• Light-Front Holography

\[ \psi(x, k_{\perp}) = \psi(\zeta) \]

• Light Front Wavefunctions:
Schrödinger Wavefunctions of Hadron Physics

\[ \Psi_n(x_i, k_{\perp i}, \lambda_i) \]
Light-Front QCD

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_{i} \left[ \frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

$$H_{LF}^{int}$$: Matrix in Fock Space

$$H_{LF}^{QCD} |\Psi_h >= \mathcal{M}^2_h |\Psi_h >$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Physical gauge: $$A^+ = 0$$

Stan Brodsky, SLAC
Applications of AdS/CFT to QCD

in collaboration with Guy de Teramond

Changes in physical length scale mapped to evolution in the 5th dimension z

Novel QCD Phenomena

UTSM May 19, 2011

Stan Brodsky, SLAC
Scale Transformations

• Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu \nu} dx^\mu dx^\nu - dz^2), \]

\[ x^\mu \rightarrow \lambda x^\mu, \ z \rightarrow \lambda z, \] maps scale transformations into the holographic coordinate $z$.

• AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

• Different values of $z$ correspond to different scales at which the hadron is examined.

\[ x^2 \rightarrow \lambda^2 x^2, \ z \rightarrow \lambda z. \]

\[ x^2 = x_\mu x^\mu: \text{invariant separation between quarks} \]

• The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
Soft-Wall Model

\[ S = \int d^4 x \, dz \, \sqrt{g} \, e^{\phi(z)} \mathcal{L}, \]

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)

Timelike \( x^2 > 0 \to \phi(z) = -\kappa^2 z^2 \)

Spacelike \( x^2 < 0 \to \phi(z) = +\kappa^2 z^2 \)

- Equation of motion for scalar field \( \mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_{\ell} \Phi \partial_{m} \Phi - \mu^2 \Phi^2) \)

\[
\left[ z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0
\]

with \( (\mu R)^2 \geq -4 \).

- Bound state Equations: \( \varphi = +\kappa^2 z^2 \). Lowest possible state \( (\mu R)^2 = -4 \)

\[
\mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2}
\]

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion!
\[ ds^2 = e^{\kappa z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2) \]

\[ ds^2 = e^{A(y)} (-dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2) + dy^2 \]
• Nonconformal metric dual to a confining gauge theory

\[ ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu \nu} dx^\mu dx^\nu - dz^2) \]

where \( \varphi(z) \to 0 \) at small \( z \) for geometries which are asymptotically AdS$_5$

• Gravitational potential energy for object of mass \( m \)

\[ V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z} \]

• Consider warp factor \( \exp(\pm \kappa^2 z^2) \)

• Plus solution: \( V(z) \) increases exponentially confining any object in modified AdS metrics to distances \( \langle z \rangle \sim 1/\kappa \)
\[
e^{\Phi(z)} = e^{+\kappa^2 z^2}
\]

**Positive-sign dilaton**

**AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:**

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

where

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

**Derived from variation of Action**

**Dilaton-Modified AdS\(_5\)**
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

(a) $S = 0$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$Pion mass automatically zero!$

$m_q = 0$
\( H_{\text{QED}} \)

\[
\begin{align*}
(H_0 + H_{\text{int}}) |\Psi\rangle &= E |\Psi\rangle \\
\left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) &= E \psi(\vec{r}) \\
\left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell + 1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) &= E \psi(r) \\
V_{\text{eff}} &\rightarrow V_C(r) = -\frac{\alpha}{r}
\end{align*}
\]

**QED atoms:** positronium and muonium

**Effective two-particle equation**

**Includes Lamb Shift, quantum corrections**

**Spherical Basis** \( r, \theta, \phi \)

**Coulomb potential**

Bohr Spectrum

**Semiclassical first approximation to QED**

**Coupled Fock states**
$H_{QCD}^{LF}$

$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle \geq M^2 |\Psi\rangle$

$\left[ \frac{k_\perp^2 + m^2}{x(1-x)} + V_{eff}^{LF} \right] \psi_{LF}(x, k_\perp) = M^2 \psi_{LF}(x, k_\perp)$

$[- \frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L)] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$

$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$

QCD Meson Spectrum

Coupled Fock states

Effective two-particle equation

$\zeta^2 = x(1-x)b_\perp^2$

Azimuthal Basis $\zeta, \phi$

Confining AdS/QCD potential

Semiclassical first approximation to QCD
• Functional relation: \[ \frac{|\phi|^2}{\zeta} = \frac{2\pi}{x(1-x)} |\psi(x, b_\perp)|^2 \]

• Invariant mass \( M^2 \) in terms of LF mode \( \phi \)

\[
M^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)
\]

\[
= \int d\zeta \, \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} \right) \phi(\zeta) + \int d\zeta \, \phi^*(\zeta) U(\zeta) \phi(\zeta)
\]

where the interaction terms are summed up in the effective potential \( U(\zeta) \) and the orbital angular momentum in \( \nabla^2 \) has the \( SO(2) \) Casimir representation \( SO(N) \sim S^{N-1} : L(L+N-2) \)

\[
- \frac{\partial^2}{\partial \phi^2} |\phi\rangle = L^2 |\phi\rangle
\]

• LF eigenvalue equation \( H_{LF} |\phi\rangle = M^2 |\phi\rangle \) is a LF wave equation for \( \phi \)

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = M^2 \phi(\zeta)
\]

• Effective light-front Schrödinger equation: relativistic, covariant and analytically tractable.
Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

\[ \psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta) \]
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta) \]

\[ \zeta^2 = x(1 - x)b_\perp^2. \]

\[ U(\zeta) = \kappa^4 \zeta^2 \]

Frame Independent

soft wall

confining potential:

G. de Teramond, sjb

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Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.
General-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \ldots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[ z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2 \right) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2 + J} e^{\kappa^2 \zeta^2 / 2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left( - \frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \ldots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \ldots \mu_J}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$
Bosonic Modes and Meson Spectrum

\[ M^2 = 4\kappa^2(n + J/2 + L/2) \rightarrow 4\kappa^2(n + L + S/2) \]

Same slope in \( n \) and \( L \)

Regge trajectories for the \( \pi \) (\( \kappa = 0.6 \) GeV) and the \( I = 1 \) \( \rho \)-meson and \( I = 0 \) \( \omega \)-meson families (\( \kappa = 0.54 \) GeV)
\[ \mathcal{M}^2 = 2 \kappa^2 (2n + 2L + S). \]

\[ S = 1 \]
AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories

\[ \alpha(t) \approx \frac{1}{2} + 0.9t \]
Hadrons in AdS/QCD correspondence

Alfredo Vega and Ivan Schmidt
Departamento de Física y Centro de Estudios Subatómicos, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile
(Received 27 November 2008; published 4 March 2009)

We present a holographical soft wall model that is able to reproduce not only Regge spectra for hadrons with arbitrary integer spin, but also with spin 1/2 and 3/2, and with an arbitrary number of constituents. The model includes the anomalous dimension of operators that create hadrons, together with a dilaton, whose form is suggested by Einstein equations and the AdS metric used.
Baryons in Ads/CFT

- Action for massive fermionic modes on AdS\(_5\):
  \[
  S[\Psi, \bar{\Psi}] = \int d^4x \, dz \sqrt{g} \bar{\Psi}(x, z) \left( i \Gamma^\ell D_\ell - \mu \right) \Psi(x, z)
  \]

- Equation of motion:
  \[
  \left( i \Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0
  \]

\[
  \left[ i \left( z \eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0
  \]

- Solution (\(\mu R = \nu + 1/2\))
  \[
  \Psi(z) = C z^{5/2} \left[ J_\nu(z M) u_+ + J_{\nu+1}(z M) u_- \right]
  \]

- Hadronic mass spectrum determined from IR boundary conditions \(\psi_\pm (z = 1/\Lambda_{QCD}) = 0\)
  \[
  M^+ = \beta_{\nu,k} \Lambda_{QCD}, \quad M^- = \beta_{\nu+1,k} \Lambda_{QCD}
  \]

  with scale independent mass ratio

- Obtain spin-\(J\) mode \(\Phi_{\mu_1 \cdots \mu_{J-1/2}}\), \(J > \frac{1}{2}\), with all indices along 3+1 from \(\Psi\) by shifting dimensions
Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0,\]

in terms of the matrix-valued operator \(\Pi\)

\[\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),\]

and its adjoint \(\Pi^\dagger\), with commutation relations

\[\left[ \Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.\]

- Solutions to the Dirac equation

\[\psi_+(\zeta) \sim z^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),\]
\[\psi_-(\zeta) \sim z^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1}(\kappa^2 \zeta^2).\]

- Eigenvalues

\[\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).\]
\( M^2 \)

\[
\left( \frac{\omega_B}{\omega_M} \right)^2 = \frac{5}{8}
\]

\[ 4\kappa^2 \text{ for } \Delta n = 1 \]
\[ 4\kappa^2 \text{ for } \Delta L = 1 \]
\[ 2\kappa^2 \text{ for } \Delta S = 1 \]


\( L \)

Parent and daughter 56 Regge trajectories for the \( N \) and \( \Delta \) baryon families for \( \kappa = 0.5 \text{ GeV} \)
E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD


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Other Applications of Light-Front Holography

- Light baryon spectrum
- Light meson spectrum
- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form factors of composite hadrons
- $n$-parton holographic mapping
- Heavy flavor mesons

hep-th/0501022
hep-ph/0602252
arXiv:0707.3859
arXiv:0802.0514
arXiv:0804.0452
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

However, \( J/\psi \to \rho \pi \) is largely a two-body hadron decays.

Small value for \( \psi \to \rho \pi \)

\[ \rho \pi \]

Data Compilation
Baldini, Kloe and Volmer

de Taramond, sjb
See also: Radyushkin

Soft Wall: Harmonic Oscillator Confinement

Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

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Photon-to-pion transition form factor

Lepage, sjb

\[ Q^2 F_{\pi\gamma}(Q^2 \to \infty) = 2 f_\pi. \]

\[ Q^2 \left( Q^2 \pi\gamma \right) (\text{GeV}) \]

\[ Q^2 (\text{GeV}^2) \]

F.-G. Cao, G. de Teramond, sjb

\( (\text{Chern-Simons}) \)

\[ \text{BaBar} \]

\[ \text{CLEO} \]

\[ \text{CELLO} \]

Free current; Twist 2

Dressed current; Twist 2

Dressed current; Twist 2+4

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Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

\[ F_+(Q^2) = g_+ \int d\zeta \ J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_-(Q^2) = g_- \int d\zeta \ J(Q, \zeta) |\psi_-(\zeta)|^2, \]

where the effective charges \( g_+ \) and \( g_- \) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \( S^z = +1/2 \). The two AdS solutions \( \psi_+(\zeta) \) and \( \psi_-(\zeta) \) correspond to nucleons with \( J^z = +1/2 \) and \( -1/2 \).

- For \( SU(6) \) spin-flavor symmetry

\[ F_1^p(Q^2) = \int d\zeta \ J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \ J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \]

where \( F_1^p(0) = 1, \ F_1^n(0) = 0. \)
Scaling behavior for large $Q^2$: $Q^4 F_1^P(Q^2) \rightarrow \text{constant}$

Proton $\tau = 3$

- Scaling behavior for large $Q^2$: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$

Neutron $\tau = 3$

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

\[ F_2^p (Q^2) \]

Harmonic Oscillator
Confinement
Normalized to anomalous moment

\[ k = 0.49 \text{ GeV} \]

G. de Teramond, sjb

Preliminary
Form Factors in AdS/QCD

\[ F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2, \]

\[ F(Q^2) = \frac{1}{(1 + \frac{Q^2}{\mathcal{M}_\rho^2})(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2})}, \quad N = 3, \]

\[ \cdots \]

\[ F(Q^2) = \frac{1}{(1 + \frac{Q^2}{\mathcal{M}_\rho^2})(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}) \cdots (1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2})}, \quad N, \]

Positive Dilaton Background \exp(+\kappa^2 y^2)

\[ \mathcal{M}_n^2 = 4\kappa^2 \left(n + \frac{1}{2}\right) \]

\[ F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \quad Q^2 \rightarrow \infty \]

Constituent Counting
Nucleon Transition Form Factor

\[ F_{1N\rightarrow N^*}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{M_P^2}\right) \left(1 + \frac{Q^2}{M_{\rho'}^2}\right) \left(1 + \frac{Q^2}{M_{\rho''}^2}\right)} \]
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k^2_\perp) \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

Increases PQCD prediction for \( F_\pi(Q^2) \) by 16/9

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Second Moment of Pion Distribution Amplitude

\[
\langle \xi^2 \rangle = \int_{-1}^{1} d\xi \, \xi^2 \phi(\xi)
\]

\[
\xi = 1 - 2x
\]

\[
\langle \xi^2 \rangle_\pi = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1 - x)
\]

\[
\langle \xi^2 \rangle_\pi = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1 - x)}
\]

Lattice (I) \( \langle \xi^2 \rangle_\pi = 0.28 \pm 0.03 \)

Lattice (II) \( \langle \xi^2 \rangle_\pi = 0.269 \pm 0.039 \)
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite $N_c = 3$: Baryons built on 3 quarks -- Large $N_c$ limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
Generalized parton distributions in AdS/QCD

Alfredo Vega\textsuperscript{1}, Ivan Schmidt\textsuperscript{1}, Thomas Gutsche\textsuperscript{2}, Valery E. Lyubovitskij\textsuperscript{2}\textsuperscript{*}

\textsuperscript{1}Departamento de Física y Centro Científico y Tecnológico de Valparaíso, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaiso, Chile

\textsuperscript{2}Institut für Theoretische Physik, Universität Tübingen, Kepler Center for Astro and Particle Physics, Auf der Morgenstelle 14, D–72076 Tübingen, Germany

(Dated: January 19, 2011)

\[ u(x = 0.1, \vec{b}_\perp) \]

\[ d(x = 0.1, \vec{b}_\perp) \]
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

\[
\alpha_s^q(Q)/\pi = e^{-Q^2/4\kappa^2}
\]

\(\kappa = 0.54 \text{ GeV}\)

Deur, de Teramond, sb
\[ \beta^{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4 \kappa^2} e^{-Q^2/4\kappa^2} \]

**Figure:**

- **Lattice QCD (2007)**
- **From \( \alpha_{F3} \)**
- **Lattice QCD (2004)**
- **AdS**
- **Modified AdS**
- **From \( \alpha_{g1} \)** Hall A/CLAS
- **From \( \alpha_{g1} \)** CLAS
- **From GDH sum rule constraint on \( \alpha_{g1} \)**
- **From \( \alpha_{g1} \)** (pQCD)
\[
\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}\]

\[-\frac{d}{d\zeta^2} \equiv \frac{k_\perp^2}{x(1-x)}\]

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

\[-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}\]
Result: Soft-Wall LFWF for massive constituents

\[ \psi(x, k_\perp) = \frac{4\pi c}{\kappa \sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2} \left( \frac{k_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right)} \]

LFWF in impact space: soft-wall model with massive quarks

\[ \psi(x, b_\perp) = \frac{c \kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-\frac{1}{2} \kappa^2 x(1-x)b_\perp^2 - \frac{1}{2\kappa^2} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]} \]

\[ z \rightarrow \zeta \rightarrow \chi \]

\[ \chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right] \]
\( J/\psi \)

LFWF peaks at

\[ x_i = \frac{m_{\perp i}}{\sum_j n_j m_{\perp j}} \]

where

\[ m_{\perp i} = \sqrt{m^2 + k_i^2} \]

minimum of LF energy denominator

\[ \kappa = 0.375 \text{ GeV} \]

\( \psi J/\psi(x, b) \)

\[ m_a = m_b = 1.25 \text{ GeV} \]

\[ b \text{[GeV}^{-1}] \]
We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.
Our starting point is the action in AdS\textsubscript{d+1} for a spin-\(J\) field \(\Phi_J = \Phi_{M_1 \ldots M_J}(x, z)\) – a symmetric, traceless tensor used in the LFH approach [1], where we perform two modifications: 1) use a positive dilaton profile \(\phi(z) = k^2 z^2\); 2) include a nontrivial \(z\)-dependence of the mass term coefficient \(\mu_J^2 \rightarrow \mu_J^2(z)\) due to the interaction of the dilaton field with the matter field:

\[
S_\Phi = \frac{(-1)^J}{2} \int d^d x dz \sqrt{g} e^{-\phi(z)} \left( \partial_N \Phi_J \partial^N \Phi^J - \mu_J^2(z) \Phi_J \Phi^J \right),
\]

(2.1)

\[
M_{nJ}^2 = \int_0^\infty d\zeta \Phi_{nJ}(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J - 1) \right) \Phi_{nJ}(\zeta)
\]

\[
+ \int_0^1 dx \left( \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right) f^2(x, m_1, m_2).
\]

\[
M_{nJ}^2 = 4\kappa^2 \left( n + \frac{L+J}{2} \right) + \int_0^1 dx \left( \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right) f^2(x, m_1, m_2) - \frac{64\alpha_s^2 m_1 m_2}{9(n+L+1)^2} + \frac{32\pi \alpha_s}{9} \beta_S v \mu_{12}.
\]
Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega

### Table 1: Masses of light mesons

<table>
<thead>
<tr>
<th>Meson</th>
<th>$n$</th>
<th>$L$</th>
<th>$S$</th>
<th>Mass [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>140</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>0</td>
<td>1355</td>
</tr>
<tr>
<td>$K$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>495</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>0</td>
<td>544</td>
</tr>
<tr>
<td>$f_0[\bar{\eta}n]$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>1114</td>
</tr>
<tr>
<td>$f_0[\bar{s}s]$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>1304</td>
</tr>
<tr>
<td>$a_0(980)$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>1114</td>
</tr>
<tr>
<td>$\rho(770)$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1</td>
<td>804</td>
</tr>
<tr>
<td>$\rho(770)$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>1</td>
<td>804</td>
</tr>
<tr>
<td>$\omega(782)$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1</td>
<td>804</td>
</tr>
<tr>
<td>$\omega(782)$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>1</td>
<td>804</td>
</tr>
<tr>
<td>$\phi(1020)$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1</td>
<td>1019</td>
</tr>
<tr>
<td>$a_1(1260)$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>1358</td>
</tr>
</tbody>
</table>
Table 4: Decay constants $f_P$ in MeV of pseudoscalar mesons

<table>
<thead>
<tr>
<th>Meson</th>
<th>Data [13]</th>
<th>Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$</td>
<td>$130.4 \pm 0.03 \pm 0.2$</td>
<td>131</td>
</tr>
<tr>
<td>$K^-$</td>
<td>$156.1 \pm 0.2 \pm 0.8$</td>
<td>155</td>
</tr>
<tr>
<td>$D^+$</td>
<td>$206.7 \pm 8.9$</td>
<td>167</td>
</tr>
<tr>
<td>$D_s^+$</td>
<td>$257.5 \pm 6.1$</td>
<td>170</td>
</tr>
<tr>
<td>$B^-$</td>
<td>$193 \pm 11$</td>
<td>139</td>
</tr>
<tr>
<td>$B_s^0$</td>
<td>$253 \pm 8 \pm 7$</td>
<td>144</td>
</tr>
<tr>
<td>$B_c$</td>
<td>$489 \pm 5 \pm 3$ [14]</td>
<td>159</td>
</tr>
</tbody>
</table>

Table 5: Decay constants $f_V$ in MeV of vector mesons

<table>
<thead>
<tr>
<th>Meson</th>
<th>Data [13]</th>
<th>Our</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^+$</td>
<td>$210.5 \pm 0.6$</td>
<td>170</td>
</tr>
<tr>
<td>$D^*$</td>
<td>$245 \pm 20_{-2}^{+3}$ [15]</td>
<td>167</td>
</tr>
<tr>
<td>$D_s^*$</td>
<td>$272 \pm 16_{-20}^{+3}$ [16]</td>
<td>170</td>
</tr>
<tr>
<td>$B^*$</td>
<td>$196 \pm 24_{-2}^{+39}$ [15]</td>
<td>139</td>
</tr>
<tr>
<td>$B_s^*$</td>
<td>$229 \pm 20_{-16}^{+41}$ [15]</td>
<td>144</td>
</tr>
</tbody>
</table>

$\rho^0$, $\omega$, $\phi$, $J/\psi$.
Table 2: Masses of heavy–light mesons

<table>
<thead>
<tr>
<th>Meson</th>
<th>$J^P$</th>
<th>$n$</th>
<th>$L$</th>
<th>$S$</th>
<th>Mass [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D(1870)$</td>
<td>$0^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1857 2435 2696 2905</td>
</tr>
<tr>
<td>$D^*(2010)$</td>
<td>$1^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>1</td>
<td>2015 2547 2797 3000</td>
</tr>
<tr>
<td>$D_s(1969)$</td>
<td>$0^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1963 2621 2883 3085</td>
</tr>
<tr>
<td>$D_{s}^*(2107)$</td>
<td>$1^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>1</td>
<td>2113 2725 2977 3173</td>
</tr>
<tr>
<td>$B(5279)$</td>
<td>$0^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>5279 5791 5964 6089</td>
</tr>
<tr>
<td>$B^*(5325)$</td>
<td>$1^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>5336 5843 6015 6139</td>
</tr>
<tr>
<td>$B_s(5366)$</td>
<td>$0^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>0</td>
<td>5360 5941 6124 6250</td>
</tr>
<tr>
<td>$B_{s}^*(5413)$</td>
<td>$1^-$</td>
<td>0</td>
<td>0,1,2,3</td>
<td>1</td>
<td>5416 5992 6173 6298</td>
</tr>
</tbody>
</table>

Table 3: Masses of heavy quarkonia $c\bar{c}, b\bar{b}$ and $c\bar{b}$

<table>
<thead>
<tr>
<th>Meson</th>
<th>$J^P$</th>
<th>$n$</th>
<th>$L$</th>
<th>$S$</th>
<th>Mass [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_c(2980)$</td>
<td>$0^-$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>0</td>
<td>2997 3717 3962 4141</td>
</tr>
<tr>
<td>$\psi(3097)$</td>
<td>$1^-$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1</td>
<td>3097 3798 4038 4213</td>
</tr>
<tr>
<td>$\chi_{c0}(3415)$</td>
<td>$0^+$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>3635 3885 4067 4226</td>
</tr>
<tr>
<td>$\chi_{c1}(3510)$</td>
<td>$1^+$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>3718 3963 4141 4297</td>
</tr>
<tr>
<td>$\chi_{c2}(3555)$</td>
<td>$2^+$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>3798 4038 4213 4367</td>
</tr>
<tr>
<td>$\eta_b(9390)$</td>
<td>$0^-$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>0</td>
<td>9428 10190 10372 10473</td>
</tr>
<tr>
<td>$\Upsilon(9460)$</td>
<td>$1^-$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>1</td>
<td>9460 10219 10401 10502</td>
</tr>
<tr>
<td>$\chi_{b0}(9860)$</td>
<td>$0^+$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>10160 10343 10444 10521</td>
</tr>
<tr>
<td>$\chi_{b1}(9893)$</td>
<td>$1^+$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>10190 10372 10473 10550</td>
</tr>
<tr>
<td>$\chi_{b2}(9912)$</td>
<td>$2^+$</td>
<td>0,1,2,3</td>
<td>1</td>
<td>1</td>
<td>10219 10401 10502 10579</td>
</tr>
<tr>
<td>$B_c(6276)$</td>
<td>$0^-$</td>
<td>0,1,2,3</td>
<td>0</td>
<td>0</td>
<td>6276 6911 7092 7209</td>
</tr>
</tbody>
</table>

Valery E. Lyubovitskij, Tanja Branz, Thomas Gutsche, Ivan Schmidt, Alfredo Vega
\[ |\pi^+ > = |ud\bar{d} > \]
\[ m_u = 2 \text{ MeV} \quad m_d = 5 \text{ MeV} \]

\[ |D^+ > = |cd\bar{d} > \]
\[ m_c = 1.25 \text{ GeV} \]

\[ |B^+ > = |u\bar{b} > \]
\[ m_b = 4.2 \text{ GeV} \]

\[ |K^+ > = |u\bar{s} > \]
\[ m_s = 95 \text{ MeV} \]

\[ |\eta_c > = |c\bar{c} > \]

\[ |\eta_b > = |b\bar{b} > \]
\[ \kappa = 375 \text{ MeV} \]

Stan Brodsky, SLAC
Deur, Korsch, et al.

![Graph showing the evolution of the QCD gluon coupling with Q.]

- **α_s/π (JLab)**
- **α_s/π (GDH limit)**
- **α_s/π (Fit)**
- **α_s/π (pQCD evol. eq.)**

**Lattice QCD**
- **Cornwall**
- **Godfrey-Isgur**
- **Bloch et al.**
- **Bhagwat et al.**
- **Maris-Tandy**
- **Fischer et al.**

**DSE gluon couplings**

**Q (GeV)**

---

**Fig:** Infrared confinement or malwindow from Deur et al., arXiv:0803.4119

F from Strling to Things, INT, Seattle, April 10, 2008
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$.
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Chiral Features of Soft-Wall AdS/QCD Model

- Boost Invariant
- Trivial LF vacuum.
- Massless Pion
- Hadron Eigenstates have LF Fock components of different $L^z$
- Proton: equal probability
  \[ S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1 \]
  \[ J^z = +1/2 : < L^z >= 1/2, < S_q^z = 0 > \]
- Self-Dual Massive Eigenstates: Proton is its own chiral partner.
- Label State by minimum $L$ as in Atomic Physics
- Minimum $L$ dominates at short distances
- AdS/QCD Dictionary: Match to Interpolating Operator Twist at $z=0$. 
An analytic first approximation to QCD

**AdS/QCD + Light-Front Holography**

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable $\zeta$ conjugate to invariant mass squared
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales: Essential for Gauge Link phenomena
- Hadron Spectroscopy and Dynamics from one parameter $\kappa$
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates: Zero cosmological constant!
- Systematically improvable with DLCQ Methods
"One of the gravest puzzles of theoretical physics"

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

\[(\Omega_\Lambda)_{\text{QCD}} \sim 10^{45}\]
\[\Omega_\Lambda = 0.76(\text{expt})\]
\[(\Omega_\Lambda)_{\text{EW}} \sim 10^{56}\]

QCD Problem Solved if Quark and Gluon condensates reside
within hadrons, not vacuum!

R. Shrock, sjb
"Condensates in Quantum Chromodynamics and the Cosmological Constant."
New perspectives on the quark condensate

Stanley J. Brodsky,¹,² Craig D. Roberts,³,⁴ Robert Shrock,⁵ and Peter C. Tandy⁶

¹SLAC National Accelerator Laboratory, Stanford University, Stanford, California 94309, USA
²Centre for Particle Physics Phenomenology: CP³-Origins, University of Southern Denmark, Odense 5230 M, Denmark
³Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
⁴Department of Physics, Peking University, Beijing 100871, China
⁵C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA
⁶Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA

(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.
Gell-Mann Oakes Renner Formula in QCD

\[ m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 > \]

QCD: composite pion
Bethe-Salpeter Eq.

\[ m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi > \]

current algebra:

effective pion field

vacuum condensate actually is an “in-hadron condensate”

Maris, Roberts, Tandy
Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator

\[ M^2 = 4\kappa^2(n + L + S/2) \]

light-quark meson spectra

\[ \kappa \simeq 0.5 \text{ GeV} \]

\[ R_{e^+e^-}(s) = N_c \sum_q e_q^2 \left( 1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots \right) \]

mimics dimension-4 gluon condensate

\[ \langle 0 | \frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0) | 0 \rangle \]

\[ e^+e^- \rightarrow X, \tau \text{ decay, } Q\bar{Q} \text{ phenomenology} \]
Summary on QCD `Condensates'

- Condensates do not exist as space-time-independent phenomena

- Property of hadron wavefunctions: Bethe-Salpeter or Light-Front: “In-Hadron Condensates"

\[
\frac{<0|\bar{q}q|0>}{f_\pi} - <0|i\bar{q}\gamma_5 q|\pi> = \rho_\pi
\]

- Find:

\[
<0|i\bar{q}gamma_5 q|\pi> \text{ similar to } <0|i\bar{q}gamma^\mu gamma_5 q|\pi>
\]

- Zero contribution to cosmological constant!
  Included in hadron mass
Quark and Gluon condensates reside within hadrons, not vacuum

- Bound-State Dyson Schwinger Equations
- AdS/QCD
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict
QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary
Key Experiment for the ILC

W anomalous moments and the polarization asymmetry zero in $\gamma e \rightarrow W \nu$

Stanley J. Brodsky and Thomas G. Rizzo

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309

Ivan Schmidt

Universidad Federico Santa María, Casilla 100-V, Valparaiso, Chile

(Received 5 June 1995)

We show from general principles that there must be a center-of-mass energy $\sqrt{s_0}$ where the polarization asymmetry $A = \Delta \sigma(\gamma e \rightarrow W \nu)/\sigma(\gamma e \rightarrow W \nu)$ for circularly polarized photon and electron beams vanishes. In the case of the standard model, the crossing point where the asymmetry changes sign occurs in Born approximation at $\sqrt{s_{\gamma e}} = 3.1583 \ldots M_W \simeq 254$ GeV. We demonstrate the sensitivity of the position of the polarization asymmetry zero to modification of the SM trilinear $\gamma WW$ coupling. Given reasonable assumptions for the luminosity and energy range for the Next Linear Collider with a backscattered laser beam, we show that the zero point, $\sqrt{s_0}$, of the polarization asymmetry may be determined with sufficient precision to constrain the anomalous couplings of the $W$ to better than the 1% level at 95% C.L. In addition to the fact that only a limited range of energy is required, the polarization asymmetry measurements have the important advantage that many of the systematic errors cancel in taking cross section ratios. The position of the zero thus provides an additional weapon in the arsenal used to probe anomalous trilinear gauge couplings.
Novel QCD Phenomena

Celebrating Ivan Schmidt and Boris Kopeliovich #65

Valparaiso International Symposium on Particle Physics

Stan Brodsky

SLAC

Universidad Técnica Federico Santa María

Valparaiso, Chile May 19-20, 2011
Novel QCD Phenomena

Celebrating Ivan Schmidt and Boris Kopeliovich #65

and Their Remarkable Achievements in QCD

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