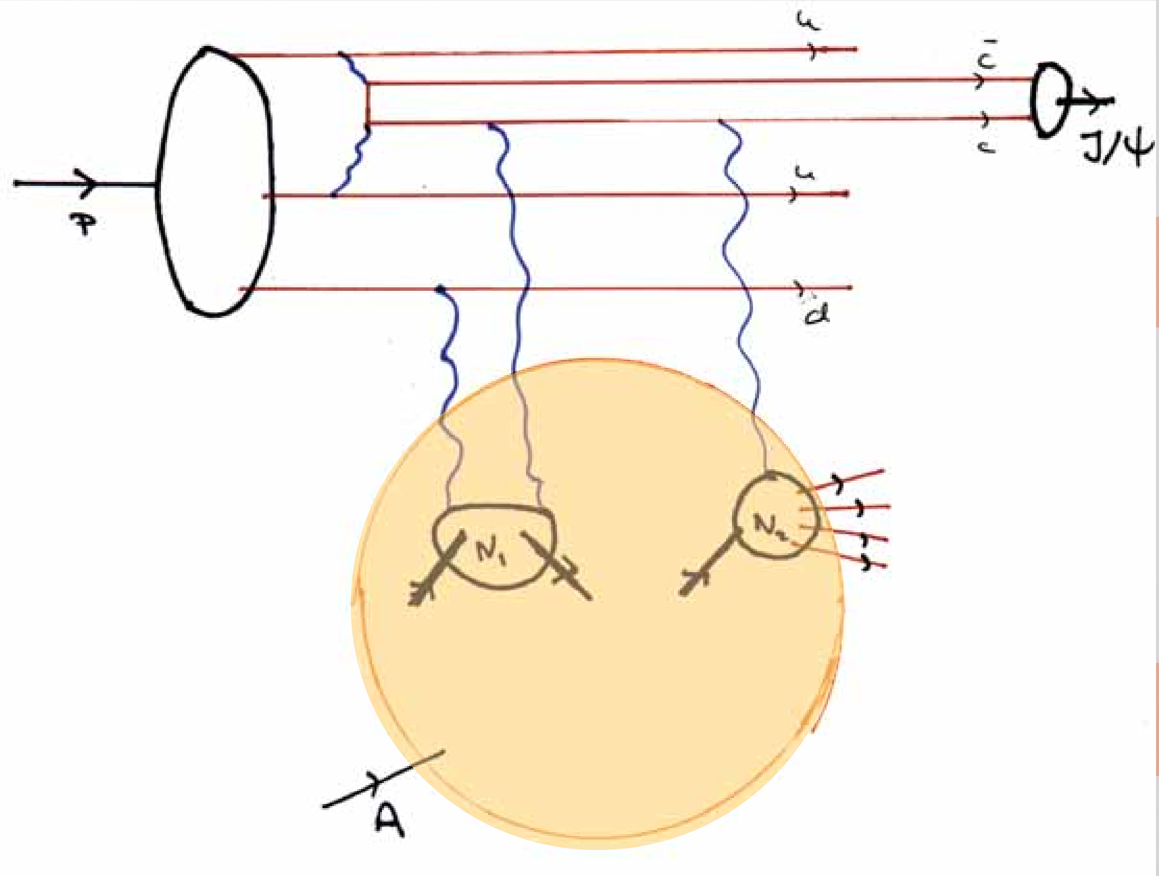
$$\frac{dP_{\rm ic}}{\prod_{i=1}^n dx_i d^2 k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n k_{T,i})\delta(1-\sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$



Production of Color - Octet IC Fock State

Coalescence of Color-Singlet Pair into Charmonium State

Scattering on Nucleon via one Gluon



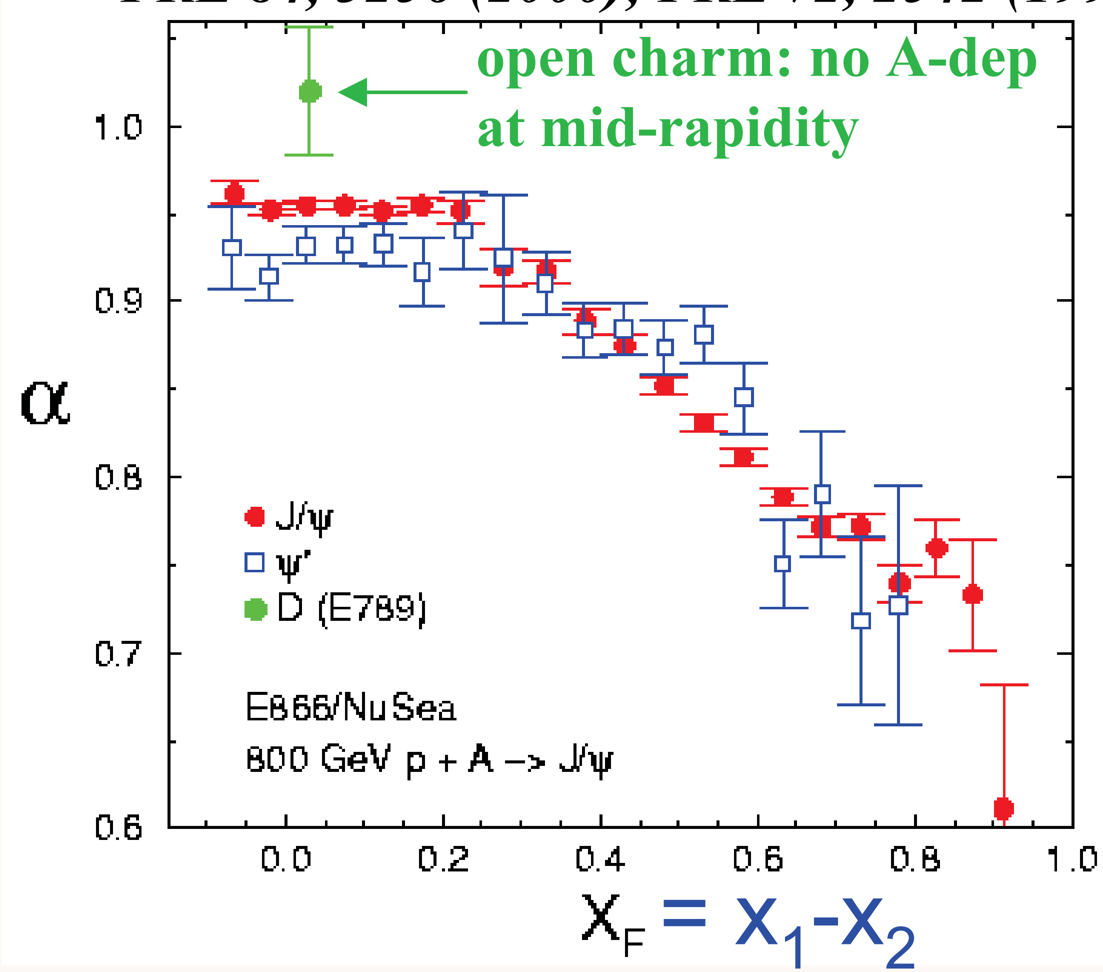
Shadowing of $pA \to J/\Psi X$

J/Ψ Production on Front Surface
No Absorption of Propagating J/Ψ
$\sigma(p + A \to J/\Psi + X) \propto A^{2/3}$

Elastic scattering of IC Fock state:
$|[uud]_{8_C}[c\bar{c}]_{8_C} > + N_1 \to |[uud]_{8_C}[c\bar{c}]_{8_C} > + N_1$
followed by:
$|[uud]_{8_C}[c\bar{c}]_{8_C} > + N_2 \to J/\Psi + X$
Depleted flux on downstream nucleons

800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^\alpha$

PRL 84, 3256 (2000); PRL 72, 2542 (1994)



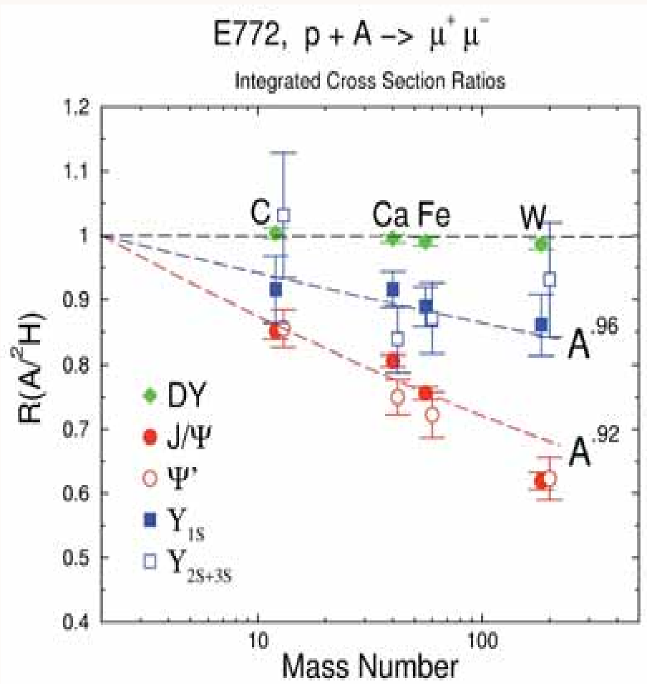
Remarkably Strong Nuclear Dependence for Fast Charmonium

M. Leitch

Nuclear effects in Quarkonium production

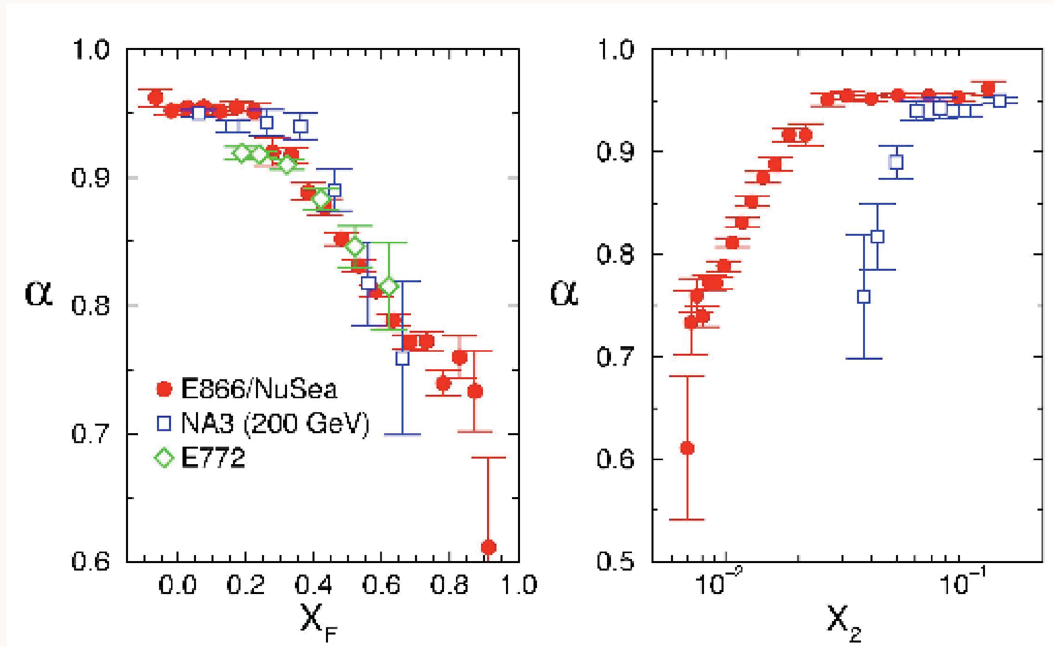
p + A at $s^{1/2}$ = 38.8 GeV

E772 data



$\sigma(p+A) = A^{\alpha}\, \sigma(p+N)$

Strong x_F - dependence



Nuclear effects scale with x_F, not x_2 !!!

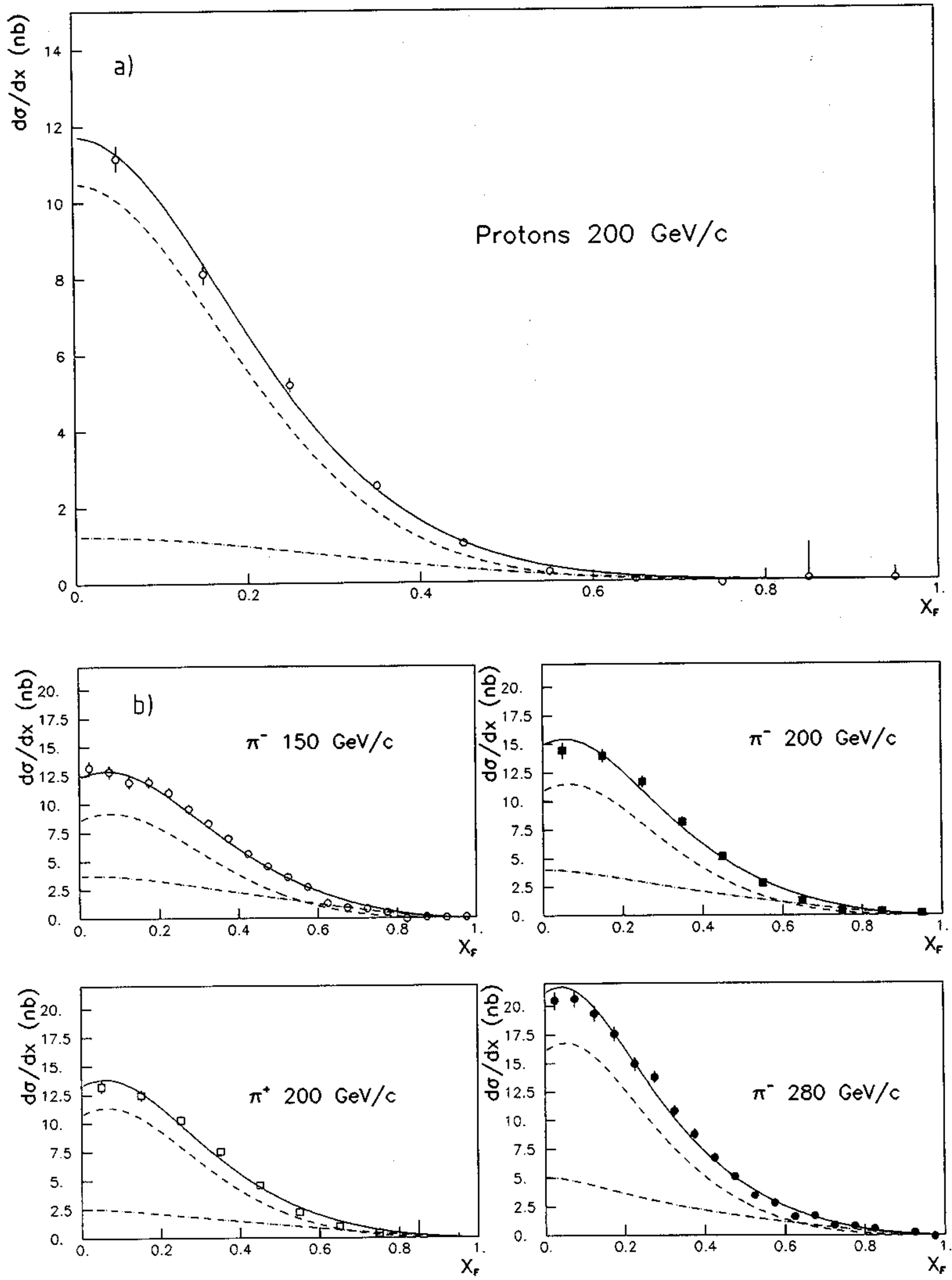
M.Leitch

Nuclear Dependence of Quarkonium Production

NA3 data for $\frac{d\sigma}{dx_F}(p(\pi)A \to J/\psi X)$: hard A^1 and "diffractive" $A^{2/3}$ components

Diffractive contribution extends to large x_F

$A^{\alpha(x_F)}$ not $A^{\alpha(x_2)}$: PQCD Factorization Violated!



Hard Component $\frac{d\sigma}{dx_F}(p(\pi)A \to J/\psi X)$

The fit: gg fusion (dashed)

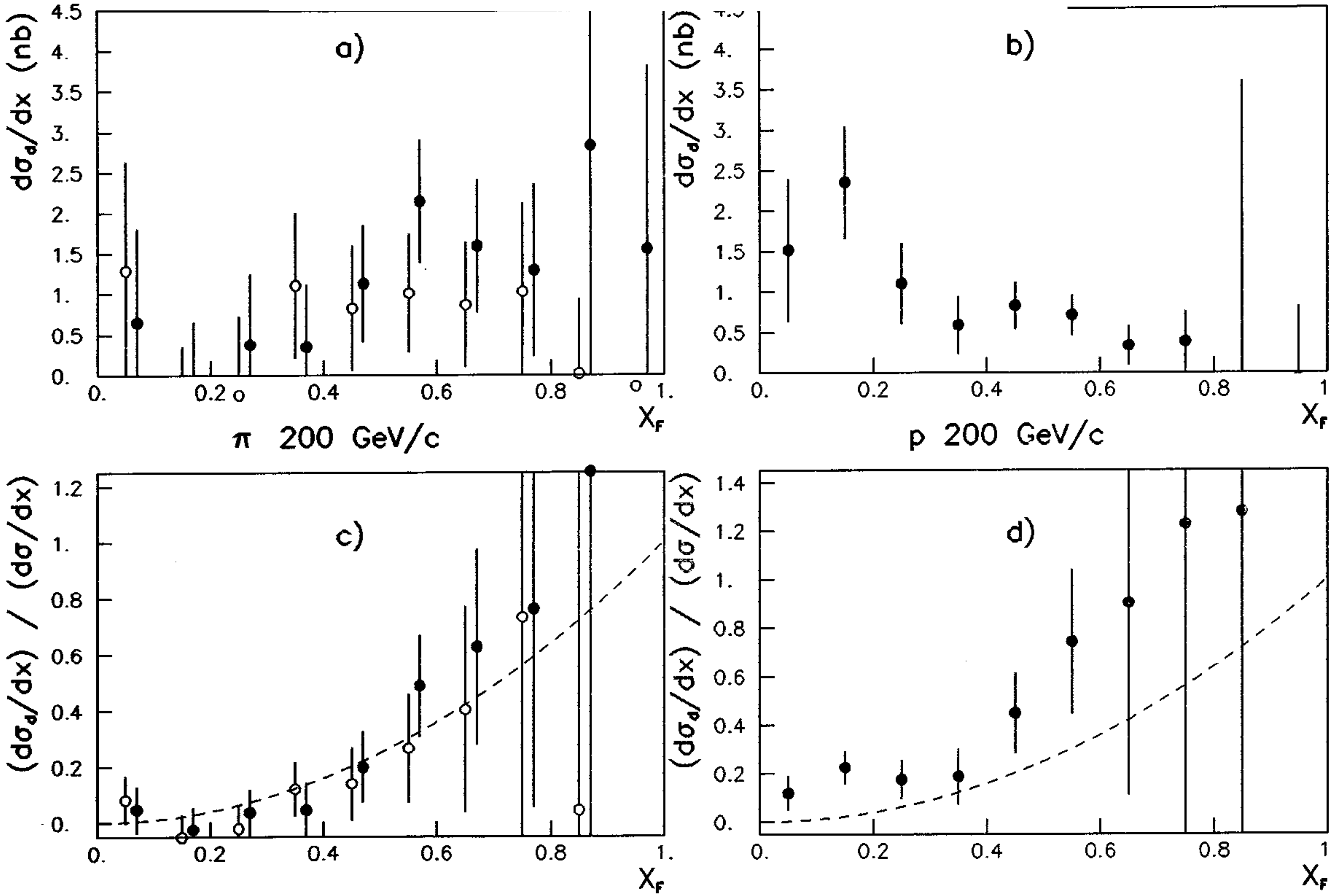
$q\bar{q}$ fusion (dashed-dot)

total (solid)

EXPERIMENTAL J/ψ HADRONIC PRODUCTION FROM 150 TO 280 GeV/c

NA3 COLLABORATION

CERN-EP/83-86
June 29th, 1983

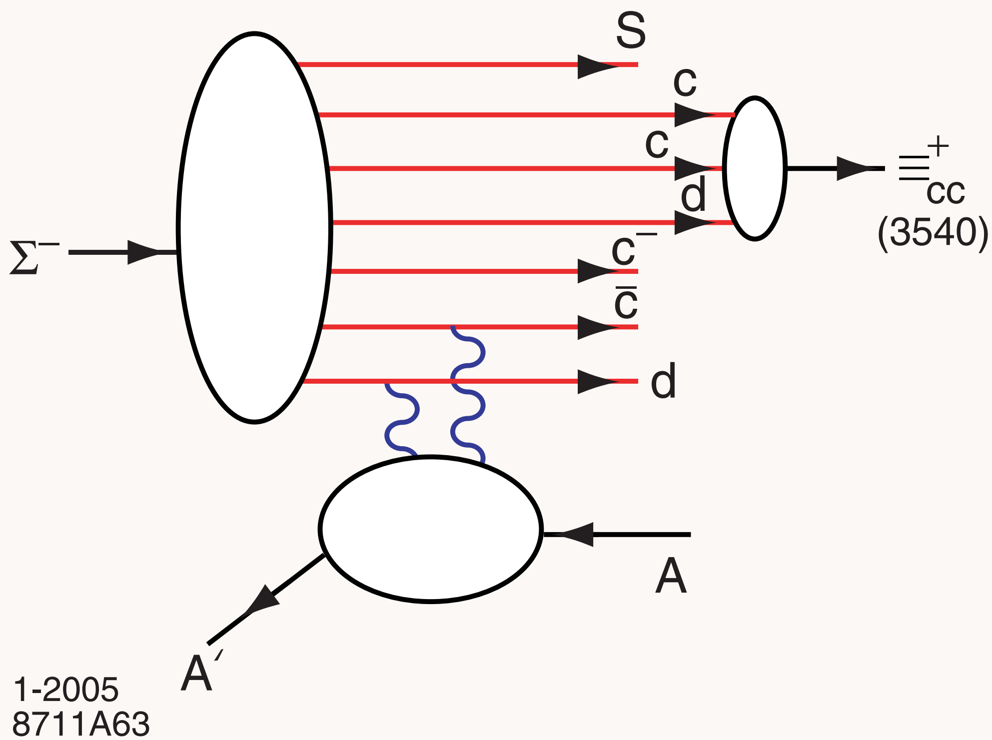


- IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \to J/\psi X$
(Mueller, Gunion, Tang, SJB)

- Color Octet IC Explains $A^{2/3}$ behavior at high x_F (NA3, Fermilab)
(Kopeliovitch, Schmidt, Soffer, SJB)

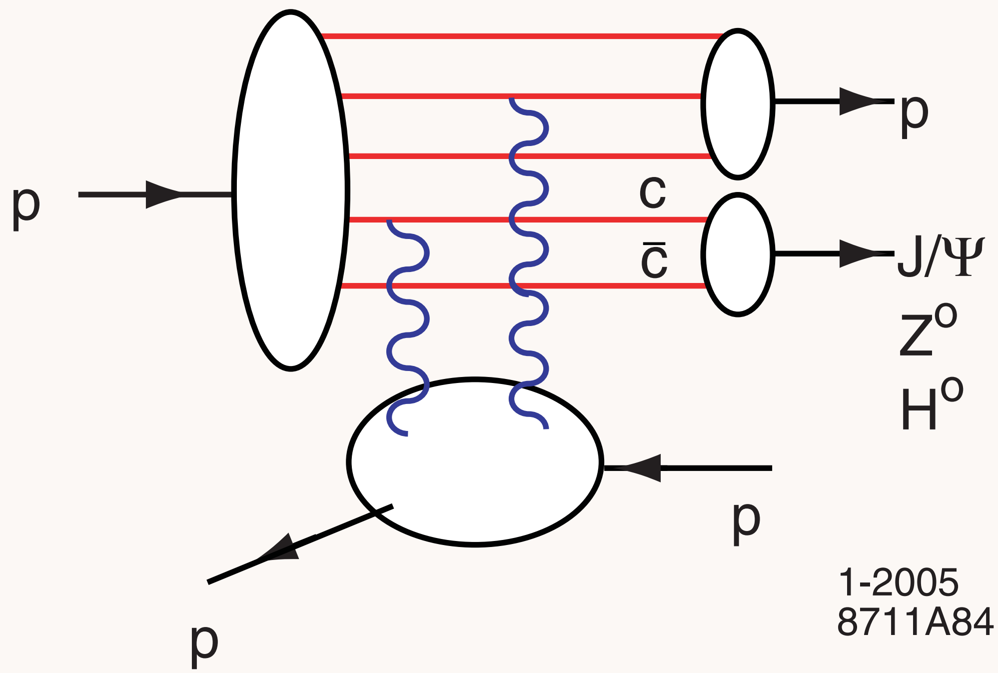
- IC Explains $J/\psi \to \rho\pi$ puzzle
(Karliner, SJB)

- IC leads to new effects in B decay
(Gardner, SJB)



Production of a Double-Charm Baryon

Intrinsic Charm Mechanism for Exclusive Diffraction Production



$$p\,p \to J/\psi\; p\; p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

Exclusive Diffractive High-X_F Higgs Production

Kopeliovitch, Schmidt, Soffer, sjb

Intrinsic $c\bar{c}$ pair formed in color octet 8_C in proton wavefunction Large Color Dipole

Collision produces color-singlet J/ψ through color exchange

RHIC Experiment

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$$x_i = \frac{k_i^+}{P^+}$$

$$H_{LF}^{QCD}|\psi> = M^2|\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

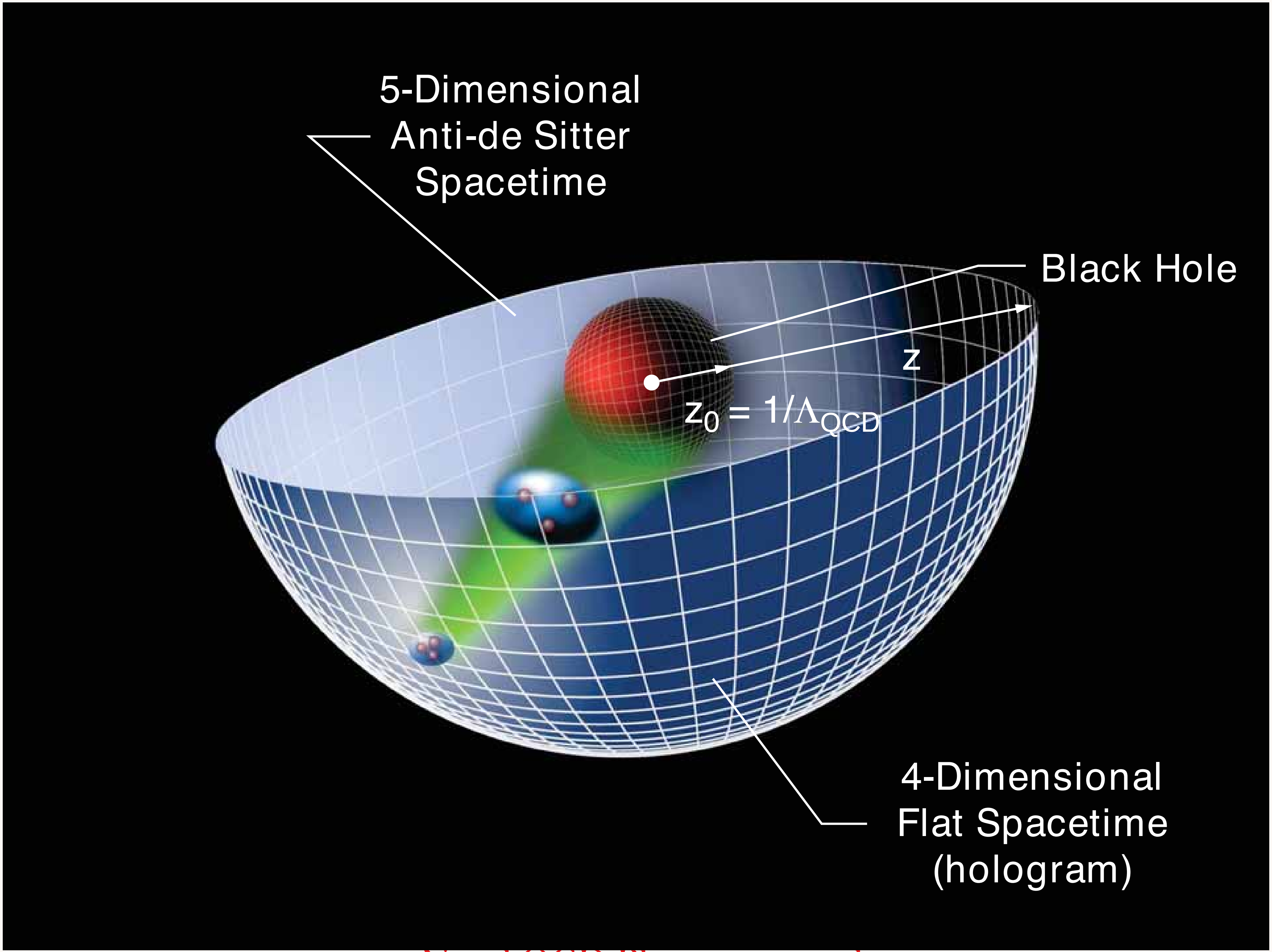
AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)

- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)

- Power-law fall-off at large transverse momentum, $x \to 1$

- Hadron Spectra, Regge Trajectories


5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
z
$z_0 = 1/\Lambda_{QCD}$
4-Dimensional
Flat Spacetime
(hologram)

- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances

- Holographic Model: Initial "classical" approximation to QCD: Remarkable agreement with light hadron spectroscopy

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing H^{LF}_{QCD}; variational methods

Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with $M^{\mu\nu}$, P^μ, D, K^μ, the generators of $SO(4,2)$.

- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops (running coupling). For $\beta = d\alpha_s(Q^2)/dlnQ^2 = 0$ (fixed point theory), PQCD is a conformal theory:
  Parisi, Phys. Lett. B **39**, 643 (1972).

- Phenomenological success of dimensional scaling laws for exclusive processes $d\sigma/dt \sim 1/s^{n-2}$ (n total number of constituents), implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies (PQCD predicts powers of α_s and logs).

- Theoretical and empirical evidence that $\alpha_s(Q^2)$ has an IR fixed point (constant in the IR):
  Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Brodsky, Menke, Merino and Rathsman, hep-ph/0212078; .

Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space $SO(1,5)$

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu,\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Guy de Teramond
SJB

AdS/CFT

z= R²/r

- Use mapping of SO(4,2) to AdS5
- Scale Transformations represented by wavefunction Ψ(r) in 5th dimension $x_\mu^2 \to \lambda^2 x_\mu^2 \quad \equiv \quad r \to \frac{r}{\lambda} \quad \equiv \quad z \to \lambda z$
- Holographic model: Confinement at large distances and conformal symmetry at short distances

$$0 < z < z_0 = \frac{1}{\Lambda_{QCD}}, \; r > r_0 = \Lambda_{QCD} R^2$$

- Match solutions at large r to conformal dimension of hadron wavefunction at short distances

$$\psi(r) \to r^{-\Delta} \text{ at large } r, \text{ small } z$$

- Truncated space simulates "bag" boundary conditions

$$\psi(z_0) = \psi(r_0) = 0$$

$$r = \frac{R^2}{z}$$

Meson Spectrum

- Vector meson interpolating operator with twist-dimension minus spin-two, and conformal dimension $\Delta = 3 + L$

$$\mathcal{O}^{\mu}_{3+L} = \overline{\psi} \gamma^{\mu} D_{\{\ell_1} \ldots D_{\ell_m\}} \psi,$$

- AdS wave equation with effective 5-dim mass μ. Solution is a vector field Φ_μ with polarization along Poincaré coordinates:

$$\left[z^2 \, \partial_z^2 - (d-1) z \, \partial_z + z^2 \, \mathcal{M}^2 - (\mu R)^2 + d - 1\right] f_\mu(z) = 0,$$

with $\Phi_\mu(x, z) = e^{-iP \cdot x} \, f_\mu(z)$ and $P_\mu P^\mu = \mathcal{M}^2$ ($\Phi_z = 0$ gauge).

- Normalizable AdS vector mode:

$$\Phi_\mu(x, z) = C e^{-iP \cdot x} z^{\frac{d}{2}} J_{\Delta - \frac{d}{2}} (z \mathcal{M}) \, \epsilon_\mu .$$

with $\Delta = d - 1 + L$ and $(\mu R)^2 = L(L + d - 2)$.

Match fall-off at small z to Conformal Dimension of State at short distances

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

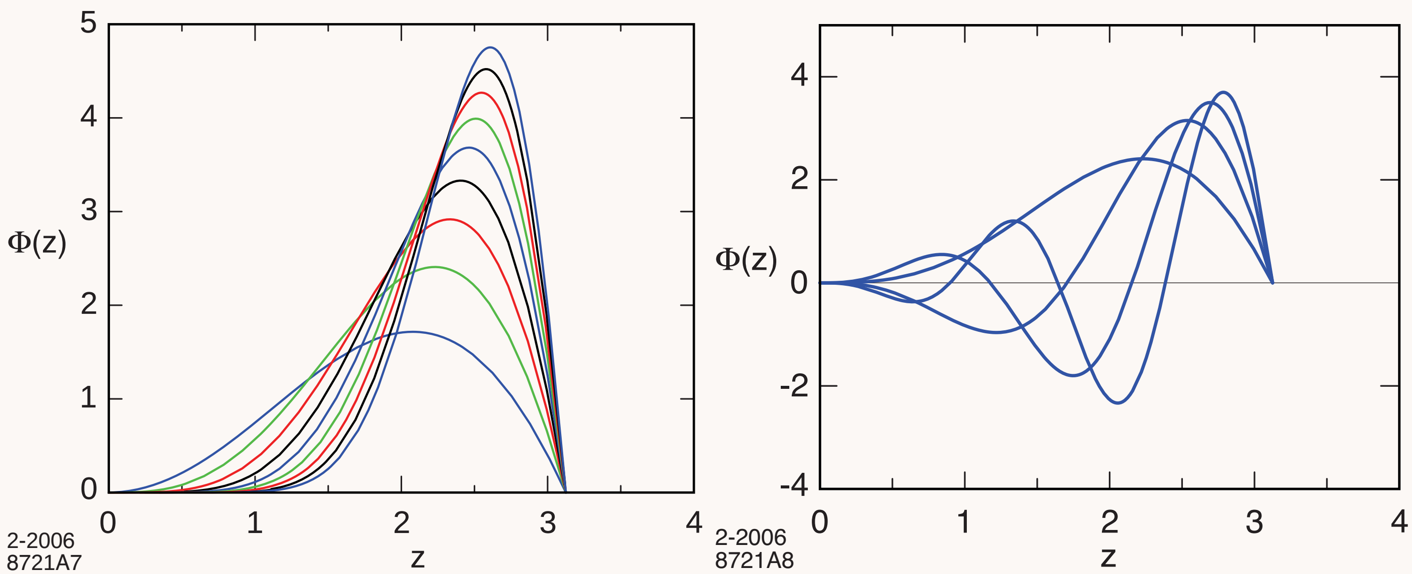


Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

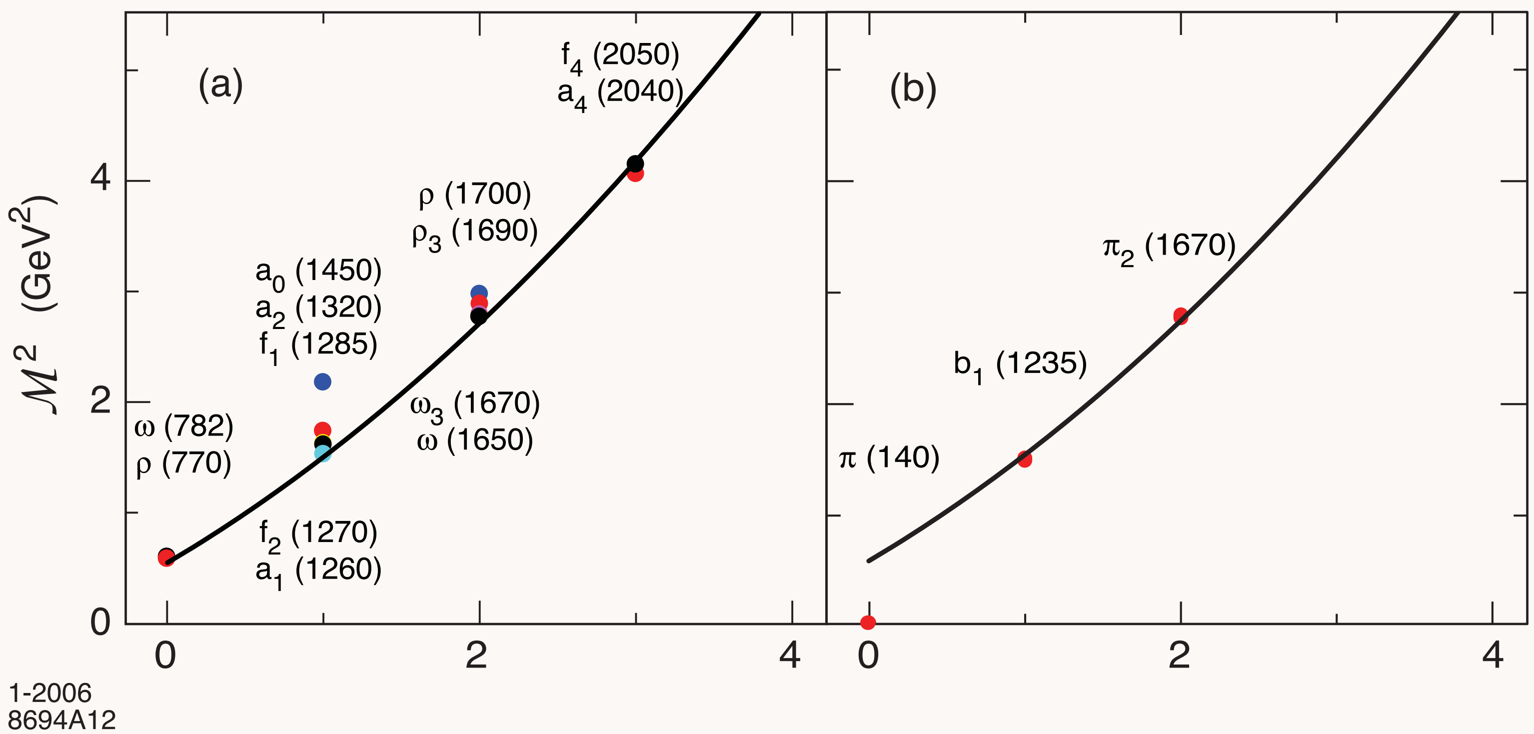


Fig: Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

- Three-quark baryon described by wave equation ($d = 4$, $\kappa = 0$)

$$\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2\mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0,$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = Ce^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P)\Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k}\Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k}\Lambda_{QCD}$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

- $SU(6)$ multiplet structure for N and Δ orbital states, including internal spin S and L.

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^+(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^+(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ $N\frac{13}{2}^-$

Predictions of AdS/CFT

Entire light quark baryon spectrum

Only one parameter!

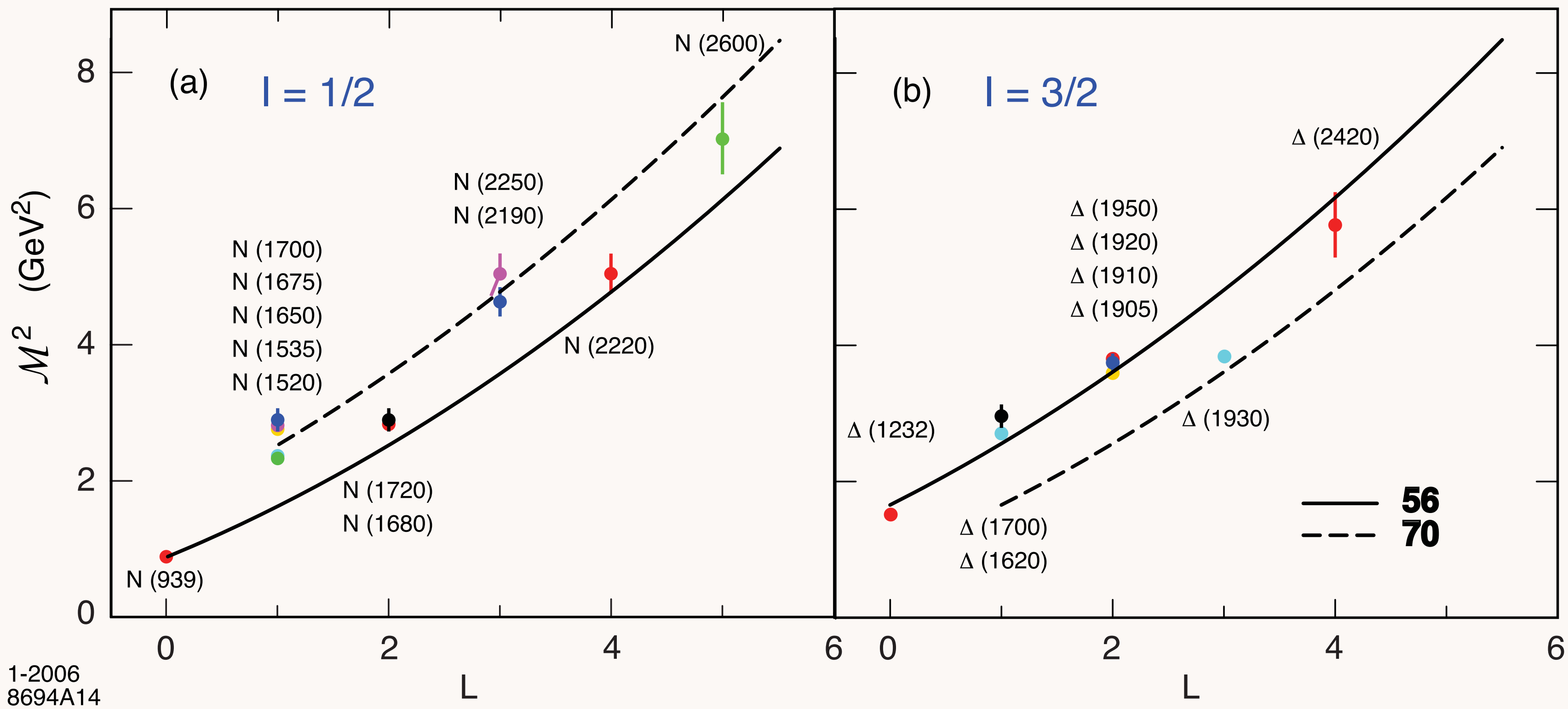


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

Guy de Teramond
SJB

AdS/CFT and Light-Front Wavefunctions

- Light-Front Wavefunctions can be determined by matching functional dependence in fifth dimension to scaling in impact space.

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

$$z \to \zeta = b\sqrt{x(1-x)}$$

- High transverse momentum behavior matches PQCD LFWF with orbital: Belitsky, Ji, Yuan

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron Φ_I and Φ_F and the non-normalizable mode J, dual to the external source:

$$F(Q^2)_{I\to F} = R^3 \int_0^\infty \frac{dz}{z^3}\, e^{3A(z)} \Phi_F(z)\, J(Q,z)\, \Phi_I(z)$$
$$\simeq R^3 \int_0^{z_o} \frac{dz}{z^3}\, \Phi_F(z)\, J(Q,z)\, \Phi_I(z).$$

- At large enough Q, the interaction occurs in the small-z conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

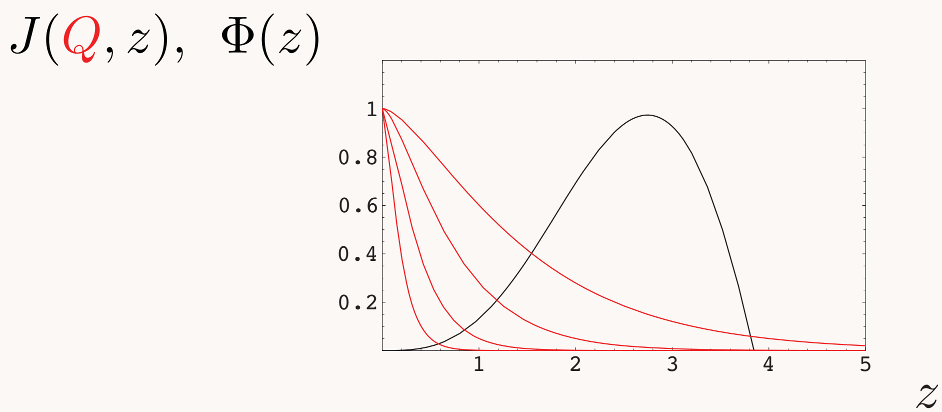


Fig: Suppression of external perturbations for large Q inside AdS

- At small z, Φ scales as $\Phi \sim z^{\Delta}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\Delta - 1},$$

Hard scattering behavior for spinless constituents ($n = \Delta$) !

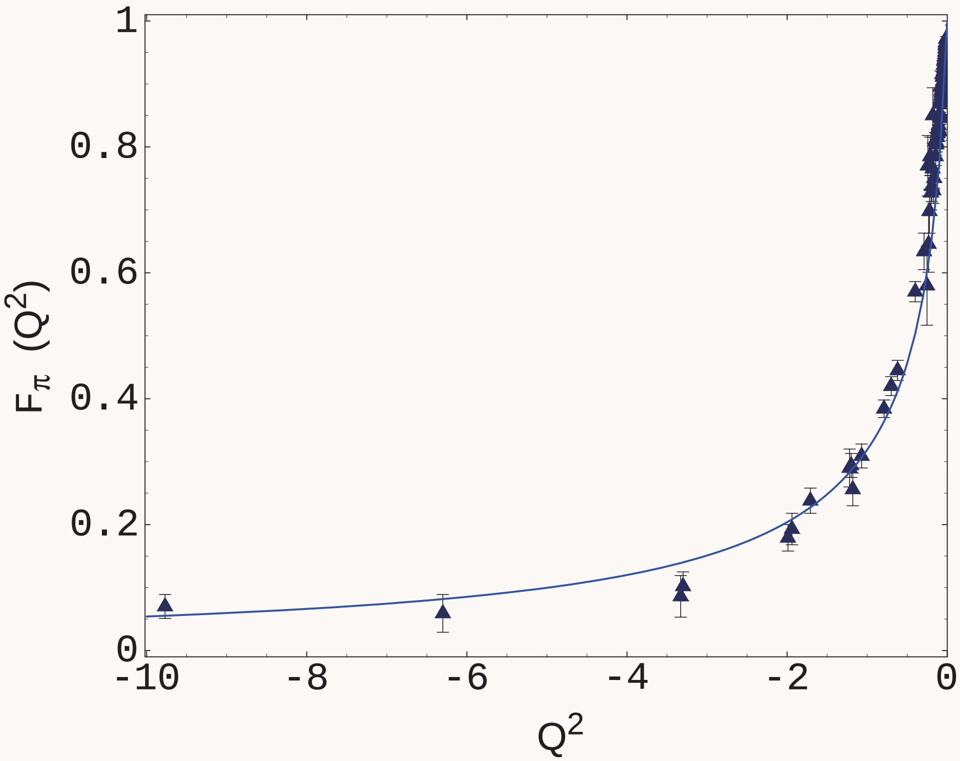
- For partons with spin σ there is an additional kinematical factor p^{σ} from the boost of the W.F.

- Spin carrying constituents ($\Delta \to \tau$):

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau - 1},$$

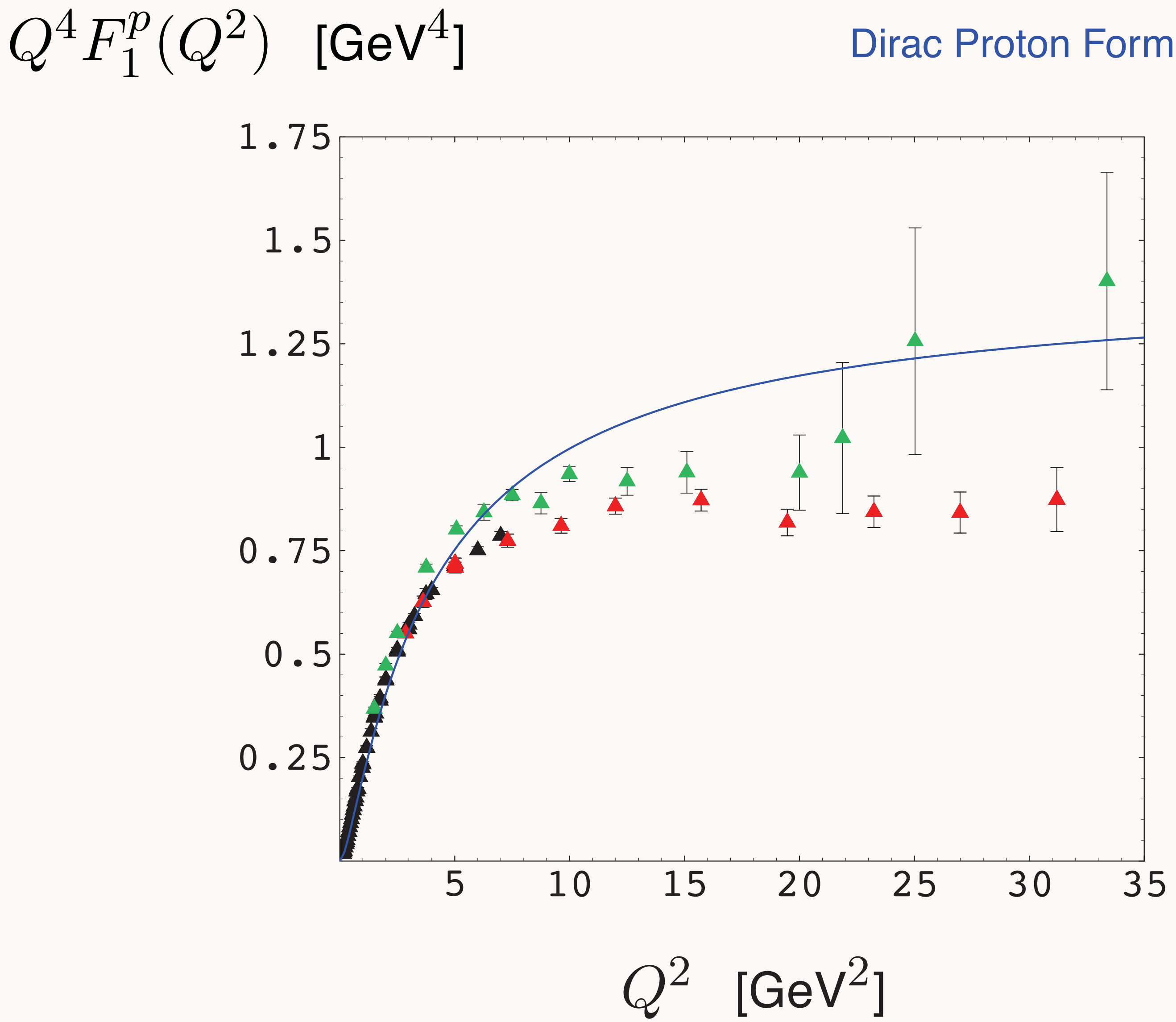
where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$.

- The twist is equal to the number of partons, $\tau = n$.



Space-like pion form factor in holographic model ($\Lambda_{QCD} = 0.2$ GeV)

Dirac Proton Form Factor F_1^p

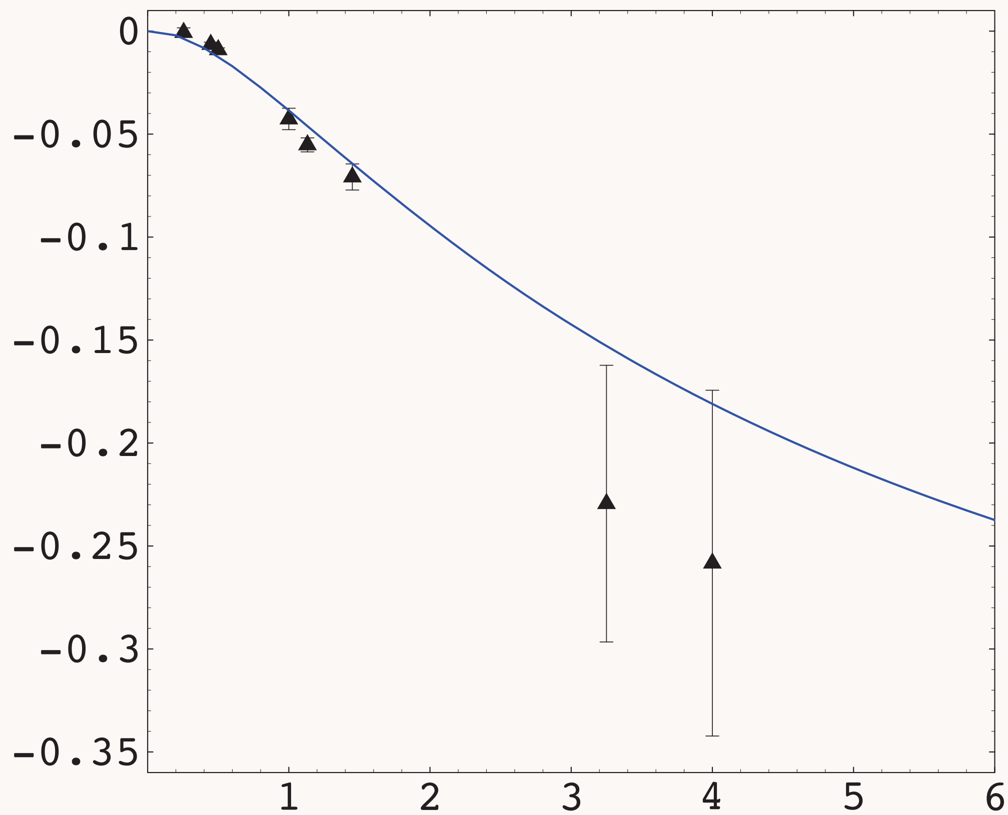


Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the infinite wall approximation.

▲ P. N. Kirk *et al.*, Phys. Rev. D **8** (1973) 63. ▲ A. F. Sill *et al.*, Phys. Rev. D **48** (1993) 29.

Dirac Neutron Form Factor F_1^n

$Q^4 F_1^n(Q^2)$ [GeV^4]

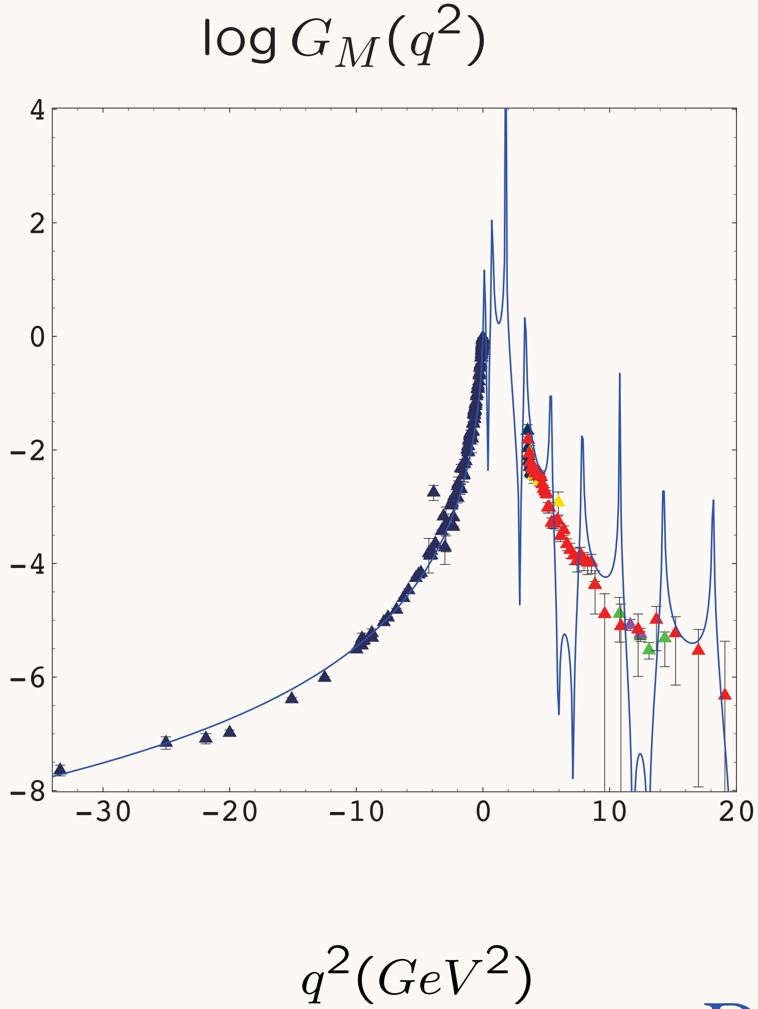


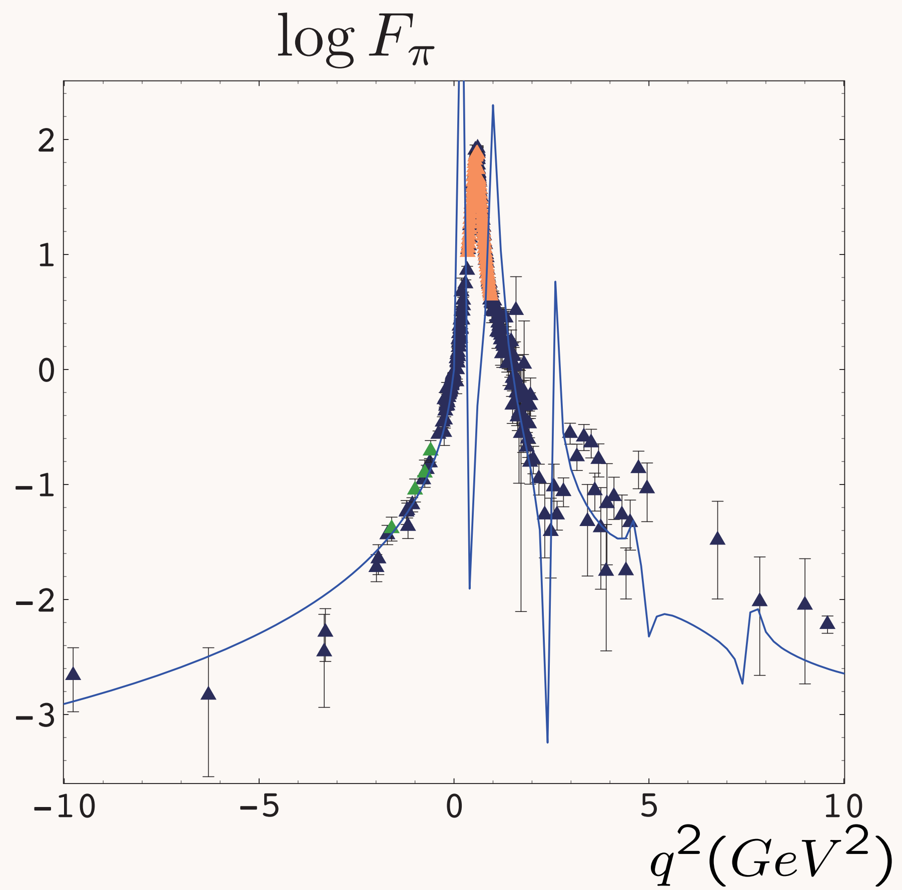
Q^2 [GeV^2]

Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\mathrm{QCD}} = 0.21$ GeV in the infinite wall approximation.

Prediction of AdS/CFT Holographic Model

Guy F. de Téramond and sjb





Dressed Current

Holographic Model for QCD Light-Front Wavefunctions

SJB and GdT in preparation

- Drell-Yan-West form factor in the light-cone (two-parton state)

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi^*_{P'}(x, \vec{k}_\perp - x\vec{q}_\perp)\, \psi_P(x, \vec{k}_\perp).$$

- Fourrier transform to impact parameter space $\vec{b}_\perp$

$$\psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2\vec{b}_\perp\, e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \widetilde{\psi}(x, \vec{b}_\perp)$$

- Find $(b = |\vec{b}_\perp|)$:

$$\begin{aligned} F(q^2) &= \int_0^1 dx \int d^2\vec{b}_\perp\, e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x, b)\right|^2 \\ &= 2\pi \int_0^1 dx \int_0^\infty b\, db\, J_0\,(bqx)\, \left|\widetilde{\psi}(x, b)\right|^2, \end{aligned}$$

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary Q. Find:

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

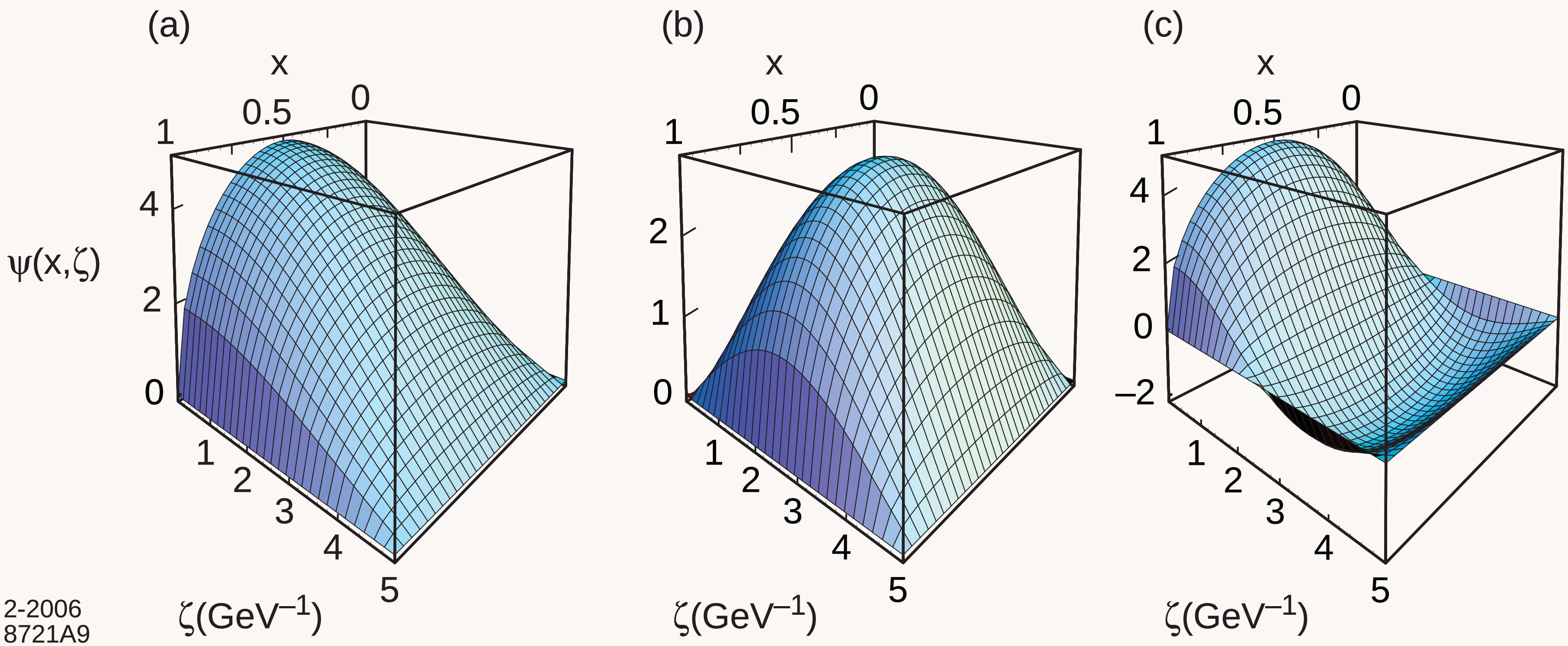
the solution for the electromagnetic potential in AdS space, and

$$\boxed{\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)}$$

the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!

$$\boxed{z = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \right|}$$



Two-quark holographic LFWF in impact space $\psi(x,\zeta)$: $(a)\ \ell = 0,\ k = 1;\ (b)\ \ell = 1,\ k = 1;$ $(c)\ \ell = 0,\ k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

AdS/CFT prediction for meson LFWF

Guy de Teramond
SJB

$$\psi_L(x,\vec{k}_\perp) = \frac{C}{4\pi} \int_0^{\Lambda^{-1}_{\rm QCD}} d\zeta \, J_0\left(\frac{\zeta|\vec{k}_\perp|}{\sqrt{x(1-x)}}\right) \, J_{1+L}\left(\zeta \mathcal{M}\right).$$

At large $k_\perp$ the LFWF has the scaling behavior

$$\psi(x,\vec{k}_\perp) \to \left[\frac{|\vec{k}_\perp|}{\sqrt{x(1-x)}}\right]^L \left[\frac{x(1-x)}{\vec{k}_\perp^2}\right]^{1+L},$$

Agrees with PQCD

Ji, Yuan
Lepage, sjb

Exact Holographic Mapping for Light-Front n-Parton State

- Define transverse position coordinates

$$x_i \vec{r}_{\perp i} = x_i \vec{R}_\perp + \vec{b}_{\perp i},$$

so that

$$\sum_{i=1}^{n} b_{\perp i} = 0, \quad \sum_{i=1}^{n} x_i \vec{r}_{\perp i} = \vec{R}_\perp.$$

- DYW result for the form factor takes the convenient form:

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, |\psi_n(x_j, \vec{b}_\perp)|^2 e^{i\vec{q}_\perp \cdot \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}},$$

corresponding to a change of transverse momentum $x_j \vec{q}_\perp$ for each of the $n-1$ spectators.

- Struck constituent with momentum fraction x and $n-1$ spectators with total longitudinal momentum $1-x$.

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

- Our final result: hadronic QCD transverse density $\tilde{\rho}$ is determined by the modes Φ in AdS space!

$$\boxed{\tilde{\rho}(x,\zeta) = \frac{R^3}{2\pi}\,\frac{x}{1-x}\,e^{3A(\zeta)}\,\frac{|\Phi(\zeta)|^2}{\zeta^4}}$$

- The variable ζ, $0 \le \zeta \le \Lambda^{-1}_{\rm QCD}$, is related to the average transverse separation between spectator constituents, and it is also the holographic variable z, $\zeta = z$.

- For the two-particle case

$$\tilde{\rho}(x,\zeta) = \frac{1}{(1-x)^2}\,|\psi\,(x,\zeta)|^2\,,$$

and we recover our previous results

$$|\psi(x,\zeta)|^2 \simeq \frac{R^3}{2\pi}\,x(1-x)\,\frac{|\Phi(\zeta)|^2}{\zeta^4}\theta\left(\zeta^2 \le \Lambda^{-2}_{\rm QCD}\right).$$

Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

G. de Teramond and sjb

General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda_{\rm QCD}^{-2}}{x(1-x)}\right),$$

Features of Holographic Model

de Teramond sjb

- Ratio of proton to Delta trajectories= ratio of zeroes of Bessel functions.

- Scale Λ_{QCD} determines hadron spectrum (slightly different for mesons and baryons)

- Covariant version of bag model: confinement plus conformal symmetry

- Pion decay constant

Evaluation of QCD Matrix Elements: Example f_π

- Pion decay constant defined by the matrix element of EW current J_W^+:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ (1-\gamma_5) \psi_d \right| \pi^- \right\rangle = i\sqrt{2} P^+ f_\pi,$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle.$$

- Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky, Phys. Rev. D **22**, 2157 (1980)

- Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\rm QCD}}{8 J_1(\beta_{0,1})} = 83.4 \;{\rm Mev},$$

for $\Lambda_{\rm QCD} = 0.2$ GeV.

Experiment: $f_\pi = 92.4$ Mev.

New Perspectives on QCD Phenomena from AdS/CFT

- AdS/CFT: Duality between string theory in Anti-de Sitter Space and Conformal Field Theory
- New Way to Implement Conformal Symmetry
- Holographic Model: Conformal Symmetry at Short Distances, Confinement at large distances
- Remarkable predictions for hadronic spectra, wavefunctions, interactions: gluonium (gg) , meson (q q), and baryon (qqq)
- Quark-interchange dominates scattering amplitudes
- AdS/CFT predicts Light-front wavefunctions: Fundamental description of hadrons at amplitude level

Outlook

- Only one scale Λ_{QCD} determines hadronic spectrum (slightly different for mesons and baryons)
- Light-cone frame is the natural frame to establish the AdS/QCD holographic duality.
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- AdS modes dual to hadrons extrapolate to valence constituents at zero separation in the AdS boundary.
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Initial good approximation for description of the structure of hadronic form factors and other observables.
- Use of holographic light-front wave functions to compute hadronic matrix elements.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model, modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.
- Precise mapping of string modes to partonic states. Modes inside AdS represent the probability amplitude for the distribution of quarks at a given scale.
- Exact holographic mapping for n-parton state determines effective QCD transverse charge density in terms of modes in AdS space.
- Holographic mapping allows deconstruction: express the eigenvalue problem in terms of 3+1 QCD degrees of freedom.

Hadron Dynamics at the Amplitude Level

- LFWFS are the universal hadronic amplitudes that underlie structure functions, GPDs, exclusive processes.
- Relation of transversity and other distributions to physics of the hadron itself.
- Connections between observables
- GPDs are not densities or probability distributions
- Parton number not conserved: n=n’ & n=n’+2 at nonzero skewness
- orbital angular momentum
- Role of FSI and ISIs--Sivers effect

Physics of Rescattering

- Diffractive DIS: New Insight into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon

Anomalous QCD Effects

- Hidden Color of Nuclear Wavefunction
- Odderon Trajectory: Charm jet asymmetry
- Anomalous Regge Behavior: J=0 Fixed Pole
- Proton-Proton Scattering:
  Color Transparency Breakdown and A_{NN}
- Non-Universality of Antishadowing
- Intrinsic Heavy Quarks at large x
- Anomalous scaling of single-particle inclusive at high p_T

Compute LFWFs from First Principles

- Very difficult using Euclidean lattice
- Discretized light-cone quantization: Diagonalize light-cone Hamiltonian
- Bethe-Salpeter Dyson-Schwinger Eqns
- Transverse lattice
- AdS/CFT guidance: Initial Approximation

Solving the LF Heisenberg Equation

- Discretized Light-Cone Quantization (DLCQ) Pauli, sjb
  Minkowski space !
- Many 1+1 model field theories completely solved using DLCQ Hornbostel, Pauli, sjb; Klebanov
- UV Regularization: 3+ 1 Pauli Villars Hiller, McCartor, sjb
- Transverse Lattice Bardeen, Peterson,Rabinovici, Burkardt, Dalley
- Bethe-Salpeter/Dyson-Schwinger at fixed LF time
- Angular Structure of Solutions known Karmanov, Hwang, sjb
- Use AdS/CFT model solutions as starting point! Vary, sjb

Essential to test QCD

- GSI antiprotons
- 12 GeV Jlab
- J-Parc
- BaBar/Belle: ISR, two-gamma, timelike DVCS
- RHIC/LHC Nuclear Collisions; LHCb
- photon-photon collider at the ILC (Zerwas)
- electron-proton, electron-nucleus collisions