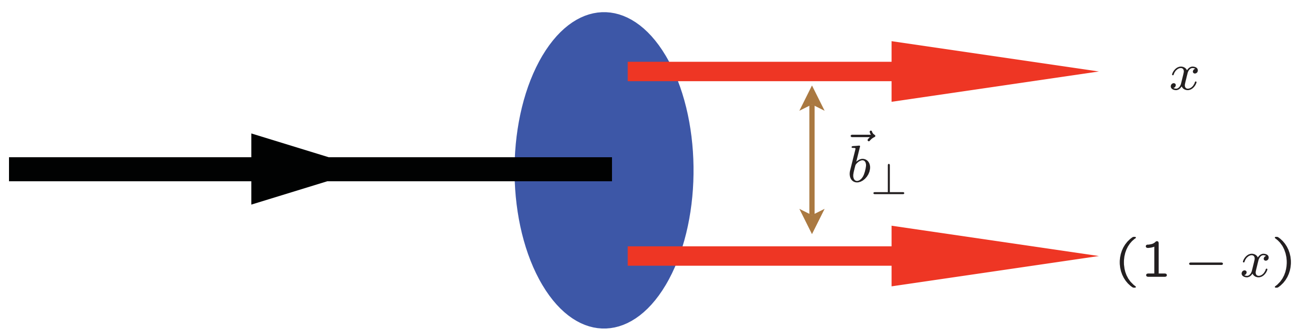
LF(3+1)   AdS$_5$

$$\psi(x, \vec{b}_\perp) \quad \longleftrightarrow \quad \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \longleftrightarrow \quad z$$



$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements

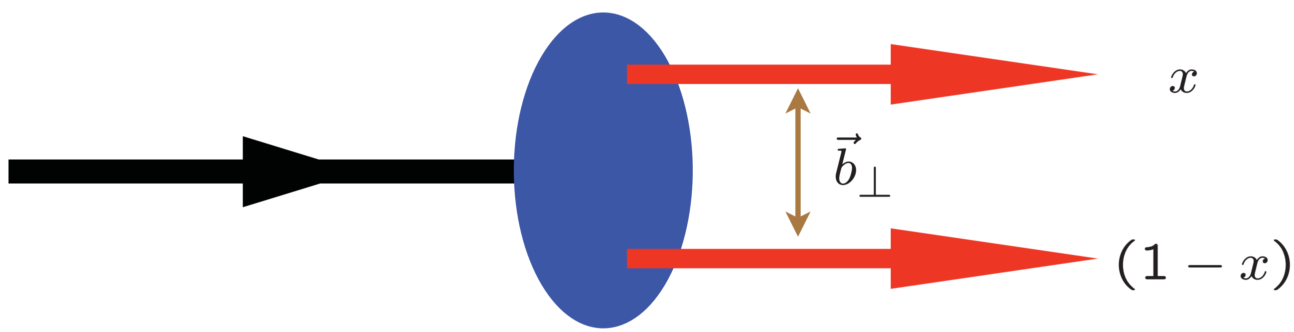
Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation — Frame Independent

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

G. de Teramond, sjb

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$



Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

Induced by conformal metric

Holographic Light-Front Representation (Hard Wall Model)

- We can represent the EOM in AdS space in light-front Lorentz invariant Hamiltonian form in physical 3+1 space-time at fixed LC time $\tau = t + z/c$

$$H_{LF}|\phi\rangle = \mathcal{M}^2|\phi\rangle.$$

- Write the AdS metric in terms of light front coordinates $x^\pm = x^0 \pm x^3$

$$ds^2 = \frac{R^2}{z^2}\left(dx^+dx^- - d\mathbf{x}_\perp^2 - dz^2\right).$$

- The AdS metric ds^2 is invariant if $\mathbf{x}_\perp^2 \to \lambda^2 \mathbf{x}_\perp^2$ and $z \to \lambda z$ at equal light-front time τ. Small z related to small transverse dimensions !

- We can identify the light-front variable ζ in 3+1 space with the fifth dimension z of AdS space, $\zeta = z$. The LF variable ζ represents the invariant transverse separation between pointlike constituents.

Example: Evaluation of QCD Matrix Elements

- Pion decay constant f_π defined by the matrix element of EW current J^+_W:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ \tfrac{1}{2}(1-\gamma_5)\psi_d \right| \pi^- \right\rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left(b^\dagger_{c\,d\downarrow} d^\dagger_{c\,u\uparrow} - b^\dagger_{c\,d\uparrow} d^\dagger_{c\,u\downarrow} \right) |0\rangle.$$

- Find light-front expression (Lepage and Brodsky '80):

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3}\, \psi_{\overline{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8}\sqrt{\frac{3}{2}}\, R^{3/2} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f_\pi^{HW} = \frac{\sqrt{3}}{8 J_1(\beta_{0,k})} \Lambda_{QCD} = 91.7\,\mathrm{MeV}, \quad f_\pi^{SW} = \frac{\sqrt{3}}{8}\kappa = 81.2\,\mathrm{MeV},$$

N-parton case

- Define effective single particle transverse density by (Soper, Phys. Rev. D **15**, 1141 (1977))

$$F(q^2) = \int_0^1 dx \int d^2\vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp)$$

- From DYW expression for the FF in transverse position space:

$$\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j \, d^2\vec{b}_{\perp j} \, \delta(1 - x - \sum_{j=1}^{n-1} x_j) \, \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{b}_{\perp j} - \vec{\eta}_\perp) |\psi_n(x_j, \vec{b}_{\perp j})|^2$$

- Compare with the the form factor in AdS space for arbitrary Q:

$$F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_{P'}(z) \, J(Q, z) \, \Phi_P(z)$$

- Holographic variable z is expressed in terms of the average transverse separation distance of the spectator constituents $\vec{\eta} = \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j}$

$$z = \sqrt{\frac{x}{1-x}} \, \Big| \sum_{j=1}^{n-1} x_j \, \vec{b}_{\perp j} \Big|$$

Current Propagation in the SW Model

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q,z) = 0.$$

- Solution: bulk-to-boundary propagator

$$J_\kappa(Q,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where $U(a,b,c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a,b,z) = \int_0^\infty e^{-zt} t^{a-1}(1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

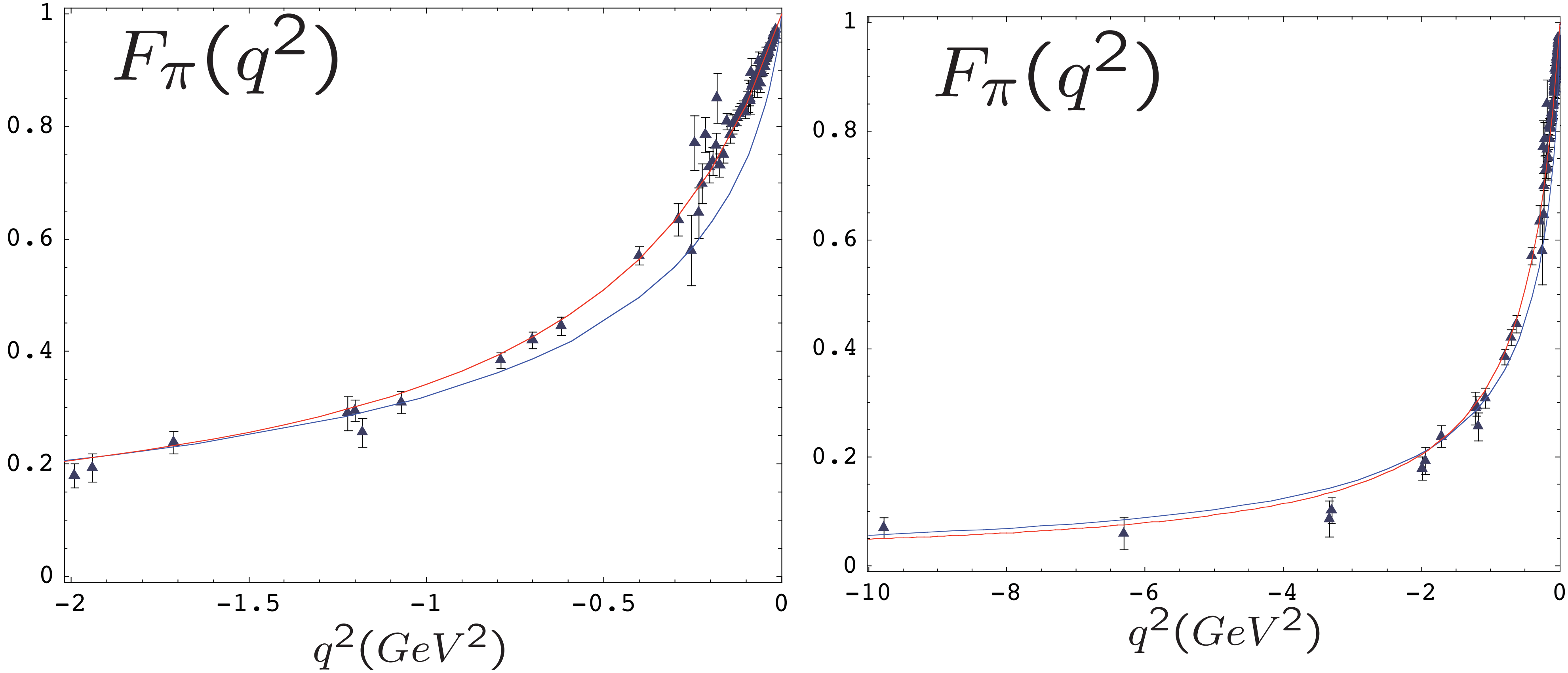
$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q,z)\Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q,z) \to zQK_1(zQ) = J(Q,z),$$

the external current decouples from the dilaton field.

Spacelike pion form factor from AdS/CFT



Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

Truncated Space Confinement

One parameter - set by pion decay constant

de Teramond, sjb
See also: Radyushkin

Space and Time-Like Pion Form Factor

- Hadronic string modes $\Phi_\pi(z) \to z^2$ as $z \to 0$ (twist $\tau = 2$)

$$\Phi_\pi^{HW}(z) = \frac{\sqrt{2}\Lambda_{QCD}}{R^{3/2} J_1(\beta_{0,1})} z^2 J_0\left(z\beta_{0,1}\Lambda_{QCD}\right),$$

$$\Phi_\pi^{SW}(z) = \frac{\sqrt{2}\kappa}{R^{3/2}} z^2.$$

- F_π has analytical solution in the SW model $F_\pi(Q^2) = \frac{4\kappa^2}{4\kappa^2+Q^2}$.

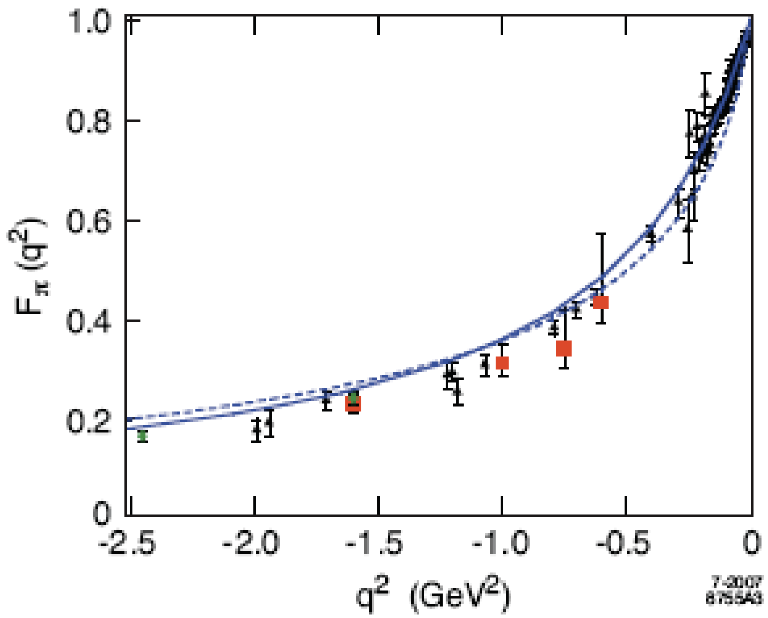


Fig: $F_\pi(q^2)$ for $\kappa = 0.375$ GeV and $\Lambda_{QCD} = 0.22$ GeV. Continuous line: SW, dashed line: HW.

- Scaling behavior for large Q^2: $Q^2 F_\pi(Q^2) \rightarrow \text{constant}$ $\boxed{\text{Pion } \tau = 2}$

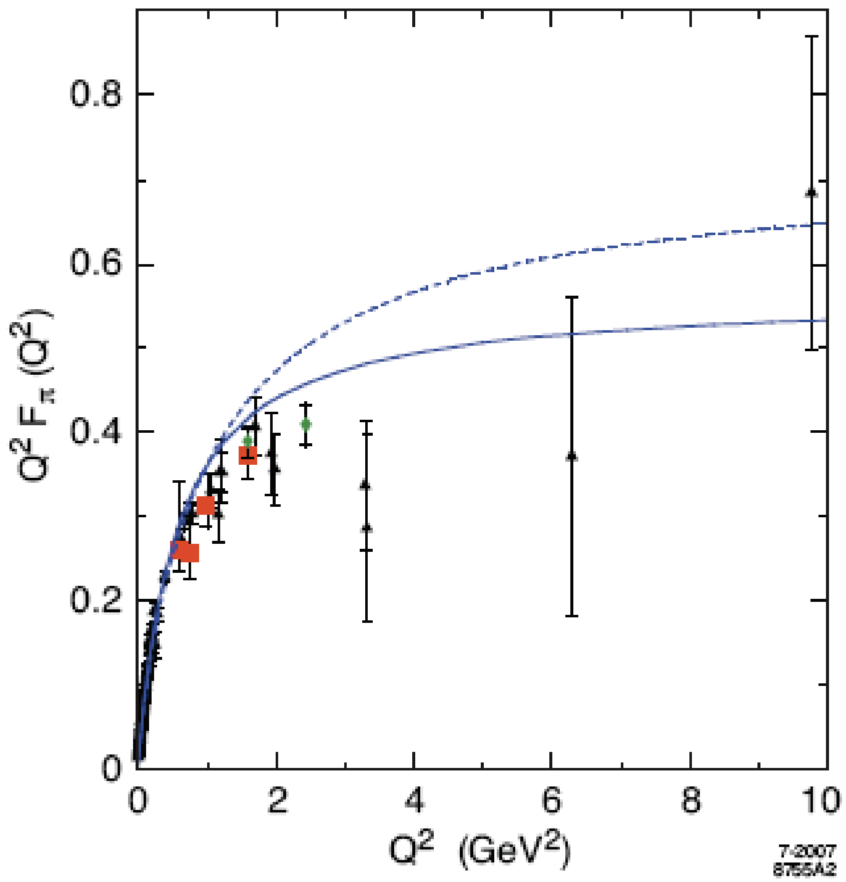
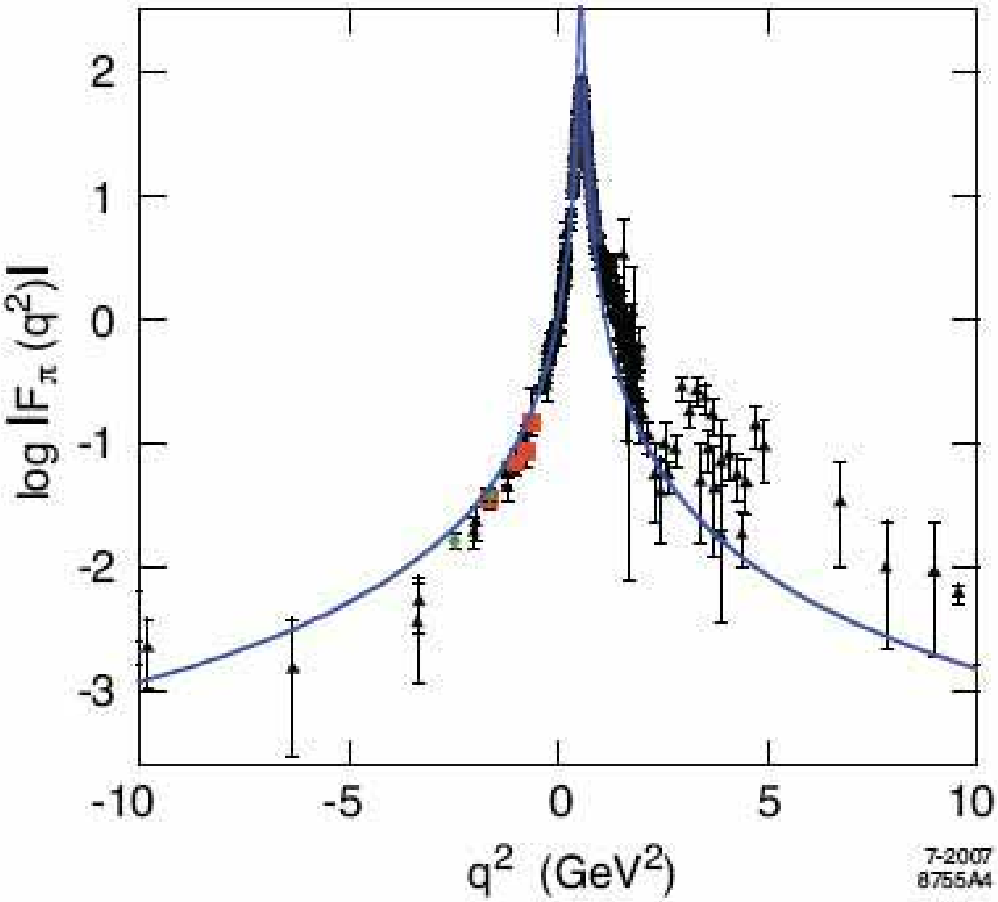


Fig: Continuous line: SW model for $\kappa = 0.375$ GeV. Dashed line: HW model for $\Lambda_{QCD} = 0.22$ GeV.

- Analytical continuation to time-like region $q^2 \to -q^2 \quad (M_\rho = 4\kappa^2 = 750$ MeV)
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

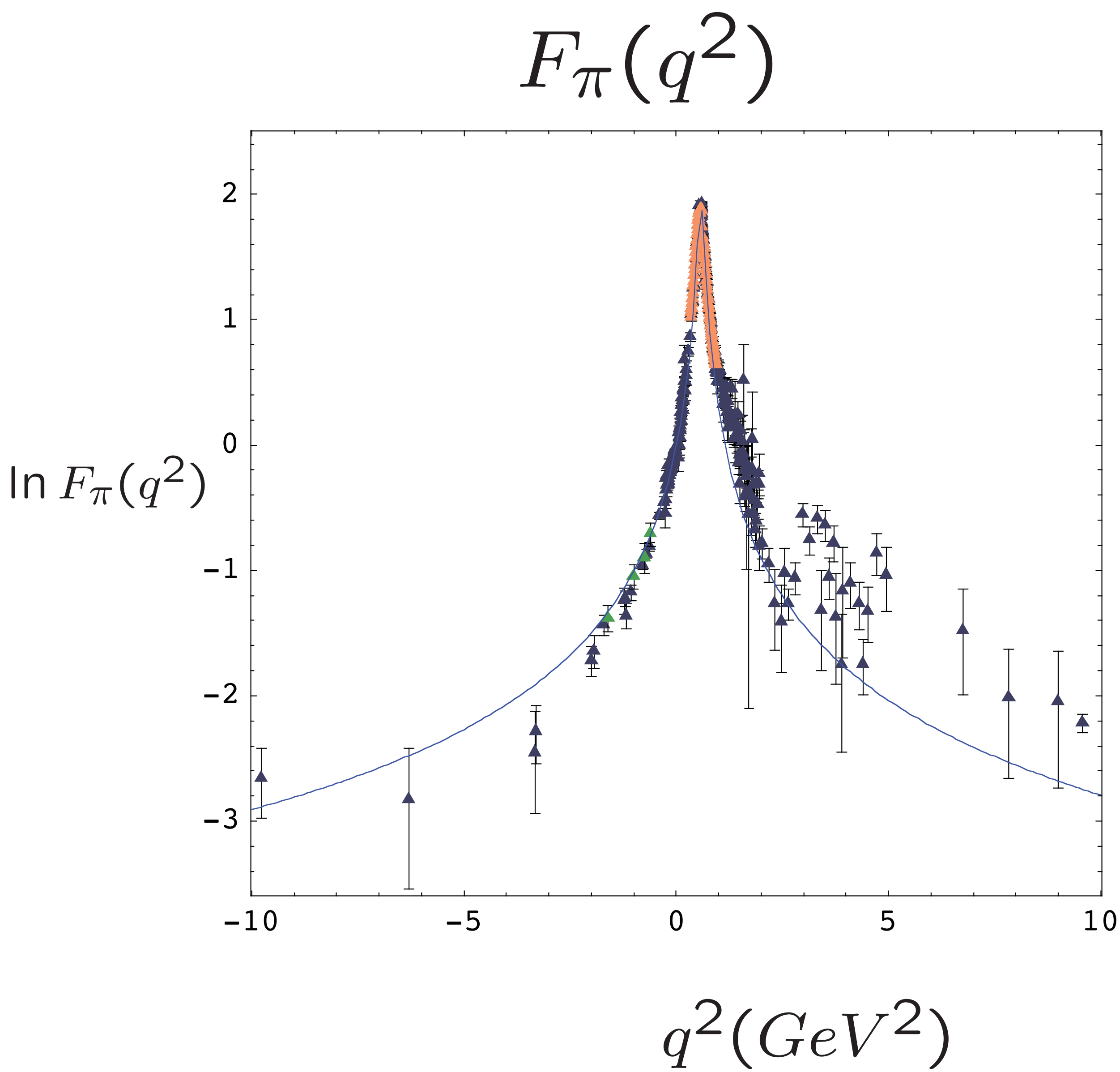


Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).

Spacelike and Timelike Pion form factor from AdS/CFT

G. de Teramond, sjb



Harmonic Oscillator Confinement

$\kappa = 0.38 \text{ GeV}$

Analytic continue to timelike momenta and introduce width

$$q^2 \to q^2 + i\epsilon \to q^2 + iM\Gamma$$

Fit to height, predict width

$\Gamma_\rho = 111 \text{ MeV}$

$\Gamma_\rho^{exp} = 150.3 \pm 1.6 \text{ MeV}$

Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension τ, Φ_τ in the SW model

$$F(Q^2) = \Gamma(\tau)\frac{\Gamma\left(1+\frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau+\frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\quad \Gamma(N+z) = (N-1+z)(N-2+z)\ldots(1+z)\Gamma(1+z)$.

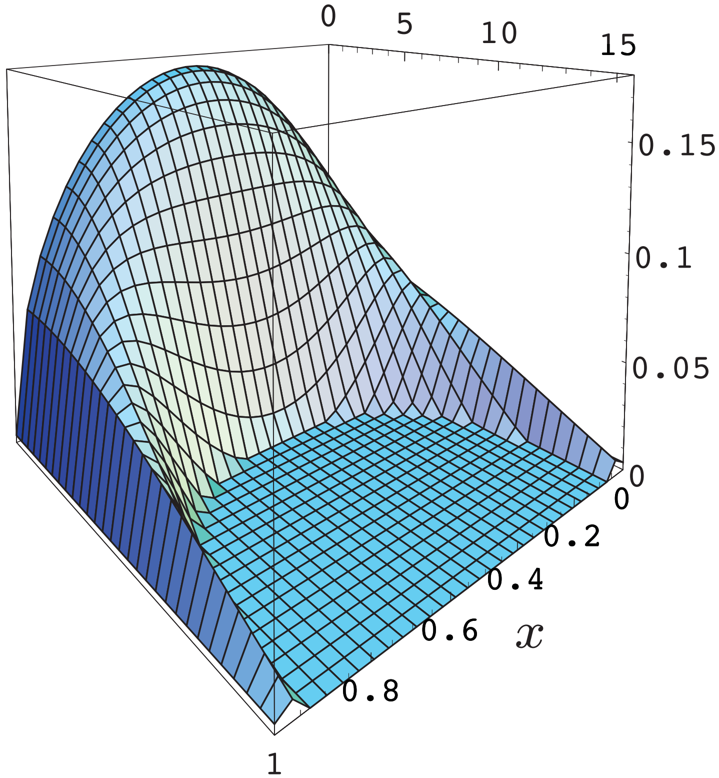
- Form factor expressed as $N-1$ product of poles

$$\begin{aligned}
F(Q^2) &= \frac{1}{1+\frac{Q^2}{4\kappa^2}}, \quad N=2, \\
F(Q^2) &= \frac{2}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)}, \quad N=3, \\
&\cdots \\
F(Q^2) &= \frac{(N-1)!}{\left(1+\frac{Q^2}{4\kappa^2}\right)\left(2+\frac{Q^2}{4\kappa^2}\right)\cdots\left(N-1+\frac{Q^2}{4\kappa^2}\right)}, \quad N.
\end{aligned}$$

- For large Q^2:

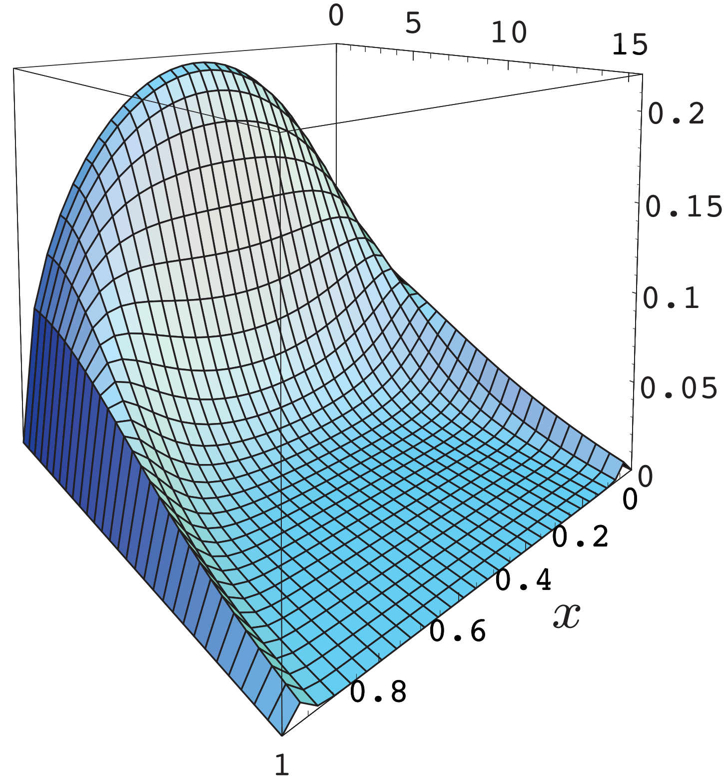
$$F(Q^2) \to (N-1)!\left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$

AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$



$\Lambda_{QCD} = 0.32$ GeV

Truncated Space



$\kappa = 0.76$ GeV

Harmonic Oscillator

Example: Pion LFWF

- Two parton LFWF bound state:

$$\widetilde{\psi}^{HW}_{\overline{q}q/\pi}(x,\mathrm{b}_\perp) = \frac{\Lambda_{\rm QCD}\sqrt{x(1-x)}}{\sqrt{\pi}J_{1+L}(\beta_{L,k})} J_L\left(\sqrt{x(1-x)}\,|\mathrm{b}_\perp|\beta_{L,k}\Lambda_{\rm QCD}\right)\theta\left(\mathrm{b}_\perp^2 \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right),$$

$$\widetilde{\psi}^{SW}_{\overline{q}q/\pi}(x,\mathrm{b}_\perp) = \frac{\kappa^{L+1}}{\sqrt{\pi}}\sqrt{\frac{n!}{(n+L)!}}\,[x(1-x)]^{\frac{1}{2}+L}|\mathrm{b}_\perp|^L e^{-\frac{1}{2}\kappa^2 x(1-x)\mathrm{b}_\perp^2} L_n^L\left(\kappa^2 x(1-x)\mathrm{b}_\perp^2\right).$$

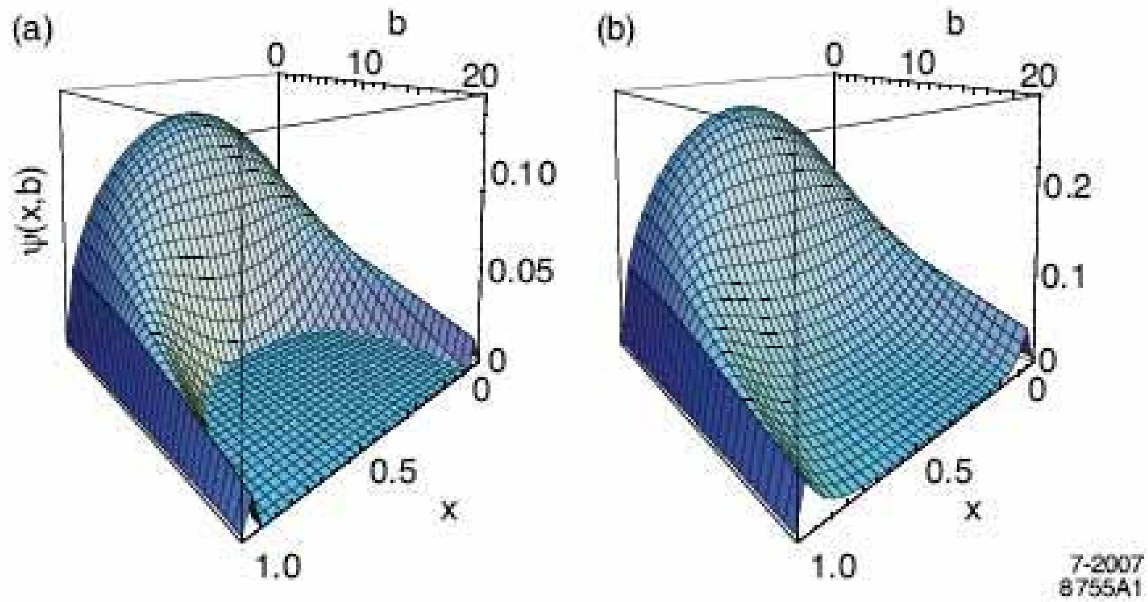
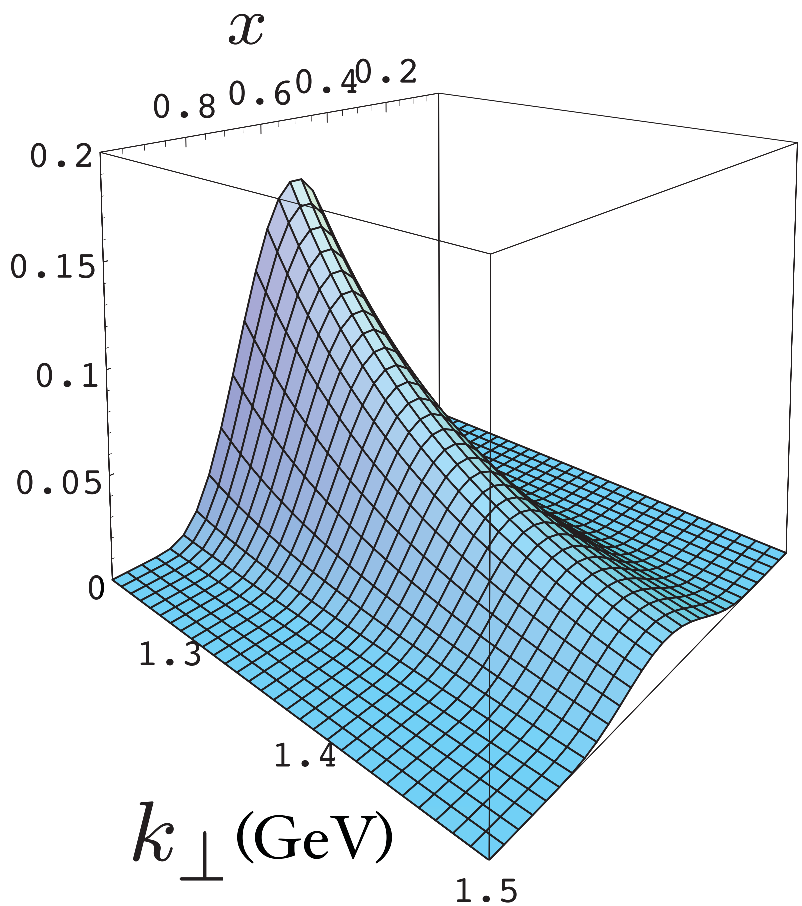


Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{\rm QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.

Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

$\psi_M(x, k_\perp^2)$



"Soft Wall" model

$\kappa = 0.375$ GeV

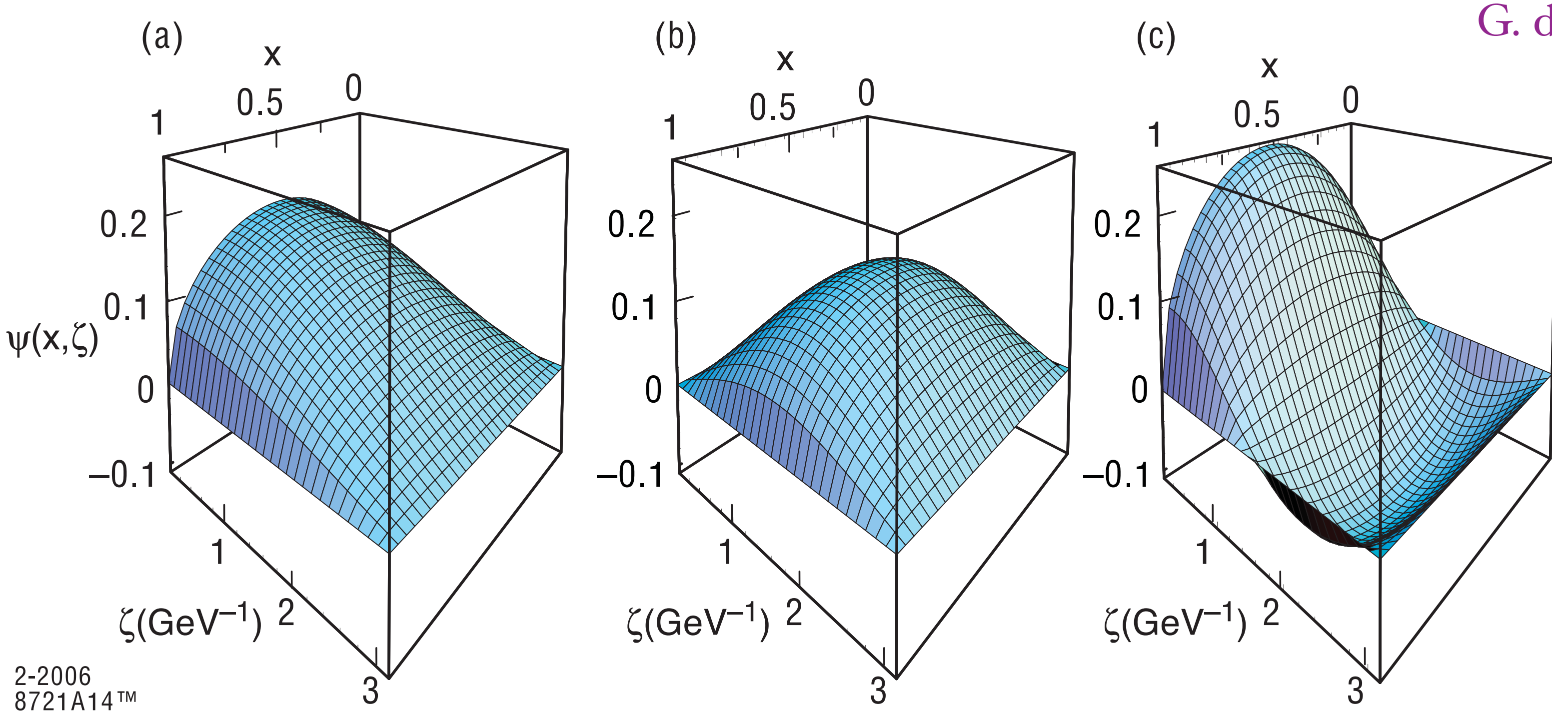
massless quarks

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

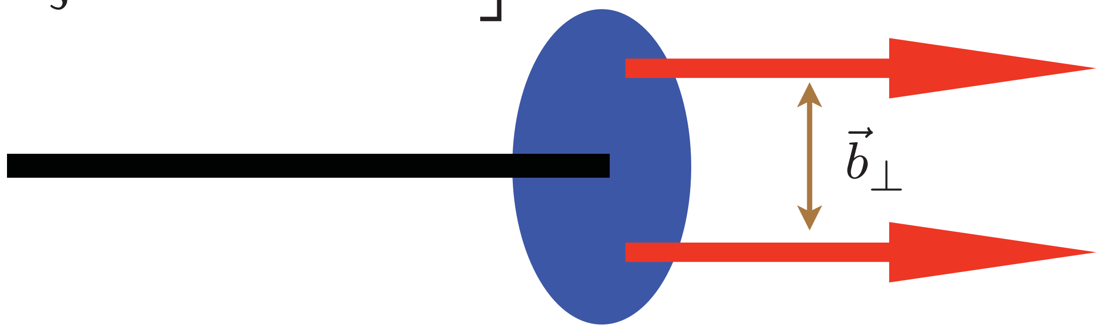
AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB



Two-parton holographic LFWF in impact space $\widetilde{\psi}(x, \zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L = 0,\ k = 1$; (b) first orbital exited state $L = 1,\ k = 1$; (c) first radial exited state $L = 0,\ k = 2$. The variable ζ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$



$$\zeta = \sqrt{x(1-x)\vec{b}^2_\perp}$$ *Holographic Variable*

$$-\frac{d}{d\zeta^2} \equiv \frac{k^2_\perp}{x(1-x)}$$ *LF Kinetic Energy in momentum space*

Conjecture for mesons with massive quarks

$$-\frac{d}{d\zeta^2} \to -\frac{d}{d\zeta^2} + \frac{m_a^2}{x} + \frac{m_b^2}{1-x} \equiv \frac{k^2_\perp + m_a^2}{x} + \frac{k^2_\perp + m_b^2}{1-x}$$

Introduction of Heavy Quark Masses

- Introduction of light quark masses in AdS/QCD involve complex dynamics, as the evolution from current to constituent quark masses should be included.
- Assume the momentum space LFWF is a function of the invariant (off-energy shell)

$$\mathcal{M}^2 - \mathcal{E} = \sum_{i=1}^{n} \left(\mathbf{k}_{\perp i}^2 + m_i^2\right) / x_i .$$

- Soft-Wall LFWF ansatz for bound state with massive constituents:

$$\psi(x, \mathbf{k}_\perp) \sim \frac{4\pi}{\kappa\sqrt{x(1-x)}}\, e^{-\frac{1}{2\kappa^2}\left(\frac{\mathbf{k}_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right)},$$

- Fourier transform to impact space

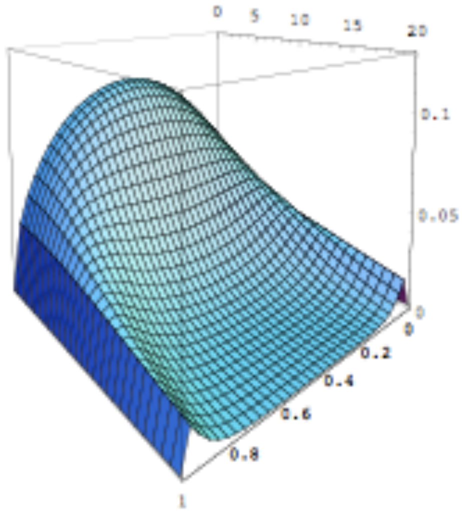
$$\widetilde{\psi}(x, \mathbf{b}_\perp) \sim \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)}\, e^{-\frac{1}{2}\kappa^2 x(1-x)\mathbf{b}_\perp^2 + \frac{1}{2\kappa^2}\left[\frac{m_1^2}{x} - \frac{m_2^2}{1-x}\right]}.$$

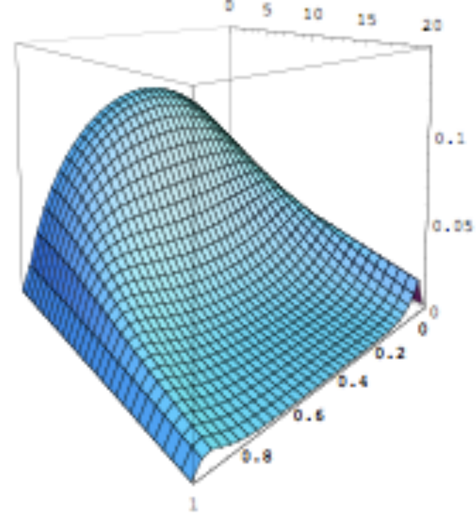
$$z \to \zeta \xrightarrow{m_i} \chi$$

$$\chi^2 = x(1-x)\mathbf{b}_\perp^2 + \frac{1}{\kappa^4}\left[\frac{m_1^2}{x} + \frac{m_2^2}{1-x}\right].$$

$|\pi^+ >= |u\bar{d}>$

$m_u = 2\ MeV$
$m_d = 5\ MeV$



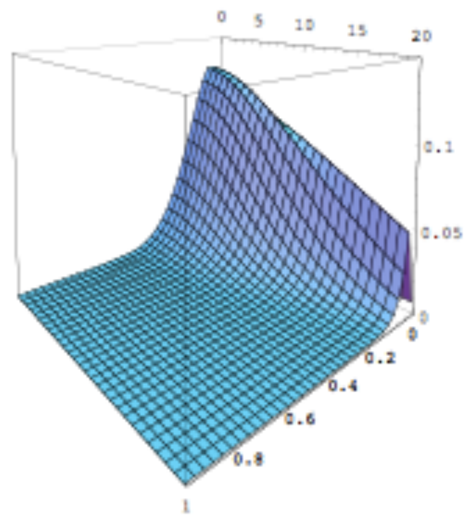


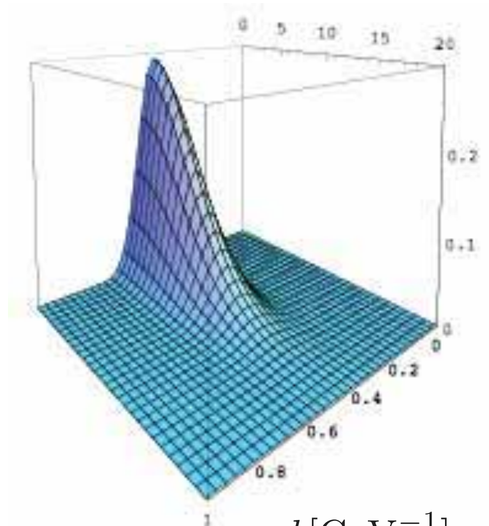
$|K^+ >= |u\bar{s}>$

$m_s = 95\ MeV$

$|D^+ >= |c\bar{d}>$

$m_c = 1.25\ GeV$

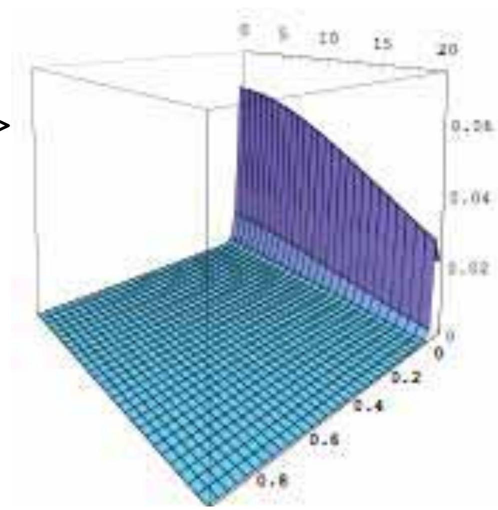


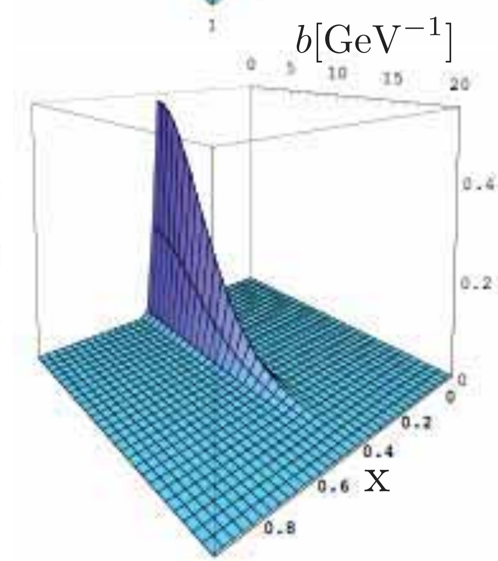


$|\eta_c >= |c\bar{c}>$

$|B^+ >= |u\bar{b}>$

$m_b = 4.2\ GeV$





$|\eta_b >= |b\bar{b}>$

$\kappa = 375\ MeV$

Perturbative CFT vs Non-Perturbative AdS Results

- Heavy quark potential: $g_s N_C \to \sqrt{g_s N_C}$ Maldacena (1998), Rey and Yee (1998).
- Distribution amplitude $\phi_M(x,Q) \sim \int^{Q^2} d^2\mathbf{k}_\perp \, \psi_{\overline{q}q/M}(x,\mathbf{k}_\perp)$.
- Second Moment of the Distribution $(\xi = 1 - 2x)$

$$\langle \xi^2 \rangle = \int_{-1}^{1} d\xi \, \xi^2 \phi(\xi) \Big/ \int_{-1}^{1} d\xi \, \phi(\xi)$$

$$\langle \xi^2 \rangle = 1/5 \qquad \phi_{\mathrm{PQCD}} \sim x(1-x),$$

$$\langle \xi^2 \rangle = 1/4 \qquad \phi_{\mathrm{AdS/QCD}} \sim \sqrt{x(1-x)}.$$

- Sachrajda lattice result: $\langle \xi^2 \rangle = 0.28 \pm 0.02$

Baryon Spectrum

- Baryon: twist-three, dimension $\frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i.$$

Wave Equation: $$\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x, z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P) \Big].$$

- 4-d mass spectrum $\Psi(x, z_o)^\pm = 0 \quad \Longrightarrow$ parallel Regge trajectories for baryons !

$$\mathcal{M}^+_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}^-_{\alpha,k} = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Prediction from AdS/QCD

Only one parameter!

Entire light quark baryon spectrum

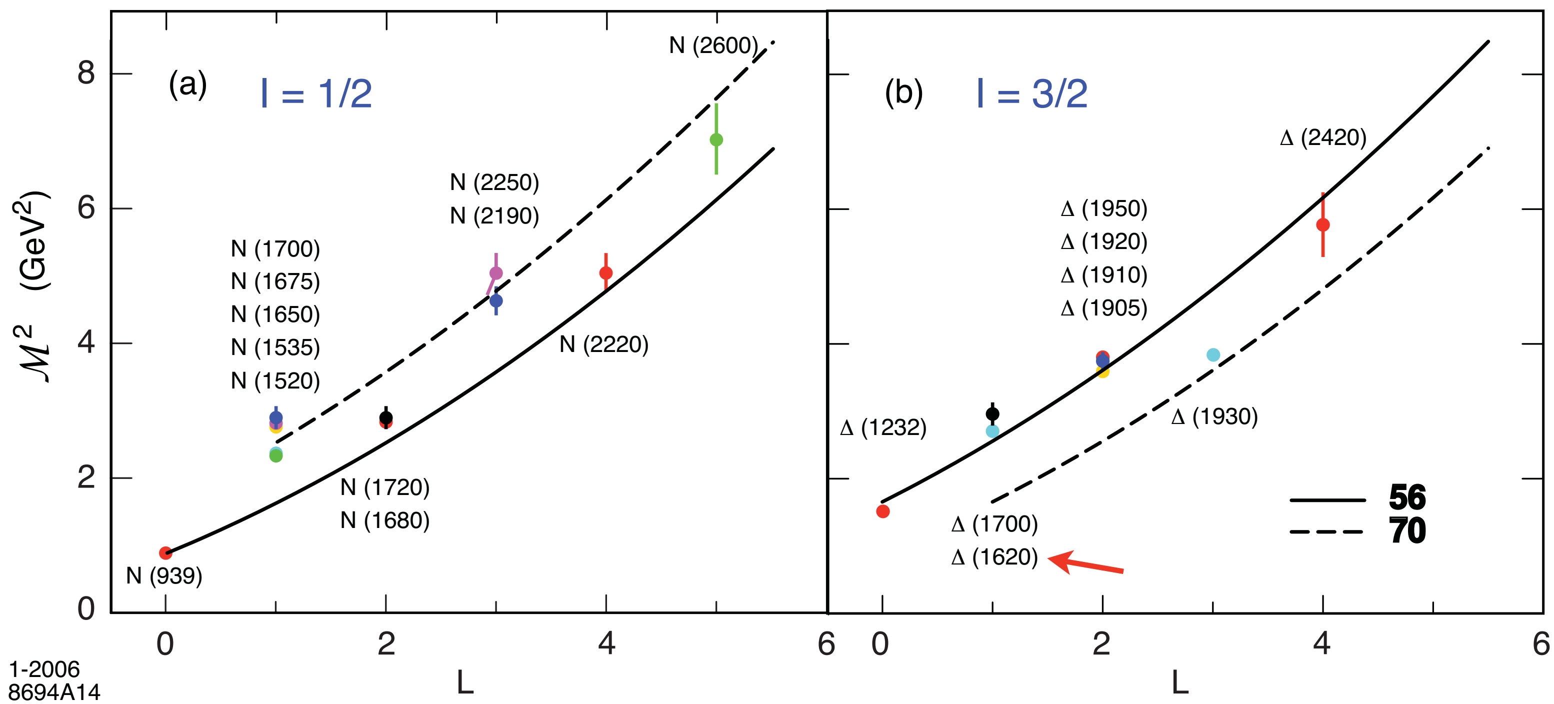


Fig: Predictions for the light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV. The **56** trajectory corresponds to L even $P = +$ states, and the **70** to L odd $P = -$ states.

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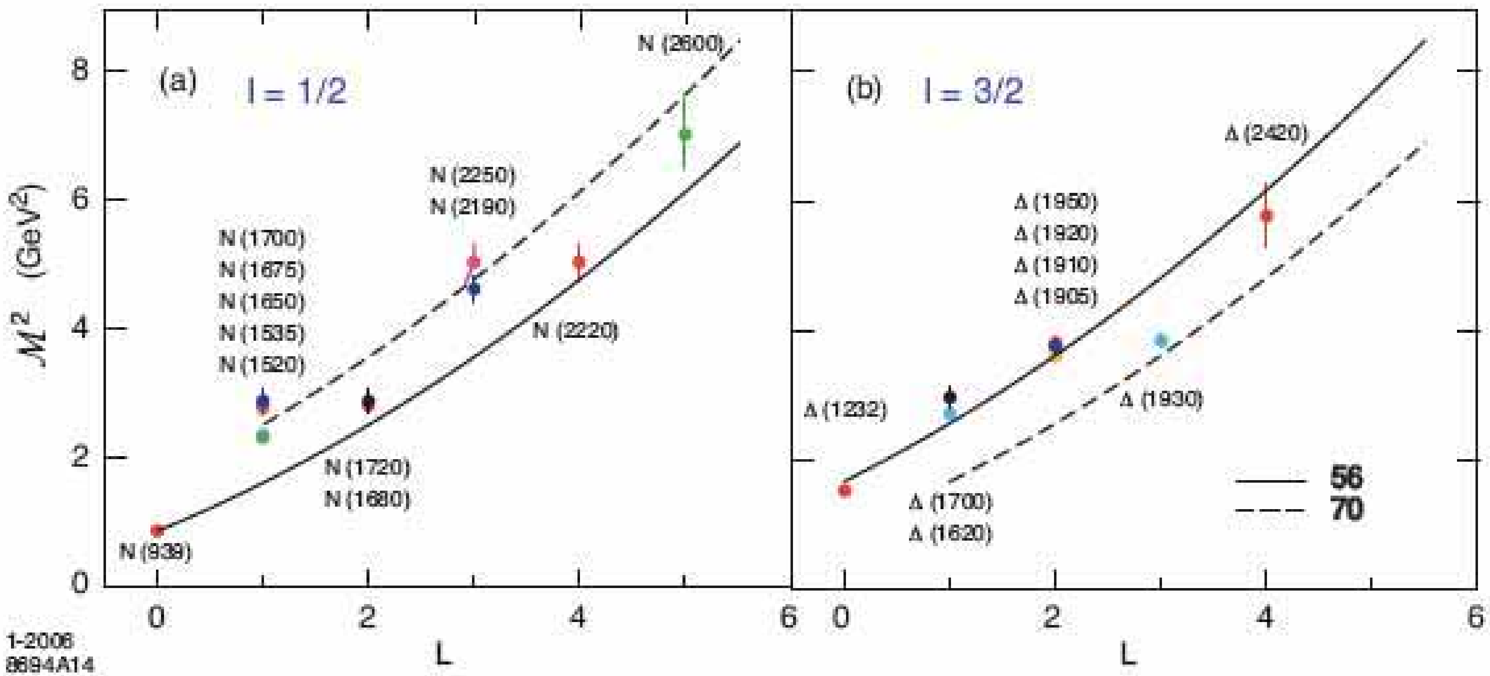


Fig: Light baryon orbital spectrum for Λ_{QCD} = 0.25 GeV in the HW model. The $\mathbf{56}$ trajectory corresponds to L even $P = +$ states, and the $\mathbf{70}$ to L odd $P = -$ states.

- New analysis: $\Delta(1930)$ as a $L = 1, N = 1, J = \frac{3}{2}$ state I. Horn *et. al.* arXiv: 0711.1138

$SU(6)$	S	L	Baryon State
56	$\frac{1}{2}$	0	$N\frac{1}{2}^{+}(939)$
	$\frac{3}{2}$	0	$\Delta\frac{3}{2}^{+}(1232)$
70	$\frac{1}{2}$	1	$N\frac{1}{2}^{-}(1535)$ $N\frac{3}{2}^{-}(1520)$
	$\frac{3}{2}$	1	$N\frac{1}{2}^{-}(1650)$ $N\frac{3}{2}^{-}(1700)$ $N\frac{5}{2}^{-}(1675)$
	$\frac{1}{2}$	1	$\Delta\frac{1}{2}^{-}(1620)$ $\Delta\frac{3}{2}^{-}(1700)$
56	$\frac{1}{2}$	2	$N\frac{3}{2}^{+}(1720)$ $N\frac{5}{2}^{+}(1680)$
	$\frac{3}{2}$	2	$\Delta\frac{1}{2}^{+}(1910)$ $\Delta\frac{3}{2}^{+}(1920)$ $\Delta\frac{5}{2}^{+}(1905)$ $\Delta\frac{7}{2}^{+}(1950)$
70	$\frac{1}{2}$	3	$N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}$
	$\frac{3}{2}$	3	$N\frac{3}{2}^{-}$ $N\frac{5}{2}^{-}$ $N\frac{7}{2}^{-}(2190)$ $N\frac{9}{2}^{-}(2250)$
	$\frac{1}{2}$	3	$\Delta\frac{5}{2}^{-}(1930)$ $\Delta\frac{7}{2}^{-}$
56	$\frac{1}{2}$	4	$N\frac{7}{2}^{+}$ $N\frac{9}{2}^{+}(2220)$
	$\frac{3}{2}$	4	$\Delta\frac{5}{2}^{+}$ $\Delta\frac{7}{2}^{+}$ $\Delta\frac{9}{2}^{+}$ $\Delta\frac{11}{2}^{+}(2420)$
70	$\frac{1}{2}$	5	$N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}(2600)$
	$\frac{3}{2}$	5	$N\frac{7}{2}^{-}$ $N\frac{9}{2}^{-}$ $N\frac{11}{2}^{-}$ $N\frac{13}{2}^{-}$

Baryons

Linear Holographic Confinement

- Compare with usual Dirac equation in AdS space $\left(x^\ell = (x^\mu, z)\right)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell\partial_m + \frac{d}{2}\Gamma_z\right) + \mu R + V(z)\right]\Psi(x^\ell) = 0.$$

in presence of a linear confining potential $V(z) = \kappa^2 z.$

- Upon substitution $\Psi(x,z) = e^{-iP\cdot x} z^2 \psi(z),\ z \rightarrow \zeta$ we find

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\,\psi(\zeta)$$

with

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu+\frac{1}{2}}{\zeta}\gamma_5 - \kappa^2\zeta\gamma_5\right), \quad \mu R = \nu + \frac{1}{2},$$

our previous result.

- Soft-wall model for baryons corresponds to a linear confining potential in the LF transverse variable ζ!

Baryons

Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha \Pi(\zeta) - \mathcal{M})\, \psi(\zeta) = 0,$$

in terms of the matrix-valued operator Π_ν

$$\Pi_\nu(\zeta) = -i \left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

- Commutation relations for fermionic generators

$$\left[\Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left(\frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5 .$$

- Solutions to the Dirac equation

$$\psi_+(\zeta) \sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2/2} L_n^\nu(\kappa^2 \zeta^2),$$
$$\psi_-(\zeta) \sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2/2} L_n^{\nu+1}(\kappa^2 \zeta^2).$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

Baryons

- Baryon: twist-dimension $3+L$ $(\nu = L+1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i.$$

- Define the zero point energy (identical as in the meson case) $\mathcal{M}^2 \to \mathcal{M}^2 - 4\kappa^2$:

$$\mathcal{M}^2 = 4\kappa^2(n+L+1).$$

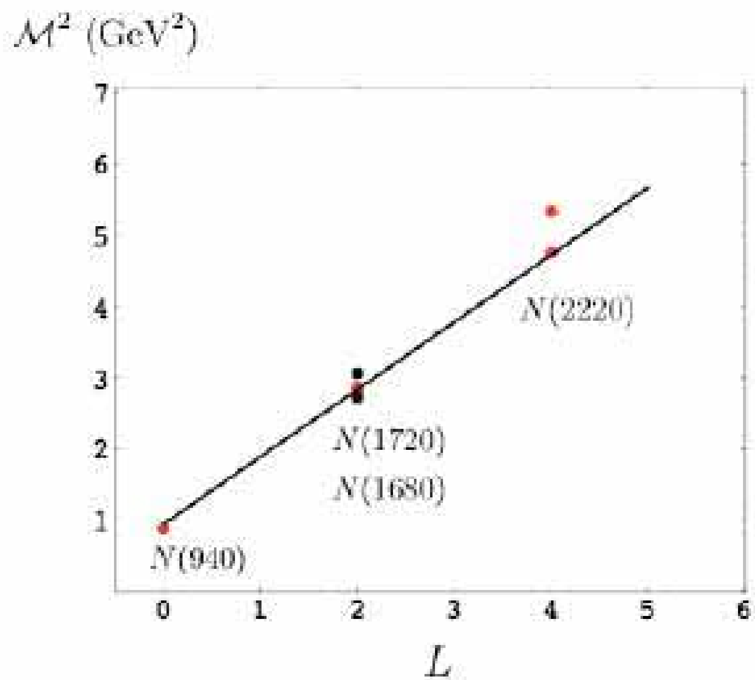


Fig: Proton Regge Trajectory $\kappa = 0.49$ GeV

- Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \qquad L = \sum_{i=1}^{m} \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C\sqrt{\zeta}\left[J_{L+1}(\zeta\mathcal{M})u_+ + J_{L+2}(\zeta\mathcal{M})u_-\right].$$

Baryonic modes propagating in AdS space have two components: orbital L and $L+1$.

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_\pm \left(\zeta = 1/\Lambda_{\rm QCD}\right) = 0,$$

given by

$$\mathcal{M}^+_{\nu,k} = \beta_{\nu,k}\Lambda_{\rm QCD}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k}\Lambda_{\rm QCD},$$

with a scale independent mass ratio.

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(z)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(z)$ are solutions of the Dirac light-front equation

$$H_{LF}|\psi\rangle = \mathcal{M}|\psi\rangle,$$

with

$$H_{LF} = \alpha\,\Pi.$$

The operator

$$\Pi_\nu(\zeta) = -i\left(\frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta}\gamma_5\right),$$

and its adjoint $\Pi^\dagger_\nu(\zeta)$ satisfy the commutation relations

$$\left[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)\right] = \frac{2\nu+1}{\zeta^2}\gamma_5.$$

- Supersymmetric QM between bosonic and fermionic modes in AdS?

Nucleon Form Factors

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_-(z)|^2,$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[|\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

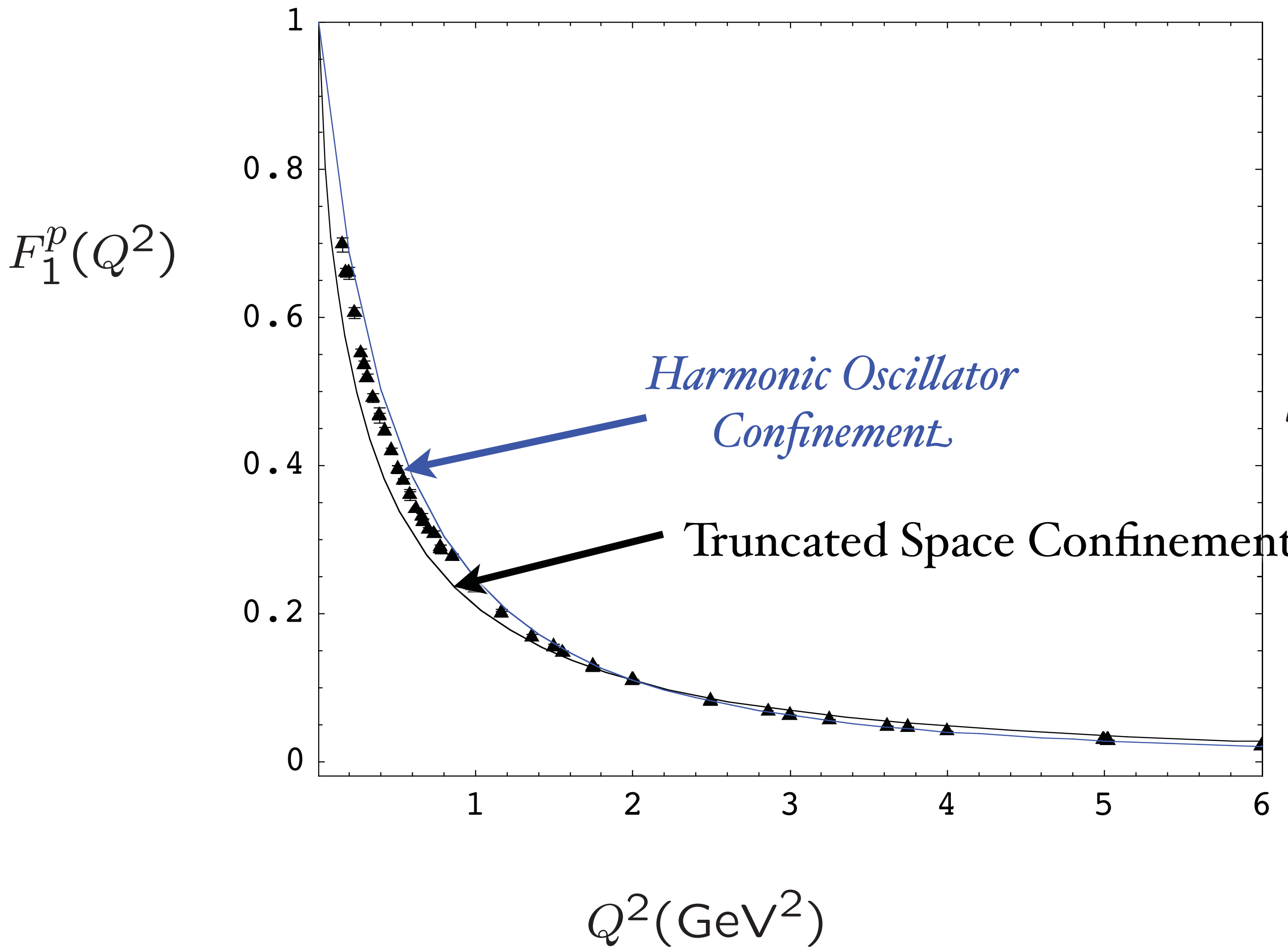
  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large Q power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.

G. de Teramond, sjb

G. de Teramond, sjb

Preliminary



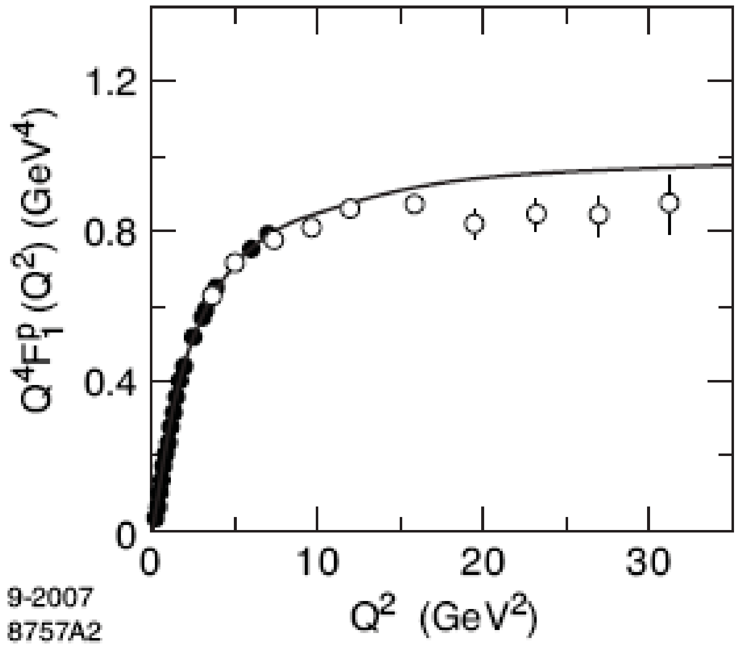
$\kappa = 0.424$ GeV

$\Lambda = 0.2$ GeV

Current modified by metric

$$F_1(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi^{\uparrow}_F(z) J(Q,z) \Phi^{\uparrow}_I(z)$$

- Scaling behavior for large Q^2: $Q^4 F_1^p(Q^2) \to \text{constant}$ $\boxed{\text{Proton } \tau = 3}$

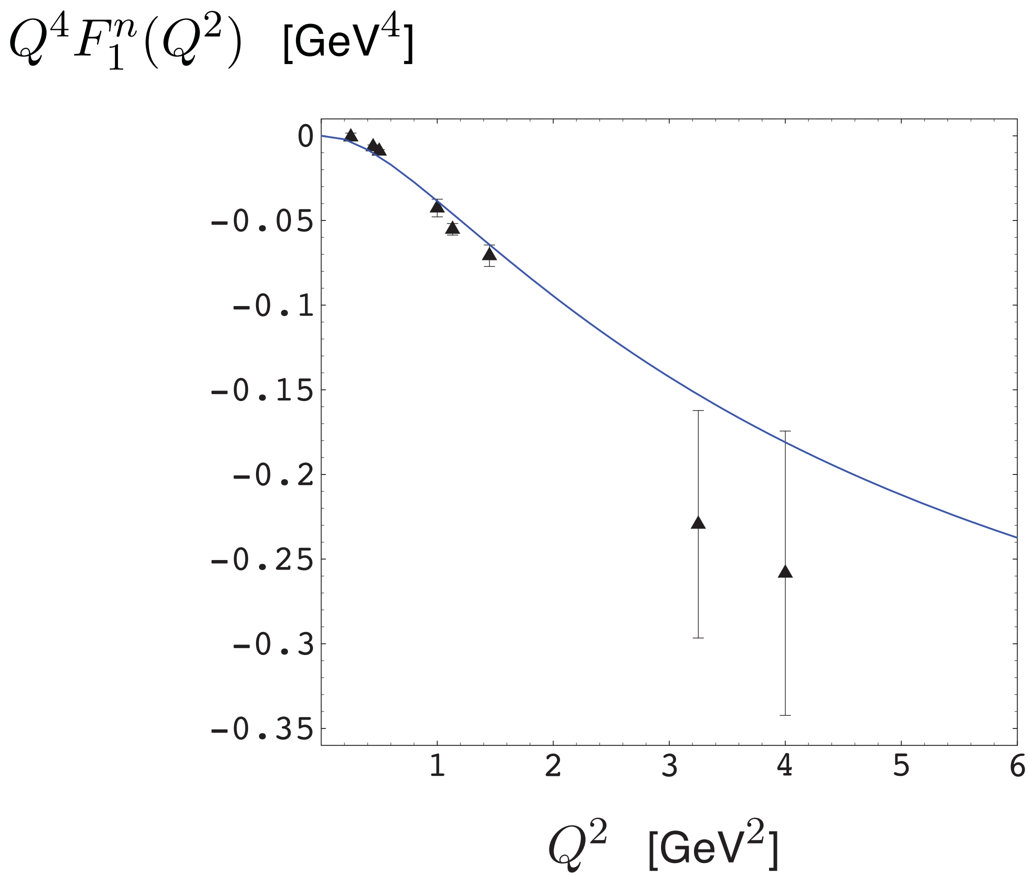


SW model predictions for $\kappa = 0.424$ GeV. Data analysis from: M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

Dirac Neutron Form Factor

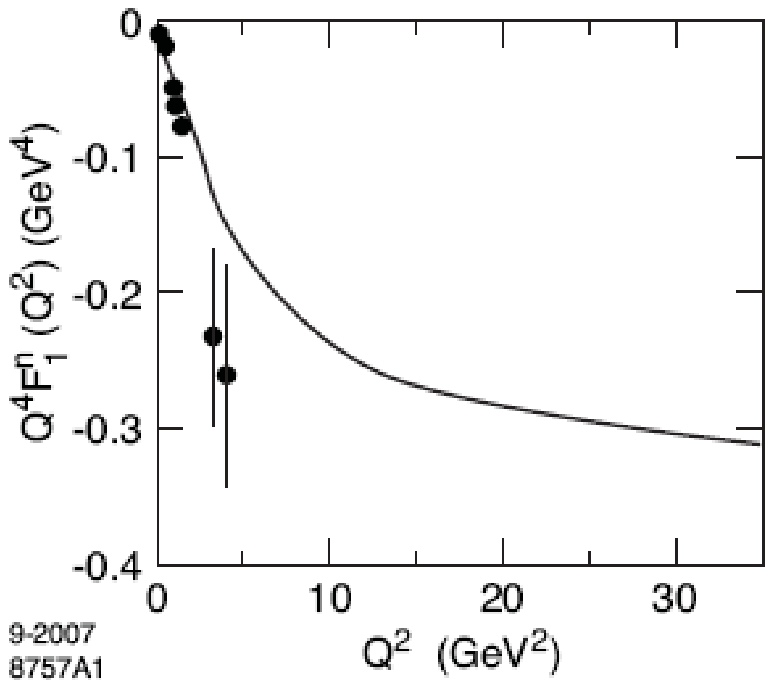
(Valence Approximation)

Truncated Space Confinement



Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).

- Scaling behavior for large Q^2: $Q^4 F_1^n(Q^2) \to \text{constant}$ $\boxed{\text{Neutron } \tau = 3}$



SW model predictions for $\kappa = 0.424$ GeV. Data analysis from M. Diehl *et al.* Eur. Phys. J. C **39**, 1 (2005).

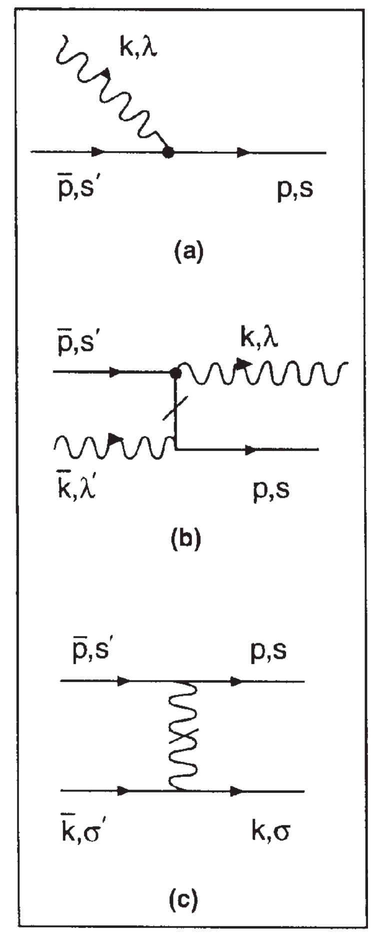
Excitation Spectrum of Baryons

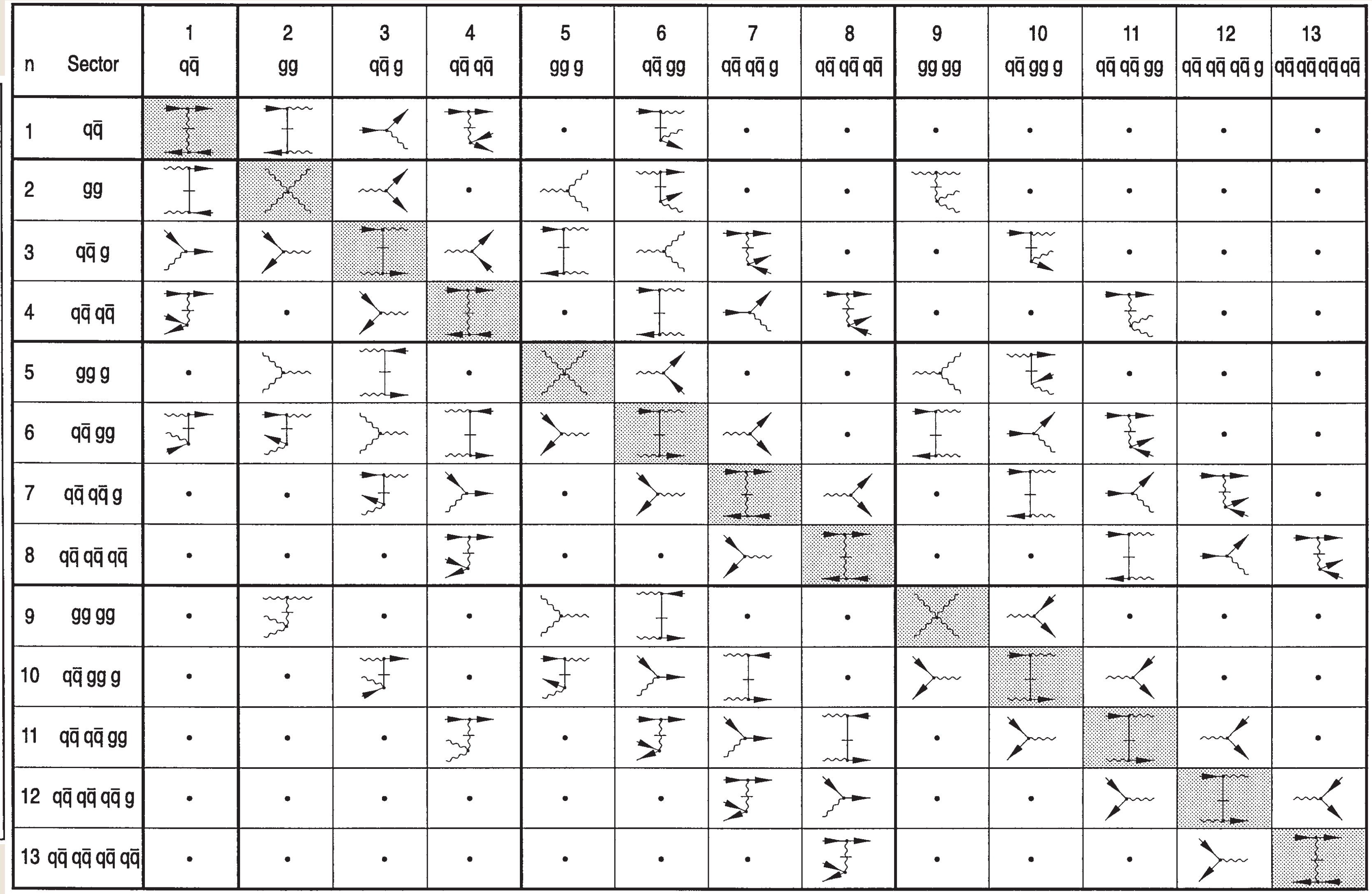
- Hard wall holographic model: $\mathcal{M}_n(L) \sim L + 2n$
- Soft wall holographic model: $\mathcal{M}_n^2(L) \sim L + n$
- Quark models (shell structure of excitations): $\mathcal{M}_n^2(L) \sim L + 2n$
- Observed same multiplicity of states for mesons and baryons!
- Natural feature of AdS/QCD and the quantized Nambu String Baker and Steinke (2002).

  See: E. Klempt (2007)

Light-Front QCD Heisenberg Equation

$$H^{QCD}_{LC} |\Psi_h\rangle = \mathcal{M}^2_h \, |\Psi_h\rangle$$



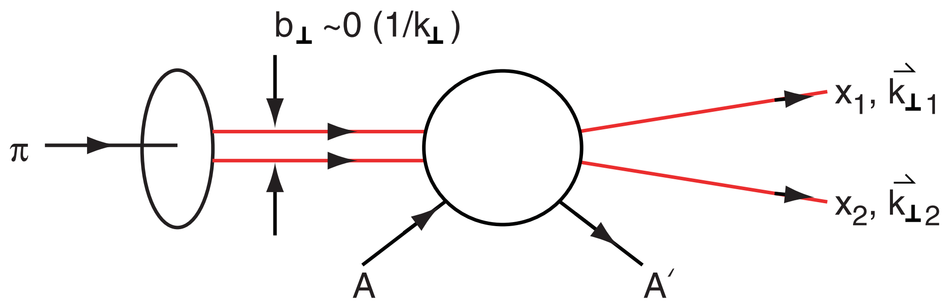


Use AdS/QCD basis functions

Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

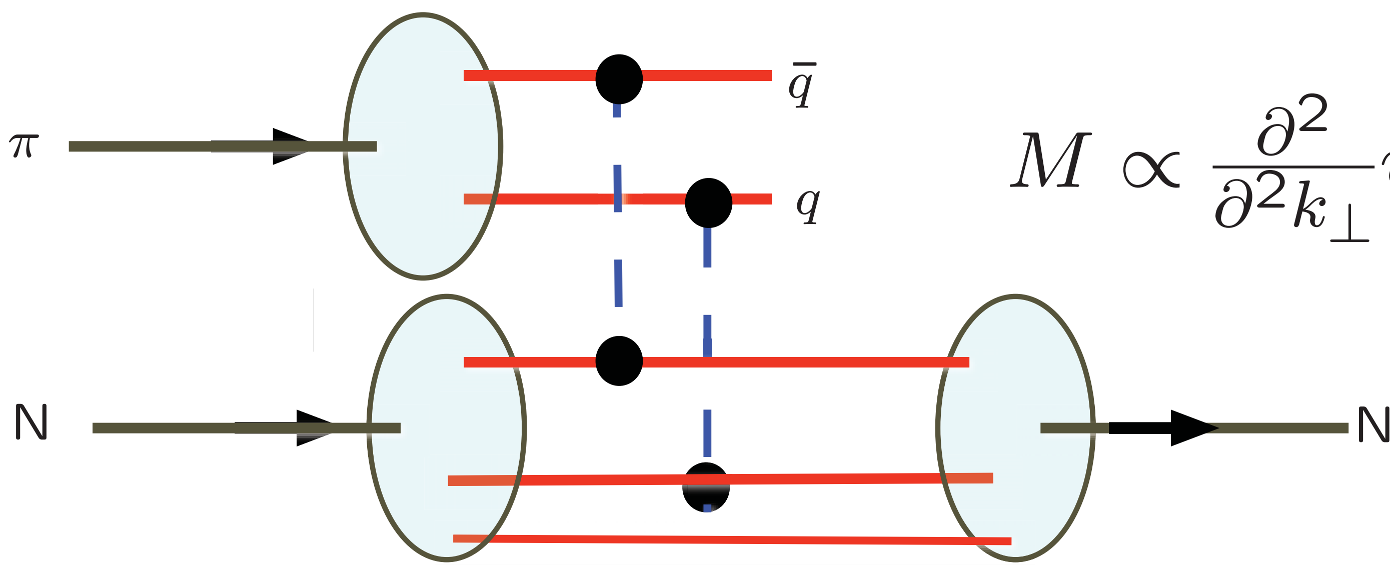
- Good initial approximant
- Better than plane wave basis — Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion — Vary, Harinandrath, Maris, sjb
- Similar to Shell Model calculations

E791 FNAL Diffractive DiJet



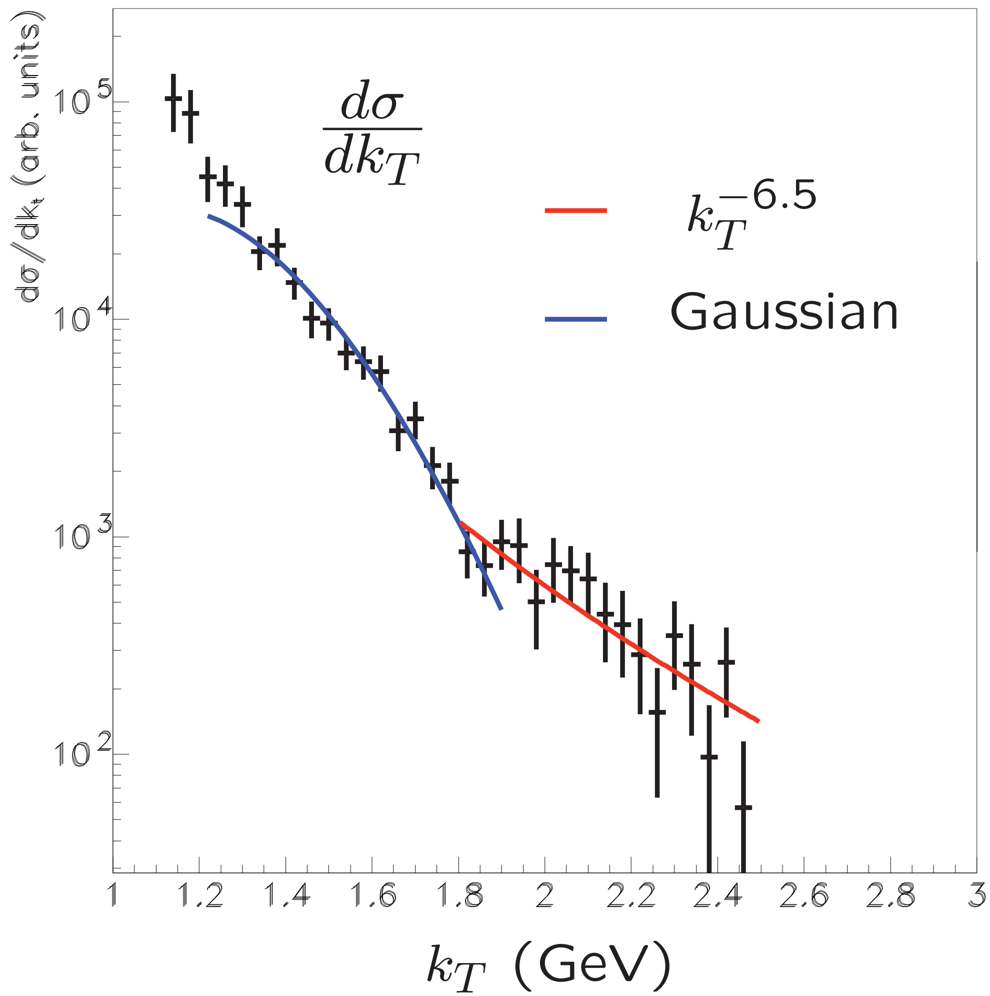
Gunion, Frankfurt, Mueller, Strikman, sjb
Frankfurt, Miller, Strikman

Two-gluon exchange measures the second derivative of the pion light-front wavefunction



$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$

E791 Diffractive Di-Jet transverse momentum distribution



Two Components

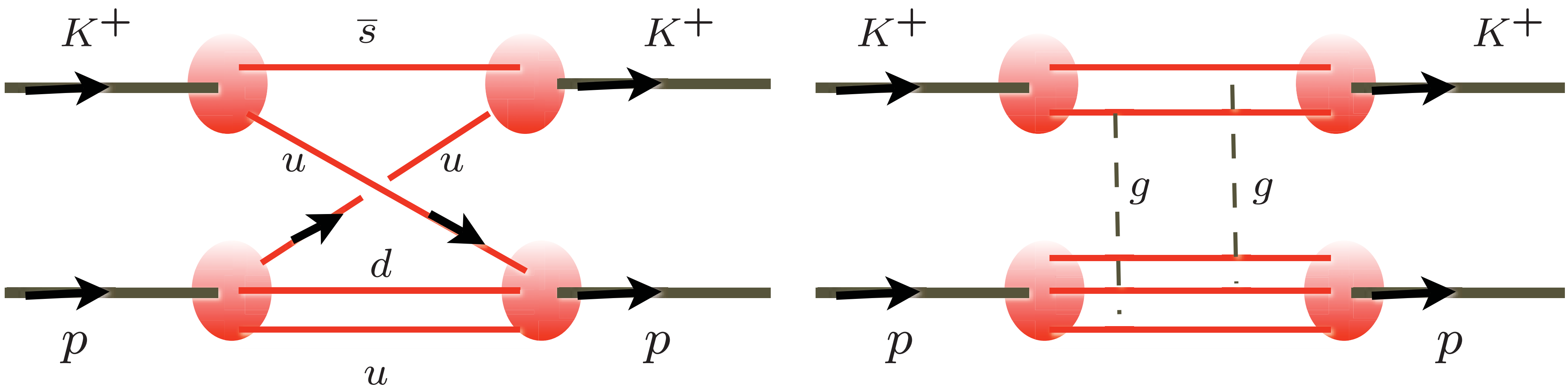
High Transverse momentum dependence $k_T^{-6.5}$ *consistent with PQCD, ERBL Evolution*

Gaussian component similar to AdS/CFT HO LFWF

New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

CIM: Blankenbecler, Gunion, sjb



Quark Interchange (Spin exchange in atom-atom scattering)

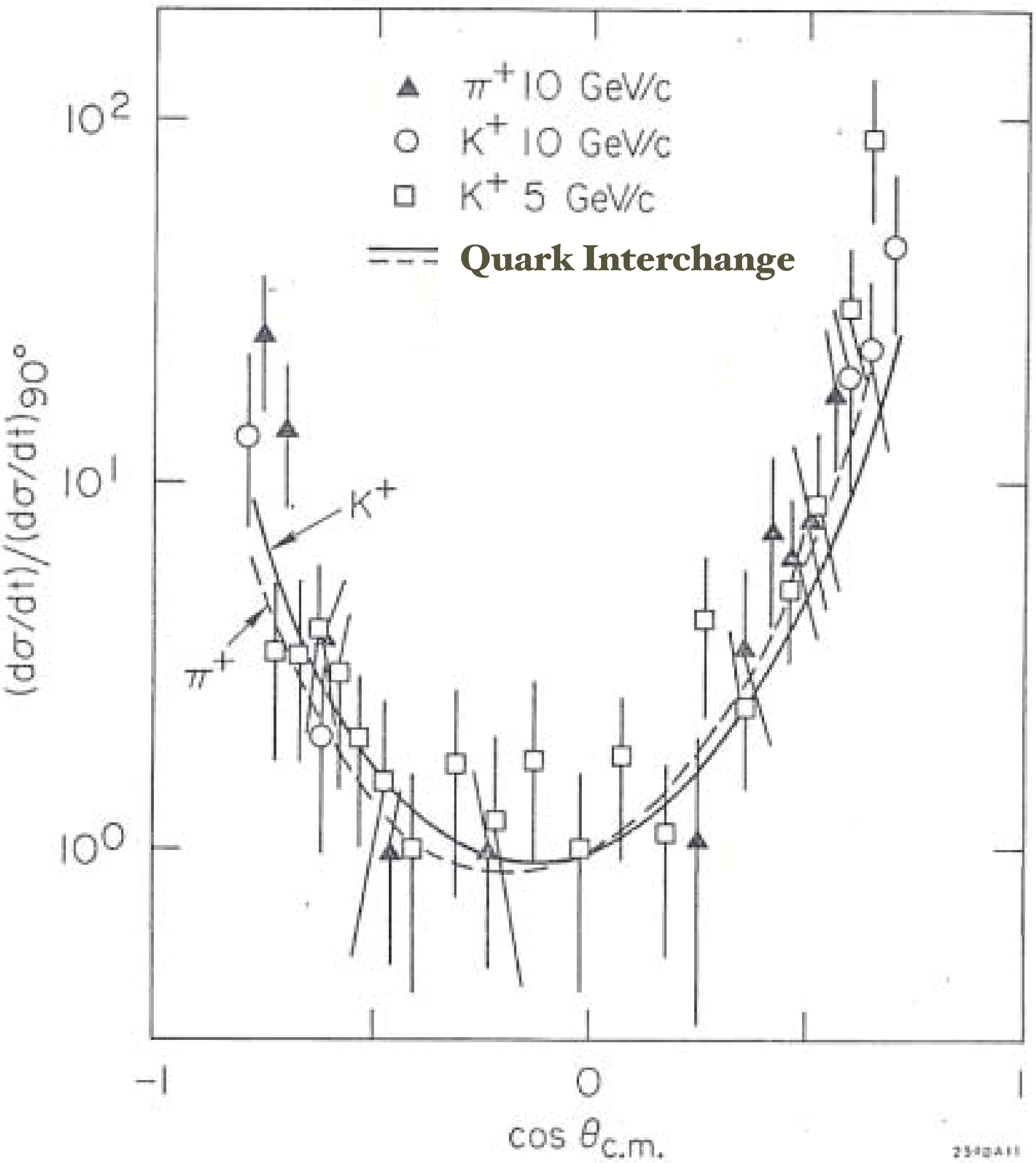
Gluon Exchange (Van der Waal -- Landshoff)

$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$

$M(s,t)_{\text{gluonexchange}} \propto sF(t)$

MIT Bag Model (de Tar), large N_C, ('t Hooft), AdS/CFT all predict dominance of quark interchange:



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

Non-linear Regge behavior:

$$\alpha_R(t) \to -1$$

Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common u quark

$M_{QIM} = \int d^2k_\perp dx\ \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of AdS_5

Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.

Comparison of Exclusive Reactions at Large t

B. R. Baller,[a] G. C. Blazey,[b] H. Courant, K. J. Heller, S. Heppelmann,[c] M. L. Marshak, E. A. Peterson, M. A. Shupe, and D. S. Wahl[d]
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and

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Southeastern Massachusetts University, North Dartmouth, Massachusetts 02747
(Received 28 October 1987; revised manuscript received 3 February 1988)

Cross sections or upper limits are reported for twelve meson-baryon and two baryon-baryon reactions for an incident momentum of 9.9 GeV/c, near 90° c.m.: $\pi^{\pm}p \rightarrow p\pi^{\pm}, p\rho^{\pm}, \pi^{+}\Delta^{\pm}, K^{+}\Sigma^{\pm}, (\Lambda^{0}/\Sigma^{0})K^{0}$; $K^{\pm}p \rightarrow pK^{\pm}$; $p^{\pm}p \rightarrow pp^{\pm}$. By studying the flavor dependence of the different reactions, we have been able to isolate the quark-interchange mechanism as dominant over gluon exchange and quark-antiquark annihilation.

$\pi^{\pm}p \rightarrow p\pi^{\pm}$,

$K^{\pm}p \rightarrow pK^{\pm}$,

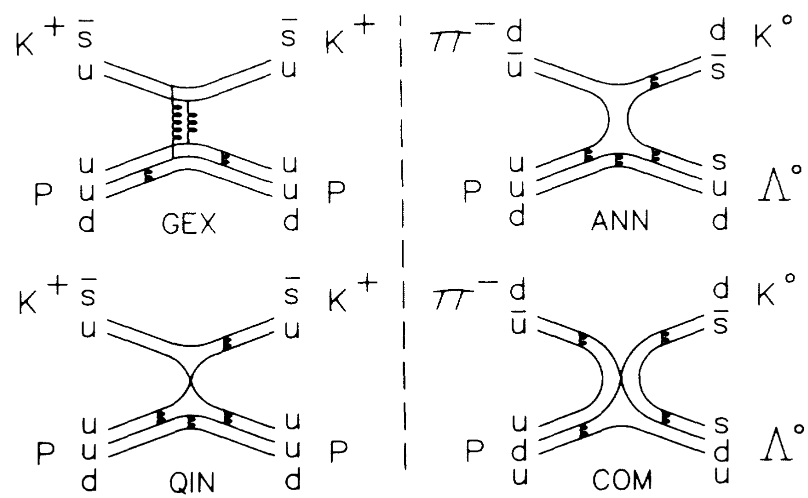
$\pi^{\pm}p \rightarrow p\rho^{\pm}$,

$\pi^{\pm}p \rightarrow \pi^{+}\Delta^{\pm}$,

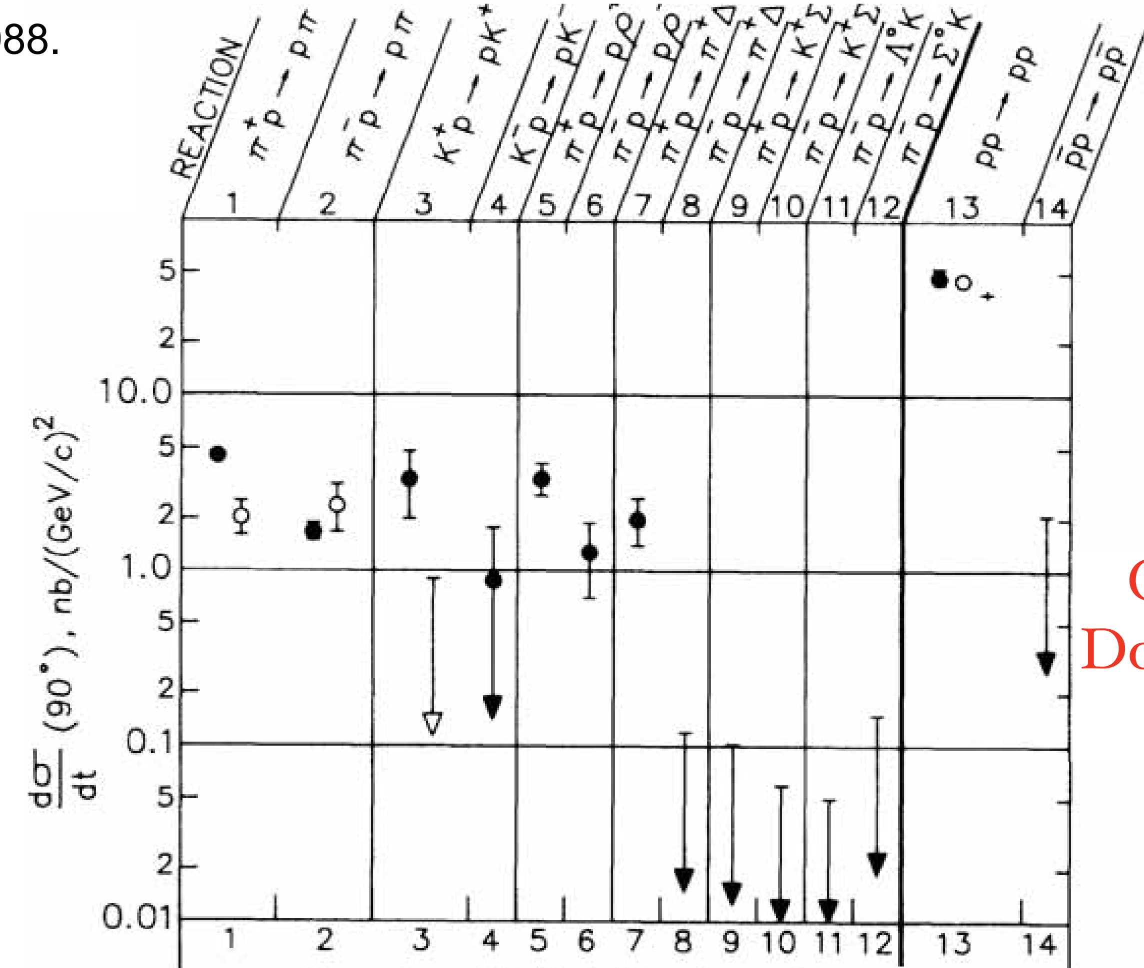
$\pi^{\pm}p \rightarrow K^{+}\Sigma^{\pm}$,

$\pi^{-}p \rightarrow \Lambda^{0}K^{0}, \Sigma^{0}K^{0}$,

$p^{\pm}p \rightarrow pp^{\pm}$.



B.R. Baller *et al.*. 1988.
Published in
Phys.Rev.Lett. 60:1118-1121,1988



Comparison of Exclusive Reactions at Large *t*

Quark Interchange: Dominant Dynamics at large t, u

Relative Rates Correct

The cross section and upper limits (90% confidence level) measured by this experiment are indicated by the filled circles and arrowheads. Values from this experiment and from previous measurements represent an average over the angular region of $-0.05 < \cos\theta_{c.m.} < 0.10$. The other measurements were obtained from the following references: π^+p and K^+p elastic, Ref. 5; $\pi^-p \to p\pi^-$, Ref. 6; $pp \to pp$, Ref. 7: Allaby, open circle; Akerlof, cross. Values for the cross sections [(Reaction), cross section in nb/(GeV/c)^2] are as follows: (1), 4.6 ± 0.3; (2), 1.7 ± 0.2; (3), 3.4 ± 1.4; (4), $0.9^{+0.7}_{-0.9}$; (5), 3.4 ± 0.7; (6), 1.3 ± 0.6; (7), 2.0 ± 0.6; (8), < 0.12; (9), < 0.1; (10), < 0.06; (11), < 0.05; (12), < 0.15; (13), 48 ± 5; (14), < 2.1.

Holographic Connection between LF and AdS/CFT

- Predictions for hadronic spectra, light-front wavefunctions, interactions
- Use AdS/CFT as basis for diagonalizing the LF Hamiltonian
- Deduce meson and baryon wavefunctions, distribution amplitude, structure function from holographic constraint
- Extension to massive quarks
- Implementation of Chiral Symmetry Breaking

Some Applications of Light-Front Wavefunctions

- Exact formulae for form factors, quark and gluon distributions; vanishing anomalous gravitational moment; edm connection to anm
- Deeply Virtual Compton Scattering, generalized parton distributions, angular momentum sum rules
- Exclusive weak decay amplitudes
- Single spin asymmetries: Role if ISI and FSI
- Factorization theorems, DGLAP, BFKL, ERBL Evolution
- Quark interchange amplitude
- Relation of spin, momentum, and other distributions to physics of the hadron itself.

Advantages of LF QCD

- Lorentz Invariant
- No fermion doubling
- Minkowski space
- Complete set of eigensolutions, bound state and continuum spectroscopy
- LFWFs, observables, simple spin properties
- Physical gauge: ghost free
- Zero modes instead of VEVS
- QED(3+1), QCD(1+1) ......