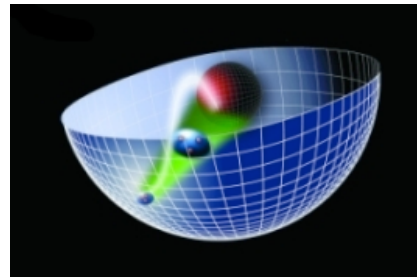


Light-Front Quantization Approach to the Gauge/Gravity Correspondence

Guy F. de Téramond

University of Costa Rica



Slac Theory Seminar

February 17, 2010



Brodsky and GdT, PRL **96**, 201601 (2006)
Brodsky and GdT, PRD **77**, 056007 (2008)
Brodsky and GdT, PRD **78**, 025032 (2008)
GdT and Brodsky, PRL**102**, 081601 (2009)
Brodsky, GdT and Deur, arXiv:1002.3948

Outline

1. Introduction: Gauge/Gravity Correspondence and QCD

Part A

2. Light-Front Quantization of QCD
 - Light-Front Partonic Representation
 - Light-Front Bound State Hamiltonian Equation
3. Semiclassical Approximation to QCD
 - Hard-Wall Model
 - Light-Front Holographic Mapping
 - Soft-Wall Model
4. Fermionic Modes
 - Hard-Wall Model
 - Soft-Wall Model

Part B

5. Holographic Mapping of Current Matrix Elements

Light-Front Holographic Mapping

Form Factors in the Soft-Wall Model

Space-Like Dirac Proton Form Factor

Part C

6. Nonperturbative QCD Coupling From Light-Front Holography

Effective couplings

Holographic β -Function

Full β -function of QCD ?

Static $\bar{Q}Q$ Potential

1 Introduction: Gauge/Gravity Correspondence and QCD

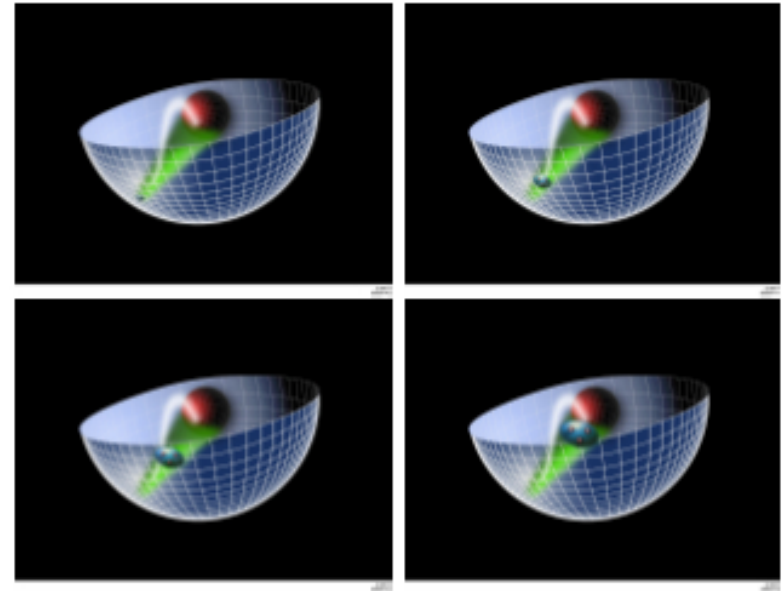
- Most challenging problem of strong interaction dynamics: determine the composition of hadrons in terms of their fundamental QCD quark and gluon degrees of freedom
- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between gravity theories in AdS space and conformal field theories in physical space-time have led to analytical insights into the confining dynamics of QCD
- Description of strongly coupled gauge theory using a dual gravity description!
- Strings describe spin- J extended objects (no quarks). QCD degrees of freedom are pointlike particles and hadrons have orbital angular momentum: how can they be related?
- Isomorphism of $SO(4, 2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS_5 , a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space

- AdS₅ metric:

$$\underbrace{ds^2}_{L_{\text{AdS}}} = \frac{R^2}{z^2} \left(\underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\text{Minkowski}}} - dz^2 \right)$$

- A distance L_{AdS} shrinks by a warp factor z/R as observed in Minkowski space ($dz = 0$):

$$L_{\text{Minkowski}} \sim \frac{z}{R} L_{\text{AdS}}$$



- Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, the variable z acts like a scaling variable in Minkowski space
- Short distances $x_\mu x^\mu \rightarrow 0$ maps to UV conformal AdS₅ boundary $z \rightarrow 0$
- Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda_{\text{QCD}}^2$ maps to large IR region of AdS₅, $z \sim 1/\Lambda_{\text{QCD}}$, thus there is a maximum separation of quarks and a maximum value of z
- Local operators like \mathcal{O} and \mathcal{L}_{QCD} defined in terms of quark and gluon fields at the AdS₅ boundary
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

2 Light-Front Quantization of QCD and AdS/CFT

- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...
- Frame-independent LF Hamiltonian equation

$$P_\mu P^\mu |\psi(P)\rangle = (P^- P^+ - \mathbf{P}_\perp^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

similar structure of AdS EOM

$$\left[z^2 \partial_z^2 - 3z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0$$

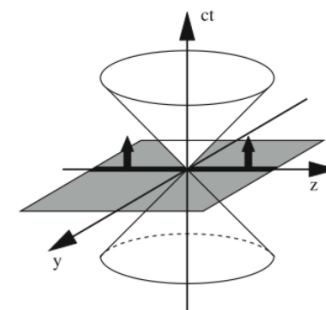
- First semiclassical approximation to the bound-state LF Hamiltonian equation in QCD is equivalent to EOM in AdS and can be systematically improved [GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

$$|\psi(P)\rangle \text{ dual to } \Phi(z)$$



Light-Front Quantization of QCD

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- *Instant form*: hypersurface defined by $t = 0$, the familiar one
- *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$



$$x^+ = x^0 + x^3 \quad \text{light-front time}$$

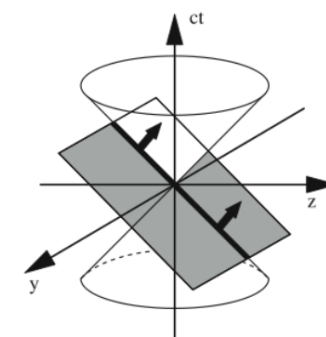
$$x^- = x^0 - x^3 \quad \text{longitudinal space variable}$$

$$k^+ = k^0 + k^3 \quad \text{longitudinal momentum} \quad (k^+ > 0)$$

$$k^- = k^0 - k^3 \quad \text{light-front energy}$$

$$k \cdot x = \frac{1}{2} (k^+ x^- + k^- x^+) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp$$

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{\mathbf{k}_\perp^2 + m^2}{k^+}$



- QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} (G^{\mu\nu} G_{\mu\nu}) + i\bar{\psi} D_\mu \gamma^\mu \psi + m\bar{\psi}\psi$$

- LF Momentum Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields ψ, \mathbf{A}_\perp

$$P^- = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \psi + \text{interactions}$$

$$P^+ = \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ i\partial^+ \psi$$

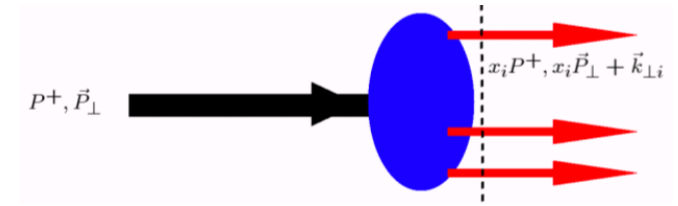
$$\mathbf{P}_\perp = \frac{1}{2} \int dx^- d^2 \mathbf{x}_\perp \bar{\psi} \gamma^+ i\nabla_\perp \psi$$

- LF Hamiltonian P^- generates LF time translations

$$[\psi(x), P^-] = i \frac{\partial}{\partial x^+} \psi(x)$$

and the generators P^+ and \mathbf{P}_\perp are kinematical

Light-Front Partonic Representation



- Dirac field ψ , expanded in terms of ladder operators on the initial surface $x^+ = x^0 + x^3$

$$\psi(x^-, \mathbf{x}_\perp)_\alpha = \sum_\lambda \int_{q^+ > 0} \frac{dq^+}{\sqrt{2q^+}} \frac{d^2 \mathbf{q}_\perp}{(2\pi)^3} \left[b_\lambda(q) u_\alpha(q, \lambda) e^{-iq \cdot x} + d_\lambda(q)^\dagger v_\alpha(q, \lambda) e^{iq \cdot x} \right]$$

- LF Generators $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of constituents with momentum $q = (q^+, q^-, \mathbf{q}_\perp)$

$$P^- = \sum_\lambda \int \frac{dq^+ d^2 \mathbf{q}_\perp}{(2\pi)^3} \left(\frac{\mathbf{q}_\perp^2 + m^2}{q^+} \right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions}$$

$$P^+ = \sum_\lambda \int \frac{dq^+ d^2 \mathbf{q}_\perp}{(2\pi)^3} q^+ b_\lambda^\dagger(q) b_\lambda(q)$$

$$\mathbf{P}_\perp = \sum_\lambda \int \frac{dq^+ d^2 \mathbf{q}_\perp}{(2\pi)^3} \mathbf{q}_\perp b_\lambda^\dagger(q) b_\lambda(q)$$

Light-Front Bound State Hamiltonian Equation

- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state system

$$P_\mu P^\mu |\psi(P)\rangle = (P^- P^+ - \mathbf{P}_\perp^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- State $|\psi(P^+, \mathbf{P}_\perp, J_z)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \begin{cases} |uud\rangle \\ |uudg\rangle \\ |uud\bar{q}q\rangle \quad \dots \end{cases}$$

where $k_i^2 = m_i^2$, $k_i = (k_i^+, k_i^-, \mathbf{k}_{\perp i})$, for each component i

- Fock components $\psi_n(x_i, \mathbf{k}_{\perp i}, \lambda_i^z)$ are independent of P^+ and \mathbf{P}_\perp and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $\mathbf{k}_{\perp i}$ and spin λ_i^z

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n \mathbf{k}_{\perp i} = 0.$$

- Compute \mathcal{M}^2 from hadronic matrix element

$$\langle \psi(P') | P_\mu P^\mu | \psi(P) \rangle = \mathcal{M}^2 \langle \psi(P') | \psi(P) \rangle$$

- Find

$$\mathcal{M}^2 = \sum_n \int [dx_i] [d^2 \mathbf{k}_{\perp i}] \sum_\ell \left(\frac{\mathbf{k}_{\perp \ell}^2 + m_\ell^2}{x_q} \right) |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 + \text{interactions}$$

- Phase space normalization of LFWFs

$$\sum_n \int [dx_i] [d^2 \mathbf{k}_{\perp i}] |\psi_n(x_i, \mathbf{k}_{\perp i})|^2 = 1$$

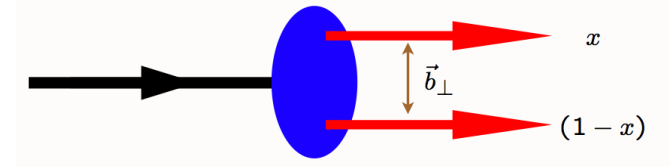
- In terms of $n-1$ independent transverse impact coordinates $\mathbf{b}_{\perp j}$, $j = 1, 2, \dots, n-1$,

$$\mathcal{M}^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} \psi_n^*(x_i, \mathbf{b}_{\perp i}) \sum_\ell \left(\frac{-\nabla_{\mathbf{b}_{\perp \ell}}^2 + m_\ell^2}{x_q} \right) \psi_n(x_i, \mathbf{b}_{\perp i}) + \text{interactions}$$

- Normalization

$$\sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \mathbf{b}_{\perp j} |\psi_n(x_j, \mathbf{b}_{\perp j})|^2 = 1$$

3 Semiclassical Approximation to QCD



- Consider a two-parton hadronic bound state in transverse impact space in the limit $m_q \rightarrow 0$

$$\mathcal{M}^2 = \int_0^1 \frac{dx}{1-x} \int d^2 \mathbf{b}_\perp \psi^*(x, \mathbf{b}_\perp) (-\nabla_{\mathbf{b}_\perp}^2) \psi(x, \mathbf{b}_\perp) + \text{interactions}$$

- Functional dependence of Fock state $|n\rangle$ given by invariant mass

$$\mathcal{M}_n^2 = \left(\sum_{a=1}^n k_a^\mu \right)^2 = \sum_a \frac{\mathbf{k}_{\perp a}^2 + m_a^2}{x_a} \rightarrow \frac{\mathbf{k}_\perp^2}{x(1-x)}$$

the off-energy shell of the bound state $\mathcal{M}^2 - \mathcal{M}_n^2$

- In impact space the relevant variable is $\zeta^2 = x(1-x)\mathbf{b}_\perp^2$
- To first approximation LF dynamics depend only on the invariant variable \mathcal{M}_n or ζ , and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring out angular φ , longitudinal $X(x)$ and transverse mode $\phi(\zeta)$

- Find ($L = |M|$)

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left(-\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the confining forces from the interaction terms is summed up in the effective potential $U(\zeta)$

- Ultra relativistic limit $m_q \rightarrow 0$ longitudinal modes $X(x)$ decouple and
- LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for ϕ

$$\left(\underbrace{\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right)}_{\text{kinetic energy of partons}} + \underbrace{U(\zeta)}_{\text{confinement}} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find n -massless partons at transverse impact separation ζ within the hadron at equal light-front time
- Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism

Hard-Wall Model

- Consider the potential (hard wall)

$$U(\zeta) = \begin{cases} 0 & \text{if } \zeta \leq \frac{1}{\Lambda_{\text{QCD}}} \\ \infty & \text{if } \zeta > \frac{1}{\Lambda_{\text{QCD}}} \end{cases}$$

- If $L^2 \geq 0$ the Hamiltonian is positive definite $\langle \phi | P_\mu P^\mu | \phi \rangle \geq 0$ and thus $\mathcal{M}^2 \geq 0$
- If $L^2 < 0$ the Hamiltonian is not bounded from below (“Fall-to-the-center” problem in Q.M.)
- Critical value of the potential corresponds to $L = 0$, the lowest possible stable state
- Solutions:

$$\phi_L(\zeta) = C_L \sqrt{\zeta} J_L(\zeta \mathcal{M})$$

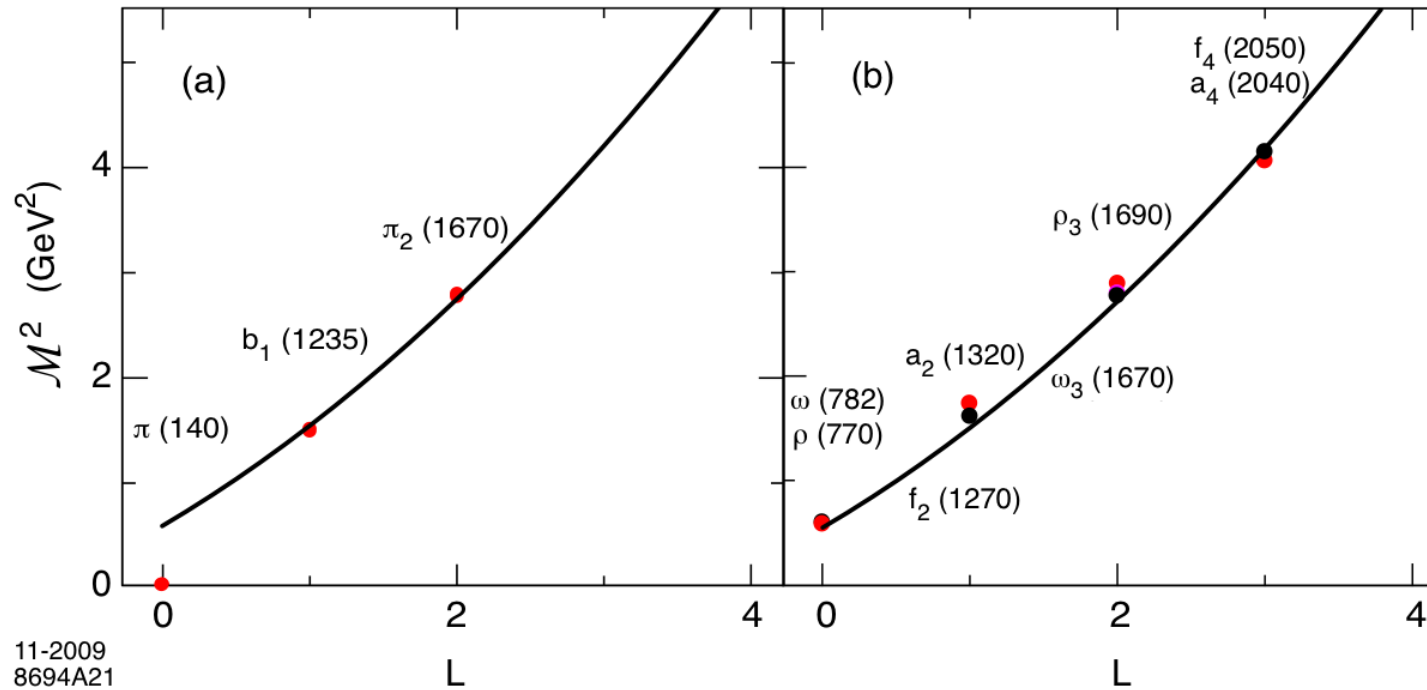
- Mode spectrum from boundary conditions

$$\phi(\zeta = 1/\Lambda_{\text{QCD}}) = 0$$

Thus

$$\mathcal{M}^2 = \beta_{Lk} \Lambda_{\text{QCD}}$$

- Excitation spectrum hard-wall model: $\mathcal{M}_{n,L} \sim L + 2n$



Light-meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Light-Front Holographic Mapping

- LF Holographic mapping found originally by matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD [Brodsky and GdT (2006, 2008)]

- Substitute $\Phi(\zeta) \sim \zeta^{3/2} \phi(\zeta)$, $\zeta \rightarrow z$ in the conformal LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

- Find:

$$\boxed{[z^2 \partial_z^2 - 3z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0}$$

with $(\mu R)^2 = -4 + L^2$, the wave equation of string mode in AdS₅ !

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$
- Scaling dimension τ of AdS mode Φ given in terms of 5-dim mass by $(\mu R)^2 = \tau(\tau - 4)$. Thus $\tau = 2 + L$ in agreement with the twist scaling dimension of a two parton object in QCD

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3-2J)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

with $(\mu R)^2 = (\tau - J)(\tau - 4 + J)$

- Upon substitution $z \rightarrow \zeta$ and

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} \Phi_J(\zeta)$$

we recover the QCD LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$

- J -decoupling in the HW model



Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce smooth cutoff which depends on a dilaton background field $\varphi(z) = \pm\kappa^2 z^2$

$$S = \int d^4x dz \sqrt{g} e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$\left[z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0$$

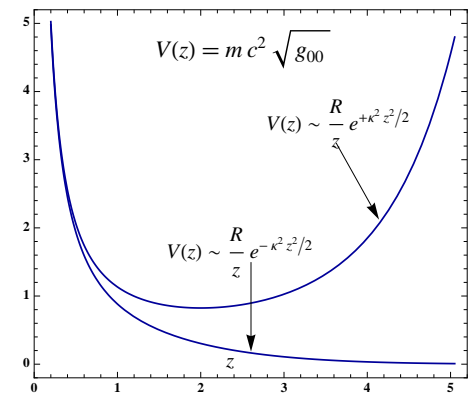
- Modified metric gives the gravitational potential

$$V(z) = mc^2 \sqrt{g_{00}} = mc^2 R e^{\pm\kappa^2 z^2/2} / z$$

Plus dilaton $\varphi = +\kappa^2 z^2$ confines any object to distances $\langle z \rangle \sim 1/\kappa$

- Lowest possible state $(\mu R)^2 = -4$ (BF bound $(\mu R)^2 \geq -4$)

$$\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}$$



A chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for Φ

$$\left[z^2 \partial_z^2 - (3 - 2J - 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1) \right) \phi_{\mu_1 \dots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \dots \mu_J}$$



with $(\mu R)^2 = -(2 - J)^2 + L^2$

- Eigenfunctions

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

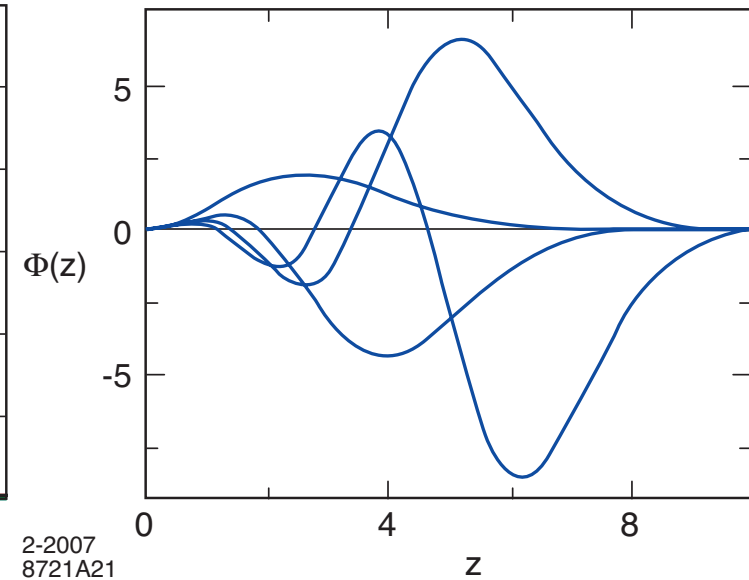
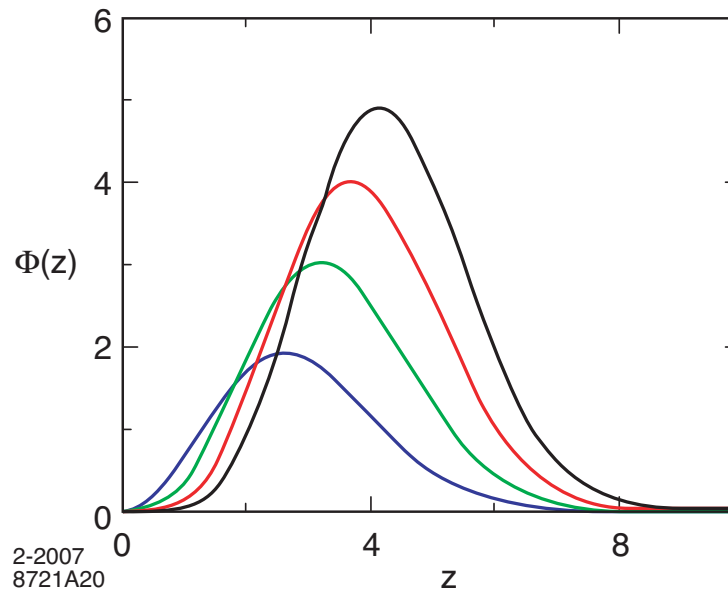
- Eigenvalues

$$\mathcal{M}_{n,L,S}^2 = 4\kappa^2 \left(n + L + \frac{S}{2} \right)$$

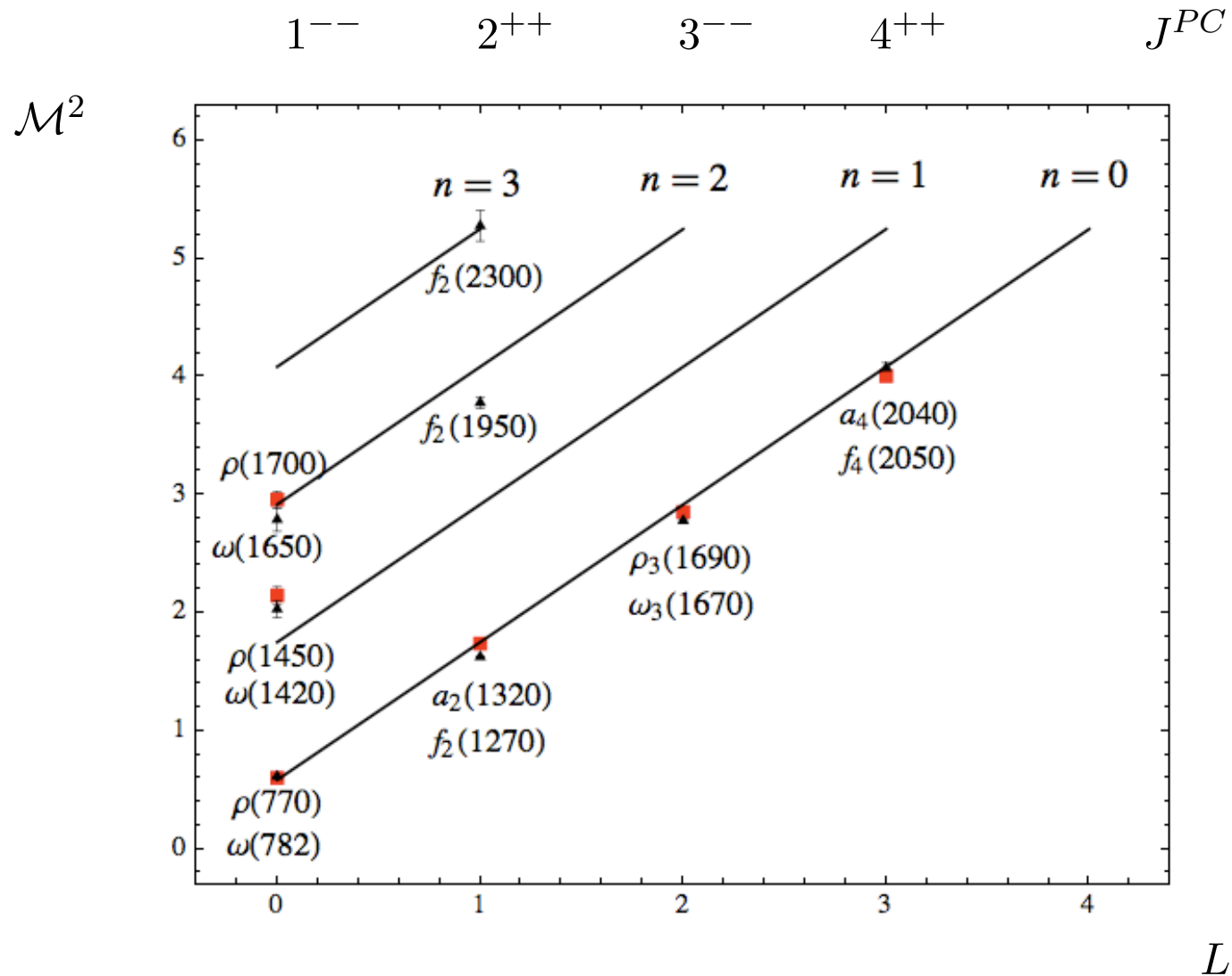
$4\kappa^2$ for $\Delta n = 1$

$4\kappa^2$ for $\Delta L = 1$

$2\kappa^2$ for $\Delta S = 1$



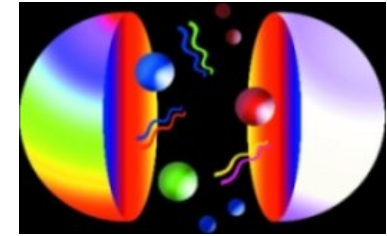
Orbital and radial states: $\langle \zeta \rangle$ increase with L and n



Parent and daughter Regge trajectories for the $I = 1$ ρ -meson family (red) and the $I = 0$ ω -meson family (black) for $\kappa = 0.54$ GeV

4 Fermionic Modes

Hard-Wall Model



From Nick Evans

- Action for massive fermionic modes on AdS₅:

$$S[\bar{\Psi}, \Psi] = \int d^4x dz \sqrt{g} \bar{\Psi}(x, z) \left(i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)$$

- Equation of motion: $(i\Gamma^\ell D_\ell - \mu) \Psi(x, z) = 0$

$$\left[i \left(z\eta^{\ell m} \Gamma_\ell \partial_m + 2\Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0$$

- Solution ($\mu R = \nu + 1/2$, $\nu = L + 1$)

$$\Psi(z) = C z^{5/2} [J_\nu(z\mathcal{M})u_+ + J_{\nu+1}(z\mathcal{M})u_-]$$

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm(z = 1/\Lambda_{\text{QCD}}) = 0$

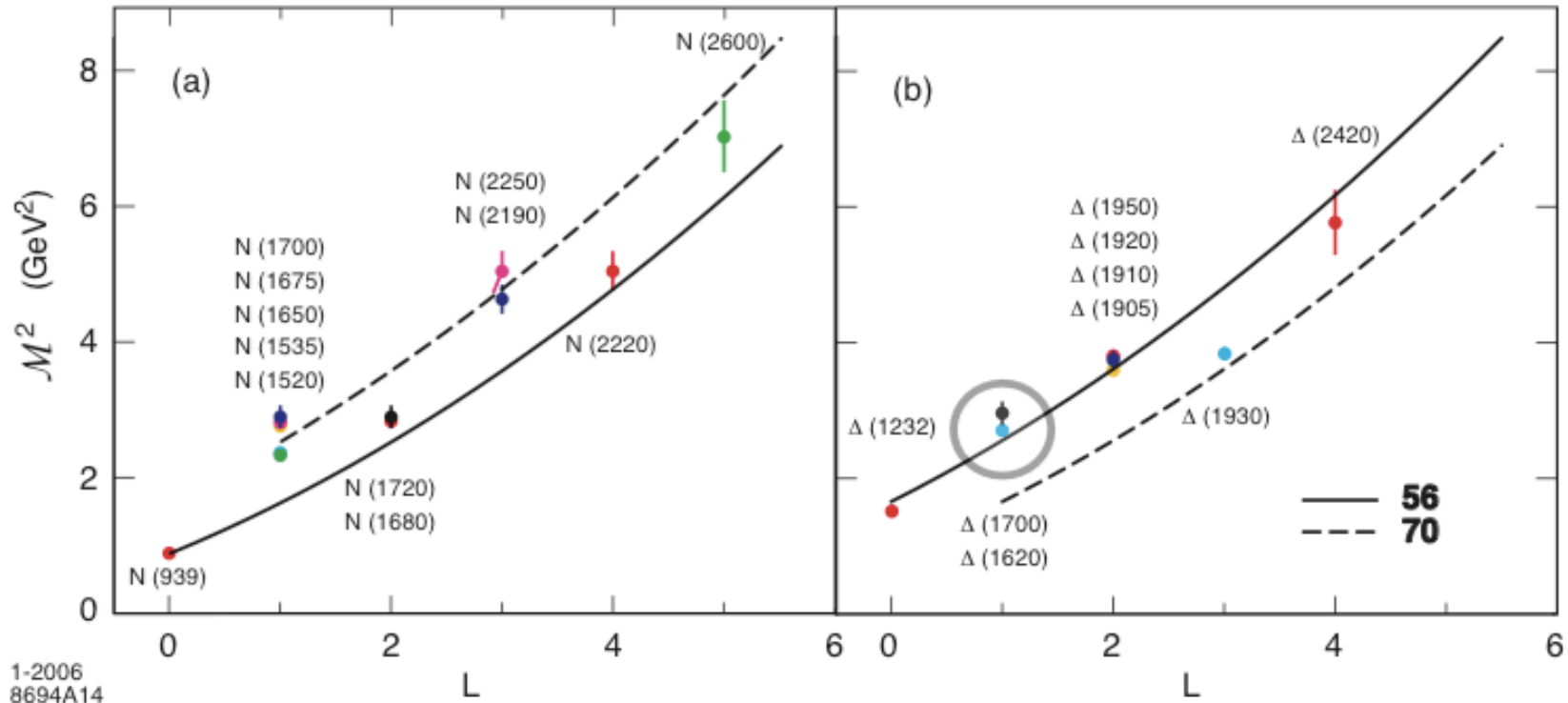
$$\mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}}$$

with scale independent mass ratio

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

SU(6)	S	L	Baryon State			
56	$\frac{1}{2}$	0	$N_{\frac{1}{2}}^{1+}$ (939)			
	$\frac{3}{2}$	0	$\Delta_{\frac{3}{2}}^{3+}$ (1232)			
70	$\frac{1}{2}$	1	$N_{\frac{1}{2}}^{1-}$ (1535) $N_{\frac{3}{2}}^{3-}$ (1520)			
	$\frac{3}{2}$	1	$N_{\frac{1}{2}}^{1-}$ (1650) $N_{\frac{3}{2}}^{3-}$ (1700) $N_{\frac{5}{2}}^{5-}$ (1675)			
	$\frac{1}{2}$	1	$\Delta_{\frac{1}{2}}^{1-}$ (1620) $\Delta_{\frac{3}{2}}^{3-}$ (1700)			
56	$\frac{1}{2}$	2	$N_{\frac{3}{2}}^{3+}$ (1720) $N_{\frac{5}{2}}^{5+}$ (1680)			
	$\frac{3}{2}$	2	$\Delta_{\frac{1}{2}}^{1+}$ (1910) $\Delta_{\frac{3}{2}}^{3+}$ (1920) $\Delta_{\frac{5}{2}}^{5+}$ (1905) $\Delta_{\frac{7}{2}}^{7+}$ (1950)			
70	$\frac{1}{2}$	3	$N_{\frac{5}{2}}^{5-}$ $N_{\frac{7}{2}}^{7-}$			
	$\frac{3}{2}$	3	$N_{\frac{3}{2}}^{3-}$ $N_{\frac{5}{2}}^{5-}$ $N_{\frac{7}{2}}^{7-}$ (2190) $N_{\frac{9}{2}}^{9-}$ (2250)			
	$\frac{1}{2}$	3	$\Delta_{\frac{5}{2}}^{5-}$ (1930) $\Delta_{\frac{7}{2}}^{7-}$			
56	$\frac{1}{2}$	4	$N_{\frac{7}{2}}^{7+}$ $N_{\frac{9}{2}}^{9+}$ (2220)			
	$\frac{3}{2}$	4	$\Delta_{\frac{5}{2}}^{5+}$ $\Delta_{\frac{7}{2}}^{7+}$ $\Delta_{\frac{9}{2}}^{9+}$ $\Delta_{\frac{11}{2}}^{11+}$ (2420)			
70	$\frac{1}{2}$	5	$N_{\frac{9}{2}}^{9-}$ $N_{\frac{11}{2}}^{11-}$			
	$\frac{3}{2}$	5	$N_{\frac{7}{2}}^{7-}$ $N_{\frac{9}{2}}^{9-}$ $N_{\frac{11}{2}}^{11-}$ (2600) $N_{\frac{13}{2}}^{13-}$			

- Excitation spectrum for baryons in the hard-wall model: $\mathcal{M} \sim L + 2n$ [GdT and Brodsky (2005)]



Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states: (a) $I = 1/2$ and (b) $I = 3/2$

Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

$$\left[i \left(z \eta^{\ell m} \Gamma_{\ell} \partial_m + 2\Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^{\ell}) = 0.$$

- Solution ($\mu R = \nu + 1/2$, $\nu = L + 1$)

$$\begin{aligned} \Psi_+(z) &\sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu}(\kappa^2 z^2) \\ \Psi_-(z) &\sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2) \end{aligned}$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

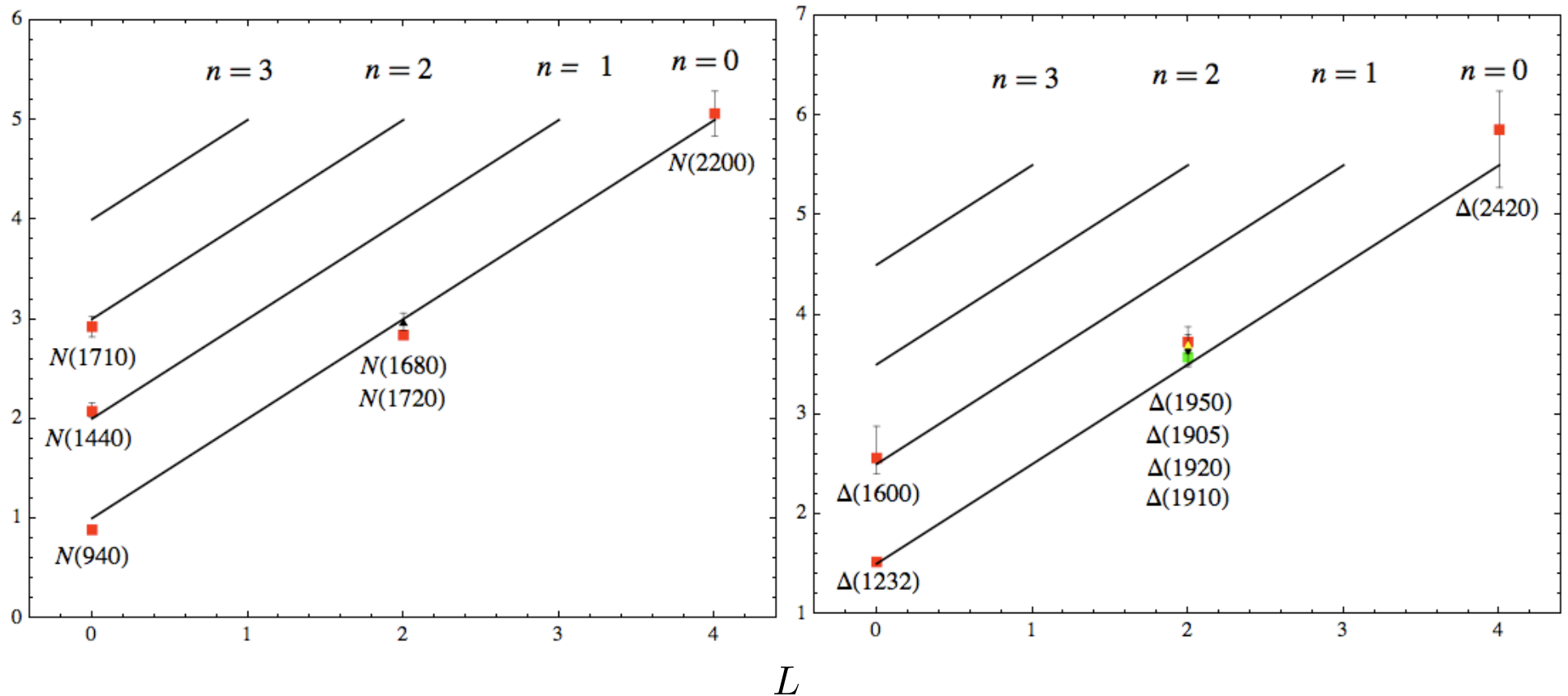
$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$

- Obtain spin- J mode $\Phi_{\mu_1 \dots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions

$$4\kappa^2 \text{ for } \Delta n = 1$$

$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$

$$\mathcal{M}^2$$


Parent and daughter **56** Regge trajectories for the N and Δ baryon families for $\kappa = 0.5 \text{ GeV}$

- Δ spectrum identical to Forkel and Klempt, Phys. Lett. B **679**, 77 (2009)

5 Holographic Mapping of Current Matrix Elements

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle, \quad Q = P' - P, \quad J^\mu = e_q \bar{q} \gamma^\mu q$$

- EM hadronic matrix element in AdS: non-local coupling with extended mode $\Phi(x, z)$

$$\int d^4x dz \sqrt{g} A^\ell(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_\ell \Phi_P(x, z)$$

- Are the transition amplitudes related ?

$$|\psi(P)\rangle \text{ dual to } \Phi(z)$$



- How to recover hard pointlike scattering out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

- Electromagnetic probe polarized along Minkowski coordinates, ($Q^2 = -q^2 > 0$)

$$A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q, z), \quad A_z = 0$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$[z^2 \partial_z^2 - z \partial_z - z^2 Q^2] J(Q, z) = 0$$

- Solution

$$J(Q, z) = zQ K_1(zQ)$$

- Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

$$\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \quad \Phi(z) \rightarrow z^\tau, \quad z \rightarrow 0$$

- Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons Φ_P and $\Phi_{P'}$, with the non-normalizable mode $J(Q, z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int_0^{\Lambda_{\text{QCD}}^{-1}} \frac{dz}{z^3} \Phi(z) J(Q, z) \Phi(z)$$

- Since $K_n(x) \simeq \sqrt{\frac{\pi}{2x}} e^{-x}$, the external source is suppressed inside AdS for large Q . Important contribution to the integral is from $z \sim 1/Q$ where $\Phi \sim z^\tau$
- For large Q^2

$$F(Q^2) \rightarrow \left[\frac{1}{Q^2} \right]^{\tau-1}$$

and the power-law ultraviolet point-like scaling is recovered [Polchinski and Susskind (2001)]

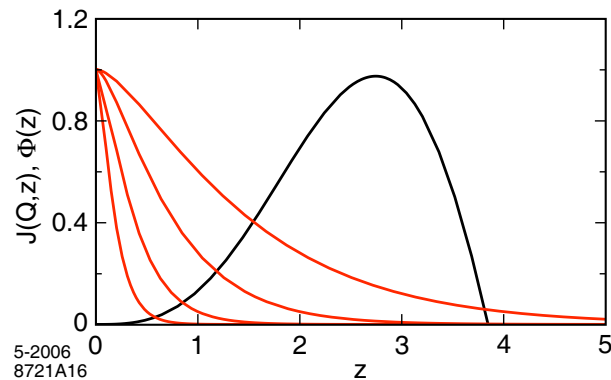


Fig: Suppression of external modes for large Q inside AdS. Red curves: $J(Q, z)$, black: $\Phi(z)$

Light-Front Holographic Mapping

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_{\perp} \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\right) \left| \tilde{\psi}_n(x_j, \mathbf{b}_{\perp j}) \right|^2$$

- Consider a two-quark π^+ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_{\perp} e^{i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}(1-x)} \left| \tilde{\psi}_{u\bar{d}/\pi}(x, \mathbf{b}_{\perp}) \right|^2$$

with normalization $F_{\pi^+}(q=0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left| \tilde{\psi}_{u\bar{d}/\pi}(x, \zeta) \right|^2$$

where $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$

- Compare with the electromagnetic FF in AdS space [Brodsky and GdT (2006)]

$$F(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2$$

where $F(Q^2=0) = 1$ and $J(Q^2, z) = zQK_1(zQ)$

- Use the integral representation

$$J(Q^2, z) = \int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right)$$

- Find

$$F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0\left(zQ \sqrt{\frac{1-x}{x}}\right) |\Phi_{\pi^+}(z)|^2$$

- Compare with the expression for the electromagnetic FF in LF QCD for arbitrary values of Q

$$\left| \tilde{\psi}_{u\bar{d}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4}$$

- Identify transverse LF variable ζ , $0 \leq \zeta \leq \Lambda_{\text{QCD}}$, with holographic variable z

Form Factors in the Soft-Wall Model

- Form factor for a string mode with twist scaling dimension τ , Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}$$

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z) \dots (1 + z)\Gamma(1 + z)$
- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)}, \quad N = 3,$$

...

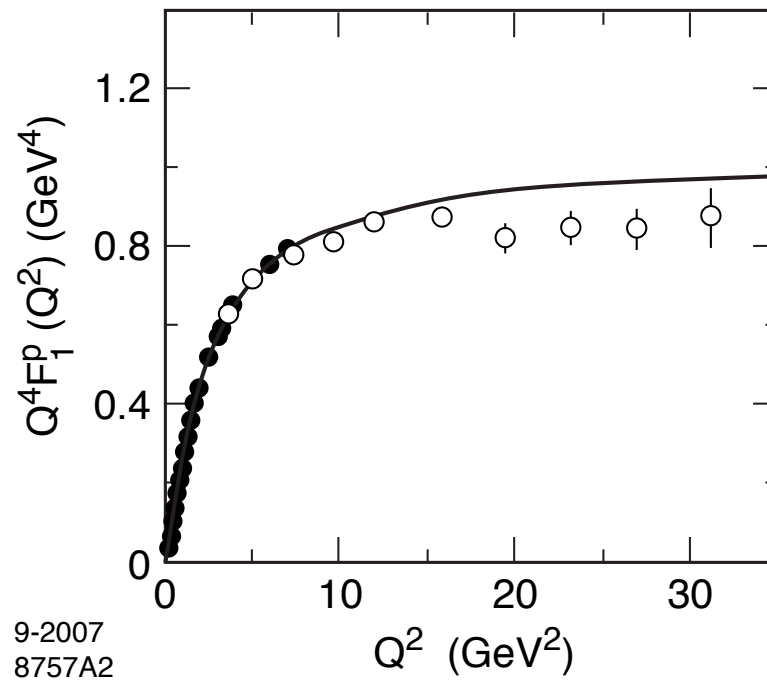
$$F(Q^2) = \frac{(N - 1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \dots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N$$

- For large Q^2

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2} \right]^{(N-1)}$$

Space-Like Dirac Proton Form Factor

- Scaling behavior for large Q^2 : $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$ Proton $\tau = 3$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005)

6 Non-Perturbative QCD Coupling From Light-Front Holography

With A. Deur and S. J. Brodsky

- Consider five-dim gauge fields propagating in AdS₅ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

- Identify

$$g_5^{-2}(z) = e^{\varphi(z)} g_5^{-2}$$

as the effective coupling of the theory at scale z

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$
- Coupling measured at momentum scale Q

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s^{AdS}(\zeta)$$

- Solution

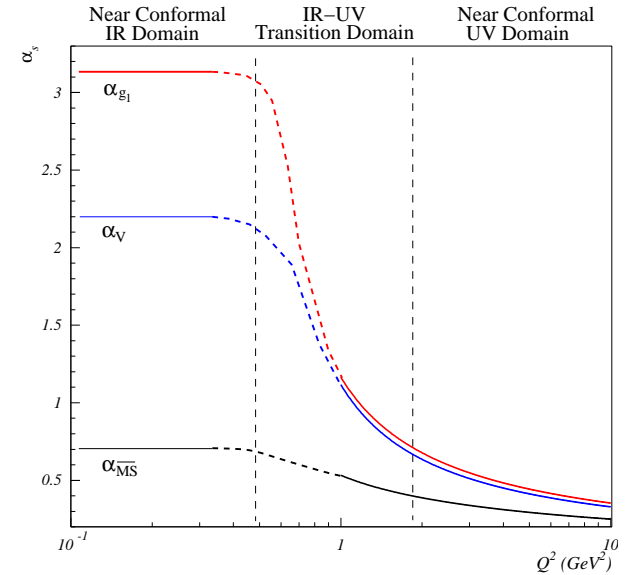
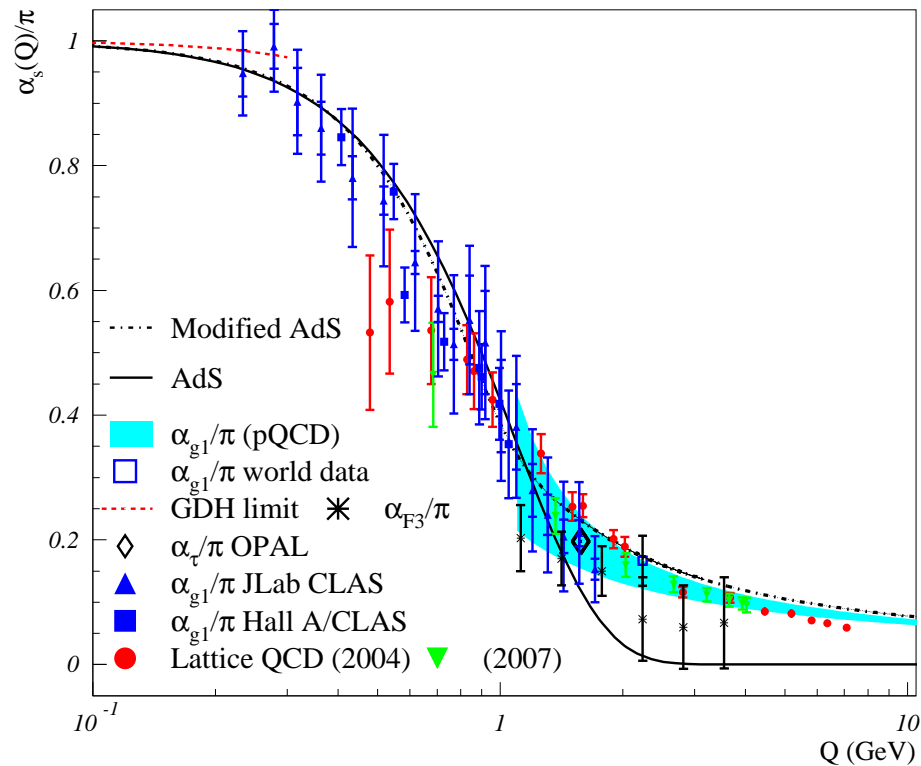
$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) e^{-Q^2/4\kappa^2}$$

- Effective charge α_s^{AdS} incorporates the non-conformal dynamics of confinement

Effective Couplings

- Nonperturbative coupling normalized to the g_1 scheme (Bjorken sum rule)

$$\alpha_s^{AdS}(Q)/\pi = e^{-Q^2/4\kappa^2}$$

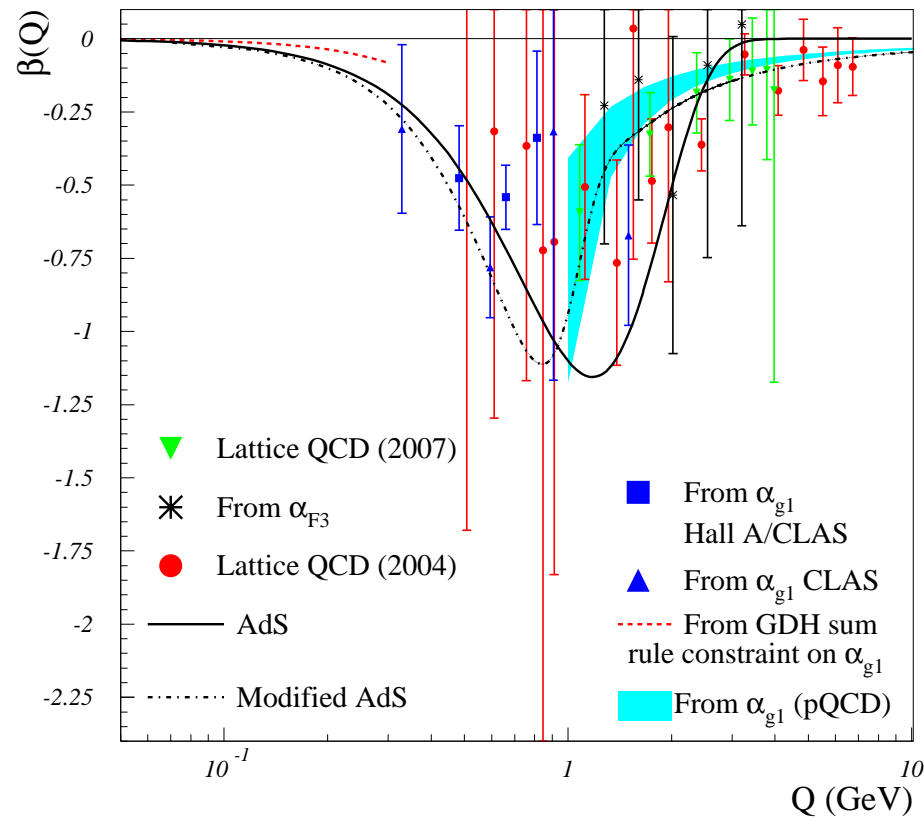


Effective coupling from LF holography for $\kappa = 0.54$ GeV

Holographic β -Function

- β -function

$$\beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2}$$



β -function from LF holography for $\kappa = 0.54$ GeV

Full β -function of QCD ?

$$\beta(Q \rightarrow 0) = \beta(Q \rightarrow \infty) = 0, \quad (1)$$

$$\beta(Q) < 0, \text{ for } Q > 0, \quad (2)$$

$$\left. \frac{d\beta}{dQ} \right|_{Q=Q_0} = 0, \quad (3)$$

$$\frac{d\beta}{dQ} < 0, \text{ for } Q < Q_0, \quad \frac{d\beta}{dQ} > 0, \text{ for } Q > Q_0. \quad (4)$$

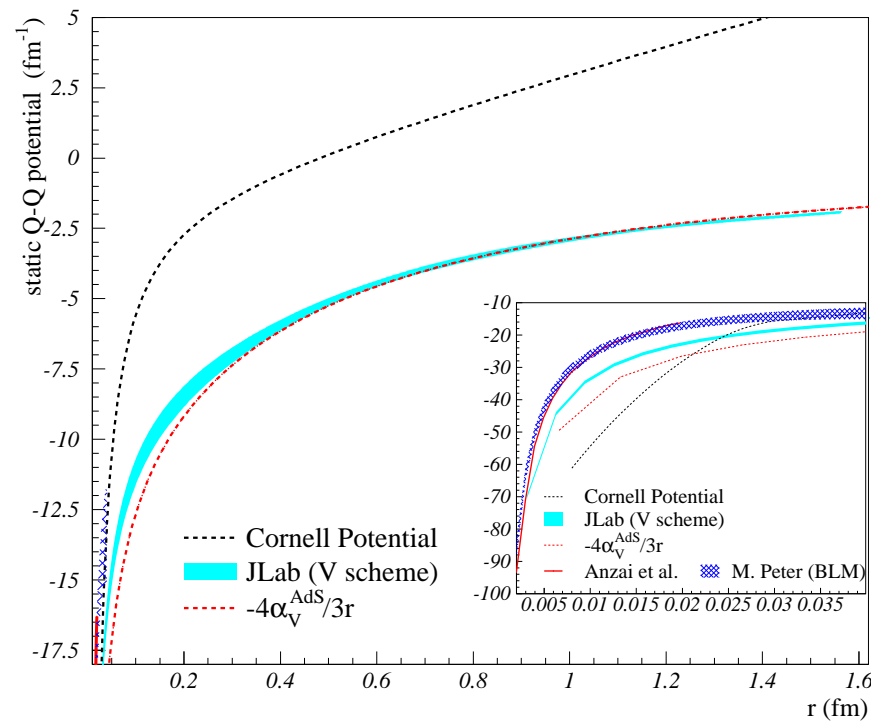
1. QCD is conformal in the far UV and deep IR
2. Anti-screening behavior of QCD which leads to asymptotic freedom
3. Hadronic-partonic transition: the minimum is an absolute minimum
4. Since there is only one transition (4) follows from the above

Static $\bar{Q}Q$ Potential

- For heavy quarks LF holographic equations reduce to NR Schrödinger equation in configuration space

$$V(r) = -\frac{4}{3} \frac{\alpha_V(r)}{r} + V_{conf}(r)$$

where $V_{conf} \simeq \frac{1}{2} m_{red} \omega^2 r^2$, $m_{red} = m_Q m_{\bar{Q}} / (m_Q + m_{\bar{Q}})$ and $\omega = \kappa^2 / (m_Q + m_{\bar{Q}})$



Comparison of Coulomb $\bar{Q}Q$ potential with Cornell potential