Summary of Supersymmetric Relations

Massless	**Massive**
$F_G + 4F_Q + (10-d)F_S = 0$	$F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$
$\Sigma_{QG}(F) \equiv \frac{d-2}{2} F_Q + F_G$	$\Sigma_{MQG}(F) \equiv \frac{d-1}{2} F_{MQ} + F_{MG}$
= simple	= simple

3 Scale Effective Charge

$$\tilde{\alpha}(a,b,c) \equiv \frac{\tilde{g}^2(a,b,c)}{4\pi}$$

(First suggested by H.J. Lu)

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\alpha_{bare}} + \frac{1}{4\pi}\beta_0\left(L(a,b,c) - \frac{1}{\varepsilon} + \cdots\right)$$

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

L(a,b,c) = 3-scale "log-like" function

L(a,a,a) = log(a)

3 Scale Log-Like Function

$$L(a,b,c) = \frac{1}{K}\left(\alpha\gamma \log a + \alpha\beta \log b + \beta\gamma \log c - abc\bar{J}(a,b,c)\right) + \Omega$$

$$K = \alpha\beta + \beta\gamma + \gamma\alpha$$

Master triangle integral can be written in terms of Clausen functions

$$Cl_2(\theta) = \mathrm{Im}\, Li_2\left(e^{i\theta}\right)$$

$$\alpha = p_1 \cdot p_2 = \frac{1}{2}(c - a - b)$$

$$\beta = p_2 \cdot p_3 = \frac{1}{2}(a - b - c)$$

$$\gamma = p_3 \cdot p_1 = \frac{1}{2}(b - c - a)$$

$$a = p_1^2$$

$$b = p_2^2$$

$$c = p_3^2$$

$$\Omega \approx 3.125$$

3 Scale Effective Scale

$$L(a,b,c) \equiv \log\left(Q^2_{eff}(a,b,c)\right) + i\,\mathrm{Im}\,L(a,b,c)$$

Governs strength of the three-gluon vertex

$$\frac{1}{\tilde{\alpha}(a,b,c)} = \frac{1}{\tilde{\alpha}(a_0,b_0,c_0)} + \frac{1}{4\pi}\beta_0\left[L(a,b,c) - L(a_0,b_0,c_0)\right]$$

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} \propto \sqrt{\tilde{\alpha}(a,b,c)}$$

Generalization of BLM Scale to 3-Gluon Vertex

Properties of the Effective Scale

$$Q^2_{eff}(a,b,c) = Q^2_{eff}(-a,-b,-c)$$

$$Q^2_{eff}(\lambda a, \lambda b, \lambda c) = |\lambda| Q^2_{eff}(a,b,c)$$

$$Q^2_{eff}(a,a,a) = |a|$$

$$Q^2_{eff}(a,-a,-a) \approx 5.54 |a|$$

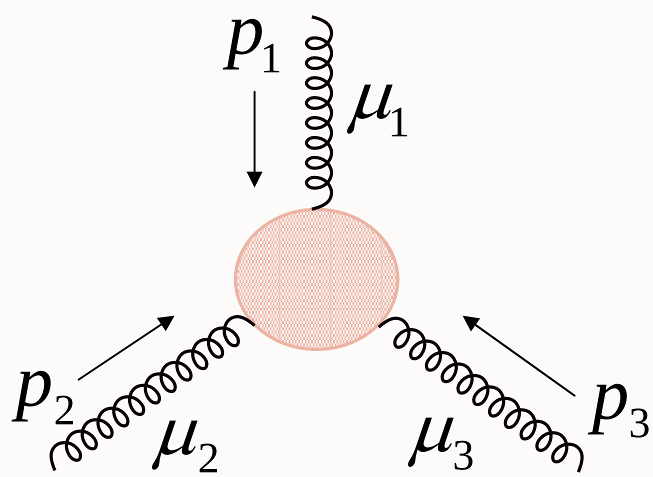
$$Q^2_{eff}(a,a,c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,-a,c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c|$$

$$Q^2_{eff}(a,b,c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c|$$

Surprising dependence on Invariants

$$\hat{\Gamma}_{\mu_1\mu_2\mu_3} =$$



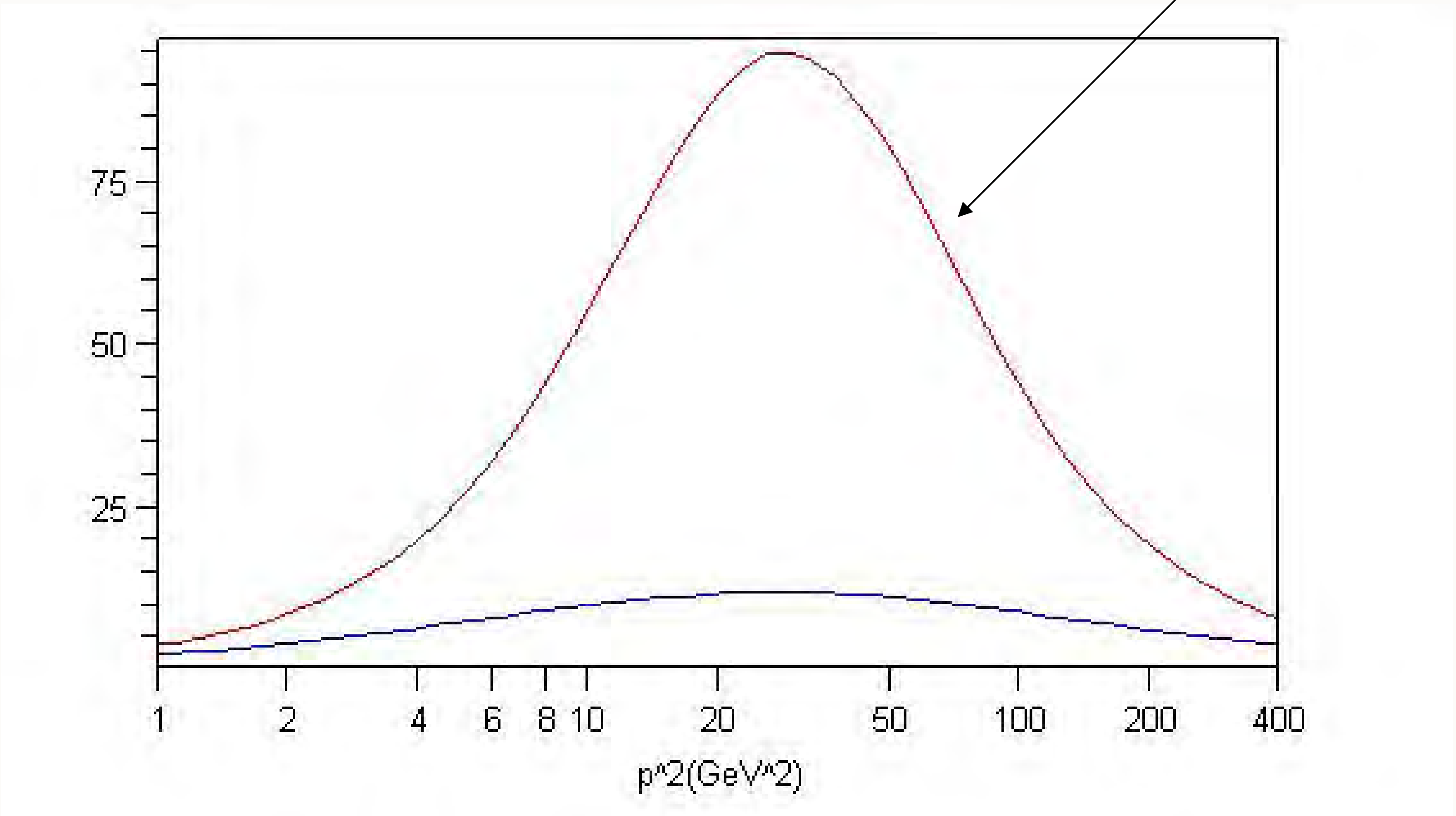
H. J. Lu

$$\mu_R^2 \simeq \frac{p^2_{min} p^2_{med}}{p^2_{max}}$$

The Effective Scale

$Q^2_{eff}\left(10\,\text{GeV}^2, 10\,\text{GeV}^2, p^2\right)$

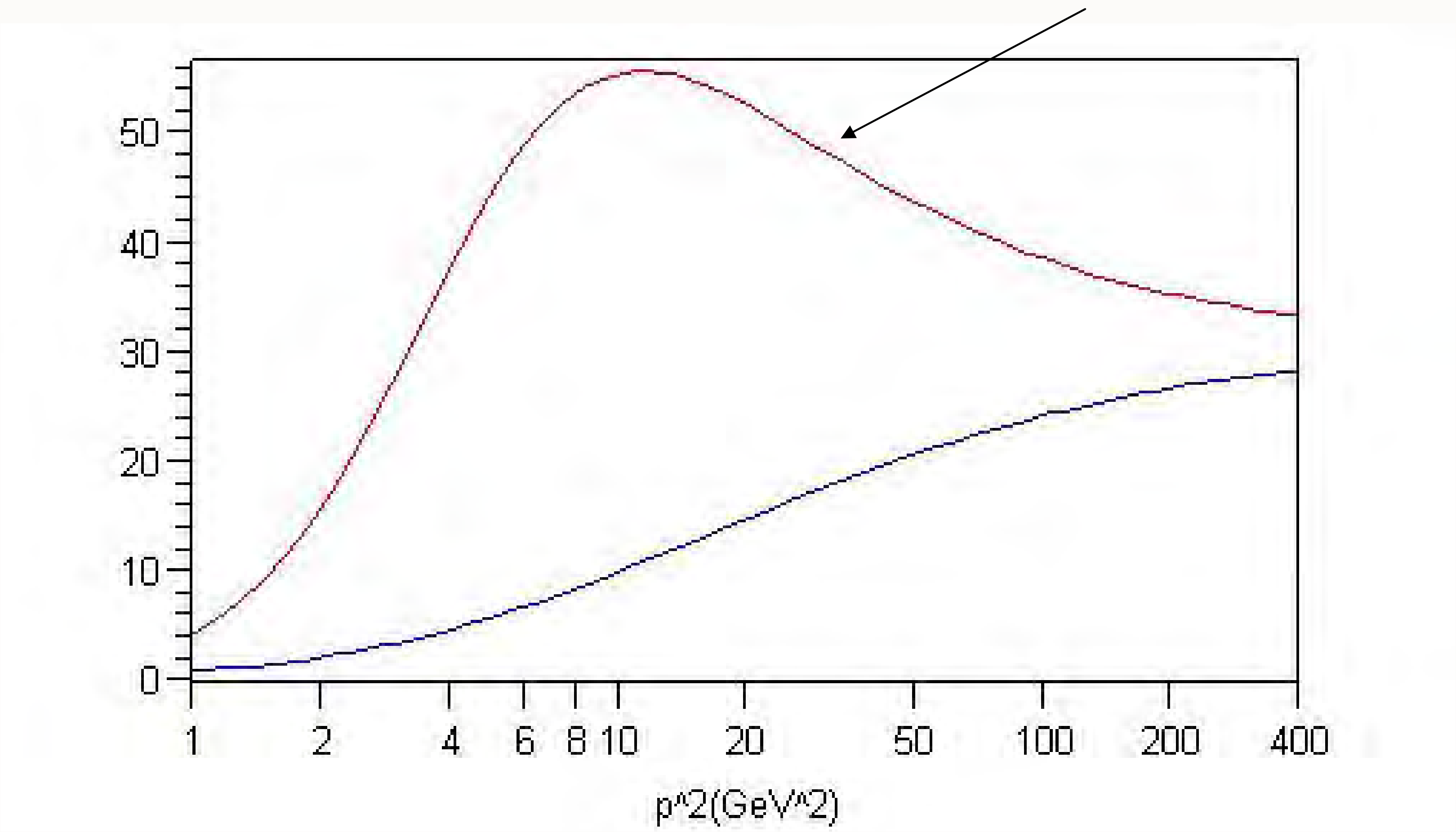
$Q^2_{eff}\left(-10\,\text{GeV}^2, -10\,\text{GeV}^2, p^2\right)$



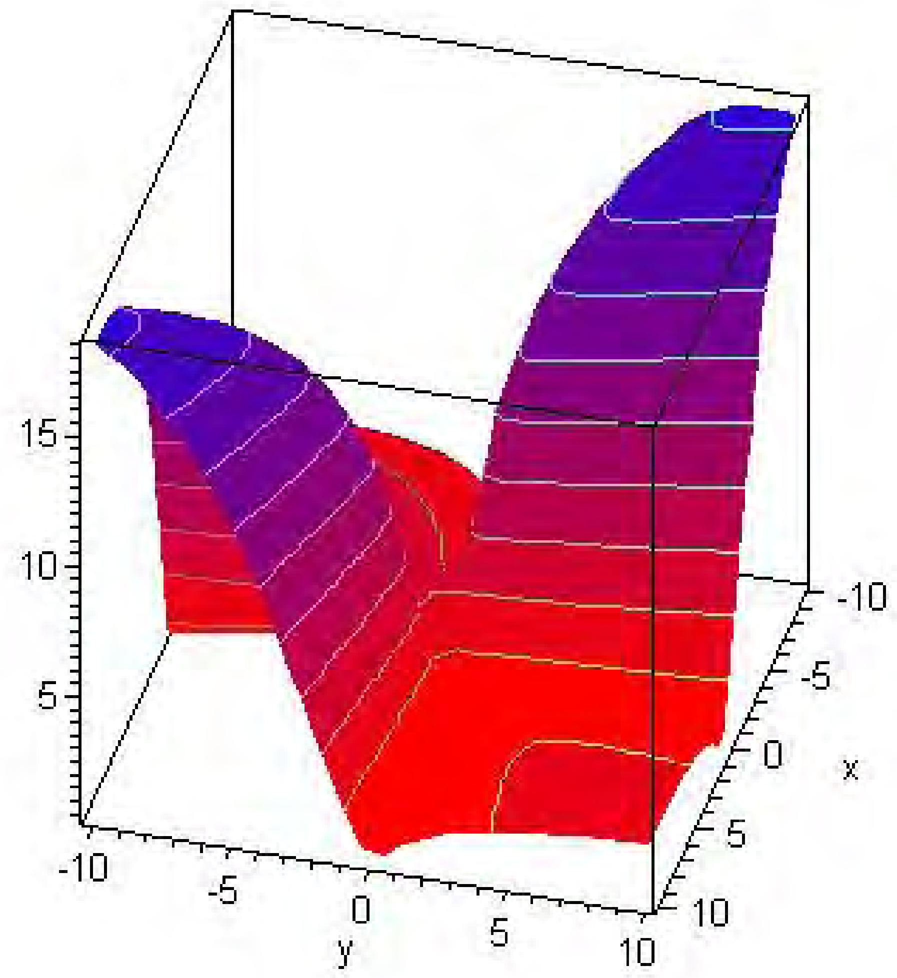
The Effective Scale

$$Q_{eff}^2\left(10\,\mathrm{GeV}^2, p^2, p^2\right)$$

$$Q_{eff}^2\left(-10\,\mathrm{GeV}^2, p^2, p^2\right)$$



The Effective Scale



$$Q^2_{eff}(1, x, y)$$

Mass Effects

Calculated for all form factors

SUSY relations $F_{MG} + 4F_{MQ} + (9-d)F_{MS} = 0$

FF of tree level tensor structure ⟺ Effective Charge

Massive "log-like" function : $L_{MQ}\left(\frac{a}{M^2}, \frac{b}{M^2}, \frac{c}{M^2}\right)$

$$L_{MQ}\left(\frac{a}{M^2}, \frac{b}{M^2}, \frac{c}{M^2}\right) \approx 5.125 \text{ for } M^2 >> |a|,|b|,|c|$$

$$L_{MQ}\left(\frac{a}{M^2}, \frac{b}{M^2}, \frac{c}{M^2}\right) \approx L(a,b,c) - \log M^2 \text{ for } M^2 << |a|,|b|,|c|$$

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Massive Log-Like Function

$$L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)=\frac{1}{K}\Big(\alpha\gamma\Lambda(a)+\alpha\beta\Lambda(b)+\beta\gamma\Lambda(c)-abc\overline{J_M}(a,b,c)\Big)+\Omega$$

$$+2M^2\left(\frac{\Lambda(a)-2}{a}+\frac{\Lambda(b)-2}{b}+\frac{\Lambda(c)-2}{c}-\overline{J_M}(a,b,c)\right)$$

$$\Lambda(a)=\begin{Bmatrix} 2v\tanh^{-1}\left(v^{-1}\right) \\ 2\bar{v}\tan^{-1}\left(\bar{v}^{-1}\right) \\ 2v\tanh^{-1}(v)-iv\pi \end{Bmatrix}\ \text{for}\ \begin{Bmatrix} a<0 \\ 0<a<4M^2 \\ a>4M^2 \end{Bmatrix}$$

$$v=\sqrt{1-\frac{4M^2}{a}}\qquad \bar{v}=\sqrt{\frac{4M^2}{a}-1}$$

Massive Master Triangle Integral (very complicated)

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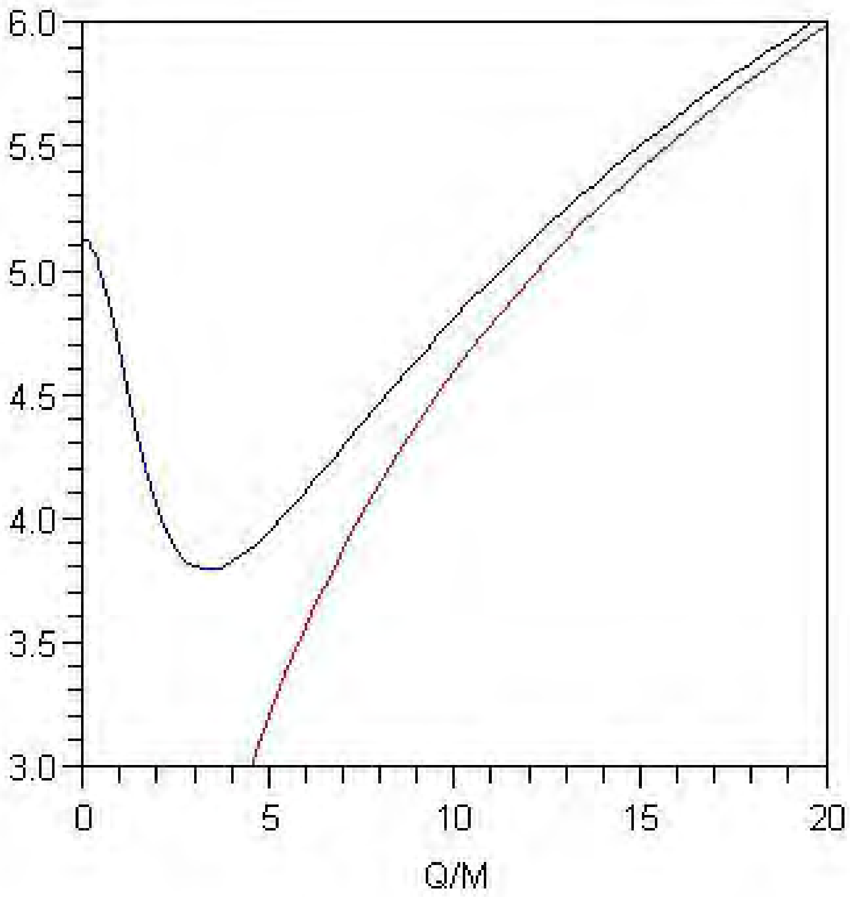
Symmetric Spacelike



$$L_{MQ}\left(-\frac{Q^2}{M^2},-\frac{Q^2}{M^2},-\frac{Q^2}{M^2}\right)$$

$$\log\left(\frac{Q^2}{M^2}\right)$$

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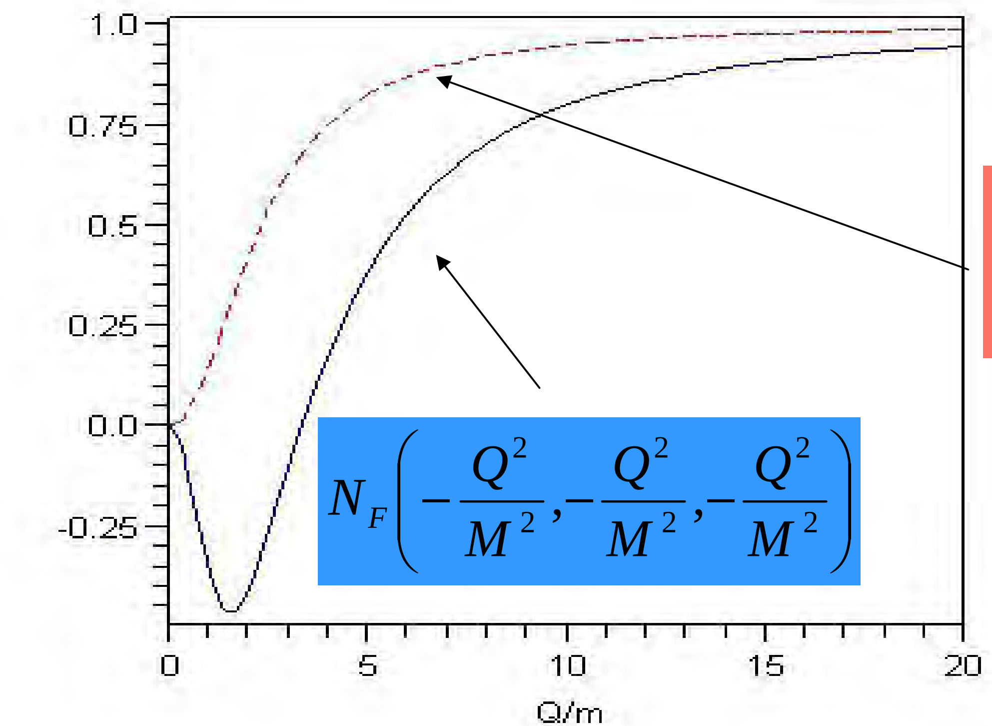
Effective Number of Flavors

$$N_F\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)=-\frac{d}{d\log M^2}L_{MQ}\left(\frac{a}{M^2},\frac{b}{M^2},\frac{c}{M^2}\right)$$

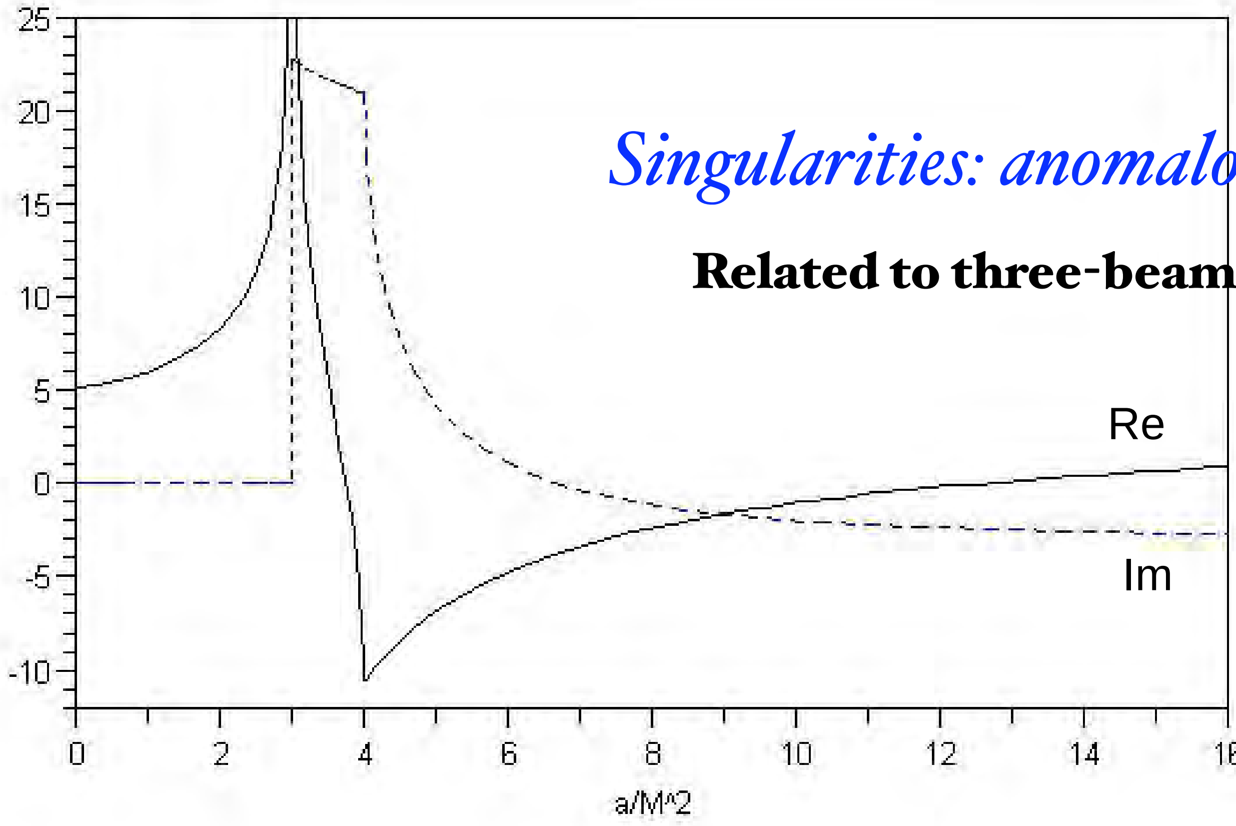


$$n_f\left(\frac{Q^2}{M^2}\right)=-\frac{d}{d\log M^2}L_{1/2}\left(\frac{Q^2}{M^2}\right)$$

$$\approx\frac{1}{1+\frac{M^2}{Q^2}e^{5/3}}$$

Symmetric Timelike

$$L_{MQ}\left(\frac{a}{M^2},\frac{a}{M^2},\frac{a}{M^2}\right)$$



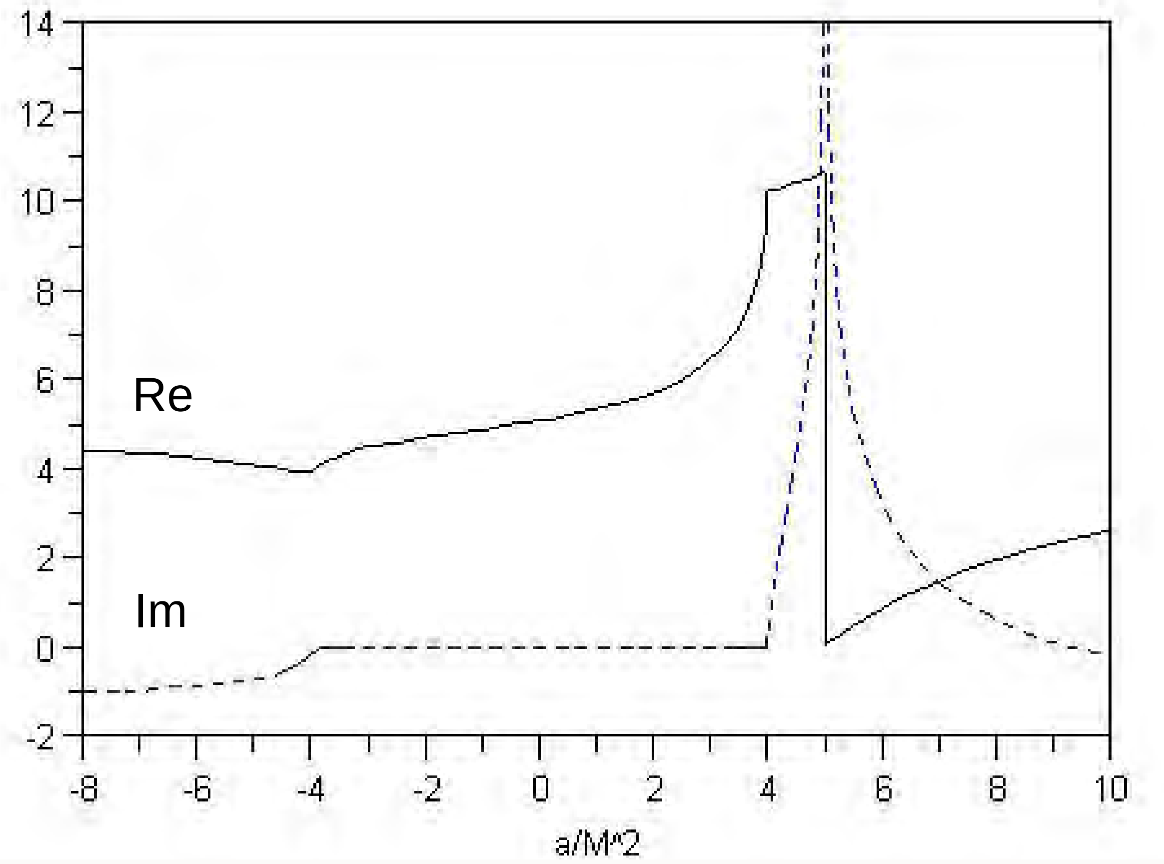
Singularities: anomalous thresholds

Related to three-beam scattering?

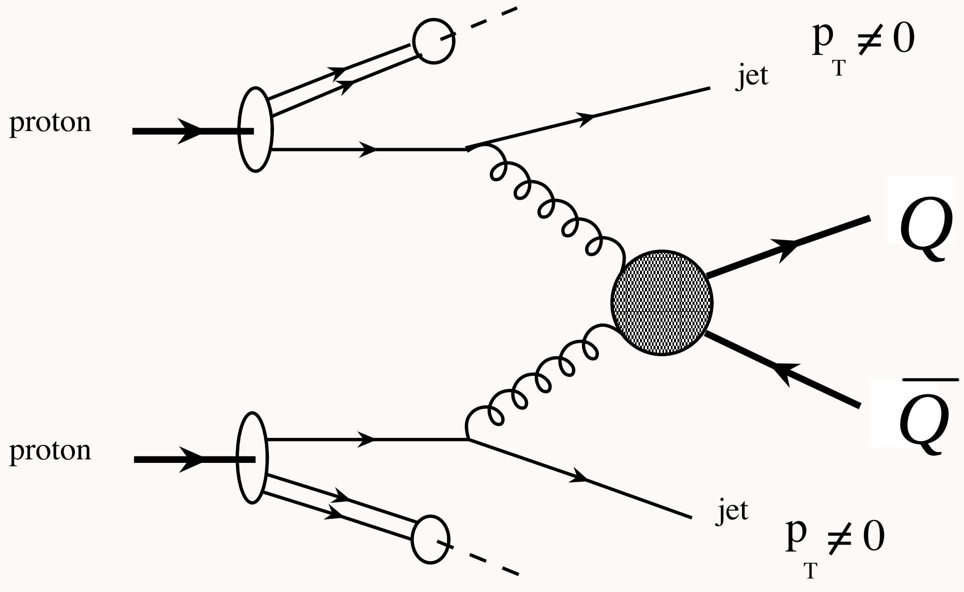
Related to three-beam scattering?

Symmetric Mixed Signature

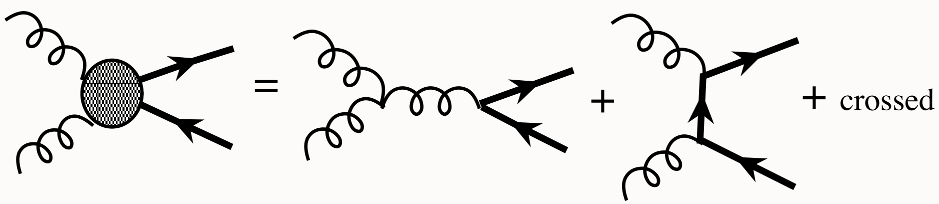
$$L_{MQ}\left(\frac{a}{M^2},\frac{a}{M^2},-\frac{a}{M^2}\right)$$



Heavy Quark Hadro-production



where



- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale

→ much larger cross section than $\overline{MS}$ with scale $\mu_R = M_{Q\overline{Q}}$ or M_Q

- Future : repeat analysis using the full mass-dependent results and include all form factors

Expect that this approach accounts for most of the one-loop corrections

Use Physical Scheme to Characterize QCD Coupling

- Use Observable to define QCD coupling or Pinch Scheme
- Analytic: Smooth behavior as one crosses new quark threshold
- New perspective on grand unification

Binger, Sjb

Unification in Physical Schemes

"PHYSICAL RENORMALIZATION SCHEMES AND GRAND UNIFICATION"
M.B. and Stanley J. Brodsky. **Phys.Rev.D69:095007,2004**

$$\alpha_i(Q) = \frac{\alpha_i(Q_0)}{1 + \hat{\Pi}_i(Q) - \hat{\Pi}_i(Q_0)}$$

i=1,2,3

$$\hat{\Pi}_i(Q) = \frac{\alpha_i}{4\pi} \sum_p \beta_i^{(p)} \left(L_{s(p)}(Q^2 / m_p^2) + \cdots \right)$$

"log-like" function:

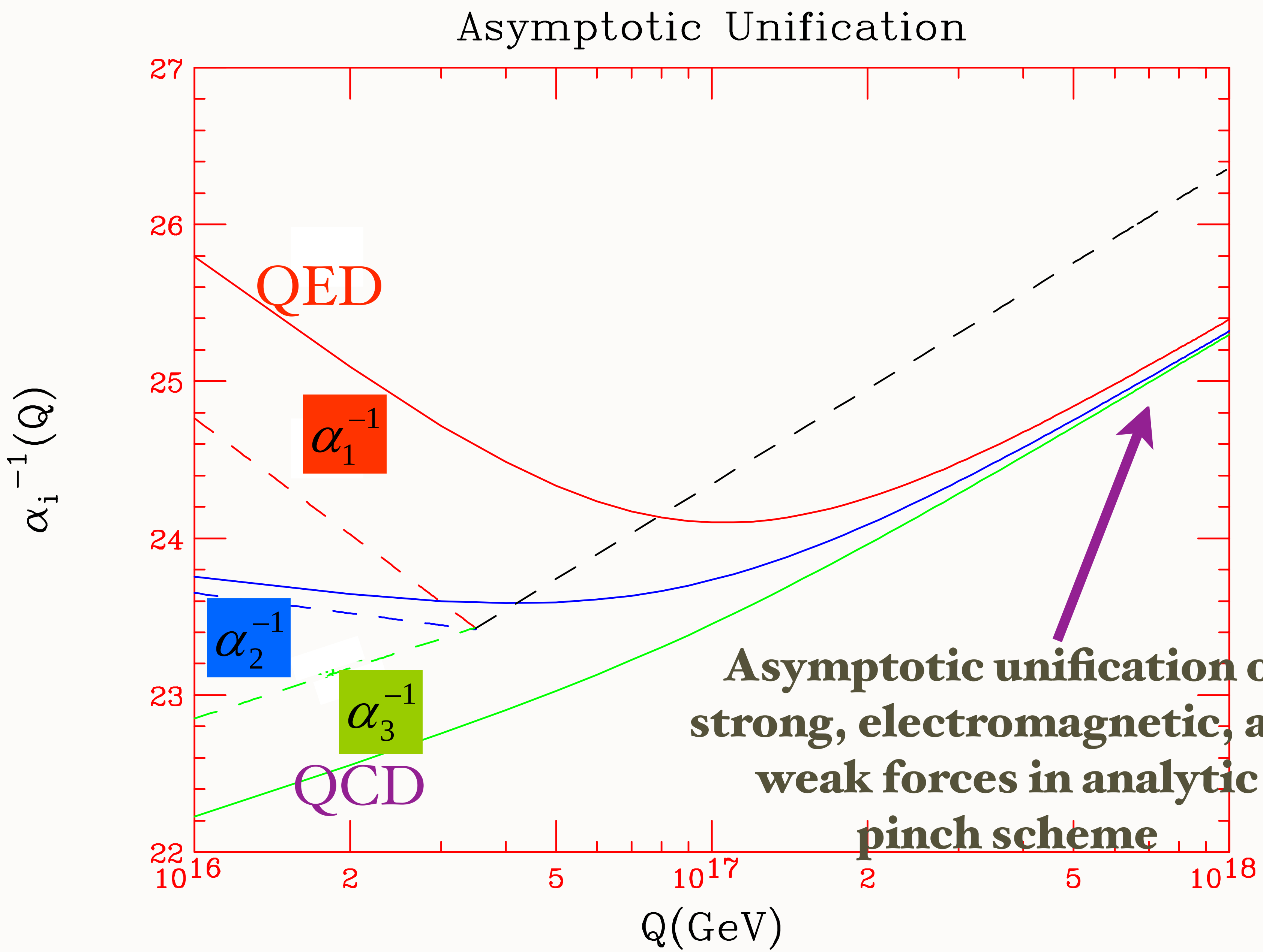
$$L_{s(p)} \approx \log(e^{\eta_p} + Q^2 / m_p^2)$$

$$\eta_p = 8/3,\ 5/3,\ 40/21$$

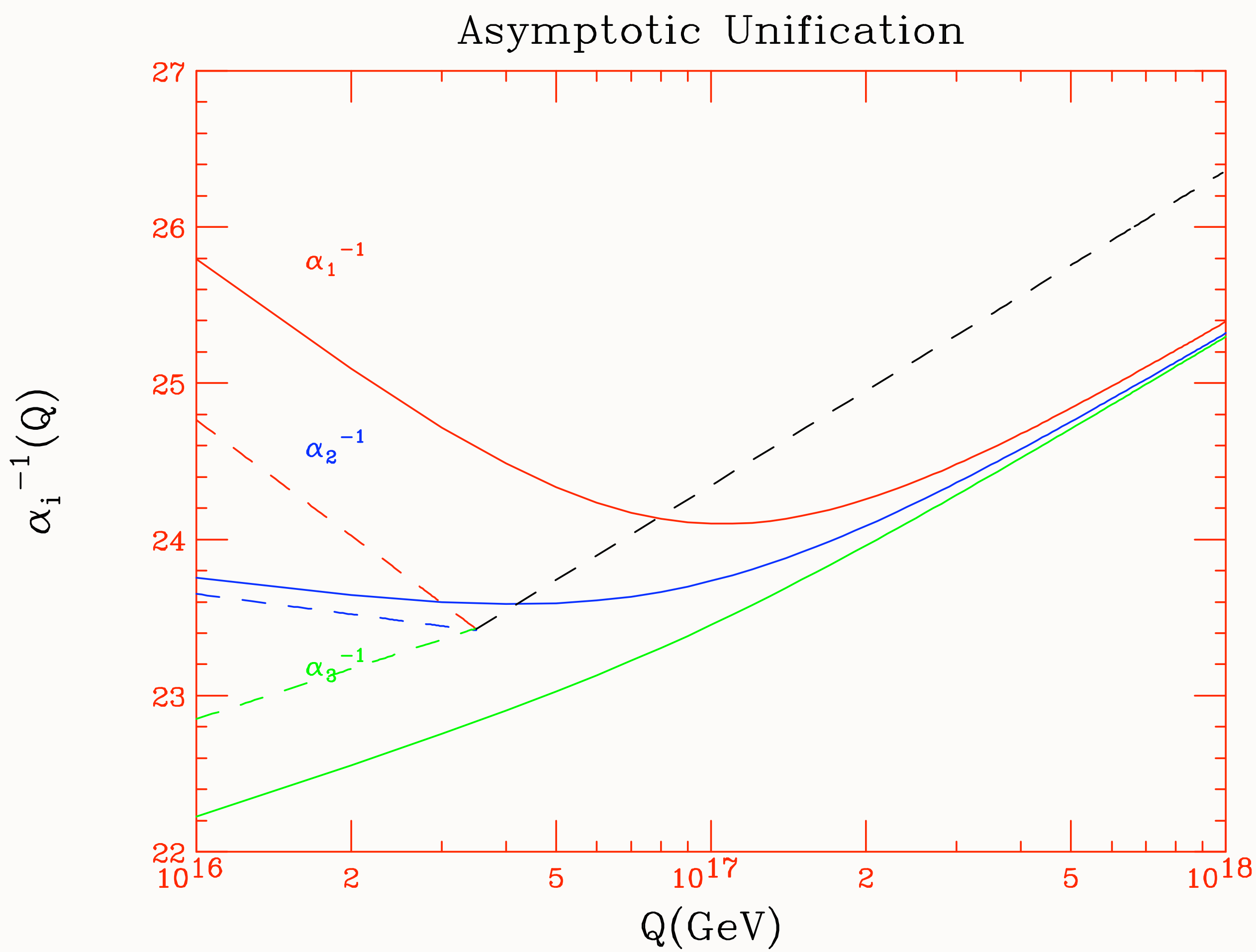
For spin s(p) = 0, ½, and 1

⇨ Elegant and natural formalism for all threshold effects

Binger, sjb



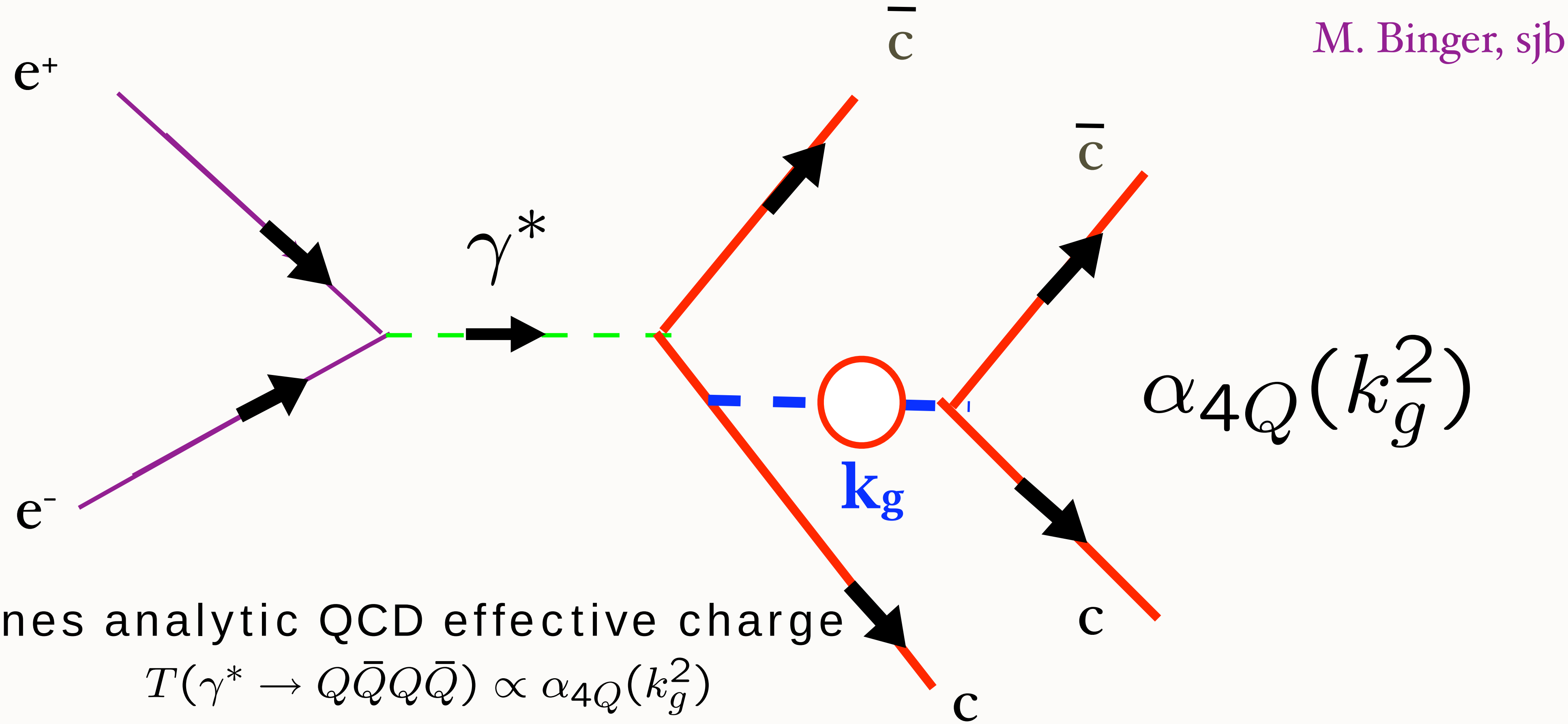
Asymptotic unification of strong, electromagnetic, and weak forces in analytic pinch scheme



Binger, sjb

Asymptotic Unification. The solid lines are the analytic $\overline{PT}$ effective couplings, while the dashed lines are the $\overline{DR}$ couplings. For illustrative purposes, $\alpha_3(M_Z)$ has been chosen so that unification occurs at a finite scale for $\overline{DR}$ and asymptotically for the $\overline{PT}$ couplings. Here $M_{SUSY} = 200\text{GeV}$ is the mass of all light superpartners except the wino and gluino which have values $\frac{1}{2}m_{\tilde{g}} = M_{SUSY} = 2m_{\tilde{w}}$. For illustrative purposes, we use $SU(5)$.

Production of four heavy-quark jets



Defines analytic QCD effective charge

$$T(\gamma^* \to Q\bar{Q}Q\bar{Q}) \propto \alpha_{4Q}(k_g^2)$$

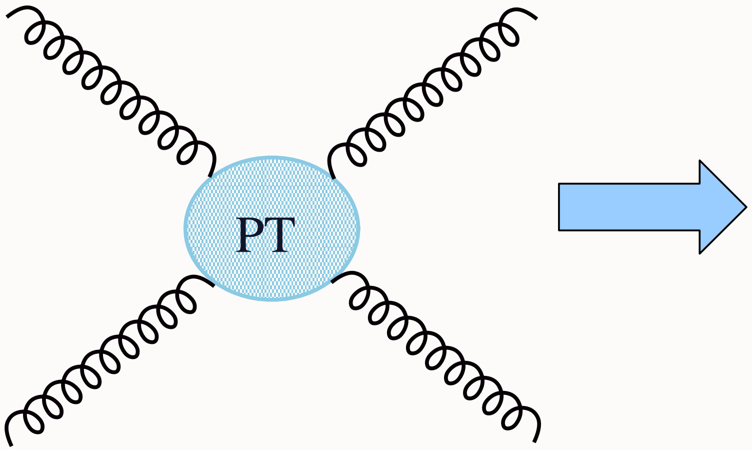
time-like values not same as space-like

coupling similar to "pinch" scheme

complex for time-like argument

Future Directions

Gauge-invariant four gluon vertex

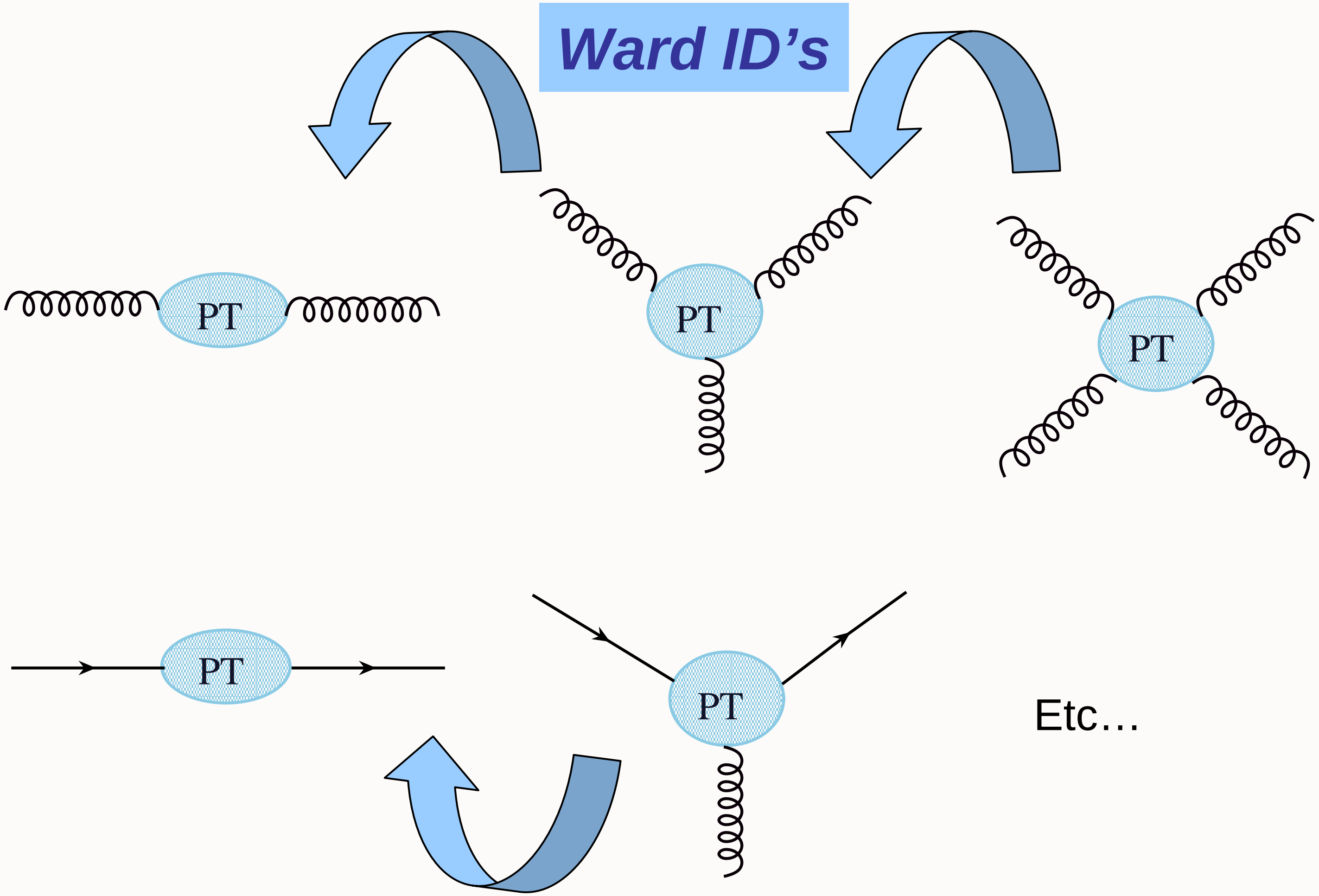


$$L_4(p_1, p_2, p_3, p_4)$$

$$Q^2_{4\,eff}(p_1, p_2, p_3, p_4)$$

Hundreds of form factors!

The Gauge-Invariant Family of Green's Functions



PT Self-Energy at Two-Loops

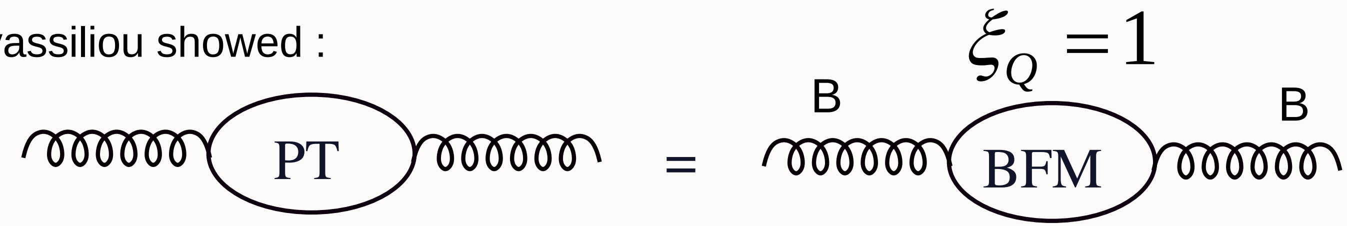
- Finite terms give relation between

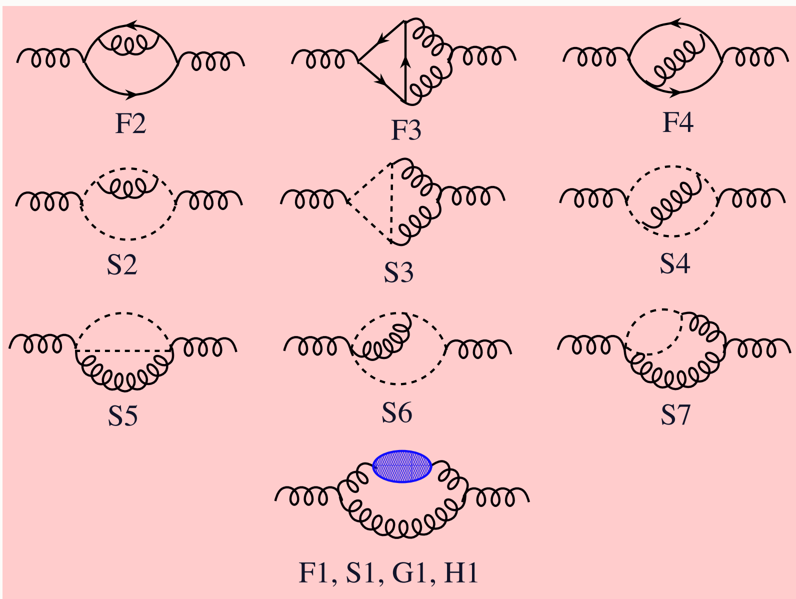
$$\alpha_{PT}(Q^2) \text{ and } \alpha_{\overline{MS}}(Q^2)$$

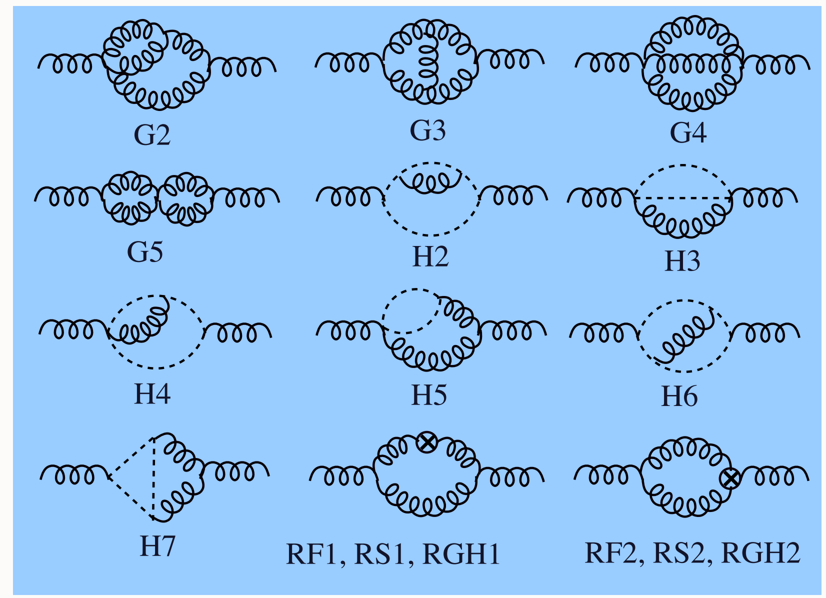
- 3-loop beta function
- 2-loop longitudinal form factors of the three-gluon vertex (via the Ward ID)
- N=4 Supersymmetry gives a non-zero but UV finite contribution

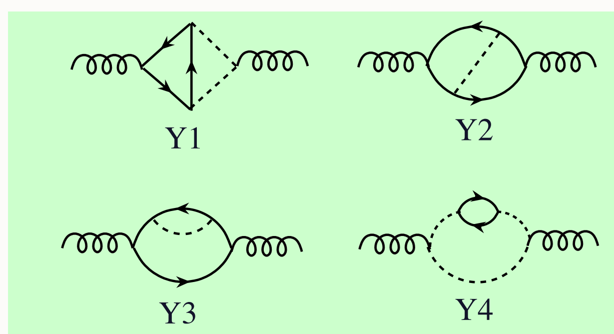
PT Self-Energy at Two-Loops

Papavassiliou showed :









Future Directions

- *Implement in Monte Carlo generator*
- *Gauge-invariant Standard Model triple gauge boson vertices*
- *Schwinger-Dyson Equations*

Summary and Future

- ***Multi-scale analytic*** renormalization based on ***physical, gauge-invariant*** Green's functions

- ***Optimal*** improvement of perturbation theory with ***no scale-ambiguity*** since physical kinematic invariants are the arguments of the (multi-scale) couplings

Conventional renormalization scale-setting method :

- Guess arbitrary renormalization scale and take arbitrary range. Wrong for QED and Precision Electroweak.

- Prediction depends on choice of renormalization scheme

- Variation of result with respect to renormalization scale only sensitive to nonconformal terms; no information on genuine (conformal) higher order terms

- FAC and PMS give unphysical results.

- Renormalization scale not arbitrary: Analytic constraint from flavor thresholds

Features of BLM Scale Setting

On The Elimination Of Scale Ambiguities In Perturbative Quantum Chromodynamics.

Lepage, Mackenzie, sjb **Phys.Rev.D28:228,1983**

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale Q* sets the number of active flavors
- Only n_f dependence required to determine renormalization scale at NLO
- Result is scheme independent: Q* has exactly the correct dependence to compensate for change of scheme
- Correct Abelian limit
- **Resulting series identical to conformal series!**
- Renormalon n! growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants

Use BLM!

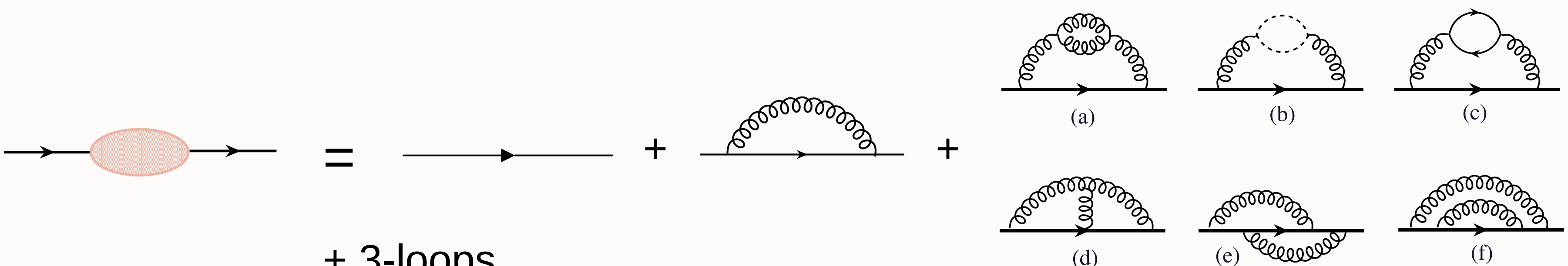
- Satisfies Transitivity, all aspects of Renormalization Group; scheme independent
- Analytic at Flavor Thresholds
- Preserves Underlying Conformal Template
- Physical Interpretation of Scales; Multiple Scales
- Correct Abelian Limit ($N_C = 0$)
- Eliminates unnecessary source of imprecision of PQCD predictions
- Commensurate Scale Relations: Fundamental Tests of QCD free of renormalization scale and scheme ambiguities
- BLM used in many applications, QED, LGTH, BFKL, ...

On Renormalons and the Structure of Perturbation Theory

Investigate the relation between :

1. Renormalons
2. BLM Scale Fixing
3. Effective Charges Running Inside of Loops

Laboratory : Higher order corrections to the quark propagator



+ 3-loops

(Gray, Broadhurst, Grafe, Schilcher and Chetyrkin, Steinhauser)

Relation between quark pole mass $\overline{MS}$ mass

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On Renormalons and the Structure of Perturbation Theory

BLM Methods

- Predicts 3-loop term with an accuracy of 3-4%
- Conformal term is very small

Not associated with running coupling

→ Expect that almost all of the loop corrections are "associated with" the running coupling

Seems to be very much in contrast to what we found using the RIA

→ Perhaps the success of BLM is not tied to a hypothetical skeleton expansion with running charges inside of loops

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Factorization scale

$$\mu_{\text{factorization}} \neq \mu_{\text{renormalization}}$$

- Arbitrary separation of soft and hard physics
- Dependence on factorization scale not associated with beta function - present even in conformal theory
- Keep factorization scale separate from renormalization scale

$$\frac{d\mathcal{O}}{d\mu_{\text{factorization}}} = 0$$

- Residual dependence when one works in fixed order in perturbation theory.