• Remarkable Test of Quark Counting Rules

• Deuteron Photo-Disintegration $\gamma d \rightarrow np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot} - 2}}$$

• $n_{tot} = 1 + 6 + 3 + 3 = 13$

Scaling characteristic of scale-invariant theory at short distances

Conformal symmetry

Hidden color: $\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$

at high $p_T$
Test of Hidden Color in Deuteron Photodisintegration

\[ R = \frac{\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++} \Delta^{--})}{\frac{d\sigma}{dt}(\gamma d \rightarrow p n)} \]

Ratio predicted to approach 2:5

Possible contribution from pion charge exchange at small t.

Ratio should grow with transverse momentum as the hidden color component of the deuteron grows in strength.
Hadronization at the Amplitude Level

Anti-Deuteron vs. double antibaryon production

$$\Upsilon \rightarrow ggg \rightarrow q\bar{q} \ q\bar{q} \ q\bar{q} \ q\bar{q} \ q\bar{q} \ q\bar{q} \rightarrow \bar{d} \ X$$

$$\Upsilon \rightarrow ggg \rightarrow q\bar{q} \ q\bar{q} \ q\bar{q} \ q\bar{q} \ q\bar{q} \ q\bar{q} \rightarrow \bar{p} \ \bar{n} \ X$$
Key Test of Hidden Color

- CLEO measurement: Upsilon decay to anti-deuteron
  \[ \Upsilon \to gg \to \bar{d}X \]

- Is ratio of deuteron production to production of anti-nucleon pairs determined by Nuclear Physics?

\[ R = \frac{\Gamma(\Upsilon \to \bar{d}X)}{\Gamma(\Upsilon \to \bar{p}\bar{n}X)} \]

\[ \frac{E}{\sigma_{\text{tot}}} \frac{d^3\sigma(d)}{d^3p} = C \left( \frac{E}{\sigma_{\text{tot}}} \frac{d^3\sigma(p)}{d^3p} \right)^2 \]

\[ C = \frac{4\pi}{3} \frac{p_0^3}{m_p} \quad p_0 \approx 130 \text{MeV} \]

Gustafson, Hakkinen
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Hadronization Mechanism

“Hidden-Color” Components \(|(uud)_{8c}(ddu)_{8c}^c >\)

New Hadronization Mechanism
Intrinsic Heavy-Quark Fock States

- Rigorous prediction of QCD, OPE
- Color-Octet Fock State

- Probability
  \[ P_{Q\bar{Q}} \propto \frac{1}{M_Q^2} \]
  \[ P_{Q\bar{Q}Q\bar{Q}} \sim \alpha_s^2 P_{Q\bar{Q}} \]
  \[ P_{cc/p} \approx 1\% \]

- Large Effect at high x

- Greatly increases kinematics of colliders such as Higgs production (Kopeliovich, Schmidt, Soffer, sjb)

- Severely underestimated in conventional parameterizations of heavy quark distributions (Pumplin)

- Many empirical tests
Fluctuation in Proton
QCD: Probability \( \frac{\Lambda_{QCD}^2}{M_Q^2} \)

Fluctuation in Positronium
QED: Probability \( \frac{(m_\ell \alpha)^4}{M_\ell^4} \)

\( |uudc\bar{c}| > \) Fluctuation in Proton
\( |e^+e^-\ell^+\ell^-| > \) Fluctuation in Positronium

OPE derivation - M. Polyakov et al.

\( < p \frac{G_{\mu\nu}^3}{m_Q^2} |p > \) vs. \( < p \frac{F_{\mu\nu}^4}{m_\ell^4} |p > \)

\( c\bar{c} \) in Color Octet

Distribution peaks at equal rapidity (velocity)
Therefore heavy particles carry the largest momentum fractions

\( \hat{x}_i = \frac{m_{\perp i}}{\sum_j m_{\perp j}} \)

**High x charm!**

Hoyer, Peterson, Sakai, sbj
Measurement of Charm Structure Function


First Evidence for Intrinsic Charm

DGLAP / Photon-Gluon Fusion: factor of 30 too small
- EMC data: $c(x, Q^2) > 30 \times \text{DGLAP}$
  $Q^2 = 75 \text{ GeV}^2$, $x = 0.42$

- High $x_F \ pp \rightarrow J/\psi X$

- High $x_F \ pp \rightarrow J/\psi J/\psi X$

- High $x_F \ pp \rightarrow \Lambda_c X$

- High $x_F \ pp \rightarrow \Lambda_b X$

- High $x_F \ pp \rightarrow \Xi(ccd)X$ \text{(SELEX)}

**IC Structure Function: Critical Measurement for COMPASS**
SELEX $\Lambda_c^+$ Studies – $p_T$ Dependence

- $\Lambda_c^+$ production by $\Sigma^-$ vs $x_F$
  shows harder spectrum at low $p_T$ - consistent with an intrinsic charm picture.


Important to measure Nuclear Dependence
Leading Hadron Production from Intrinsic Charm

Coalescence of Comoving Charm and Valence Quarks Produce $J/\psi$, $\Lambda_c$ and other Charm Hadrons at High $x_F$
Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization!

\[
\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X)
\]

800 GeV p-A (FNAL) \( \sigma_A = \sigma_p A^\alpha \)

PRL 84, 3256 (2000); PRL 72, 2542 (1994)

open charm: no A-dep at mid-rapidity

\( x_F = x_1 - x_2 \)

Violation of factorization in charm hadroproduction.


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QCD on the LF

Stan Brodsky, SLAC
Intrinsic Charm Mechanism for Exclusive Diffraction Production

\[ p \ p \rightarrow J/\psi \ p \ p \]

\[ x_{J/\psi} = x_c + x_{\bar{c}} \]

**Exclusive Diffractive High-\(X_F\) Higgs Production**

Kopeliovitch, Schmidt, Soffer, sjb

Intrinsic \(c\bar{c}\) pair formed in color octet \(8_C\) in proton wavefunction
Collision produces color-singlet \(J/\psi\) through color exchange

**IC Explains large excess of quarkonia at large \(x_F\), A-dependence**

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QCD on the LF

Stan Brodsky, SLAC
Intrinsic Charm Mechanism for Exclusive Diffractive High-\(X_F\) Higgs Production.

\[ pp \rightarrow p + H + p \]

Also: intrinsic bottom, top

Kopeliovitch, Schmidt, Soffer, sjb

Higgs can have 80% of Proton Momentum!
Intrinsic Charm and Bottom Contribution to Inclusive Higgs Production

\[ \frac{d\sigma}{dx_F}(pp \rightarrow HX)[fb] \]

\[ \frac{d\sigma}{dx_F}(pp \rightarrow HX)[fb] \]

IC

IB

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QCD on the LF

Stan Brodsky, SLAC
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

Invariant under boosts. Independent of $P^\mu$

$$H_{QCD}^{LF} |\psi > = M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
AdS/CFT Holographic Model

\[ \psi(\sigma, b_\perp) \]

\[ |b_\perp|(\text{GeV}^{-1}) \]

\[ \sigma = ct - z \]

\[ \tau = t + z/c \]

3-dimensional photograph: meson LFWF at fixed LF Time
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond

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QCD on the LF

Stan Brodsky, SLAC
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrödinger Equation for Atomic Physics

- AdS/QCD Holographic Model
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Map $\text{AdS}_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal:** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \approx \text{const}$ at small $Q^2$

- **Use mathematical mapping of the conformal group** $SO(4,2)$ to AdS$_5$ space
Conformal window
Infrared fixed-point

\[ \beta(Q^2) = \frac{d\alpha_s(Q^2)}{d \log Q^2} \rightarrow 0 \]

\[ \alpha_s(Q^2) \]

PQCD Asymptotic freedom

Schwinger-Dyson

\[ \log_{10} Q^2 (\text{GeV}^2) \]

\[ \beta(Q^2) = \frac{d\alpha_s(Q^2)}{d \log Q^2} \rightarrow 0 \]

lattice: Furui, Nakajima (MILC)

DSE: Alkofer, Fischer, von Smekal et al.

Shirkov
Gribov
Dokshitser
Siminov
Maxwell
Cornwall

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QCD on the LF

Stan Brodsky, SLAC
Define QCD Coupling from Observable

\[ R_{e^+e^- \to X(s)} \equiv 3 \Sigma q e_q^2 \left[ 1 + \frac{\alpha R(s)}{\pi} \right] \]

\[ \Gamma(\tau \to Xe\nu)(m_{\tau}^2) \equiv \Gamma_0(\tau \to u\bar{d}e\nu) \times \left[ 1 + \frac{\alpha_{\tau}(m_{\tau}^2)}{\pi} \right] \]

Effective Charges: analytic at quark mass thresholds, finite at small momenta

Deur et al: Effective Charge from Bjorken Sum Rule
QCD Effective Coupling from hadronic $\tau$ decay

Menke, Merino, Rathsman, SJB

QCD on the LF

Stan Brodsky, SLAC
Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

\[ \Gamma_{\mu}^{p,n}(Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s}{\pi} \right] \]

**QCD on the LF**  
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Conformal behavior: \( Q^2 F_\pi(Q^2) \rightarrow \text{const} \)

\[ Q^4 F_1(Q^2) \rightarrow \text{const} \]

Determination of the Charged Pion Form Factor at \( Q^2 = 1.60 \) and 2.45 (GeV/c)^2.
e-Print Archive: nucl-ex/0607005

G. Huber

Generalized parton distributions from nucleon form-factor data.
e-Print Archive: hep-ph/0408173

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QCD on the LF

Stan Brodsky, SLAC
QCD Lagrangian

\[ L_{QCD} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{n_f} m_f \bar{\psi}_f \psi_f \]

**QCD:** \( N_C = 3 \)  
Quarks: 3  
Gluons: 8

\[ \alpha_s = \frac{g^2}{4\pi} \text{ is dimensionless} \]

**Classical Lagrangian is scale invariant for massless quarks**

If \( \beta = \frac{d\alpha_s(Q^2)}{d\log Q^2} = 0 \)

then QCD is invariant under conformal transformations

Parisi
\[ M_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

\[ \alpha(t) = \frac{\alpha(0)}{1-\Pi(t)} \]

Gell Mann-Low Effective Charge for QED
QED One-Loop Vacuum Polarization

\[ \Pi(Q^2) = \frac{\alpha(0)}{3\pi} \left[ \frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2}) \sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{1 - \sqrt{1 + \frac{4m^2}{Q^2}}} \right] \]

\[ \Pi(Q^2) = \frac{\alpha(0) \log Q^2}{3\pi m^2} \quad Q^2 \gg 4M^2 \]

\[ \beta = \frac{d(\frac{\alpha}{4\pi})}{d\log Q^2} = \frac{4}{3} (\frac{\alpha}{4\pi})^2 n \ell > 0 \]

\[ \Pi(Q^2) = \frac{\alpha(0) Q^2}{15\pi m^2} \quad Q^2 << 4M^2 \quad \text{Serber-Uehling} \]

\[ \beta \propto \frac{Q^2}{m^2} \quad \text{vanishes at small momentum transfer} \]

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QCD on the LF

Stan Brodsky, SLAC
Heuristic Argument for an IR Fixed Point

\[ \alpha_s(Q^2) \simeq \text{const at small } Q^2 \]

Confinement implies effective gluon mass or maximum wavelength:
- vacuum polarization vanishes at small momentum transfer

\[ \Pi(Q^2) \propto \frac{Q^2}{m_g^2} \quad Q^2 \ll 4m_g^2 \quad \alpha_s(Q^2) \simeq \text{const} \]

Analog of Serber-Uehling vacuum polarization in QED:

\[ \Pi(Q^2) = \frac{\alpha}{15\pi} \frac{Q^2}{m_e^2} \quad Q^2 \ll 4m_e^2 \]

Decoupling of long wavelength gluonic interactions
IR Fixed-Point for QCD?

- **Dyson-Schwinger Analysis:** QCD Coupling has IR Fixed Point
  Alkofer, Fischer, von Smekal et al.

- **Evidence from Lattice Gauge Theory** Furui, Nakajima

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

- Confined or massive gluons: **Decoupling of QCD vacuum polarization at small** $Q^2$
  Serber-Uehling

  $$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2}$$

  $$Q^2 << 4m^2$$

  $\ell^+$

- **Justifies application of AdS/CFT in strong-coupling conformal window**

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QCD on the LF

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Conformal symmetry: Template for QCD

• Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses

• Eigensolutions of ERBL evolution equation for distribution amplitudes

• Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

• Fix Renormalization Scale (BLM)

• Use AdS/CFT

V. Braun et al; Frishman, Lepage, Sachrajda, sjb
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \]

\[ x^\mu \rightarrow \lambda x^\mu, \quad z \rightarrow \lambda z, \] maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

\[ x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z. \]

\[ x^2 = x_\mu x^\mu: \text{invariant separation between quarks} \]

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
AdS/CFT

- Use mapping of conformal group SO(4,2) to AdS5

- Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension
  $$x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$$

- Holographic model: Confinement at large distances and conformal symmetry in interior $0 < z < z_0$

- Match solutions at small $z$ to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

- Truncated space simulates “bag” boundary conditions
  $$\psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$
- Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

- Goal: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances


- Karch, Katz, Son, Stephanov: Linear Confinement

- Mapping of AdS amplitudes to 3+1 Light-Front equations, wavefunctions

- Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H_{\text{LFQCD}}$; variational methods
AdS Schrödinger Equation for bound state of two scalar constituents

\[
\left[ -\frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)
\]

Truncated space

\[
V(z) = -\frac{1 - 4L^2}{4z^2}
\]

\[
\Phi(z) = z^{3/2} \phi(z)
\]

Alternative: Harmonic oscillator confinement

\[
V(z) = -\frac{1 - 4L^2}{4z^2} + \kappa^4 z^2
\]

Derived from variation of Action in AdS$_5$

Karch, et al.

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QCD on the LF

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Match fall-off at small $z$ to conformal twist dimension at short distances

- Pseudoscalar mesons: $O_{3+L} = \bar{\psi} \gamma_5 D_{\ell_1} \cdots D_{\ell_m} \psi$ ($\Phi_\mu = 0$ gauge).

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $M_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.
Light meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV

Guy de Teramond
SJB

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QCD on the LF
Stan Brodsky, SLAC
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6 \text{ GeV}$.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6 \text{ GeV}$. 

Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[ \left[ -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right] \phi_S(\zeta) = \mathcal{M}^2 \phi_S(\zeta) \]

with eigenvalues \( \mathcal{M}^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube): \( M_n^2(L) = 2\pi\sigma (n + L + 1/2) \).

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Fazio, Jugeau and Nicotri (2007).
\[ \nu = L \]

\[
\left( \frac{d^2}{d\zeta^2} + \frac{1 - 4\nu^2}{4\zeta^2} - \kappa^4 \zeta^2 - 2\kappa^2 (\nu + 1) + \mathcal{M}^2 \right) \phi_\nu(\zeta) = 0
\]

**LF Hamiltonian**

\[
H^\nu_{LF} \phi_\nu = \mathcal{M}^2_\nu \phi_\nu
\]

Bilinear

\[
H^\nu_{LF} = \Pi^\dagger_\nu \Pi_\nu,
\]

where

\[
\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} - \kappa^2 \zeta \right),
\]

and its adjoint

\[
\Pi^\dagger_\nu(\zeta) = -i \left( \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta \right),
\]

with commutation relations

\[
[\Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta)] = \frac{2\nu + 1}{\zeta^2} - 2\kappa^2.
\]
**AdS/CFT LF Equation for Mesons with HO Confinement.**

\[
\left( \frac{d^2}{d\zeta^2} + \frac{1 - 4\nu^2}{4\zeta^2} - \kappa^4 \zeta^2 - 2\kappa^2 (\nu + 1) + \mathcal{M}^2 \right) \phi_\nu(\zeta) = 0
\]

Define

\[
b^\dagger_\nu = -i \Pi_\nu = \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta
\]

\[
b_\nu = \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta
\]

Ladder Operator

\[
b^{\dagger}\nu \ket{\nu} = c_\nu \ket{\nu + 1}
\]

Integrable

\[
\left( - \frac{d}{d\zeta} + \frac{\nu + \frac{1}{2}}{\zeta} + \kappa^2 \zeta \right) \phi_\nu(\zeta) = c_\nu \phi_{\nu+1}(\zeta)
\]
Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3 +1 space to AdS5 space

Conformal behavior at short distances
+ Confinement at large distance

Boost Invariant 3+1 Light-Front Wave Equations

J = 0, 1, 1/2, 3/2 plus L

Holography

Integrable!

HADRON SPECTRA, WAVEFUNCTIONS, DYNAMICS

QCD on the LF

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Current Matrix Elements in AdS Space: The Form Factor

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell)$, $x^\ell = (x^\mu, z)$

$$i g_5 \int d^4x \, dz \, \sqrt{g} \, A^\ell(x, z) \Phi_P^* (x, z) \overleftrightarrow{\partial} \ell \Phi_P (x, z).$$

- Electromagnetic probe polarized along Minkowski coordinates ($Q^2 = -q^2 > 0$)

$$A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q, z), \quad A_z = 0.$$  

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[ z^2 \partial_z^2 - z \partial_z - z^2 Q^2 \right] J(Q, z) = 0,$$

subject to boundary conditions $J(Q = 0, z) = J(Q, z = 0) = 1$.

- Solution

$$J(Q, z) = z Q K_1(z Q).$$

- Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

$$\Phi_P (x, z) = e^{-iP \cdot x} \Phi(z), \quad \Phi(z) \to z^\Delta, \quad z \to 0.$$
Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons $\Phi_P$ and $\Phi_{P'}$, with the non-normalizable mode $J(Q, z)$ dual to external source [Polchinski and Strassler (2002)].

$$F(Q^2) = R^3 \int_0^{\Lambda_{QCD}^{-1}} \frac{dz}{z^3} \Phi(z) J(Q, z) \Phi(z).$$

Since $K_n(x) \sim \sqrt{\frac{\pi}{2x}} e^{-x}$, the external source is suppressed inside AdS for large $Q$. Important contribution to the integral is from $z \sim 1/Q$, where $\Phi \sim z^\Delta$.

For large $Q^2$

$$F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\Delta^{-1}},$$

and the power-law ultraviolet point-like scaling is recovered [Polchinski and Susskind (2001)].

Fig: Suppression of external modes for large $Q$ inside AdS. Red curves: $J(Q, z)$, black: $\Phi(z)$. 

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QCD on the LF  
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Holographic Model for QCD Light-Front Wavefunctions

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi^*_P(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find (b = \( |\vec{b}_\perp| \)):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^{\infty} b \, db \, J_0(bqx) \, |\tilde{\psi}(x, b)|^2, \quad \text{Soper} \]
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[
F(q^2) = 2\pi \int_0^1 dx \frac{1 - x}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1 - x}{x}} \right) \tilde{\rho}(x, \zeta),
\]

with \(\tilde{\rho}(x, \zeta)\) QCD effective transverse charge density.

- Transversality variable

\[
\zeta = \sqrt{\frac{x}{1 - x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|.
\]

- Compare AdS and QCD expressions of FFs for arbitrary \(Q\) using identity:

\[
\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1 - x}{x}} \right) = \zeta Q K_1(\zeta Q),
\]

the solution for \(J(Q, \zeta) = \zeta Q K_1(\zeta Q)\)!
Identical DYW and AdS5 Formulae: Two-parton case

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{\text{max}} = \Lambda_{\text{QCD}}^{-1}} \zeta \, d\zeta \, J_0 \left( \frac{\zeta Q x}{\sqrt{x(1-x)}} \right) |\tilde{\psi}(x, \zeta)|^2,$$

- Compare with AdS form factor for arbitrary $Q$. Find:

$$J(Q, \zeta) = \int_0^1 dx J_0 \left( \frac{\zeta Q x}{\sqrt{x(1-x)}} \right) = \zeta Q K_1(\zeta Q), \quad \zeta \leftrightarrow z$$

the solution for the electromagnetic potential in AdS space, and

$$\tilde{\psi}(x, \vec{b}_\perp) = \frac{\Lambda_{\text{QCD}}}{\sqrt{\pi} J_1(\beta_{0,1}) \sqrt{x(1-x)} J_0 \left( \sqrt{x(1-x)} |\vec{b}_\perp| \beta_{0,1} \Lambda_{\text{QCD}} \right) \theta \left( \frac{\vec{b}_\perp^2}{x(1-x)} \right)}$$

the holographic LFWF for the valence Fock state of the pion $\psi_{qq}/\pi$.

- The variable $\zeta$, $0 \leq \zeta \leq \Lambda_{\text{QCD}}^{-1}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$!