

# Why is Conformal Theory Relevant?

- Dimensional scaling of exclusive processes implies QCD is approximately conformal
- PQCD is conformal when  $\beta = 0$
- Evaluate gluon exchange at small effective scales where  $\alpha_s$  is approximately constant: IR fixed point
- Apply AdS/CFT

# Define QCD Coupling from Observable

Grunberg

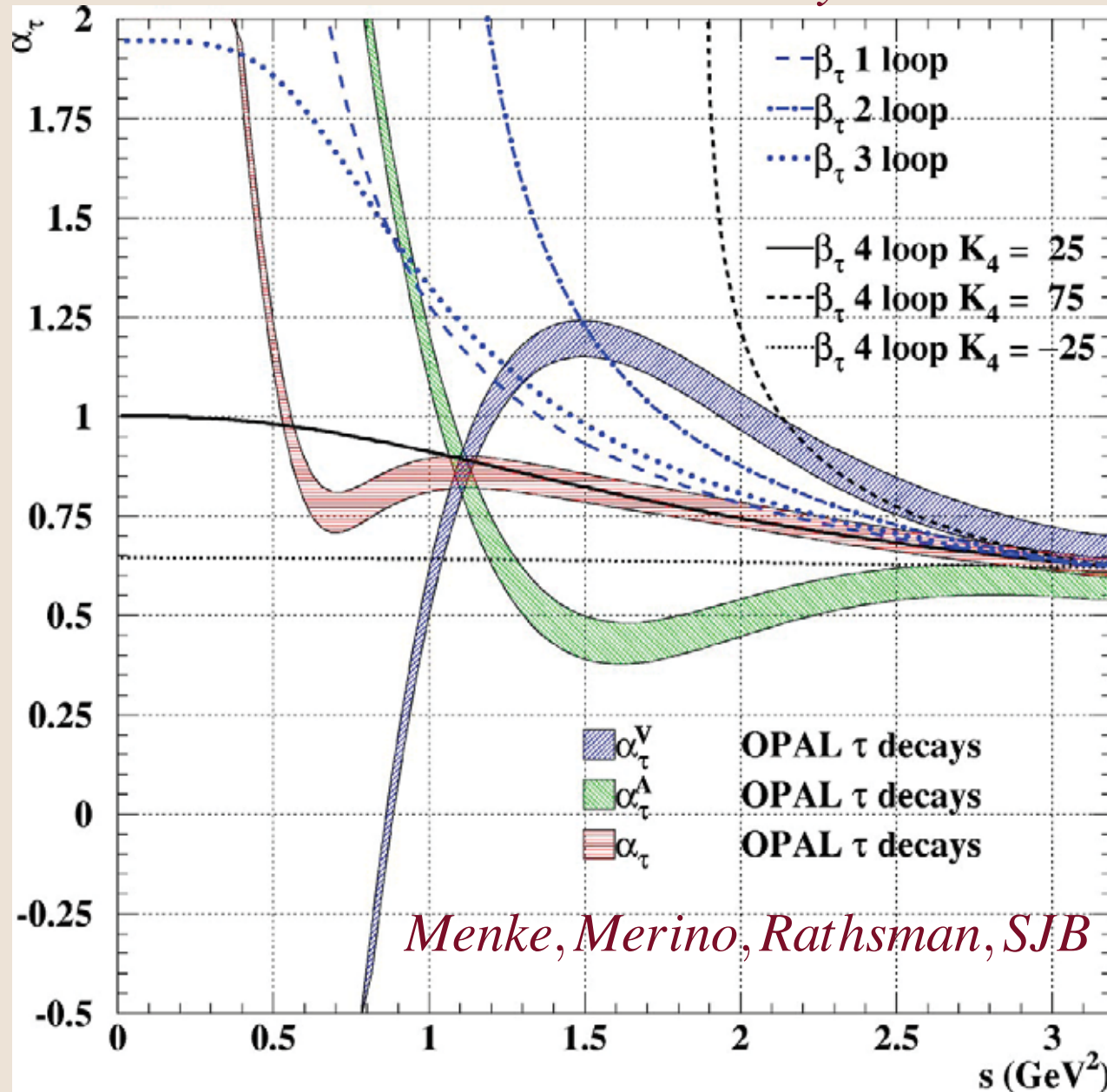
$$R_{e^+e^- \rightarrow X}(s) \equiv 3 \sum_q e_q^2 \left[ 1 + \frac{\alpha_R(s)}{\pi} \right]$$

$$\Gamma(\tau \rightarrow X e \nu)(m_\tau^2) \equiv \Gamma_0(\tau \rightarrow u \bar{d} e \nu) \times \left[ 1 + \frac{\alpha_\tau(m_\tau^2)}{\pi} \right]$$

Relate observable to observable at  
commensurate scales

H.Lu, sjb

# QCD Effective Coupling from *hadronic $\tau$ decay*



**Insights for QCD  
from AdS/CFT**

# Conformal symmetry: Template for QCD

- Initial approximation to PQCD; correct for non-zero beta function
- Commensurate scale relations: relate observables at corresponding scales
- Infrared fixed-point for  $\alpha_s$
- Effective Charges: analytic at quark mass thresholds
- Eigensolutions of Evolution Equations

# Strongly Coupled Conformal QCD and Holography

- Conformal Theories are invariant under the Poincaré and conformal transformations with  $M^{\mu\nu}$ ,  $P^\mu$ ,  $D$ ,  $K^\mu$ , the generators of  $SO(4, 2)$ .
- QCD appears as a nearly-conformal theory in the energy regimes accessible to experiment. Invariance of conformal QCD is broken by quark masses and quantum loops. For  $\beta = d\alpha_s(Q^2)/dQ^2$ , QCD is a conformal theory: Parisi, Phys. Lett. B **39**, 643 (1972).
- Growing theoretical and empirical evidence that  $\alpha_s(Q^2)$  has an IR fixed point: von Smekal, Alkofer and Hauck, arXiv:hep-ph/9705242; Alkofer, Fischer and Llanes-Estrada, hep-th/0412330; Deur, Burkert, Chen and Korsch, hep-ph/0509113 ...
- Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Brodsky and Farrar, Phys. Rev. Lett. **31**, 1153 (1973); Matveev *et al.*, Lett. Nuovo Cim. **7**, 719 (1973).



## Scale Transformations

- Isomorphism of  $SO(4, 2)$  of conformal QCD with the group of isometries of AdS space  $SO(1, 5)$

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$ , maps scale transformations into the holographic coordinate  $z$ .

- AdS mode in  $z$  is the extension of the hadron wf into the fifth dimension.
- Different values of  $z$  correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$ : invariant separation between quarks

- The AdS boundary at  $z \rightarrow 0$  correspond to the  $Q \rightarrow \infty$ , UV zero separation limit.

# AdS/CFT and QCD

## Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:  
Polchinski and Strassler, hep-th/0109174.
- Deep inelastic structure functions at small  $x$ :  
Polchinski and Strassler, hep-th/0209211.
- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:  
Brodsky and de Téramond, hep-th/0310227.
- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:  
Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ( $\eta/s = 1/4\pi$ ):

Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 ...

- **Polchinski & Strassler:** AdS/CFT builds in conformal symmetry at short distances, counting, rules for form factors and hard exclusive processes; non-perturbative derivation
- **Goal:** Use AdS/CFT to provide models of hadron structure: confinement at large distances, near conformal behavior at short distances
- **Holographic Model:** Initial “classical” approximation to QCD: Remarkable agreement with light hadron spectroscopy
- Use AdS/CFT wavefunctions as expansion basis for diagonalizing  $H^{\text{LF}}_{\text{QCD}}$ ; variational methods

# AdS/CFT

- Use mapping of conformal group  $SO(4,2)$  to  $AdS_5$
- Scale Transformations represented by wavefunction  $\psi(z)$  in 5th dimension
 
$$x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$$
- Holographic model: Confinement at large distances and conformal symmetry in interior  $0 < z < z_0$
- Match solutions at large  $r$  to conformal dimension of hadron wavefunction at short distances  $\psi(z) \sim z^\Delta$  at  $z \rightarrow 0$
- Truncated space simulates “bag” boundary conditions

$$\psi(z_0) = 0$$

$$z_0 = \frac{1}{\Lambda_{QCD}}$$

## Match fall-off at small $z$ to Conformal Dimension of State at short distances

- Pseudoscalar mesons:  $\mathcal{O}_{3+L} = \bar{\psi} \gamma_5 D_{\{\ell_1 \dots \ell_m\}} \psi$  ( $\Phi_\mu = 0$  gauge).
- 4- $d$  mass spectrum from boundary conditions on the normalizable string modes at  $z = z_0$ ,  $\Phi(x, z_0) = 0$ , given by the zeros of Bessel functions  $\beta_{\alpha,k}$ :  $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$
- Normalizable AdS modes  $\Phi(z)$

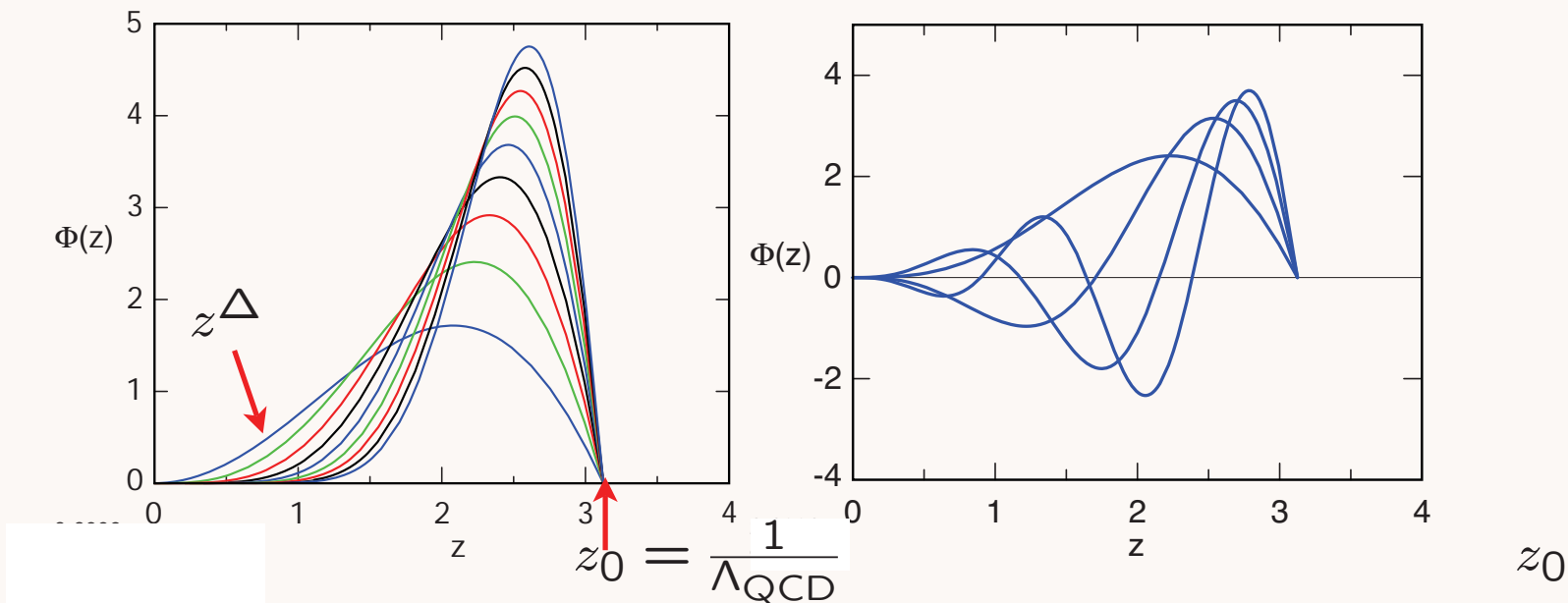


Fig: Meson orbital and radial AdS modes for  $\Lambda_{QCD} = 0.32$  GeV.

# Meson Spectrum

- Vector meson interpolating operator with twist-dimension minus spin-two, and conformal dimension  $\Delta = 3 + L$

$$\mathcal{O}_{3+L}^\mu = \bar{\psi} \gamma^\mu D_{\{\ell_1 \dots \ell_m\}} \psi. \quad \begin{array}{l} z \rightarrow 0 \\ x^2 \rightarrow 0 \end{array}$$

- AdS wave equation with effective 5-dim mass  $\mu$ . Solution is a vector field  $\Phi_\mu$  with polarization along Poincaré coordinates:

$$\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 + d - 1 \right] f_\mu(z) = 0,$$

with  $\Phi_\mu(x, z) = e^{-iP \cdot x} f_\mu(z)$  and  $P_\mu P^\mu = \mathcal{M}^2$  ( $\Phi_z = 0$  gauge).  $d = 4$

- Normalizable AdS vector mode:

$$\Phi_\mu(x, z) = C e^{-iP \cdot x} z^{\frac{d}{2}} J_{\Delta - \frac{d}{2}}(z\mathcal{M}) \epsilon_\mu. \quad \sim z^\Delta \text{ at } z \rightarrow 0$$

with  $\Delta = d - 1 + L$  and  $(\mu R)^2 = L(L + d - 2)$ . (Casimir)

## Confinement

- QCD is a confining theory in the infrared with mass gap  $\Lambda_{QCD}$  and a well-defined spectrum of color-singlet states.
- There is a maximum separation of quarks and a maximum value of  $z$ .
- AdS space should end at a finite value  $z_0 = 1/\Lambda_{QCD}$ .
- Cutoff at  $z_0$  breaks conformal invariance and allows the introduction of the QCD scale.
- Non-conformal metric dual to a confining gauge theory (Polchinski and Strassler):

$$ds^2 = \frac{R^2}{z^2} e^{2A(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) + ds_X^2,$$

where  $A(z) \rightarrow 0$  as  $z \rightarrow 0$ . Asymptotically:  $AdS \times X$ .

( $X$  is a 5-dim compact manifold if 4-dim gauge theory is dual to a critical (10-dim) string)

- Simplified model: metric factor  $e^{2A(z)}$  is a step function. Analog to the MIT bag model, but with boundary conditions on the holographic coordinate. **Alternative:  $z^2$  Potential**
- Truncated AdS/CFT model: conformal behavior at short distances and confinement at large distances.

AdS solution:

$$\Phi(z) = C e^{-iP \cdot x} z^2 J_\alpha(zM)$$

At large argument of  
the Bessel function

$$\Phi(x, z) = C e^{-iP \cdot x} z^{\frac{d}{2}} \sqrt{\frac{2}{\pi z \mathcal{M}}} \cos \left( z \mathcal{M} - \frac{\pi}{4} \sqrt{d^2 + 4l(l+4)} - \frac{\pi}{4} \right)$$

Dirichlet  
boundary  
condition:

$$\Phi(x, z = z_0 = \frac{1}{\Lambda_{QCD}}) = 0$$

$$M(n, l) = \frac{\pi}{2} \left[ \frac{1}{2} \left( 1 + \sqrt{d^2 + 4l(l+d)} \right) + (2n+1) \right] \Lambda_{QCD}$$

Quadratic Regge  
RelationIn the large  $\ell$  limit:

$$M^2 = \frac{\pi^2}{4} \ell^2 \Lambda_{QCD}^2$$

Independent of  $n, d$

## Introduction of Twist (Spin 0 and 1 AdS Modes)

- For spin-carrying constituents:  $\Delta \rightarrow \tau = \Delta - \sigma$ ,  $\sigma = \sum_{i=1}^n \sigma_i$ .
- For a two quark state  $\Delta \rightarrow \Delta - 1$ . Change compensated in  $\mu$  by the shift  $L \rightarrow L - 1$ .
- Lowest state corresponds to  $(\mu R)^2 = -1$ . Thus  $-1 \leq (\mu R)^2$ : Breitenlohner-Freedman stability bound for a 1-form.
- Two-quark vector meson described by wave equation (d=4)

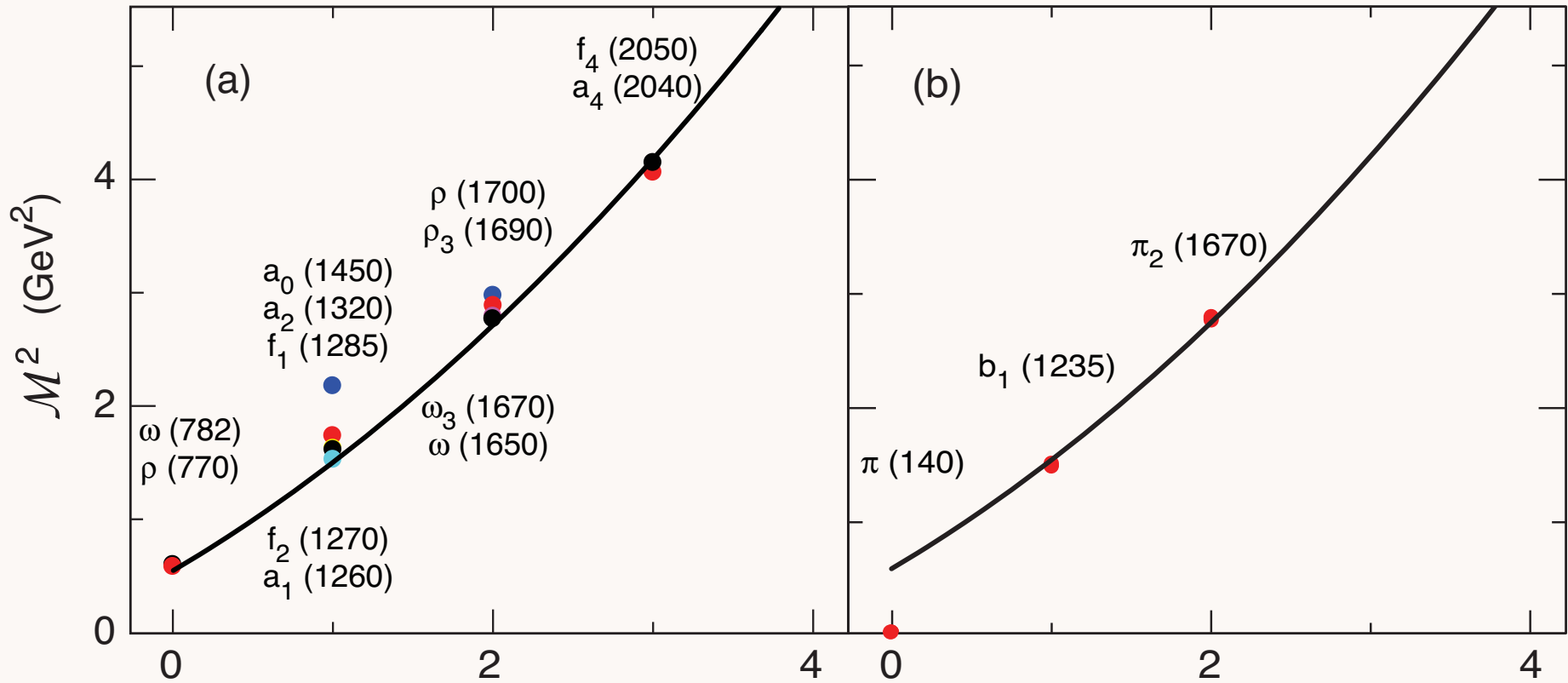
$$\left[ z^2 \partial_z^2 - 3z \partial_z + z^2 \mathcal{M}^2 - L^2 + 4 \right] f_\mu(z) = 0$$

with solution

$$\Phi_\mu(x, z) = C e^{-iP \cdot x} z^2 J_L(z\mathcal{M}) \epsilon_\mu.$$

- Same equation for  $\Delta = 4$ ,  $\tau = 2$  glueball 0-form with  $-4 \leq (\mu R)^2$  and solution

$$\Phi(x, z) = C e^{-iP \cdot x} z^2 J_L(z\mathcal{M}).$$



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Light meson orbital spectrum  $\Lambda_{QCD} = 0.32 \text{ GeV}$

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## Baryon Spectrum

- Baryon: twist-three, dimension  $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$

- Solve full 10-dim Dirac Eq.,  $\mathcal{D}\hat{\Psi} = 0$ , since baryons are charged under  $SU(4) \sim SO(6)$ .  
Baryon number conservation?
- $\hat{\Psi}$  is expanded in terms of eigenfunctions  $\eta_\kappa(y)$  of the Dirac operator on compact space  $X$  with eigenvalues  $\lambda_\kappa$ :

$$\hat{\Psi}(x, z, y) = \sum_{\kappa} \Psi_\kappa(x, z) \eta_\kappa(y).$$

- From the 10-dim Dirac equation,  $\mathcal{D}\hat{\Psi} = 0$ :

$$\left[ z^2 \partial_z^2 - d z \partial_z + z^2 \mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2} \left( \frac{d}{2} + 1 \right) + (\lambda_\kappa + \mu) R \hat{\Gamma} \right] f(z) = 0,$$

$$i\mathcal{D}_X \eta(y) = \lambda \eta(y),$$

where  $\Psi(x, z) = e^{-iP \cdot x} f(z)$ ,  $P_\mu P^\mu = \mathcal{M}^2$  and  $\hat{\Gamma} u_\pm = \pm u_\pm$ .

See: Muck and Viswanathan, hep-ph/9805945.

- Normalizable AdS baryon mode:

$$\Psi(x, z) = C e^{-iP \cdot x} z^{\frac{d+1}{2}} \left[ J_{(\mu+\lambda_\kappa)R-\frac{1}{2}}(z\mathcal{M}) u_+(P) + J_{(\mu+\lambda_\kappa)R+\frac{1}{2}}(z\mathcal{M}) u_-(P) \right].$$

with  $\Delta = \frac{d}{2} + |(\mu + \lambda_\kappa)R|$ .

- For  $d = 4$ ,  $\hat{\Gamma} = \gamma_5$  and spinors  $u_\pm(P)$  are defined along 4-dim coordinates.
- $\mu$  determined asymptotically by spectral comparison with orbital excitations in the boundary:  $\mu = L/R$  and  $\lambda_\kappa$  are the eigenvalues of the Dirac equation on  $S^{d+1}$ :

$$\lambda_\kappa R = \pm \left( \kappa + \frac{d}{2} + \frac{1}{2} \right), \quad \kappa = 0, 1, 2, \dots$$

See: Camporesi and Higuchi: gr-gc/9505009.

- Spin- $\frac{3}{2}$  Rarita-Schwinger eq. in AdS similar to spin- $\frac{1}{2}$  in the  $\Psi_z = 0$  gauge for polarization along Minkowski coordinates,  $\Psi_\mu$ . See: Volovich, hep-th/9809009.

## Introduction of Twist (Spin $\frac{1}{2}$ and $\frac{3}{2}$ AdS Modes)

- For spin-carrying constituents:  $\Delta \rightarrow \tau = \Delta - \sigma$ ,  $\sigma = \sum_{i=1}^n \sigma_i$ .
- For a three quark state  $\Delta \rightarrow \Delta - 3/2$ . Change compensated in  $\mu$  by the shift  $L \rightarrow L - 1$  and  $\Psi(z) \rightarrow z^{-\frac{1}{2}} \Psi(z)$ .
- Three-quark baryon described by wave equation ( $d = 4$ ,  $\kappa = 0$ )

$$\boxed{[z^2 \partial_z^2 - 3z \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_{\pm}^2 + 4] f_{\pm}(z) = 0}$$

with  $\mathcal{L}_+ = L + 1$ ,  $\mathcal{L}_- = L + 2$ , and solution

$$\Psi(x, z) = C e^{-iP \cdot x} z^2 \left[ J_{1+L}(z\mathcal{M}) u_+(P) + J_{2+L}(z\mathcal{M}) u_-(P) \right].$$

- 4- $d$  mass spectrum  $\Psi(x, z_o)^{\pm} = 0 \implies$  parallel Regge trajectories for baryons !

$$\mathcal{M}_{\alpha,k}^+ = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}_{\alpha,k}^- = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

# Predictions of AdS/CFT

Only one  
parameter!

Entire light  
quark baryon  
spectrum

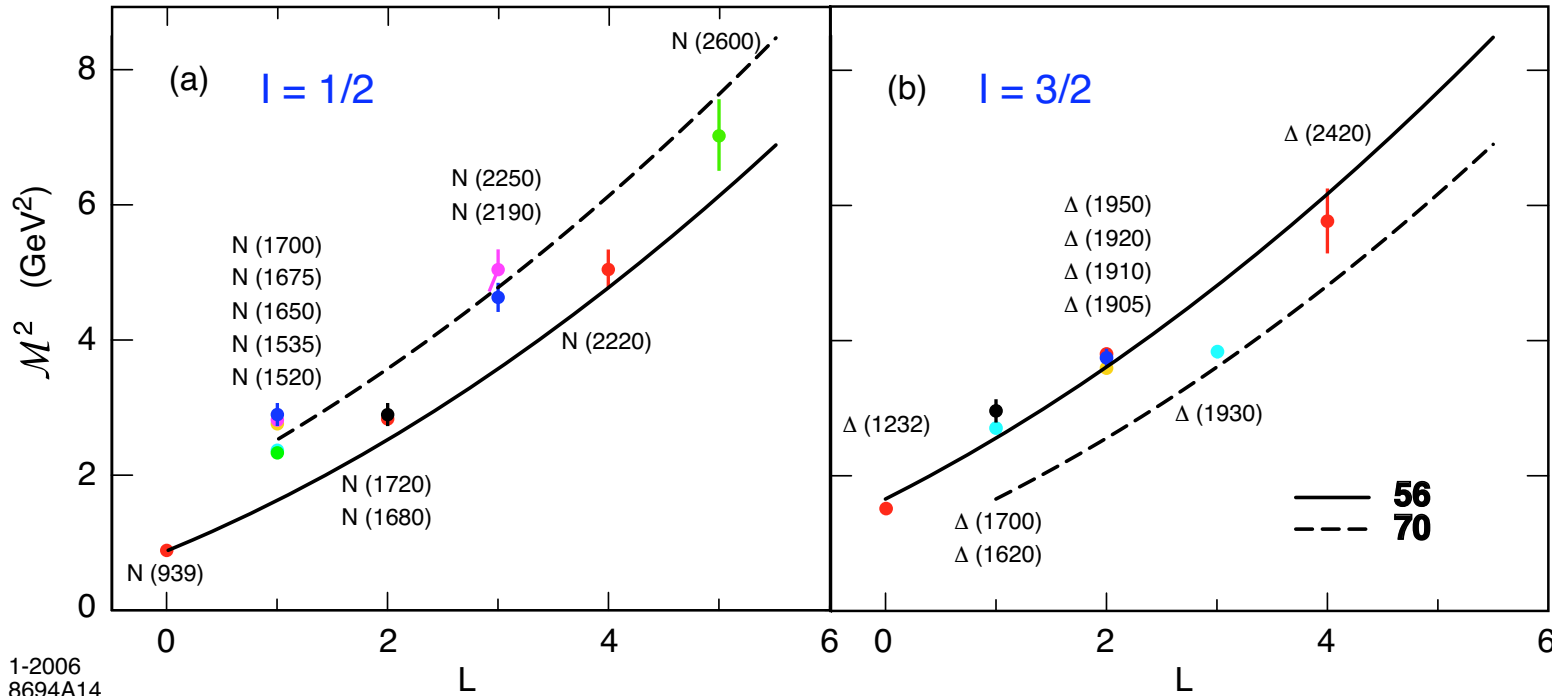


Fig: Predictions for the light baryon orbital spectrum for  $\Lambda_{QCD} = 0.25$  GeV. The **56** trajectory corresponds to  $L$  even  $P = +$  states, and the **70** to  $L$  odd  $P = -$  states.

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Insights for QCD  
from AdS/CFT

Technion  
5-1-06

- $SU(6)$  multiplet structure for  $N$  and  $\Delta$  orbital states, including internal spin  $S$  and  $L$ .

$SU(6)$	$S$	$L$	Baryon State
<b>56</b>	$\frac{1}{2}$	0	$N \frac{1}{2}^+$ (939)
	$\frac{3}{2}$	0	$\Delta \frac{3}{2}^+$ (1232)
<b>70</b>	$\frac{1}{2}$	1	$N \frac{1}{2}^-$ (1535) $N \frac{3}{2}^-$ (1520)
	$\frac{3}{2}$	1	$N \frac{1}{2}^-$ (1650) $N \frac{3}{2}^-$ (1700) $N \frac{5}{2}^-$ (1675)
	$\frac{1}{2}$	1	$\Delta \frac{1}{2}^-$ (1620) $\Delta \frac{3}{2}^-$ (1700)
<b>56</b>	$\frac{1}{2}$	2	$N \frac{3}{2}^+$ (1720) $N \frac{5}{2}^+$ (1680)
	$\frac{3}{2}$	2	$\Delta \frac{1}{2}^+$ (1910) $\Delta \frac{3}{2}^+$ (1920) $\Delta \frac{5}{2}^+$ (1905) $\Delta \frac{7}{2}^+$ (1950)
<b>70</b>	$\frac{1}{2}$	3	$N \frac{5}{2}^-$ $N \frac{7}{2}^-$
	$\frac{3}{2}$	3	$N \frac{3}{2}^-$ $N \frac{5}{2}^-$ $N \frac{7}{2}^-$ (2190) $N \frac{9}{2}^-$ (2250)
	$\frac{1}{2}$	3	$\Delta \frac{5}{2}^-$ (1930) $\Delta \frac{7}{2}^-$
<b>56</b>	$\frac{1}{2}$	4	$N \frac{7}{2}^+$ $N \frac{9}{2}^+$ (2220)
	$\frac{3}{2}$	4	$\Delta \frac{5}{2}^+$ $\Delta \frac{7}{2}^+$ $\Delta \frac{9}{2}^+$ $\Delta \frac{11}{2}^+$ (2420)
<b>70</b>	$\frac{1}{2}$	5	$N \frac{9}{2}^-$ $N \frac{11}{2}^-$
	$\frac{3}{2}$	5	$N \frac{7}{2}^-$ $N \frac{9}{2}^-$ $N \frac{11}{2}^-$ (2600) $N \frac{13}{2}^-$

# Features of Holographic Model

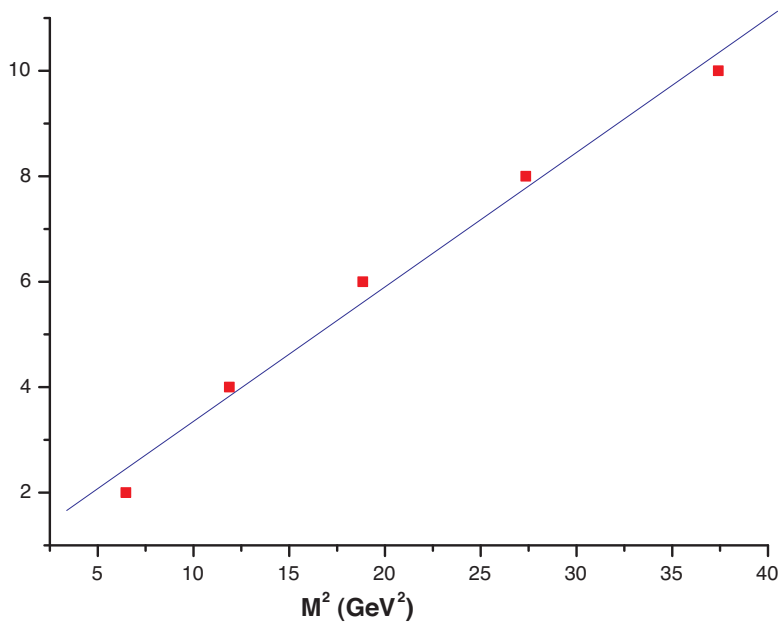
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- Ratio of proton to Delta trajectories= ratio of zeroes of Bessel functions.
- One scale  $\Lambda_{\text{QCD}}$  determines hadron spectrum (slightly different for mesons and baryons)
- Only quark-antiquark, qqq, and g g hadrons appear at classical level
- Covariant version of bag model: confinement+conformal symmetry

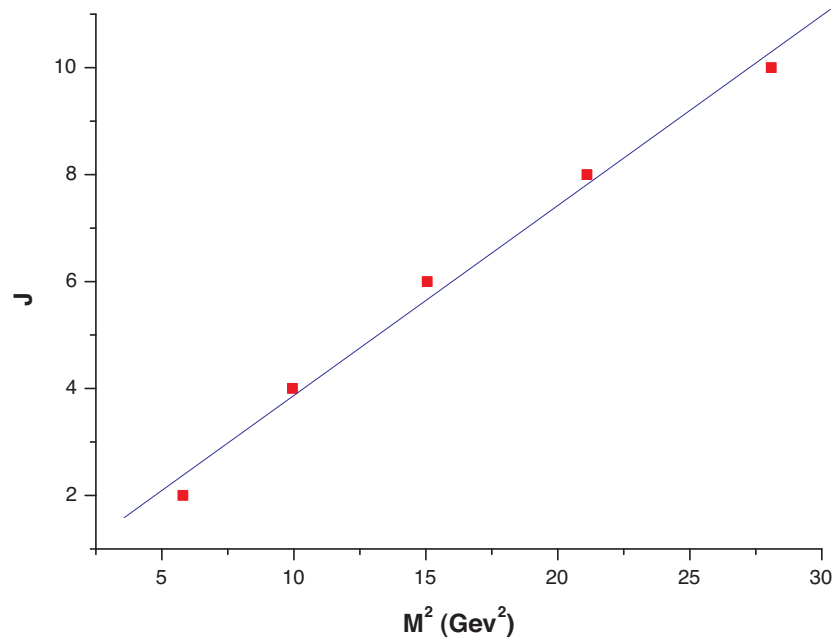
# Glueball Regge trajectories from gauge/string duality and the Pomeron

Henrique Boschi-Filho,<sup>\*</sup> Nelson R. F. Braga,<sup>†</sup> and Hector L. Carrion<sup>‡</sup>

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Neumann Boundary Conditions



Dirichlet Boundary Conditions

- Gluonium spectrum (top-bottom):

Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115.

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- Other aspects of high energy scattering in warped spaces:

S. B. Giddings, hep-th/0203004; Andreev and Siegel, arXiv:hep-th/0410131; Kang and Nastase, hep-th/0410173; Nastase, hep-th/0501039; hep-th/0501068.

- Branes in Minkowski space:

Siopsis, hep-th/0503245.

# Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron  $\Phi_I$  and  $\Phi_F$  and the non-normalizable mode  $J$ , dual to the external source (hadron spin  $\sigma$ ):

$$\begin{aligned} F(Q^2)_{I \rightarrow F} &= R^{3+2\sigma} \int_0^\infty \frac{dz}{z^{3+2\sigma}} e^{(3+2\sigma)A(z)} \Phi_F(z) J(Q, z) \Phi_I(z) \\ &\simeq R^{3+2\sigma} \int_0^{z_0} \frac{dz}{z^{3+2\sigma}} \Phi_F(z) J(Q, z) \Phi_I(z), \end{aligned}$$

- $J(Q, z)$  has the limiting value 1 at zero momentum transfer,  $F(0) = 1$ , and has as boundary limit the external current,  $A^\mu = \epsilon^\mu e^{iQ \cdot x} J(Q, z)$ . Thus:

$$\lim_{Q \rightarrow 0} J(Q, z) = \lim_{z \rightarrow 0} J(Q, z) = 1.$$

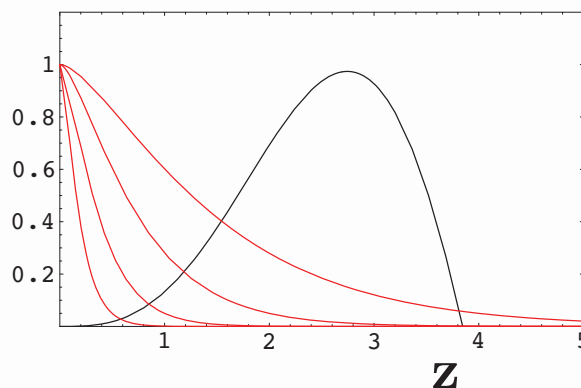
- Solution to the AdS Wave equation with boundary conditions at  $Q = 0$  and  $z \rightarrow 0$ :

$$J(Q, z) = zQ K_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

- Propagation of external perturbation suppressed inside AdS.
- At large enough  $Q \sim r/R^2$ , the interaction occurs in the large- $r$  conformal region. Important contribution to the FF integral from the boundary near  $z \sim 1/Q$ .

$\mathbf{J(Q, z), \Phi(z)}$

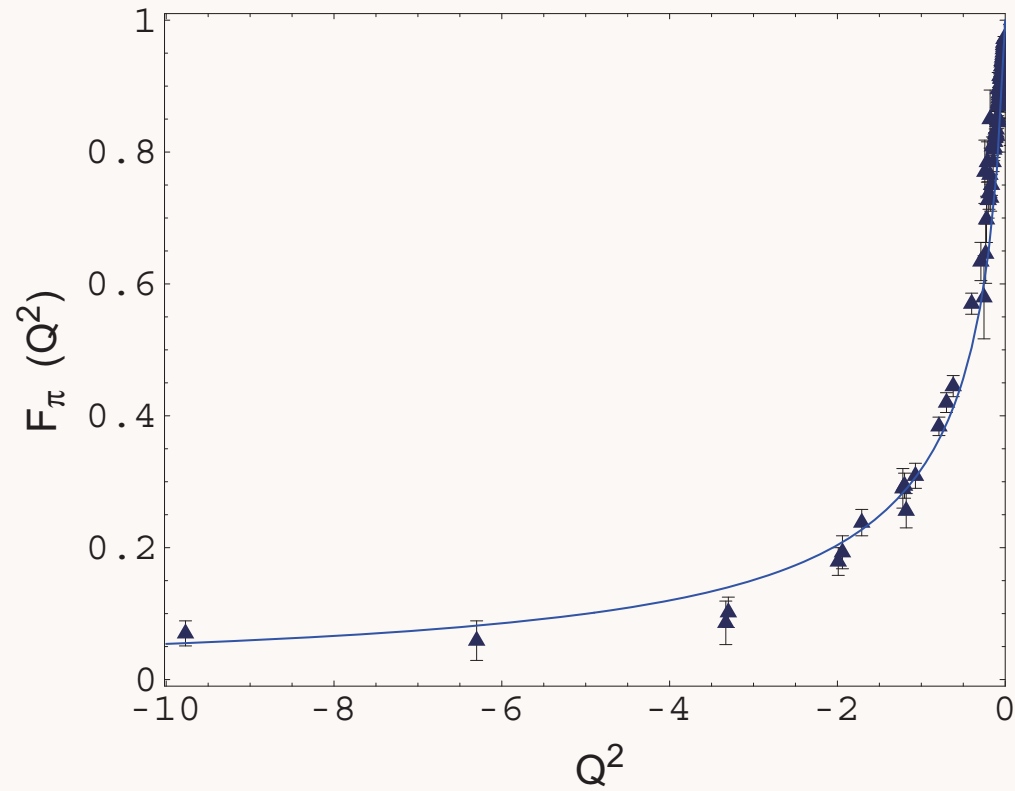


- Consider a specific AdS mode  $\Phi^{(n)}$  dual to an  $n$  partonic Fock state  $|n\rangle$ . At small  $z$ ,  $\Phi^{(n)}$  scales as  $\Phi^{(n)} \sim z^{\Delta_n}$ . Thus:

$$F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1},$$

General result from  
AdS/CFT

where  $\tau = \Delta_n - \sigma_n$ ,  $\sigma_n = \sum_{i=1}^n \sigma_i$ . The twist is equal to the number of partons,  $\tau = n$ .



Space-like pion form factor in holographic model for  $\Lambda_{QCD} = 0.2$  GeV.

# Drell-Yan West formula for Form Factor of meson

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{P'}^*(x, \vec{k}_\perp + (1-x)\vec{q}_\perp) \psi_P(x, \vec{k}_\perp).$$

## In Impact Space:

$$\begin{aligned} F(q^2) &= 4\pi \int_0^1 dx \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \\ &= 8\pi^2 \int_0^1 dx \int_0^\infty b db J_0(bqx) |\tilde{\psi}(x, b)|^2, \end{aligned}$$

For a two-parton state, the light-front wave function is effectively cutoff at

$$\vec{b}_\perp^2 \simeq \frac{1}{x(1-x)\Lambda_{\text{QCD}}^2},$$

Change Integration variable to:  $\zeta = |\vec{b}_\perp| \sqrt{x(1-x)}$

$$F_{IS}(Q^2) = 8\pi^2 \int_0^1 \frac{dx}{x(1-x)} \int_0^{\Lambda_{\text{QCD}}^{-1}} \zeta d\zeta J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) |\tilde{\psi}(x, \zeta)|^2,$$

- Change the integration variable  $\zeta = |\vec{b}_\perp| \sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max} = \Lambda_{\text{QCD}}^{-1}} \zeta d\zeta J_0 \left( \frac{\zeta Q x}{\sqrt{x(1-x)}} \right) |\tilde{\psi}(x, \zeta)|^2,$$

- Compare with AdS form factor for arbitrary  $Q$ . Find:

$$J(Q, \zeta) = \int_0^1 dx J_0 \left( \frac{\zeta Q x}{\sqrt{x(1-x)}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for the electromagnetic potential in AdS space, and

$$\tilde{\psi}(x, \vec{b}_\perp) = \frac{\Lambda_{\text{QCD}}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0 \left( \sqrt{x(1-x)} |\vec{b}_\perp| \beta_{0,1} \Lambda_{\text{QCD}} \right) \theta \left( \vec{b}_\perp^2 \leq \frac{\Lambda_{\text{QCD}}^{-2}}{x(1-x)} \right)$$

the holographic LFWF for the valence Fock state of the pion  $\psi_{\bar{q}q/\pi}$ .

- The variable  $\zeta$ ,  $0 \leq \zeta \leq \Lambda_{\text{QCD}}^{-1}$ , represents the scale of the invariant separation between quarks and is also the holographic coordinate  $\zeta = z$ !

# Mapping between $LF(3+1)$ and $AdS_5$

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp)$$

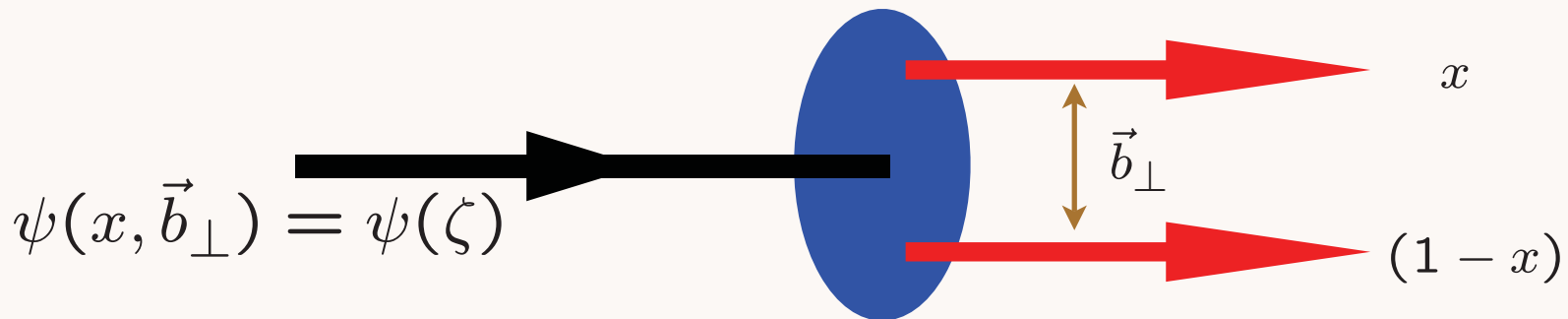


$$\phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$



$$z$$



Insights for QCD  
from AdS/CFT

## Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x) \mathbf{b}_\perp^2.$$

Effective conformal potential:

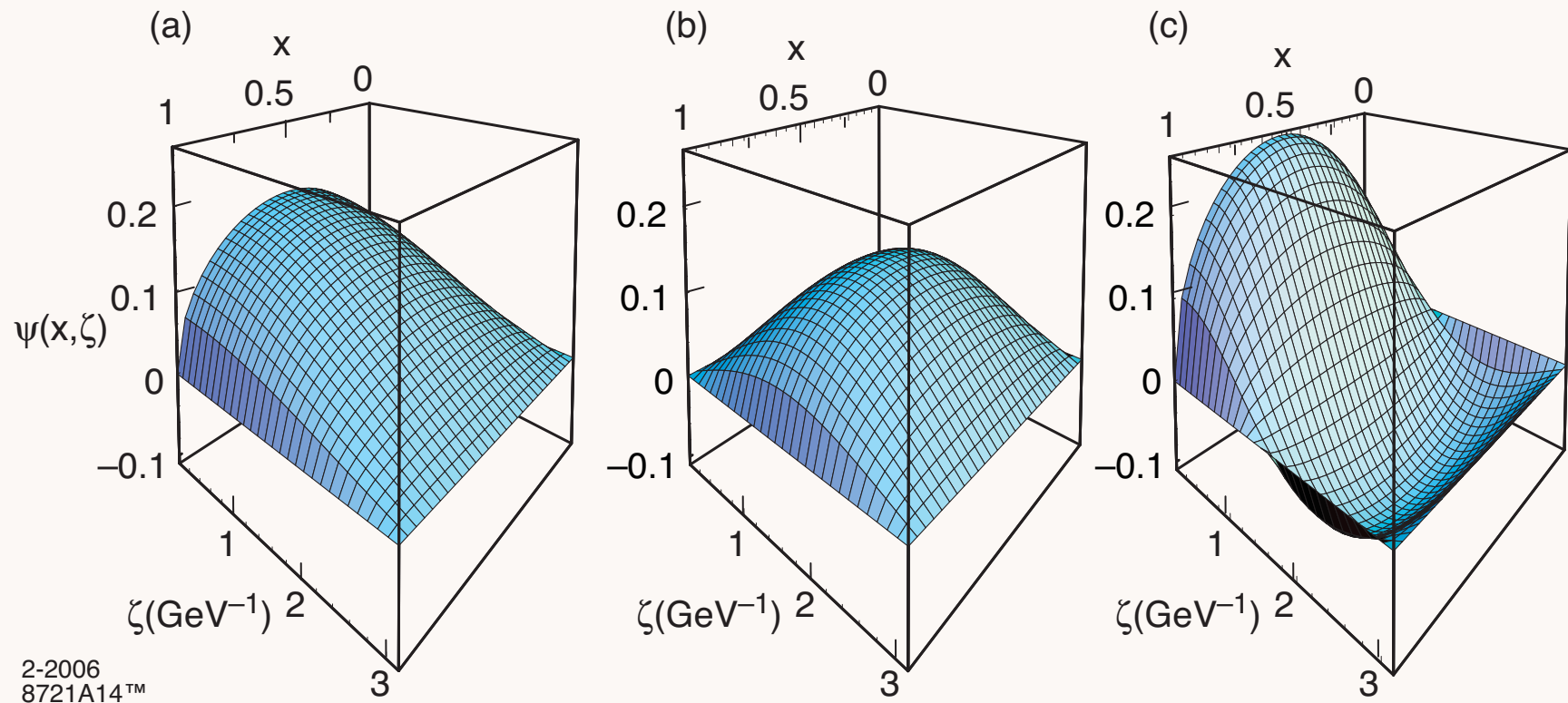
$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

General solution:

$$\tilde{\psi}_{L,k}(x, \vec{b}_\perp) = B_{L,k} \sqrt{x(1-x)}$$

$$\mathbf{X} \quad J_L \left( \sqrt{x(1-x)} |\vec{b}_\perp| \beta_{L,k} \Lambda_{\text{QCD}} \right) \theta \left( \vec{b}_\perp^2 \leq \frac{\Lambda_{\text{QCD}}^{-2}}{x(1-x)} \right),$$

# *AdS/CFT Prediction for Meson LFWF*



Two-parton holographic LFWF in impact space  $\tilde{\psi}(x, \zeta)$  for  $\Lambda_{QCD} = 0.32$  GeV: (a) ground state

$L = 0, k = 1$ ; (b) first orbital excited state  $L = 1, k = 1$ ; (c) first radial excited state  $L = 0, k = 2$ .

The variable  $\zeta$  is the holographic variable  $z = \zeta = |b_{\perp}| \sqrt{x(1-x)}$ .

## Evaluation of QCD Matrix Elements: Example $f_\pi$

- Pion decay constant defined by the matrix element of EW current  $J_W^+$ :

$$\langle 0 | \bar{\psi}_u \gamma^+ (1 - \gamma_5) \psi_d | \pi^- \rangle = i\sqrt{2}P^+ f_\pi,$$

with

$$|\pi^-\rangle = |d\bar{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left( b_{c d\downarrow}^\dagger d_{c u\uparrow}^\dagger - b_{c d\uparrow}^\dagger d_{c u\downarrow}^\dagger \right) |0\rangle.$$

- Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \psi_{\bar{q}q/\pi}(x, k_\perp).$$

Lepage and Brodsky, Phys. Rev. D **22**, 2157 (1980)

- Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\text{QCD}}}{8J_1(\beta_{0,1})} = 83.4 \text{ Mev},$$

for  $\Lambda_{\text{QCD}} = 0.2 \text{ GeV}$ .

Experiment:  $f_\pi = 92.4 \text{ Mev}$ .

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\vec{b}_{\perp j} \delta(1 - x - \sum_{j=1}^{n-1} x_j) \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary  $Q$ :

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) J(Q, z) \Phi_P(z)$$

- Holographic variable  $z$  is expressed in terms of the average transverse separation distance of the spectator constituents  $\vec{\eta} = \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j}$

$$z = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} \right|$$

**Insights for QCD  
from AdS/CFT**

- Our final result: hadronic QCD transverse density  $\tilde{\rho}$  is determined by the modes  $\Phi$  in AdS space!

$$\tilde{\rho}(x, \zeta) = \frac{R^3}{2\pi} \frac{x}{1-x} e^{3A(\zeta)} \frac{|\Phi(\zeta)|^2}{\zeta^4}$$

- The variable  $\zeta$ ,  $0 \leq \zeta \leq \Lambda_{\text{QCD}}^{-1}$ , is related to the average transverse separation between spectator constituents, and it is also the holographic variable  $z$ ,  $\zeta = z$ .
- For the two-particle case

$$\tilde{\rho}(x, \zeta) = \frac{1}{(1-x)^2} |\psi(x, \zeta)|^2,$$

and we recover our previous results

$$|\psi(x, \zeta)|^2 \simeq \frac{R^3}{2\pi} x(1-x) \frac{|\Phi(\zeta)|^2}{\zeta^4} \theta\left(\zeta^2 \leq \Lambda_{\text{QCD}}^{-2}\right).$$

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_+(z)|^2,$$

$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_-(z)|^2,$$

where the effective charges  $g_+$  and  $g_-$  are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have  $S^z = +1/2$ . The two AdS solutions  $\psi_+(z)$  and  $\psi_-(z)$  correspond to nucleons with  $J^z = +1/2$  and  $-1/2$ .
- For  $SU(6)$  spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_+(z)|^2,$$

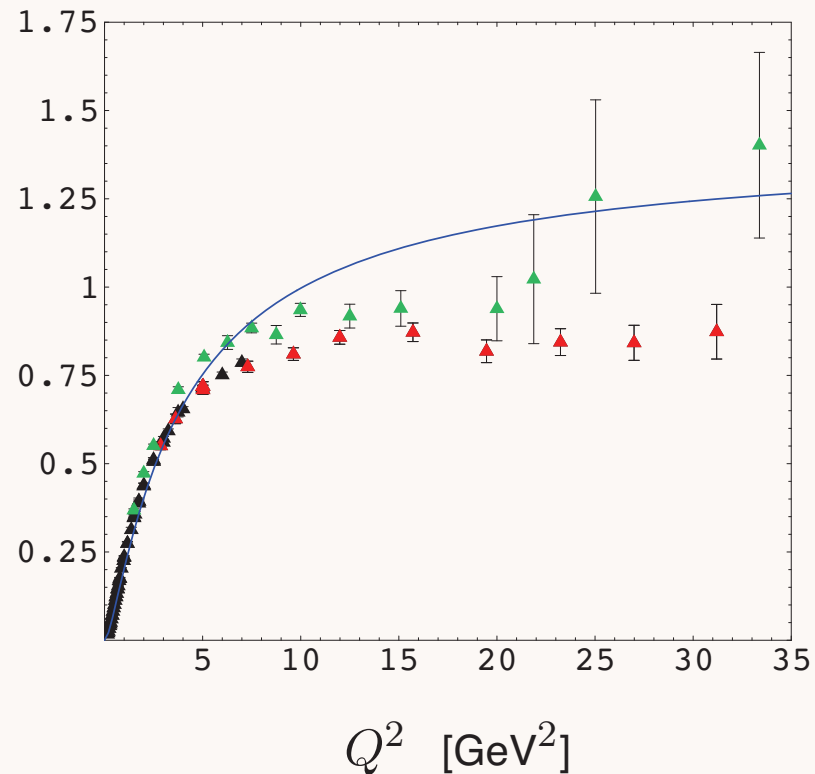
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q, z) [|\psi_+(z)|^2 - |\psi_-(z)|^2],$$

where  $F_1^p(0) = 1$ ,  $F_1^n(0) = 0$ .

- Large  $Q$  power scaling:  $F_1(Q^2) \rightarrow [1/Q^2]^2$ .

# Dirac Proton Form Factor $F_1^p$

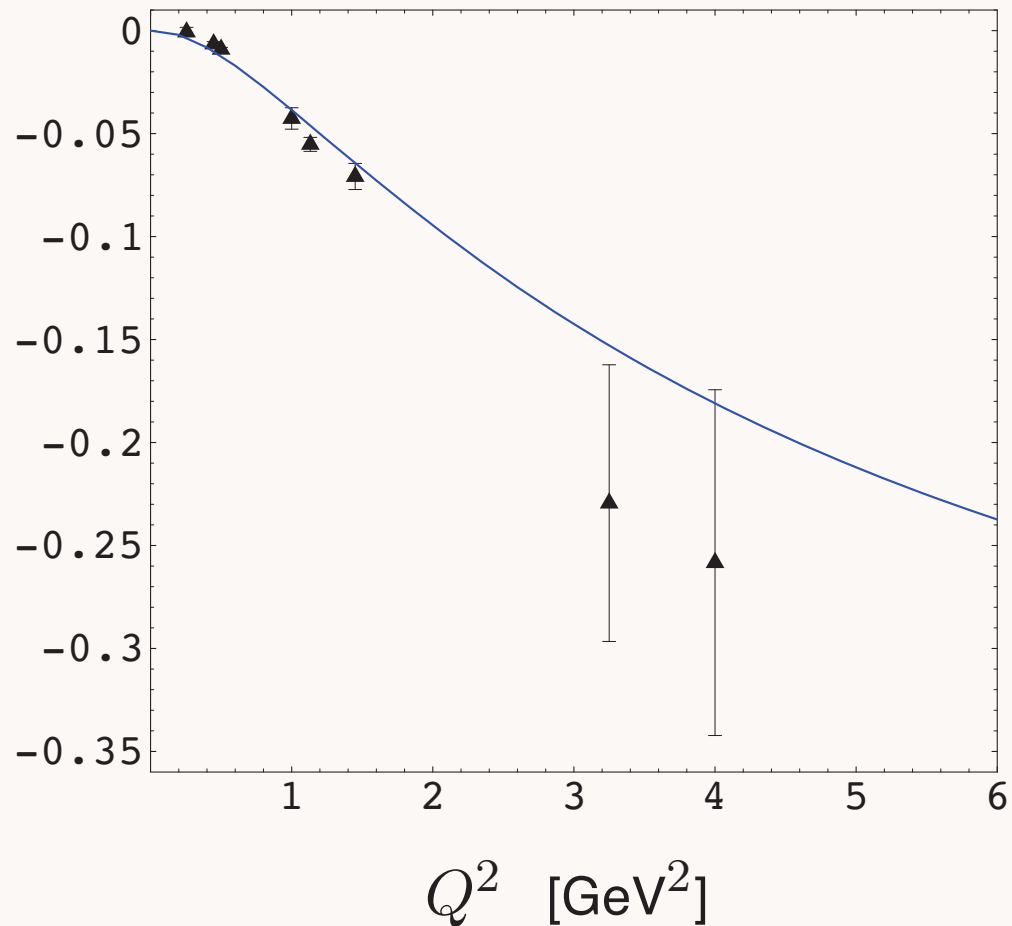
$$Q^4 F_1^p(Q^2) \text{ [GeV}^4\text{]}$$



Prediction for  $Q^4 F_1^p(Q^2)$  for  $\Lambda_{\text{QCD}} = 0.21 \text{ GeV}$  in the infinite wall approximation including the data from Kirk (superimposed green points assuming  $G_E^p = G_M^p$ ): P. N. Kirk *et al.*, Phys. Rev. D **8** (1973) 63.

$Q^4 F_1^n(Q^2)$  [GeV<sup>4</sup>]

Dirac Neutron Form Factor  $F_1^n$



Prediction for  $Q^4 F_1^n(Q^2)$  for  $\Lambda_{\text{QCD}} = 0.21$  GeV in the infinite wall approximation.

# *New Perspectives for QCD from AdS/CFT*

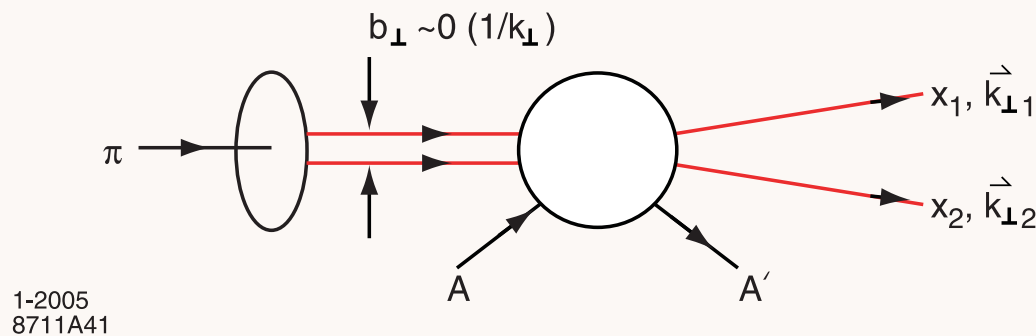
- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support  $0 < x < 1$ .
- Quark Interchange dominant force at short distances

# Use Diffraction to Resolve Hadron Substructure

- Measure Light-Front Wavefunctions
- Test AdS/CFT predictions
- Novel Aspects of Hadron Wavefunctions:  
Intrinsic Charm, Hidden Color, Color  
Transparency/Opaqueness
- Diffractive Di-Jet Production
- Nuclear Shadowing and Antishadowing
- New Mechanism for Higgs Production

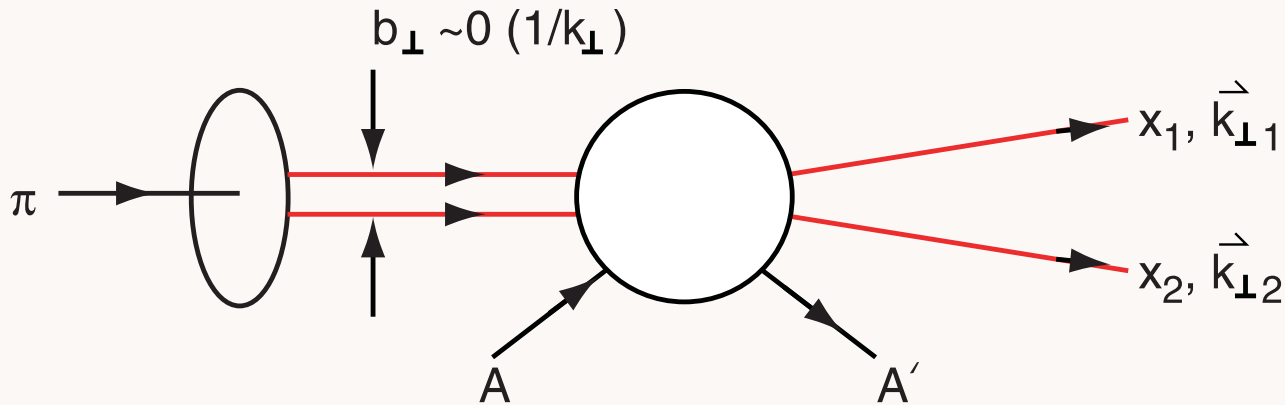
# Diffractive Dissociation of Pion

E791 Ashery et al.

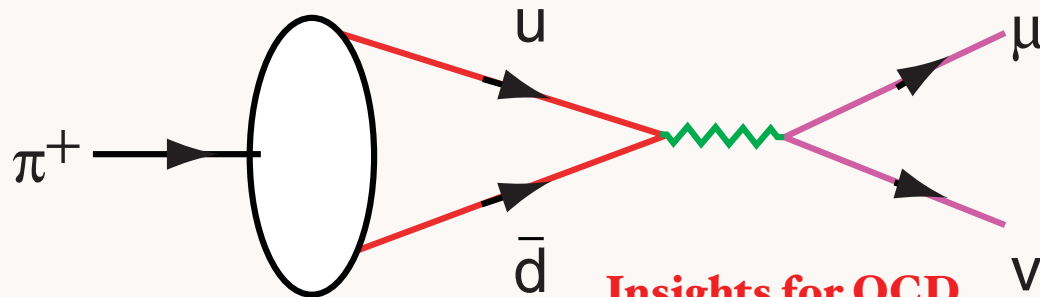


Measure Light-Front Wavefunction of Pion  
Two-gluon Exchange  
Minimal momentum transfer to nucleus  
Nucleus left Intact

# Fluctuation of a Pion to a Compact Color Dipole State



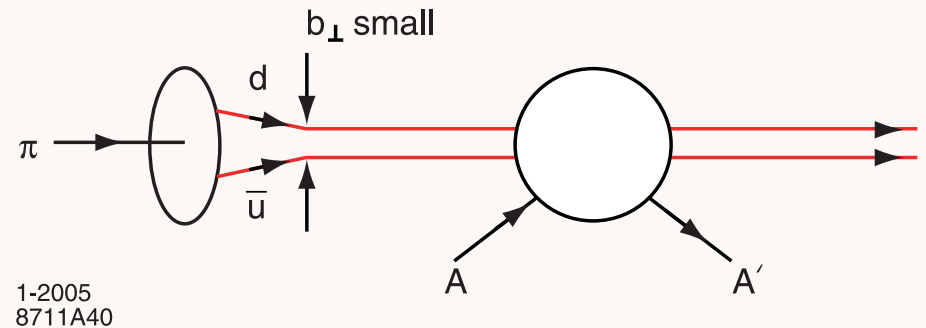
**Color-Transparent** Fock State For High Transverse Momentum Di-Jets



Same Fock State Determines Weak Decay

# Fluctuation of a Pion to a Compact Color Dipole State

Small Size Pion Can Interact Coherently on Each Nucleon of Nucleus



Diffractive Dijet Cross Section Color Transparent

$$M(\pi A \rightarrow \text{JetJet}A') = A^1 M(\pi N \rightarrow \text{JetJet}N') F_A(t)$$

$$d\sigma/dt(\pi A \rightarrow \text{JetJet}A') =$$

$$A^2 d\sigma/dt(\pi N \rightarrow \text{JetJet}N') |F_A(t)|^2$$

$$\sigma \propto \frac{A^2}{R_A^2} \sim A^{4/3}$$

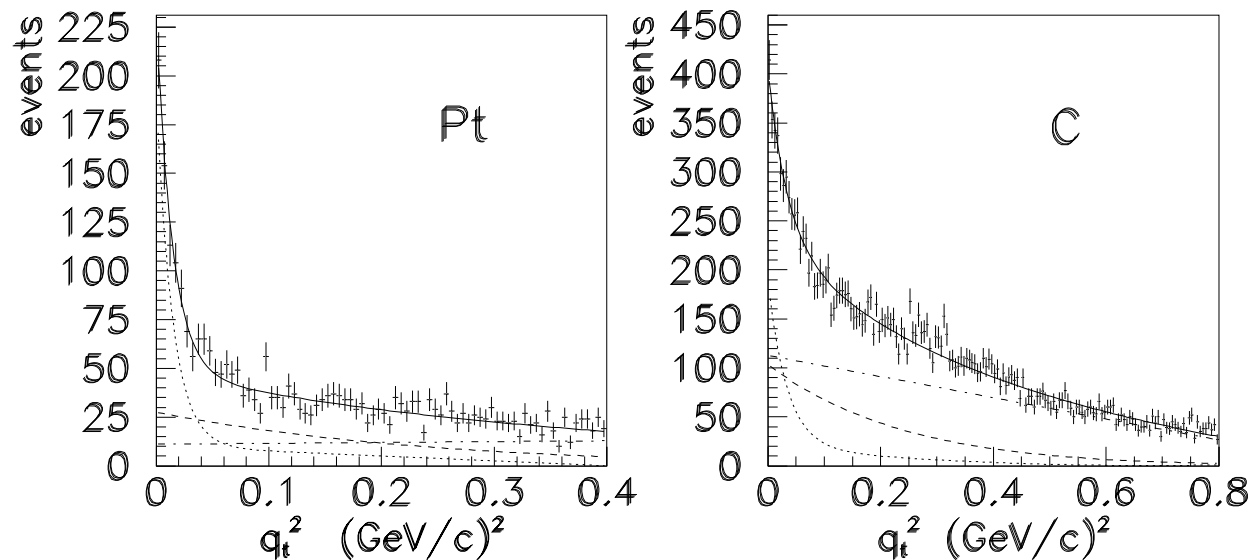
- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(A) = A \cdot \mathcal{M}(N)$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$

E791 Collaboration, E. Aitala *et al.*, Phys. Rev. Lett. 86, 4773 (2001)



**Insights for QCD  
from AdS/CFT**

# Ashery E791: Measure pion LFWF in diffractive dijet production Confirms color transparency !

Mueller, sjb; Bertsch et al; Frankfurt, Miller, Strikman

A-Dependence results:  $\sigma \propto A^\alpha$

<u><math>k_t</math> range (GeV/c)</u>	<u><math>\alpha</math></u>	<u><math>\alpha</math> (CT)</u>
$1.25 < k_t < 1.5$	$1.64 +0.06 -0.12$	1.25
$1.5 < k_t < 2.0$	$1.52 \pm 0.12$	1.45
$2.0 < k_t < 2.5$	$1.55 \pm 0.16$	1.60

$\alpha$  (Incoh.) =  $0.70 \pm 0.1$

Conventional Glauber  
Theory Ruled Out !

Factor of 7

FermiLab E791  
Ashery et al

***Technion***  
**5-1-06**

**Insights for QCD  
from AdS/CFT**

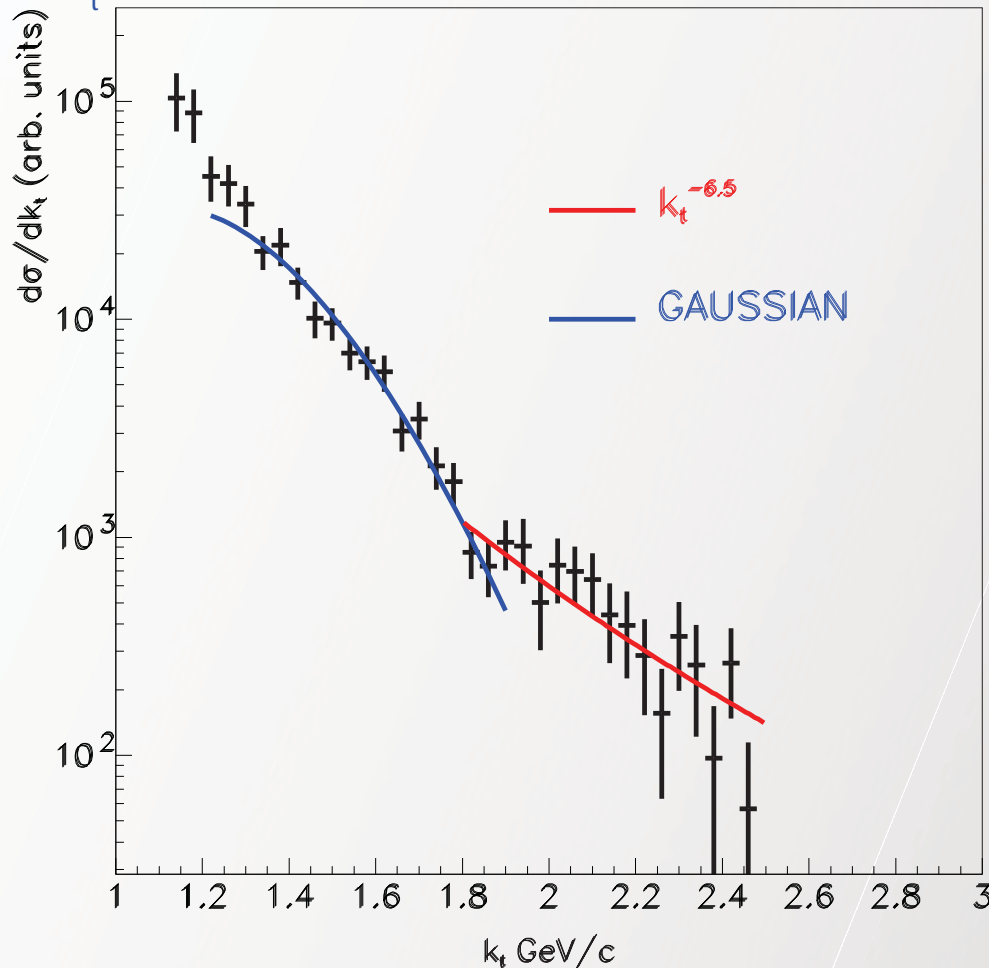
121

Stan Brodsky, SLAC

# THE $k_t$ DEPENDENCE OF DI-JETS YIELD

$$\frac{d\sigma}{dk_t^2} \propto |\alpha_s(k_t^2)G(x, k_t^2)|^2 \left| \frac{\partial^2}{\partial k_t^2} \psi(u, k_t) \right|^2$$

With  $\psi \sim \frac{\phi}{k_t^2}$ , weak  $\phi(k_t^2)$  and  $\alpha_s(k_t^2)$  dependences and  $G(x, k_t^2) \sim k_t^{1/2}$  :  $\frac{d\sigma}{dk_t} \sim k_t^{-6}$



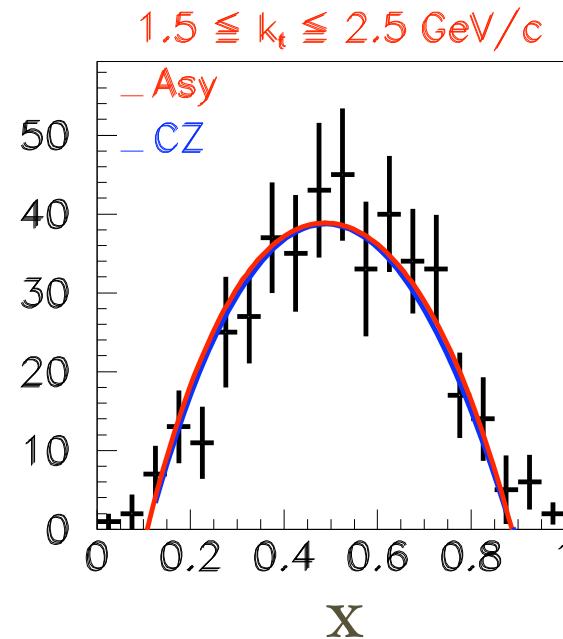
High Transverse  
momentum dependence  
consistent with PQCD/  
AdS/CFT

# Diffractive Dissociation of a Pion into Dijets

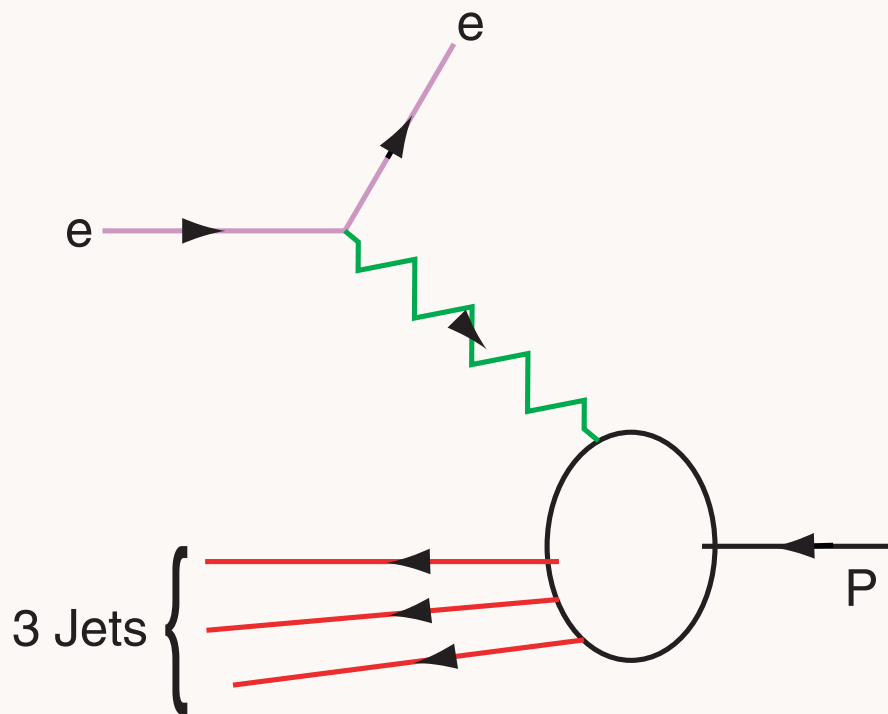
$$\pi A \rightarrow \text{Jet Jet } A'$$

$$\psi_{q\bar{q}}^{\pi}(x, \vec{k}_{\perp})$$

- E789 Fermilab Experiment  
Ashery et al
- 500 GeV pions collide on nuclei keeping it intact
- Measure momentum of two jets
- Study momentum distributions of pion LF wavefunction



# Coulomb-Dissociate Proton to Three Jets at HERA



Frankfurt  
Strikman  
Miller

Measure  $\Psi_{qqq}(x_i, \vec{k}_{\perp i})$  valence wavefunction of proton

# New Perspectives on QCD from AdS/CFT

- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- AdS/CFT predicts Light-front wavefunctions:  
Fundamental description of hadrons at amplitude level
- AdS/CFT: gluonium ( $gg$ ), meson ( $q \bar{q}$ ), and baryon ( $qqq$ ) spectra
- Quark-interchange dominates scattering amplitudes
- No  $ggg$  bound states

# Why is quark-interchange dominant over gluon exchange?

Example:  $M(K^+_p \rightarrow K^+_p) \propto \frac{1}{ut^2}$

Exchange of common  $u$  quark

$$M_{QIM} = \int d^2k_\perp dx \psi_C^\dagger \psi_D^\dagger \Delta \psi_A \psi_B$$

Holographic model (Classical level):

Hadrons enter 5th dimension of  $AdS_5$

Quarks travel freely within cavity as long as separation  $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

# Formula for quark interchange using LFWFs

Blankenbecler, Gunion, sjb; Sivers

$$\begin{aligned} M_{FI} &= \langle \psi_F | E - K | \psi_I \rangle \\ &\equiv \langle \psi_F | \Delta | \psi_I \rangle \\ &= \frac{1}{2(2\pi)^3} \int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2} \Delta \psi_C(\vec{k}_\perp - x\vec{r}_\perp, x) \psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x) \psi_B(\vec{k}_\perp, x) \end{aligned}$$

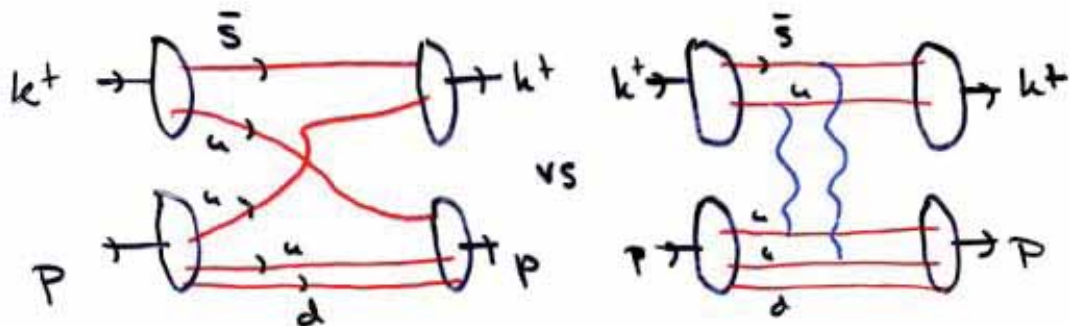
where

$$\begin{aligned} \Delta &= s - M_A^2 - M_B^2 - K_a - K_b - K_c - K_d \\ &= M_A^2 + M_B^2 - S_A(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp, x) - S_B(\vec{k}_\perp, x) \\ &= M_C^2 + M_D^2 - S_C(\vec{k}_\perp - x\vec{r}_\perp, x) - S_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) . \end{aligned}$$

Angular Distribution  $-t/s = \frac{1}{2}(1 - \cos\theta_{cm})$

$$\frac{d\sigma}{dt} = \frac{1}{s^{n_{tot}-2}} F(t/s)$$

determined by scattering mechanism



Quark Interchange

gluon exchange

↑  
Analogous to spin exchange  
in atom-atom scattering

Van der Waals

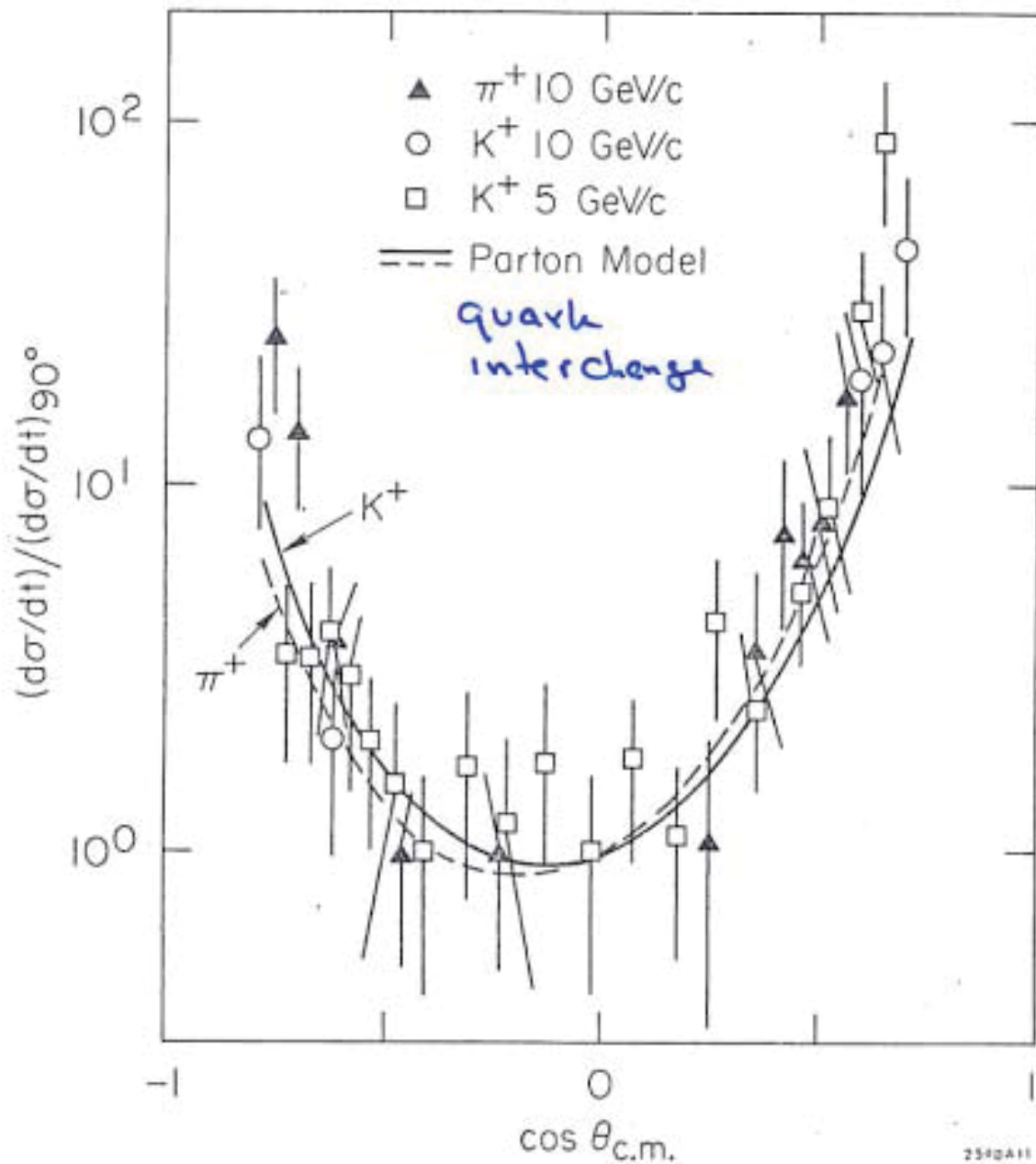
Large  $N_c$ : Quark Interchange Dominant

$$M \sim \frac{1}{s} \frac{1}{t^2}$$

f. loop limit, AdS/CFT

Blankenbecler, Gunion, sjb

MIT Bag Model  
predicts dominance of quark  
interchange: deTar

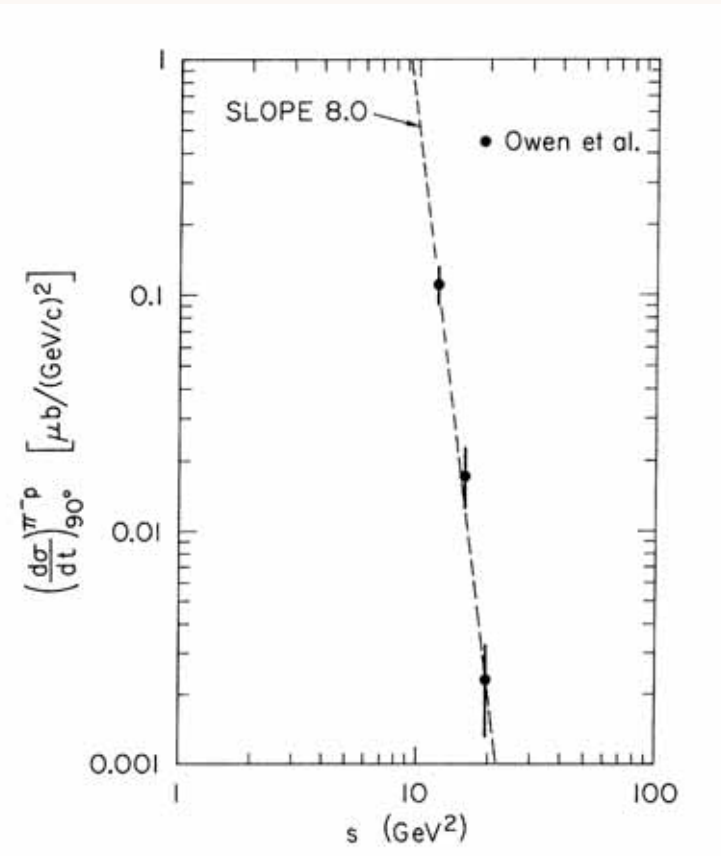
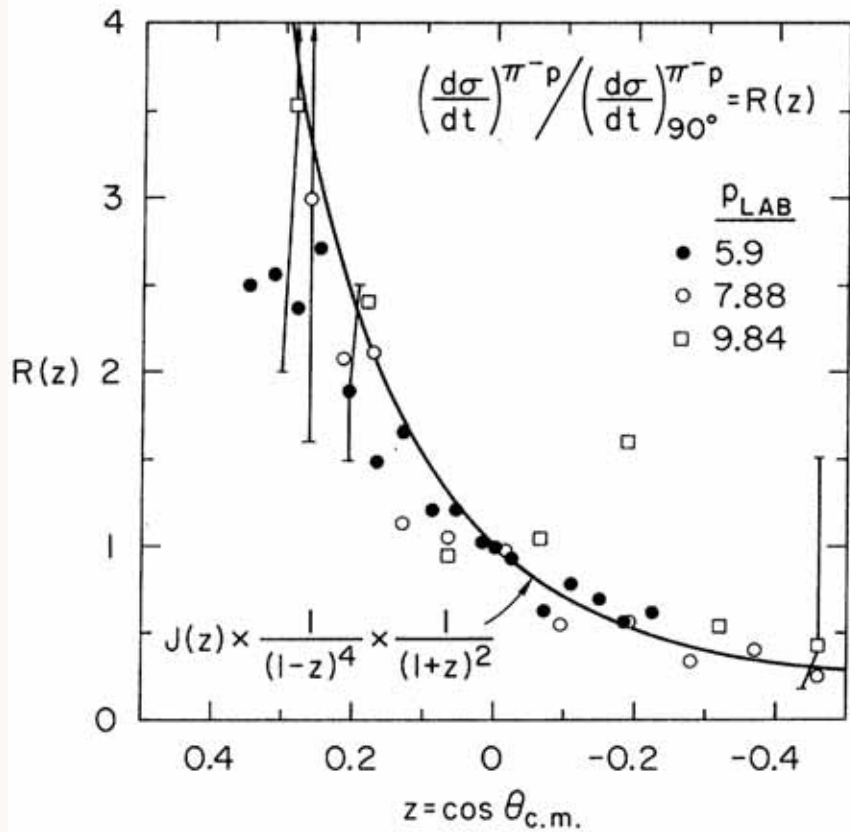


AdS/CFT explains  
 why quark  
 interchange is  
 dominant interaction  
 at high momentum  
 transfer in exclusive  
 reactions

$$\begin{aligned}
M_{FI} &= \langle \psi_F | E - K | \psi_I \rangle \\
&\equiv \langle \psi_F | \Delta | \psi_I \rangle \\
&= \frac{1}{2(2\pi)^3} \int d^3k \int_0^1 \frac{dx}{x^2(1-x)^2} \Delta \psi_C(\vec{k}_\perp - x\vec{r}_\perp, x) \psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x) \psi_B(\vec{k}_\perp, x),
\end{aligned}$$

where

$$\begin{aligned}
\Delta &= s - M_A^2 - M_B^2 - K_a - K_b - K_c - K_d \\
&= M_A^2 + M_B^2 - S_A(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp, x) - S_B(\vec{k}_\perp, x) \\
&= M_C^2 + M_D^2 - S_C(\vec{k}_\perp - x\vec{r}_\perp, x) - S_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x).
\end{aligned}$$



# Outlook

- Only one scale  $\Lambda_{QCD}$  determines hadronic spectrum (slightly different for mesons and baryons).
- Light-cone frame is the natural frame to establish the AdS/QCD holographic duality.
- Ratio of Nucleon to Delta trajectories determined by zeroes of Bessel functions.
- AdS modes dual to hadrons extrapolate to valence constituents at zero separation in the AdS boundary.
- Non-zero orbital angular momentum and higher Fock-states require introduction of quantum fluctuations.
- Initial good approximation for description of the structure of hadronic form factors and other observables, but disagreement with data at very high  $Q^2$  may indicate shortcomings of the hard wall approximation.

# *Features of Holographic Model*

- Use of holographic light-front wave functions to compute hadronic matrix elements and other observables.
- Dominance of quark-interchange in hard exclusive processes emerges naturally from the classical duality of the holographic model, modified by gluonic quantum fluctuations.
- Covariant version of the bag model with confinement and conformal symmetry.
- Precise mapping of string modes to partonic states. String modes inside AdS represent the probability amplitude for the distribution of quarks at a given scale.
- Exact holographic mapping for  $n$ -parton state determines effective QCD transverse charge density in terms of modes in AdS space.
- Holographic mapping allows deconstruction: express the eigenvalue problem in terms of 3+1 QCD degrees of freedom.

# Light-Front QCD Phenomenology

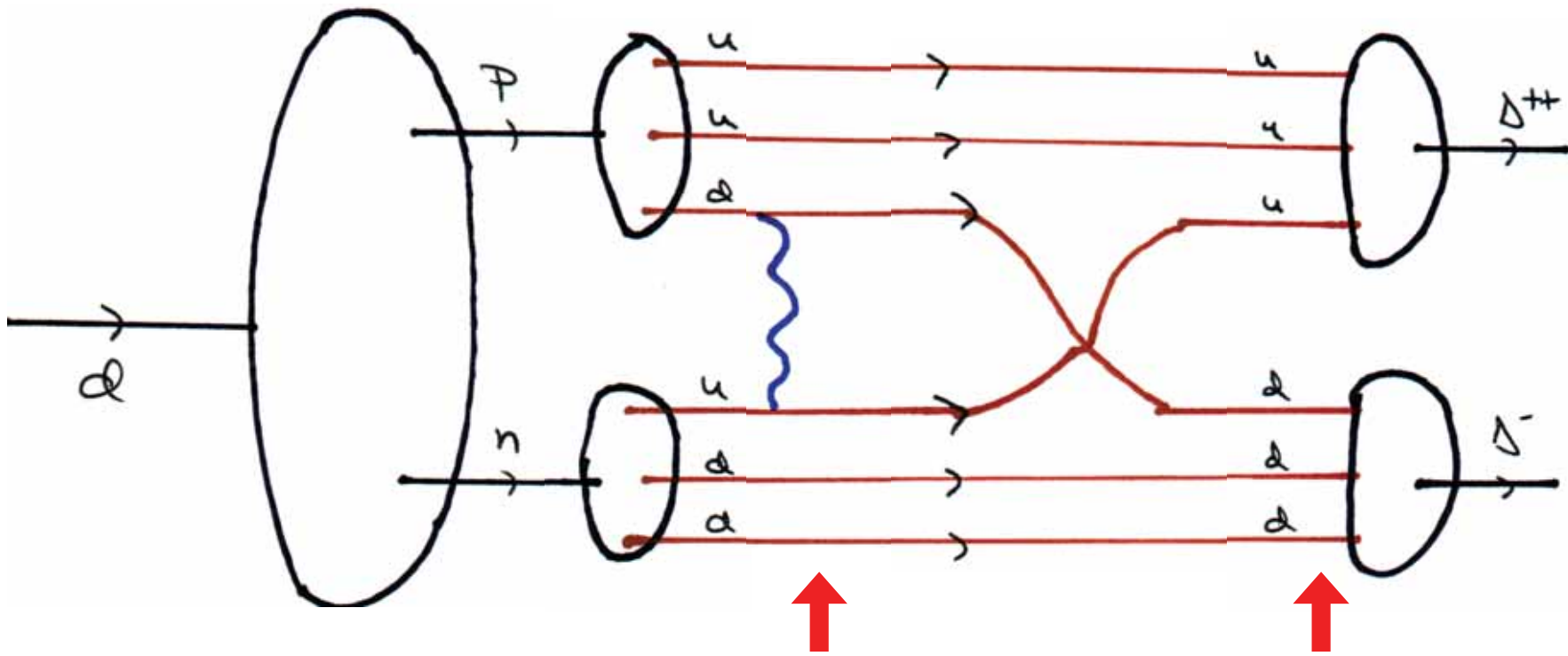
- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

# Hidden Color

- Deuteron six quark wavefunction: Lepage, Ji, sjb
- 5 color-singlet combinations of 6 color-triplets -- one state is  $|n\ p\rangle$
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- **Predict**  $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$  at high  $Q^2$

**Ratio = 2/5 for asymptotic wf**

# Structure of Deuteron in QCD



Hidden Color  
Fock State

Delta-Deuteron  
Fock State

Insights for QCD  
from AdS/CFT

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions  $x_i$  ( $i=1, 2, \dots, 6$ ) can be obtained from a generalization of the proton (three-quark) case.<sup>2</sup> A nontrivial extension is the calculation of the color factor,  $C_d$ , of six-quark systems<sup>5</sup> (see below). Since in leading order only pairwise interactions, with transverse momentum  $Q$ , occur between quarks, the evolution equation for the six-quark system becomes  $\{[dy] = \delta(1 - \sum_{i=1}^6 y_i) \prod_{i=1}^6 dy_i, C_F = (n_c^2 - 1)/2n_c = \frac{4}{3}, \beta = 11 - \frac{2}{3}n_f, \text{ and } n_f \text{ is the effective number of flavors}\}$

$$\prod_{k=1}^6 x_k \left[ \frac{\partial}{\partial \xi} + \frac{3C_F}{\beta} \right] \tilde{\Phi}(x_i, Q) = - \frac{C_d}{\beta} \int_0^1 [dy] V(x_i, y_i) \tilde{\Phi}(y_i, Q),$$

$$\xi(Q^2) = \frac{\beta}{4\pi} \int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2} \alpha_s(k^2) \sim \ln \left( \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right).$$

$$V(x_i, y_i) = 2 \prod_{k=1}^6 x_k \sum_{i \neq j}^6 \theta(y_i - x_i) \prod_{l \neq i, j}^6 \delta(x_l - y_l) \frac{y_j}{x_j} \left( \frac{\delta_{h_i \bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i} \right)$$

where  $\delta_{h_i \bar{h}_j} = 1$  (0) when the helicities of the constituents  $\{i, j\}$  are antiparallel (parallel). The infrared singularity at  $x_i = y_i$  is cancelled by the factor  $\Delta \tilde{\Phi}(y_i, Q) = \tilde{\Phi}(y_i, Q) - \tilde{\Phi}(x_i, Q)$  since the deuteron is a color singlet.

# Quantum Chromodynamic Predictions for the Deuteron Form Factor

$$F_d(Q^2) = \int_0^1 [dx][dy] \varphi_d^\dagger(y, Q) \times T_H^{6q+\gamma^* \rightarrow 6q}(x, y, Q) \varphi_d(x, Q), \quad (1)$$

where the hard-scattering amplitude

$$T_H^{6q+\gamma^* \rightarrow 6q} = [\alpha_s(Q^2)/Q^2]^5 t(x, y) \times [1 + O(\alpha_s(Q^2))] \quad (2)$$

gives the probability amplitude for scattering six quarks collinear with the initial to the final deuteron momentum and

$$\varphi_d(x_i, Q) \propto \int^{k_{\perp i} < Q} [d^2 k_{\perp}] \psi_{qqq qqq}(x_i, \vec{k}_{\perp i}) \quad (3)$$

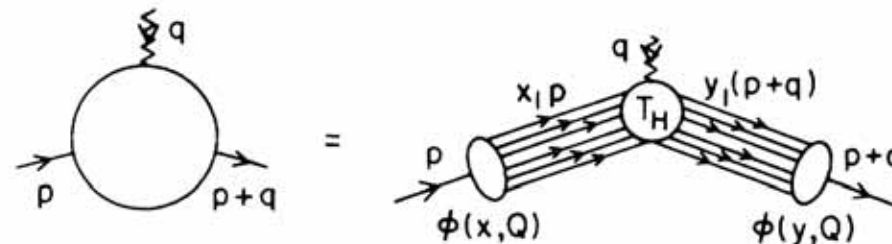


FIG. 1. The general structure of the deuteron form factor at large  $Q^2$ .

Ji, Lepage, sjb

# QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[ \frac{\alpha_s(Q^2)}{Q^2} \right]^5 \sum_{m,n} d_{mn} \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-\gamma_n^d - \gamma_m^d} \left[ 1 + \mathcal{O} \left( \alpha_s(Q^2), \frac{m}{Q} \right) \right]$$

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^2(Q^2/4)} \cdot$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left( \ln \frac{Q^2}{\Lambda^2} \right)^{-(2/5) C_F/B}$$

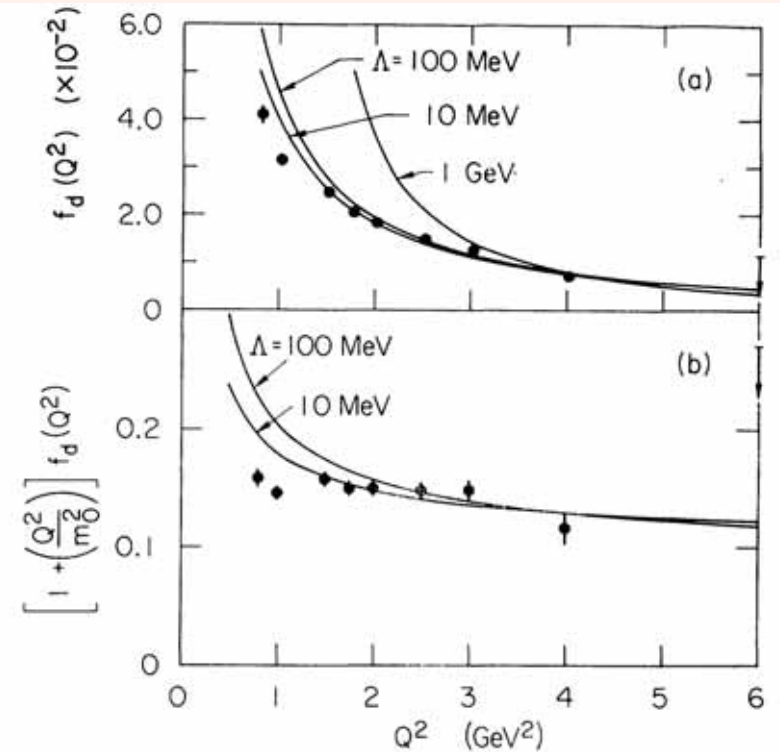
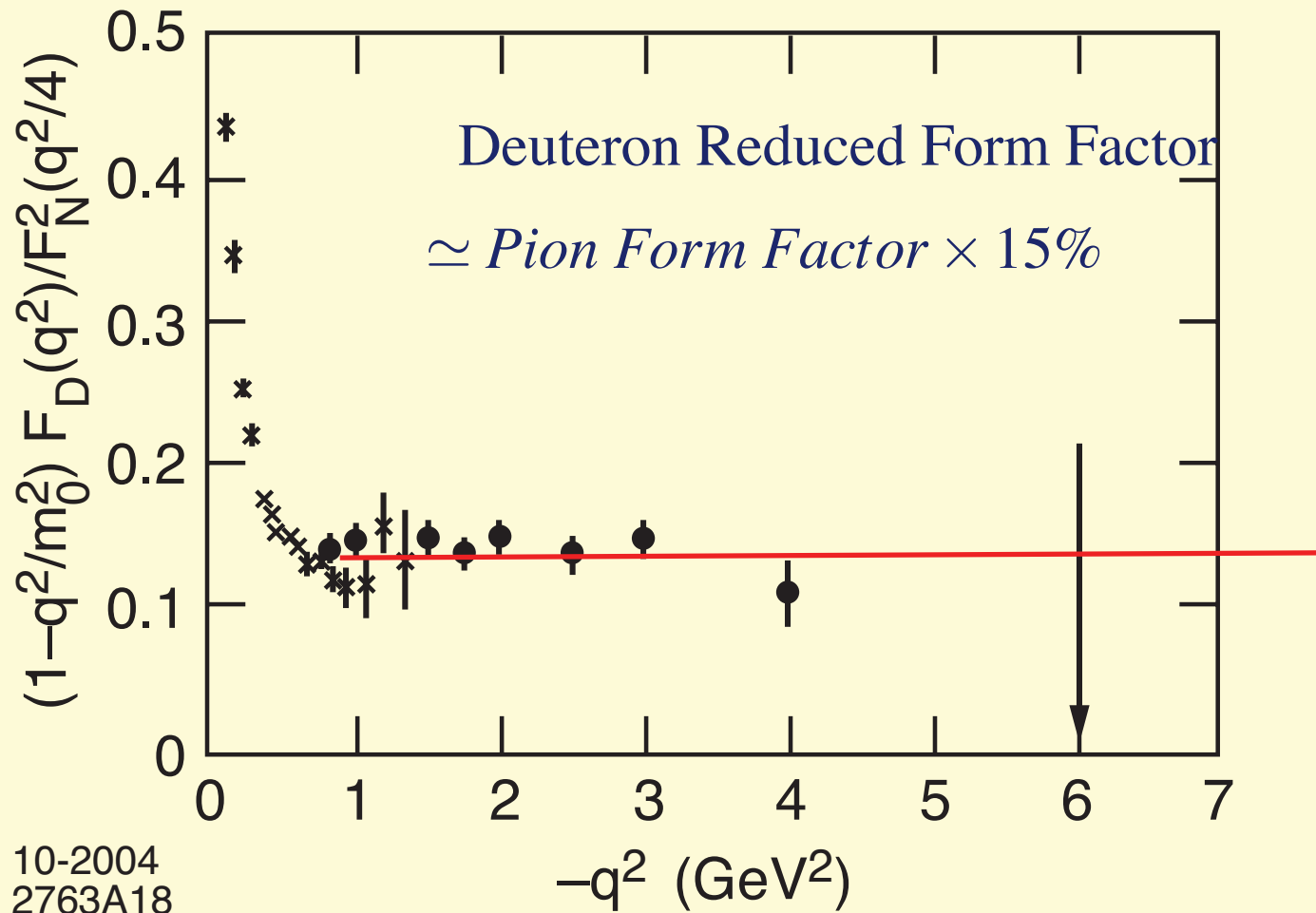


FIG. 2. (a) Comparison of the asymptotic QCD prediction  $f_d(Q^2) \propto (1/Q^2) [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/B}$  with final data of Ref. 10 for the reduced deuteron form factor, where  $F_N(Q^2) = [1 + Q^2/(0.71 \text{ GeV}^2)]^{-2}$ . The normalization is fixed at the  $Q^2 = 4 \text{ GeV}^2$  data point. (b) Comparison of the prediction  $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/B}$  with the above data. The value  $m_0^2 = 0.28 \text{ GeV}^2$  is used (Ref. 8).

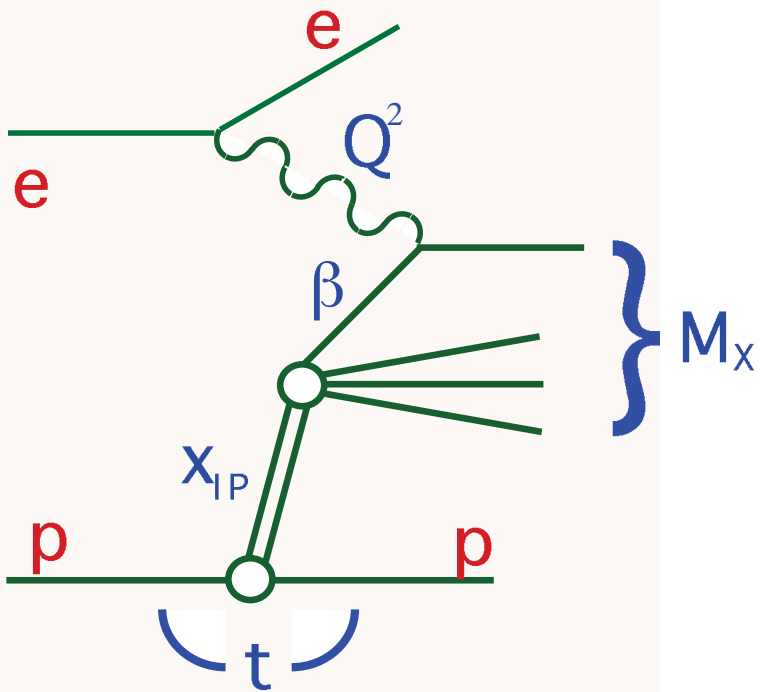


- 15% Hidden Color in the Deuteron

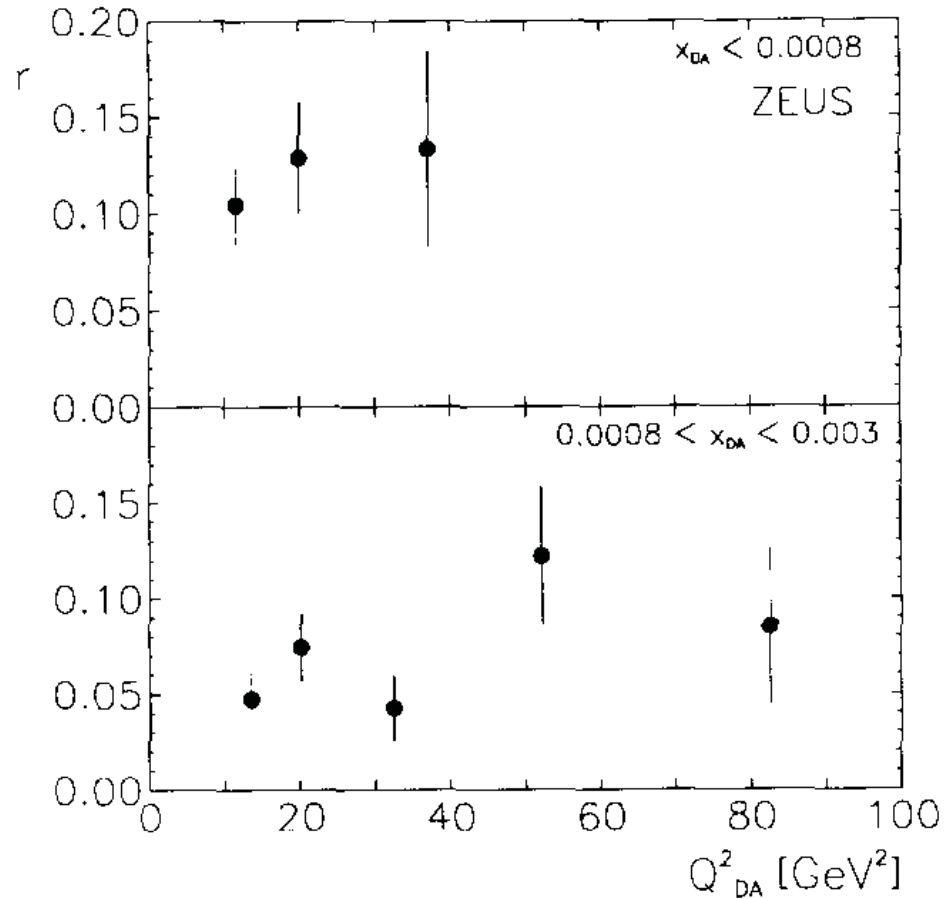
# Test Hidden Color of Deuteron

- Diffractive, Coulomb Dissociation to  $\Delta^{++} \Delta^{-}$
- Photodisintegration of Deuteron to  $\Delta^{++} \Delta^{-}$
- Connection to EMC
- Deuteron not simply  $n + p$

# Remarkable observation at HERA



10% of DIS events are diffractive !



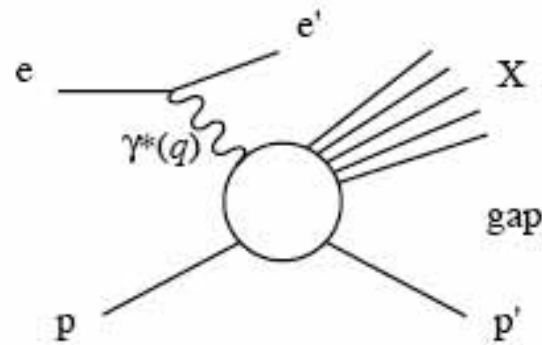
Fraction  $r$  of events with a large rapidity gap,  $\eta_{\max} < 1.5$ , as a function of  $Q^2_{DA}$  for two ranges of  $x_{DA}$ . No acceptance corrections have been applied.

M. Derrick et al. [ZEUS Collaboration], Phys. Lett. B 315, 481 (1993).

# Hard Diffraction from Rescattering

- Diffractive DIS: New Insight into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions
- T-odd Single-Spin Asymmetries
- Diffractive dijets/ trijets
- Color Transparency, Color Opaqueness

# DDIS

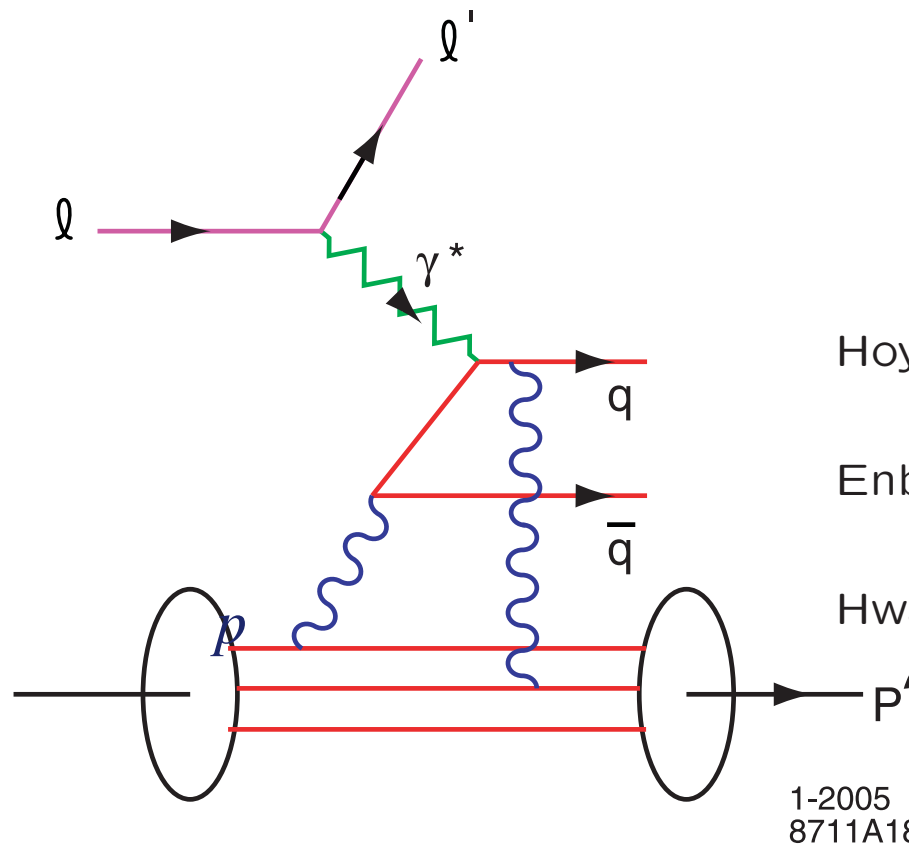


- In a large fraction ( $\sim 10\text{--}15\%$ ) of DIS events, the proton escapes intact, keeping a large fraction of its initial momentum
- This leaves a large *rapidity gap* between the proton and the produced particles
- The  $t$ -channel exchange must be *color singlet*  $\rightarrow$  a pomeron??

Enberg

## Diffractive Deep Inelastic Lepton-Proton Scattering

# Final State Interaction Produces Diffractive DIS



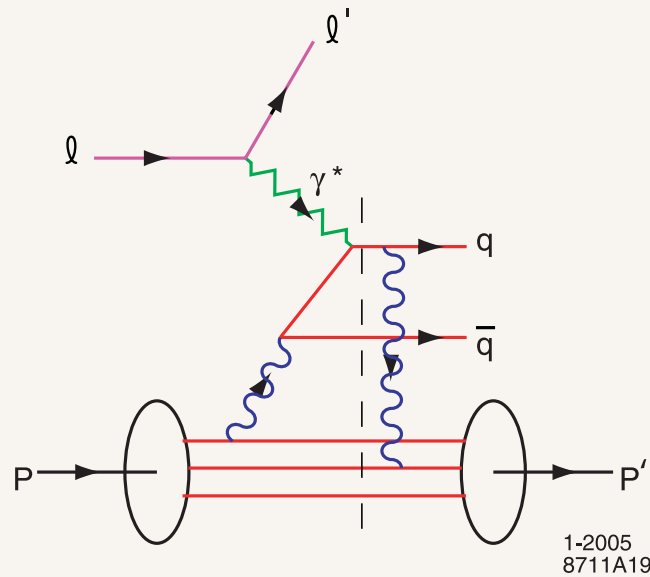
## Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB (BHM)

Enberg, Hoyer, Ingelman, SJB

Hwang, Schmidt, SJB

1-2005  
8711A18



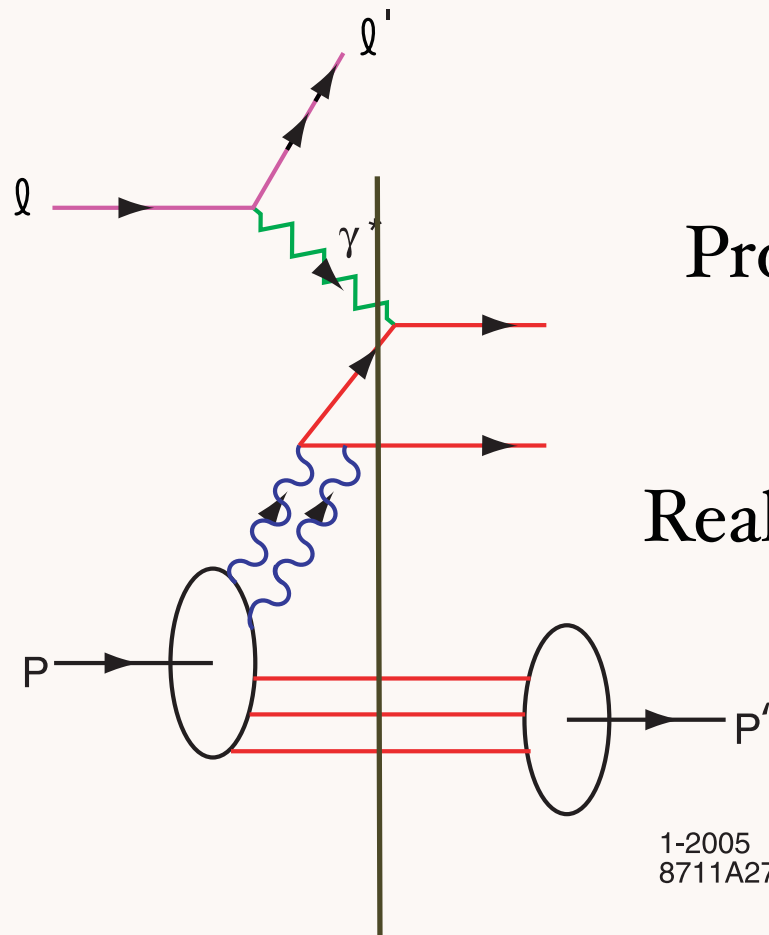
Integration over on-shell domain produces phase  $i$

Need Imaginary Phase to Generate  
Pomeron

Need Imaginary Phase to Generate  
T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Pomeron is not  
a constituent  
of proton!

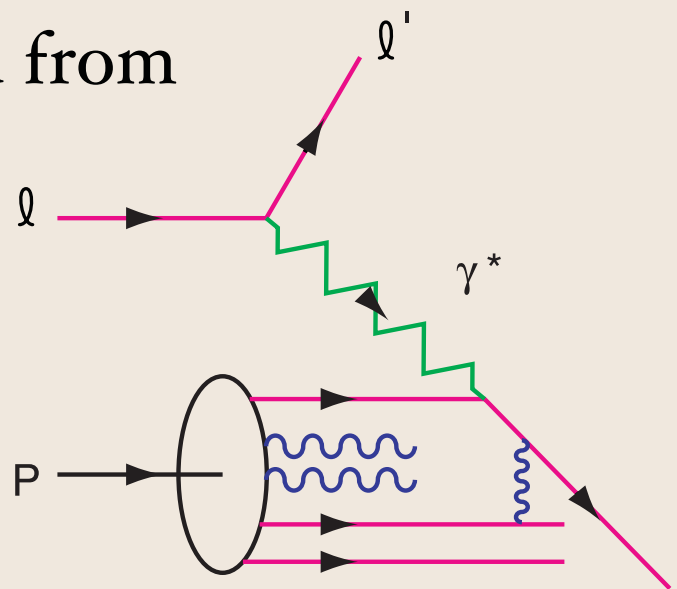


Problem: Wrong Phase

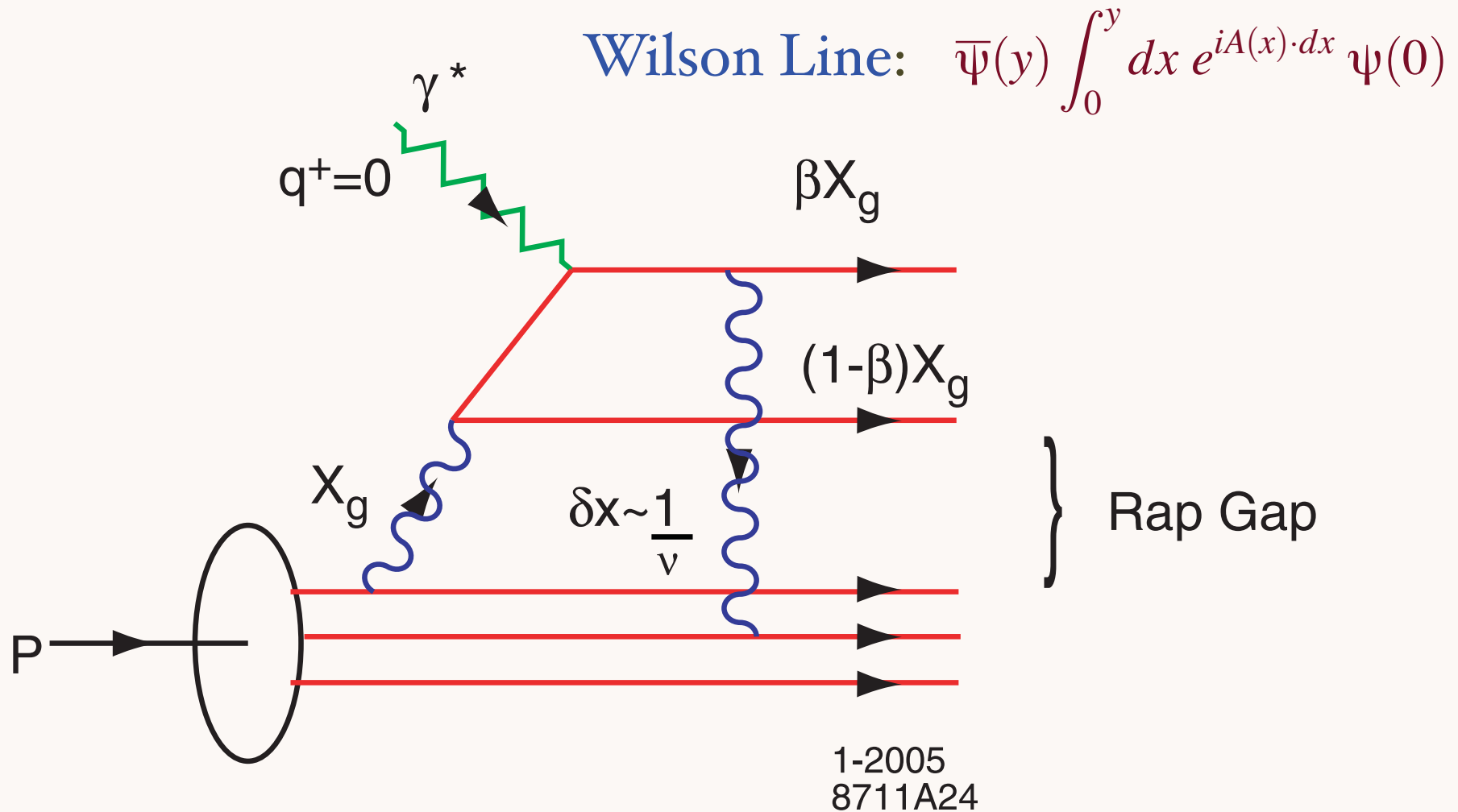
Real; should be imaginary

Need Final State Interactions !

- Quarks Reinteract in Final State
- Analogous to Coulomb phases, but not unitary
- Observable effects: DDIS, SSI, shadowing, antishadowing
- Structure functions cannot be computed from LFWFs computed in isolation
- Wilson line not 1 even in lcg



# QCD Mechanism for Rapidity Gaps



# Physics of Rescattering

- Diffractive DIS: New Insight into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- **Novel Effects: Color Transparency, Color Opacity, Intrinsic Charm, Odderon**

# *New Perspectives for QCD from AdS/CFT*

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support  $0 < x < 1$ .
- Quark Interchange dominant force at short distances

# QCD at The Amplitude Level

- Light-Front Fock Expansions
- LFWFs boost invariant
- Direct connection to form factors, structure functions, distribution amplitudes, GPDs
- Higher Twist Correlations
- Orbital Angular Momentum
- Validated in QED, Bethe-Salpeter
- AdS/CFT Holographic Model

# *Essential to test QCD*

- GSI antiprotons
- 12 GeV Jlab
- J-PARC
- BaBar/Belle: ISR, two-gamma, timelike DVCS
- RHIC/LHC Nuclear Collisions; LHCb; forward heavy quarks, higgs
- photon-photon collider at the ILC
- electron-proton, electron-nucleus collisions