Higher Curvature Effects in the ADD and RS Models

Work in progress

- ADD + RS (classic) have many common features:
  - (i) Localized SM fields on a boundary
  - (ii) Bulk has constant curvature: ADD (Minkowskian), RS (AdS_5)
  - (iii) Gravity in bulk described by Einstein-Hilbert action

$$S = \frac{M^{D-2}}{2} \int d^D x \sqrt{g} \left\{ R \, (+ \text{constant}) \right\}$$

fundamental scale — Ricci scalar — possible bulk cosm. const.

∴ How is ADD/RS phenomenology altered if we give up (iii) + consider more general actions ?? i.e,

$\boxed{R \to F}$ ... a well-behaved function of invariants

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* Why do this ??

- in both models $\sqrt{s} \sim M_{(eff)}$ are probed + we know EH is an effective theory $< M$... so 'correction' terms should be present..

- strings predict such terms sub-leading in $1/M^2$ etc

- such terms have been considered for other reasons e.g, cosmology/dark energy issues

Here, to be tractable, we restrict ourselves to

$$F(R, P, Q) \quad : \quad P \equiv R_{AB} R^{AB} \; ; \; Q \equiv R_{ABCD} R^{ABCD}$$

- a fairly general case .. Ricci tensor Curvature tensor

.. which has been considered in cosmo studies...

* What do we want to know ???

→ graviton KK properties : masses, wave functions, matter couplings, propagators, ...

(not, e.g, self-couplings) of the gravitons

↑↑ forces us to consider cubic etc terms

Then, to obtain these quantities in a <u>constant curvature background</u>[#] (which we have here)

* It is sufficient to expand F to second order in the invariants:

background value

$$\boxed{F = F_0 + \sum_i (x_i - x_{i0}) F_{x_i} + \frac{1}{2}\sum_{ij} (x_i - x_{i0})(x_j - x_{j0}) \cdot F_{x_i x_j} + \text{higher order dropped terms}}$$

$x_i = (R, P, Q)$

$\partial_{x_i} F \big|_{\text{background}}$

background value

$$\boxed{S_{eff} \rightarrow \frac{M^{D-2}}{2} \int d^D x \sqrt{g} \left\{ \Lambda + a_1 R + a_2 R^2 + a_3 C + a_4 GB \right\}}$$

Weyl scalar $\equiv C_{ABCD} C^{ABCD}$

Gauss-Bonnet term

... where $\{\Lambda, a_i\}$ are functions of F_{x_i}, $F_{x_i x_j}$, F_0 +

$$R_0 \left(= \langle R \rangle_{\text{background}}\right) \begin{cases} = 0 & \text{in ADD} \\ = -20 k^2 & \text{in interval RS} \end{cases}$$

[Next]

$$\Rightarrow \boxed{GB \equiv R^2 - 4 R_{AB} R^{AB} + R_{ABCD} R^{ABCD} \equiv \underline{R^2 - 4P + Q}}$$

\# { to be precise, in a maximally symmetric background }

Without making any further assumptions we obtain

$$
\begin{aligned}
\Lambda &= F_0 - R_0 F_R + R_0^2(F_{RR}/2 - \sigma F_P - \tau F_Q) + R_0^3(\sigma F_{PR} + \tau F_{QR}) \\
&+ R_0^4(2\sigma\tau F_{PQ} + \sigma^2 F_{PP} + \tau^2 F_{QQ})/2 \\
a_1 &= F_R - R_0 F_{RR} - R_0^2(\sigma F_{RP} + \tau F_{RQ}) \\
a_2 &= \beta F_P + \epsilon F_Q + F_{RR}/2 - R_0(\beta F_{RP} + \epsilon F_{RQ}) - R_0^2[(\tau\beta + \epsilon\sigma)F_{PQ} + \sigma\beta F_{PP} + \tau\epsilon F_{QQ}] \\
a_3 &= \alpha F_P + \delta F_Q - R_0(\alpha F_{RP} + \delta F_{RQ}) - R_0^2[(\tau\alpha + \delta\sigma)F_{PQ} + \sigma\alpha F_{PP} + \tau\delta F_{QQ}] \\
a_4 &= -\alpha F_P + \gamma F_Q + R_0(\alpha F_{RP} - \gamma F_{RQ}) + R_0^2[(\tau\alpha - \gamma\sigma)F_{PQ} + \sigma\alpha F_{PP} - \tau\gamma F_{QQ}], \qquad (11
\end{aligned}
$$

where we have defined $\sigma = (n+4)^{-1}$, $\tau = 2(n+4)^{-1}(n+3)^{-1}$, $\delta = 4\alpha = (n+2)/(n+1)$ $4\beta = (n+4)/(n+3)$, $\gamma = -(n+1)^{-1}$ and $\epsilon = (n+3)^{-1}$. For the case of $n = 0$ this reproduces th

¶Note that the quadratic terms in the Taylor expansion naturally involve factors of P^2 and Q^2 which are actuall fourth order in the (dynamical) curvature; we drop these terms for consistency in the discussion which follows.

Field Content (in D-dimensions!)

- massless tensor field (usual gravitons etc)
- massive tensor ghost (Yikes!)
- massive scalar (tachyonic?)

.. many ways to see this ...

$\Longrightarrow$ Consider gravitons being exchanged between localized SM sources (4D): $\boxed{T_{\mu\nu}}$. Then, eg, in

ADD (before KK-sums) (n extra dims)

$$\mathcal{A} = \text{"}\ \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+2)}{k^2 - m_n^2}\ ^{**}$$

usual KK masses

this is the usual "graviton" exchange structure (eg, GRW) [gravitons + graviscalars]

ghost!! wrong sign!

$$- \frac{T_{\mu\nu}T^{\mu\nu} - T^2/(n+3)}{k^2 - (m_n^2 + m_T^2)}$$

$\rightarrow$ massive in bulk tensor field

$$+ \frac{T^2/(n+2)(n+3)}{k^2 - (m_n^2 + m_s^2)}\ \text{"}$$

bulk mass for scalar field

- Remove tachyons $\therefore$ $m_s^2 > 0$ (demanded)
- Remove ghosts $\therefore$ $m_T^2 \to \infty$ $\to F(R, P-4Q)$ only

$\hookrightarrow \boxed{a_3 = 0}$

** $\boxed{T = \eta_{\mu\nu} T^{\mu\nu}}$

.. a similar requirement in RS : $\boxed{F(R, P-4Q)}$

<u>ADD</u> : $\Lambda = 0 \rightarrow R_0, F_0 = 0$ (flat space)

$$F \rightarrow \underbrace{F_R}_{>0 \text{ (no KK ghosts)}} R + \left\{ -F_Q + \frac{1}{2} F_{RR} \right\} R^2 + F_Q \cdot G$$

$$\boxed{m_s^2 = \frac{(n+2) F_R}{4(n+3) \underbrace{\left(F_{RR}/2 - F_Q \right)}_{\geq 0 \text{ (no tachyons)}}}}$$

Note in "GB gravity"
$F_{RR}/2 - F_Q = 0$ so
$m_s^2 \rightarrow \infty$ (<u>removed</u>)

- m_s is naturally $O(M)$ so a <u>new</u> KK spectrum of scalars begins at ~TeV

(Demir + Tanyildız '05)

?? effect ??

Small as " $T^2 / T_{\mu\nu} T^{\mu\nu}$ " $\sim \left(m^2_{external}/s \right)^2$ <u><< 1</u> at LHC / ILC ...

$$\rightarrow \boxed{\bar{M}_{pl}^2 = V_n M^{n+2} \underline{F_R}} \quad (F_R > 0 \text{ recall})$$

from the zero mode graviton wavefunction normalization...

* <u>If</u> $V_n = (2\pi R_c)^n$, R_c <u>shifts</u> for fixed M (+ $\bar{M}_{pl}$) as input.

- For fixed M, KK masses are shifted ...

$$\bar{M}_{pl}^2 = (2\pi R_c)^n M^{n+2} F_R$$

fixed fixed

$$\rightarrow \quad R_c \rightarrow R_c F_R^{-1/n} \quad \underline{\text{so}}$$

$$\boxed{m_{KK} \rightarrow m_{KK} F_R^{+1/n}}$$

- <u>in units of M</u> graviton emission cross-sections are modified:

$$\boxed{d\sigma_{ADD} \rightarrow \underline{F_R}^{-1}\, d\sigma_{ADD}\,(M^2, s, t, u)}$$

- Similarly, graviton <u>exchange</u> amplitudes (neglecting the new scalars!) will

$$\boxed{\mathcal{A}_{KK} \Rightarrow F_R^{-1}\, \mathcal{A}_{KK}}$$

- we expect F_R to be $O(1)$ in most models ...

(<u>Note</u> $F_R = 1$ in flat, R polynomial models)

$\Rightarrow$ $O(1)$ modifications of standard ADD ..

* RS on an interval

Λ_b = bulk cosmo const.

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

$\uparrow$ warp factor

$\boxed{k \sim M}$

$$\boxed{R_0 = \langle R\rangle_{AdS_5} = -20k^2}$$

$R \to F(R, P-4Q)$

Trace of Einstein's Equation $\to$

$$\boxed{224k^4 F_Q + 8k^2 F_R + F_0 = 2\Lambda_b/M^3}$$ constraint

$\hookrightarrow k(M, \Lambda_b)$ is a derived parameter + sets the KK mass scale..

ex. $\boxed{F = R + \beta R^2/M^2}$

$$\to \quad k^2 = \frac{3M^2}{40\beta}\left\{1 \pm \left(1 + \frac{40\Lambda_b}{9M^5}\beta\right)^{1/2}\right\}$$ two roots!

$$\to \quad -\Lambda_b/6M^3 \quad \text{as } \beta \to 0 \text{ (negative root)}$$

(usual RS relationship)

$$S_{eff} = \int d^5x\,\sqrt{-g}\left\{-\Lambda_b + a_1\frac{M^3}{2}R + \frac{\alpha M}{2}G + \frac{\beta M}{2}R^2\right\}$$

dimensionless coefficients

e.g.,

$$\frac{\beta}{M^2} = \underbrace{-F_Q + \tfrac{1}{2}F_{RR}}_{\text{ADD result}} - 20k^2 F_{RQ} - 280k^4 F_{QQ} \qquad \text{etc}$$

Scalar gets a bulk mass :

$$\boxed{m_s^2 = \frac{3a_1}{16\beta} M^2}$$

$$= \frac{3}{8} \quad \frac{F_R + 20k^2 F_{RR} + 280k^4 F_{RQ}}{F_{RR} - 2F_Q - 40k^2 F_{RQ} - 560k^4 F_{QQ}}$$

KK Scalar spectrum : $(2-\nu)\, J_\nu(x_{s_n}) + x_{s_n} J_{\nu-1}(x_{s_n}) = 0$

$\nu^2 = 4 + m_s^2/k^2$ is large

$$\rightarrow \boxed{m_{s_n} = \underbrace{x_{s_n}}\, k\, e^{-\pi k r_c}} \qquad \boxed{x_{s_0} = ?}$$

- If $\beta/a_1 = 1$, $k/M = 0.05$ $\rightarrow \frac{m_s}{k} \simeq 8.7$ $\rightarrow x_{s_0} \simeq 11$

$(x_1^{grav} = 3.83)$ $\rightarrow \approx$ $\boxed{3\times \text{ heavier}}$ than 1^{st} graviton KK [Fig]

... as in ADD these scalars are more weakly coupled than gravitons by a factor

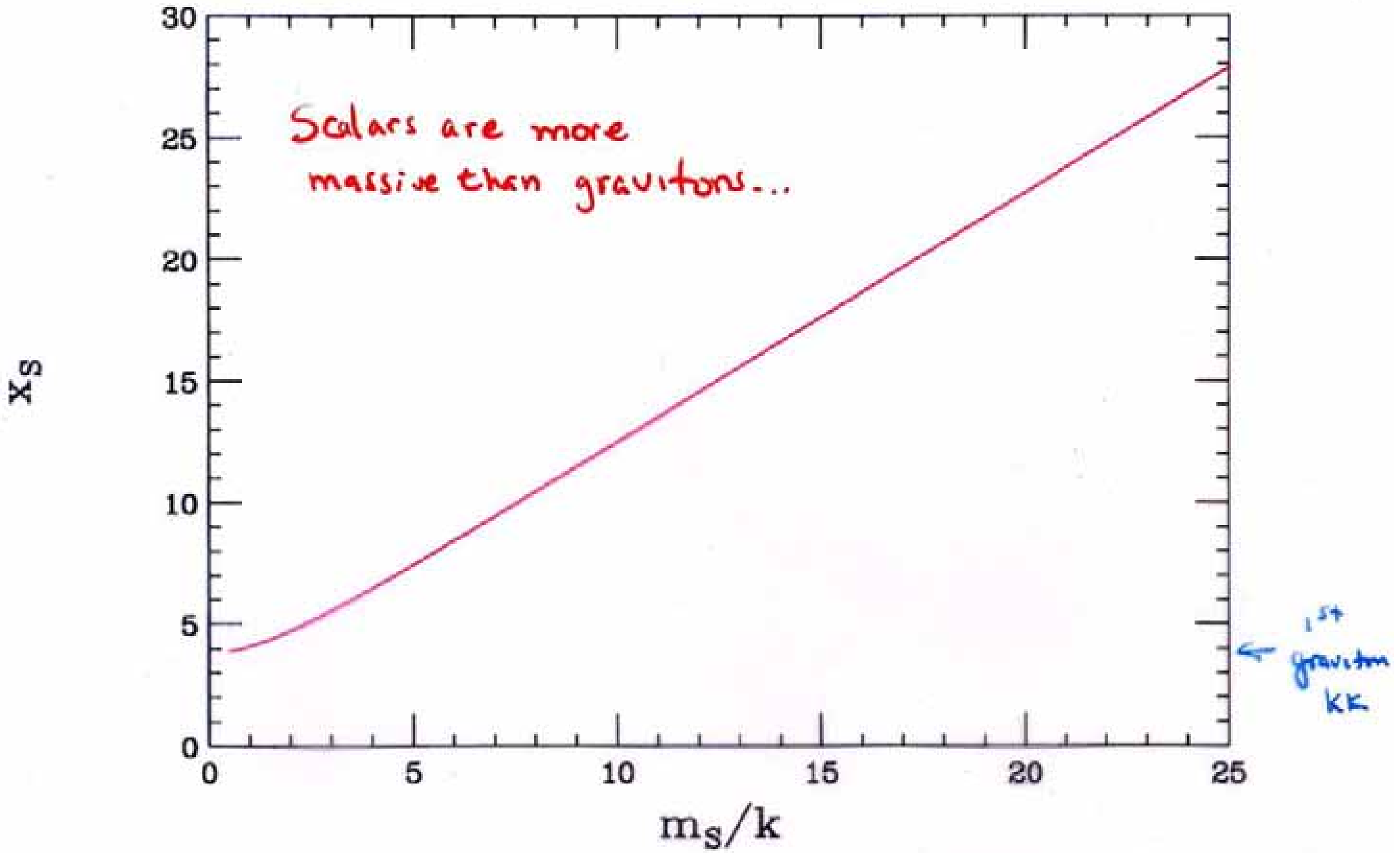
$\sim \left(m_{ext}^2/12s\right)$ in amplitude [Fig]

Graviton sector

$$\boxed{\bar{M}_{Pl}^2 = \frac{M^3}{k} \cdot \mathcal{H}}$$

$$\boxed{\mathcal{H} = F_R + 36k^2 F_Q + 1000k^4 F_{RQ} + 10080k^6 F_{QQ}}$$

Root For Lightest Scalar State

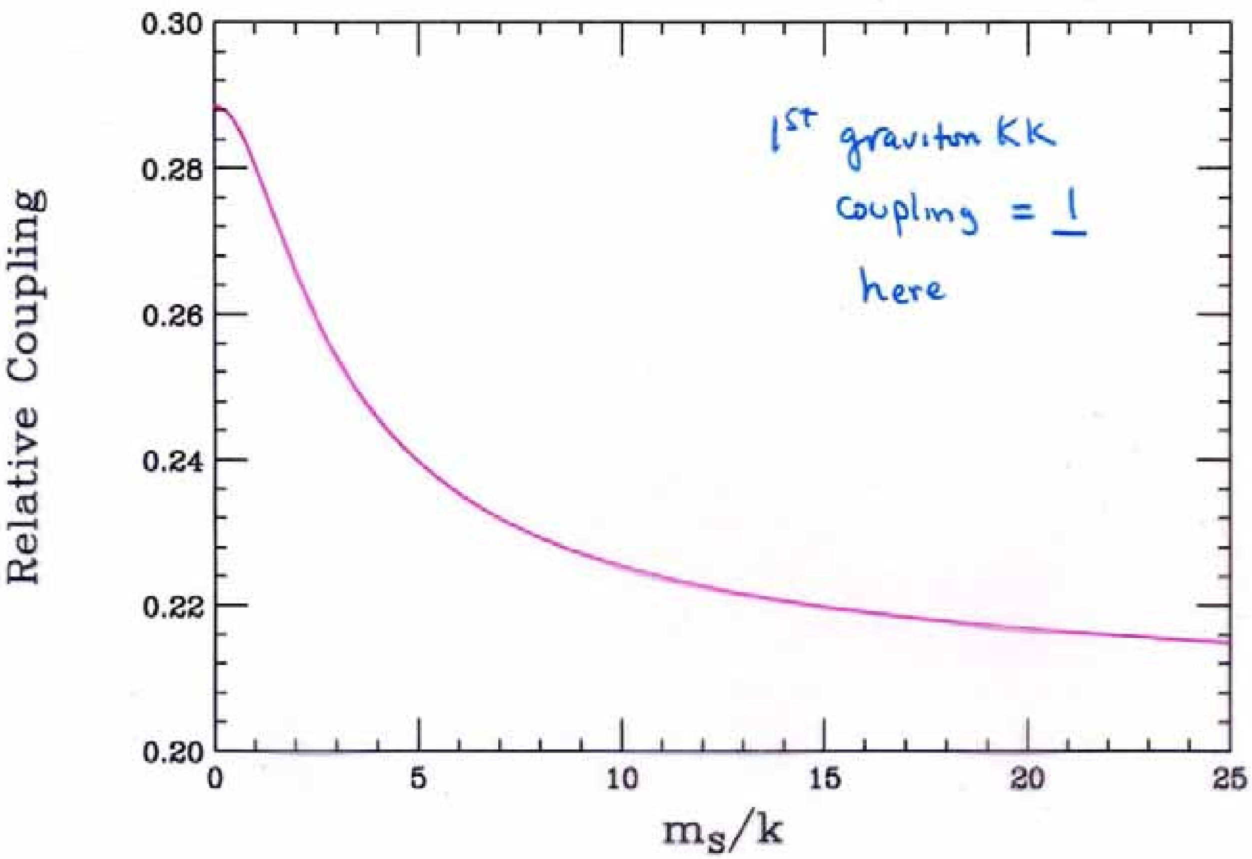


Bulk mass

Note:

- With $\bar{M}_{Pl}^2 = \frac{M^3}{k} H(k)$ and $k = k(\Lambda, F)$ the graviton KK masses expressed as $m_n = x_n k e^{-k\pi r_c}$ are left invariant ... but k changes as a function of input parameters ∴ shifting the KK masses

Scalars couple more weakly than gravitons



Bulk mass

$$\left(\mathcal{L}^{(n)}_{scalar} \equiv \frac{1}{\Lambda_\pi} (RC)\, T\, S_{(n)} \right)$$

Example of graviton KK mass shift:

$$\Rightarrow \int d^5x \sqrt{g} \left(\frac{M^3}{2} R - \Lambda_0 \right) \to \int d^5x \sqrt{g} \left[\frac{M^3}{2} \left(R + \underline{\frac{\beta}{M^2}} R^2 \right) - \underline{\Lambda} \right]$$

In standard RS: $k^2 \equiv k_0^2 = -\Lambda_0 / 6M^3$ as above.

$\Rightarrow$ $c \equiv k_0 / \overline{M}_{pl}$ is a conventional model parameter

$\simeq 0.01 - 0.10$

Then

$$R = \frac{m_{KK}}{m_{KK}^{(0)}} = \left(80 \beta c^{4/3}\right)^{-1} \left[-1 + \left(1 + 160 \beta c^{4/3}\right)^{1/2} \right]$$

usual KK mass

$\Rightarrow$ [plot]

$\Rightarrow$ Significant shifts in mass!

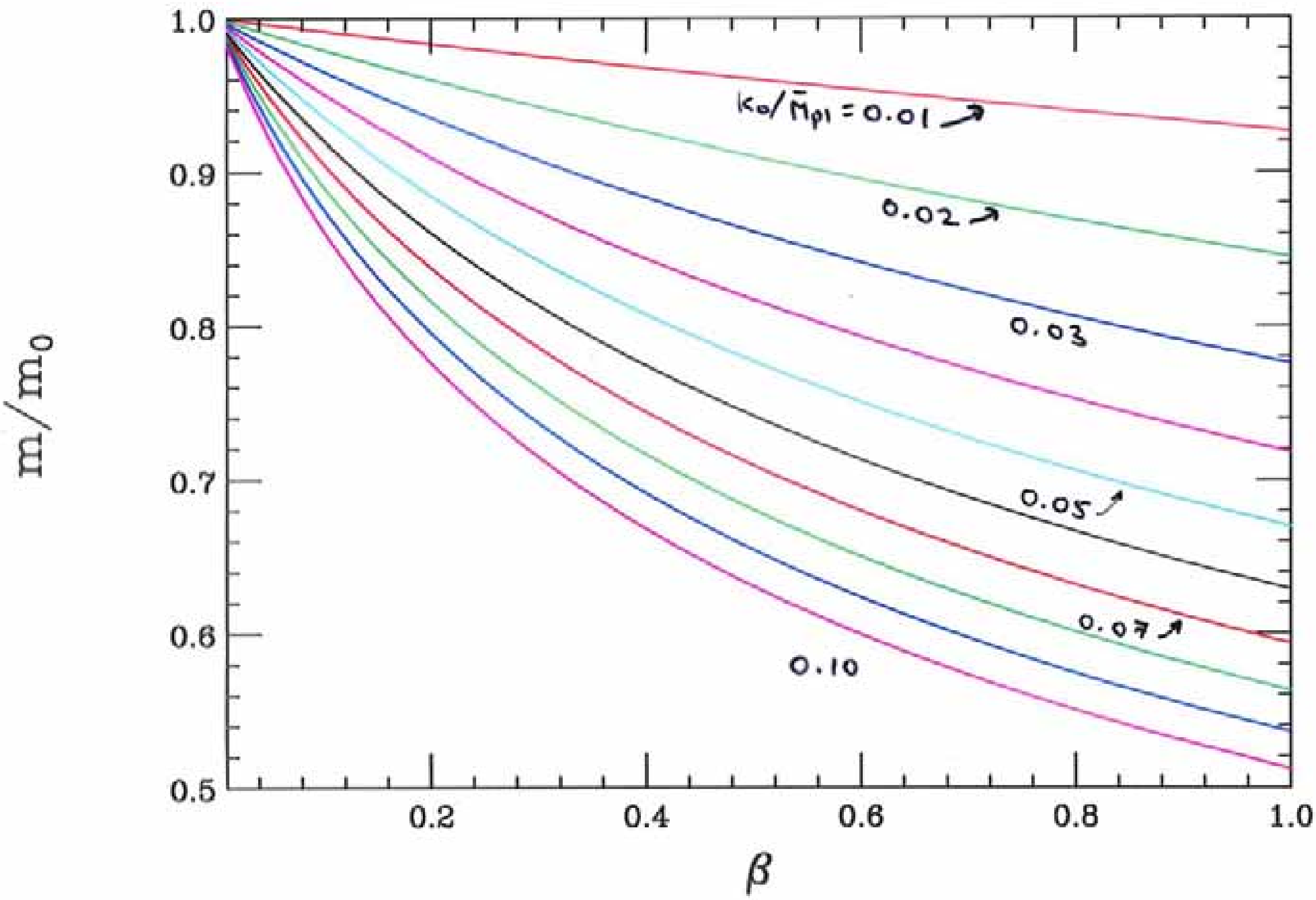
Similarly

$$\frac{\Lambda}{\Lambda_0} = -3 \left(80 \beta c^{4/3}\right)^{-1} \left[\left(1 - \frac{40}{3} \beta c^{4/3} R^2 \right)^2 - 1 \right]$$

... is also shifted to maintain consistency

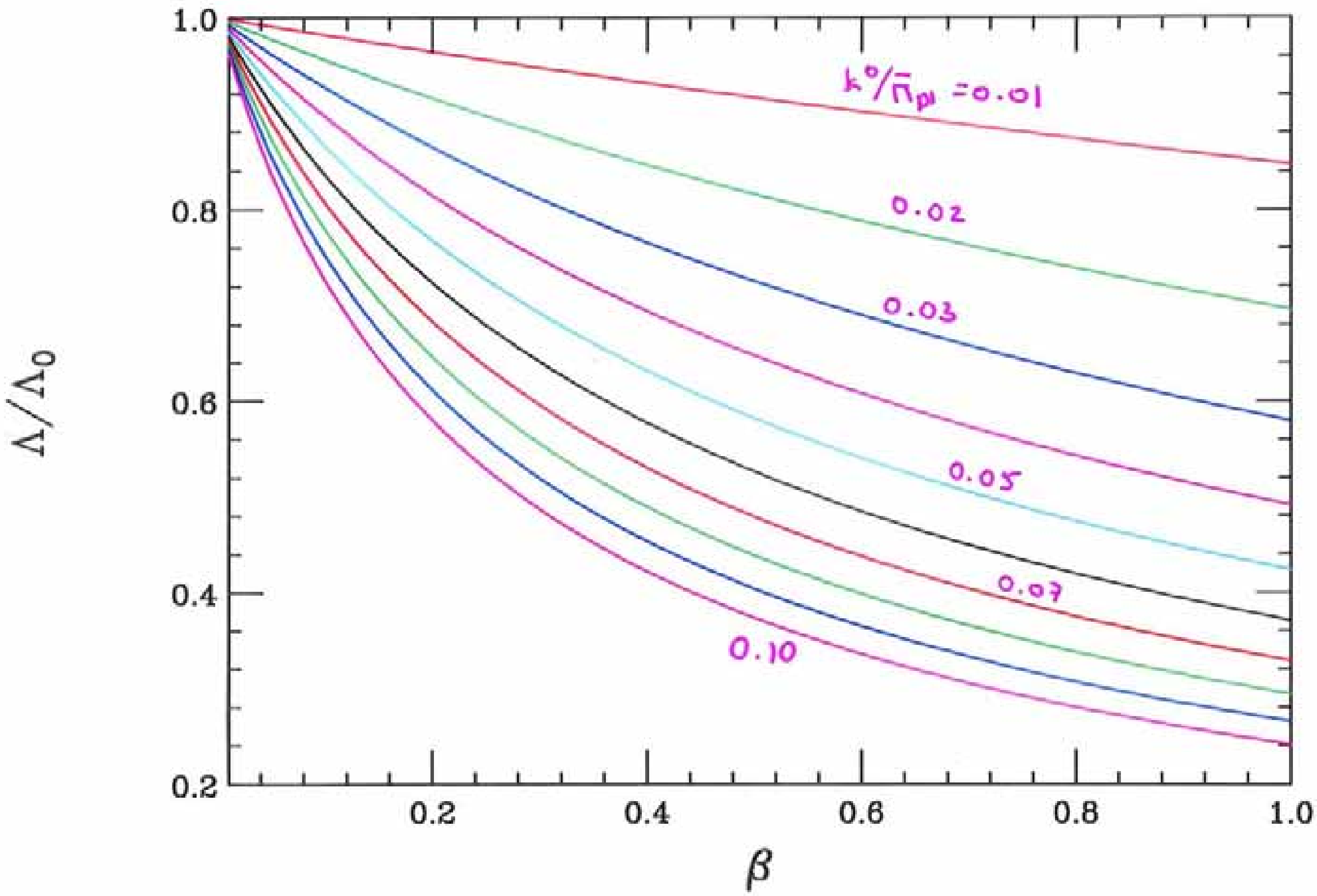
$\Rightarrow$ [plot]

KK mass shifts



... quite sizeable!

Λ shifts



very sizeable

Summary / Conclusions

- It is possible to obtain ADD + RS-like sol's from more general gravity actions
- These lead to <u>subtle</u> alterations in the model predictions → scalar KK towers (<u>not</u> Higgs KK's)

⇒ Besides alterations in model relationships, these include rescaling of "classic" predictions involving graviton KK states

- Experimental observation of any of these effects provides info on a more fundametal theory than EH...

⇒ Work in progress...