AdS/QCD

Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

\[ J(Q, z) = zQ K_1(zQ) \]

High \( Q^2 \) from small \( z \sim 1/Q \)

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi^{(n)} \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1}, \]

where \( \tau = \Delta_n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).

Polchinski, Strassler
de Teramond, sjb
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

Data Compilation from Baldini, Kloe and Volmer

Harmonic Oscillator Confinement

Truncated Space Confinement

One parameter - set by pion decay constant.

G. de Teramond, sjb
\( F_\pi(q^2) \)

Harmonic Oscillator
Confinement scale set by pion decay constant

\( \kappa = 0.38 \text{ GeV} \)
Consider the spin non-flip form factors in the infinite wall approximation

\[ F_+ (Q^2) = g_+ R^3 \int \frac{d\zeta}{\zeta^3} J(Q, \zeta) |\psi_+(\zeta)|^2, \]

\[ F_- (Q^2) = g_- R^3 \int \frac{d\zeta}{\zeta^3} J(Q, \zeta) |\psi_-(\zeta)|^2, \]

where the effective charges \( g_+ \) and \( g_- \) are determined from the spin-flavor structure of the theory.

Choose the struck quark to have \( S^z = +1/2 \). The two AdS solutions \( \psi_+(z) \) and \( \psi_-(z) \) correspond to nucleons with \( J^z = +1/2 \) and \(-1/2\).

For \( SU(6) \) spin-flavor symmetry

\[ F_1^p (Q^2) = R^3 \int \frac{dz}{z^3} J(Q, z) |\psi_+(z)|^2, \]

\[ F_1^n (Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q, z) \left[ |\psi_+(z)|^2 - |\psi_-(z)|^2 \right], \]

where \( F_1^p (0) = 1, \ F_1^n (0) = 0. \)

Large \( Q \) power scaling: \( F_1 (Q^2) \rightarrow \left[ 1/Q^2 \right]^2. \)
$$F_1^p(Q^2)$$

Harmonic Oscillator Confinement

Truncated Space Confinement

$$\kappa = 0.424 \text{ GeV}$$

$$\Lambda = 0.2 \text{ GeV}$$

Current modified by metric

$$F_1(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F^\uparrow(z) J(Q, z) \Phi_I^\uparrow(z)$$
Prediction for $Q^4 F^m_1(Q^2)$ for $\Lambda_{\text{QCD}} = 0.21$ GeV in the hard wall approximation. Data analysis from Diehl (2005).
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

Harmonic Oscillator
Confinement
Normalized to anomalous moment

$F_2^p(Q^2)$

$\kappa = 0.49 \text{ GeV}$

G. de Teramond, sjb
Light-Front Wavefunctions

\[ P^+ = p^0 + p^z \]

\[ P^+, \vec{P}_\perp \]

Fixed \( \tau = t + z/c \)

\[ \sum_i x_i = 1 \]

\[ \sum_i \vec{k}_\perp = \vec{0}_\perp \]

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

Invariant under boosts! Independent of \( P^\mu \)
• Define effective single particle transverse density by (Soper, Phys. Rev. D 15, 1141 (1977))

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{\eta}_\perp e^{i\vec{\eta}_\perp \cdot \vec{q}_\perp} \tilde{\rho}(x, \vec{\eta}_\perp) \]

• From DYW expression for the FF in transverse position space:

\[
\tilde{\rho}(x, \vec{\eta}_\perp) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 \vec{\eta}_j \delta(1 - x - \sum_{j=1}^{n-1} x_j) \delta^{(2)}(\sum_{j=1}^{n-1} x_j \vec{\eta}_j - \vec{\eta}_\perp) |\psi_n(x_j, \vec{\eta}_j)|^2
\]

• Compare with the the form factor in AdS space for arbitrary \( Q \):

\[
F(Q^2) = R^3 \int_0^\infty \frac{dz}{z^3} e^{3A(z)} \Phi_P(z) J(Q, z) \Phi_P(z)
\]

• Holographic variable \( z \) is expressed in terms of the average transverse separation distance of the spectator constituents \( \vec{\eta} = \sum_{j=1}^{n-1} x_j \vec{b}_\perp j \)

\[
z = \sqrt{\frac{x}{1-x} |\sum_{j=1}^{n-1} x_j \vec{b}_\perp j|}
\]
\[ \psi(x, b_\perp) \quad \leftrightarrow \quad \phi(z) \]

\[ \zeta = \sqrt{x(1-x)b_\perp^2} \]

\[ \psi(x, \zeta) = \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta) \]

**Holography:** Unique mapping derived from equality of LF and AdS formula for current matrix elements
Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic radial equation:

\[ \left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta) \]

\[ \zeta^2 = x(1-x)b_\perp^2. \]

Effective conformal potential:

\[ V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2}. \]

G. de Teramond, sjb

G. de Teramond, sjb
Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

\[ \left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta) \]

\[ \zeta^2 = x(1 - x)b^2_\perp. \]

Effective conformal potential:
\[ V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2. \]

Confining potential:

G. de Teramond, sjb

University of Southern Denmark
Odense May 5, 2008

Stan Brodsky, SLAC/IPPP
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k^2_\perp) \]

Harmonic Oscillator model

\[ \phi_M(x, Q_0) \propto \sqrt{x(1 - x)} \]

University of Southern Denmark
Odense May 5, 2008

AdS/QCD

Stan Brodsky, SLAC/IPPP
AdS/CFT Predictions for Meson LFWF $\psi(x, b_\perp)$

$\Lambda_{QCD} = 0.32$ GeV

$\kappa = 0.76$ GeV

Truncated Space

Harmonic Oscillator
Two-quark holographic LFWF in impact space \( \psi(x, \zeta) \): (a) \( \ell = 0, \ k = 1 \); (b) \( \ell = 1, \ k = 1 \); (c) \( \ell = 0, \ k = 2 \). The variable \( \zeta \) is the holographic variable \( z = \zeta = |b_\perp| \sqrt{x(1-x)} \).
LFWF peaks at

\[ x_i = \frac{m_{\perp i}}{\sum_j^n m_{\perp j}} \]

where

\[ m_{\perp i} = \sqrt{m^2 + k_{\perp}^2} \]

minimum of LF energy denominator

\[ \kappa = 0.375 \text{ GeV} \]

\[ m_a = m_b = 1.25 \text{ GeV} \]
\[
|\pi^+ \rangle = |ud\bar{d}\rangle \\
m_u = 2 \text{ MeV} \\
m_d = 5 \text{ MeV}
\]

\[
|K^+ \rangle = |u\bar{s}\rangle \\
m_s = 95 \text{ MeV}
\]

\[
|D^+ \rangle = |c\bar{d}\rangle \\
m_c = 1.25 \text{ GeV}
\]

\[
|\eta_c \rangle = |c\bar{c}\rangle
\]

\[
|B^+ \rangle = |u\bar{b}\rangle \\
m_b = 4.2 \text{ GeV}
\]

\[
|\eta_b \rangle = |b\bar{b}\rangle \\
\kappa = 375 \text{ MeV}
\]
Meson LFWF ($L=0$) for massive quarks

\[
\psi_{\bar{q}q}/\pi(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} \exp\left(-\frac{k^2 + m^2}{2\kappa^2 x(1-x)}\right),
\]

\[
\tilde{\psi}_{q\bar{q}}/\pi(x, b_\perp) = \frac{\kappa}{\sqrt{\pi}} \sqrt{x(1-x)} \exp\left(-\frac{1}{2} \kappa^2 x(1-x)b^2_\perp - \frac{m^2}{2\kappa^2 x(1-x)}\right)
\]

Key variable for n-parton LFWF with massive quarks:

\[
\chi^2 = \zeta^2 + \frac{1}{\kappa^4} \sum_{i=1}^{n} \frac{m_i^2}{x_i}, \quad \zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_\perp j \right|
\]
Hadronization at the Amplitude Level

$\tau = x^+$

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

$\psi(x, \vec{k}_\perp, \lambda_i)$

$e^+ e^- \gamma^* g \bar{q} q\gamma^* e q q q g g g$
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Event amplitude generator

$\tau = x^+$

e$^+$

e$^-$

$\gamma^*$

$\bar{q}$

$g$

$x, \vec{k}_\perp$

$1 - x, -\vec{k}_\perp$

$\psi(x, \vec{k}_\perp, \lambda_i)$
Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb

Formation of Relativistic Anti-Hydrogen

Coalescence of off-shell co-moving positron and antiproton.

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level
Light-Front Wavefunctions

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n k_{\perp i} = \vec{0}_{\perp} \]

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Invariant under boosts! Independent of \( P^\mu \)

Fixed \( \tau = t + z/c \)

\[ x_i P^+, x_i \vec{P}_{\perp} + \vec{k}_{\perp i} \]

\[ P^+ = P^0 + P^z \]
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs

Baryon Production

$\tau = x^+$
Jet Hadronization at the Amplitude Level

\[ \tau = t + z/c \]

Event amplitude generator

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via Light-Front Wavefunctions

AdS/QCD model

\[ \psi_\pi(y, \vec{\ell}_\perp) \]

coalesce if \( M^2 < \Lambda^2_{QCD} \)

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AdS/QCD

Stan Brodsky, SLAC/IPPP
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Higher Fock State Coalescence  \[ |uuds\bar{s}\rangle \]

Asymmetric Hadronization! \[ D_s \rightarrow p(z) \neq D_s \rightarrow \bar{p}(z) \]

B-Q Ma, sjb
\[ D_{s \to p}(z) \neq D_{s \to \bar{p}}(z) \]

\[
A_s^{p\bar{p}}(z) = \frac{D_{s \to p}(z) - D_{s \to \bar{p}}(z)}{D_{s \to p}(z) + D_{s \to \bar{p}}(z)}
\]

Consequence of \( s_p(x) \neq \bar{s}_p(x) \quad \mid uuds\bar{s} \mid \gtrsim \mid K^+ \Lambda \mid \)
AdS/CFT and QCD

- Non-Perturbative Derivation of Dimensional Counting Rules (Strassler and Polchinski)
- Light-Front Wavefunctions: Confinement at Long Distances and Conformal Behavior at short distances (de Teramond and Sjb)
- Power-law fall-off at large transverse momenta
- Hadron Spectra, Regge Trajectories
Features of Holographic Model
de Teramond sjb

• Ratio of proton to Delta trajectories = ratio of zeroes of Bessel functions.

• Scale $\Lambda_{\text{QCD}}$ determines hadron spectrum (slightly different for mesons and baryons)

• Covariant version of bag model: confinement + conformal symmetry

• Pion decay constant

• Dominance of Quark Interchange
Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus
Nucleus left Intact!

\[ M \propto \frac{\partial^2}{\partial^2 k_{\perp}} \psi_{\pi}(x, k_{\perp}) \]
Key Ingredients in E791 Experiment

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency

\[ M_A = A \, M_N \]

\[ \frac{d\sigma}{dt}(\pi A \rightarrow q\bar{q}A') = A^2 \, \frac{d\sigma}{dt}(\pi N \rightarrow q\bar{q}N') \, F^2_A(t) \]

Target left intact

Diffraction, Rapidity gap
• Fully coherent interactions between pion and nucleons.

• Emerging Di-Jets do not interact with nucleus.

\[ \mathcal{M}(A) = A \cdot \mathcal{M}(N) \]

\[ \frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0 \]

\[ \sigma \propto A^{4/3} \]

Nuclear coherence

\[ F_A^2(q_{\perp}^2) \sim e^{-\frac{1}{3} R_A^2 q_{\perp}^2} \]
Measure pion LFWF in diffractive dijet production

Confirmation of color transparency

A-Dependence results: \( \sigma \propto A^\alpha \)

<table>
<thead>
<tr>
<th>( k_t ) range (GeV/c)</th>
<th>( \alpha )</th>
<th>( \alpha ) (CT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25 &lt; ( k_t ) &lt; 1.5</td>
<td>1.64 +0.06 -0.12</td>
<td>1.25</td>
</tr>
<tr>
<td>1.5 &lt; ( k_t ) &lt; 2.0</td>
<td>1.52 ± 0.12</td>
<td>1.45</td>
</tr>
<tr>
<td>2.0 &lt; ( k_t ) &lt; 2.5</td>
<td>1.55 ± 0.16</td>
<td>1.60</td>
</tr>
</tbody>
</table>

\( \alpha \) (Incoh.) = 0.70 ± 0.1

Conventional Glauber Theory Ruled Out!

Ashery E791

Factor of 7

Mueller, sjb; Bertsch et al; Frankfurt, Miller, Strikman

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AdS/QCD

Stan Brodsky, SLAC/IPPP
A(π,dijet) data from FNAL

Coherent π⁺ diffractive dissociation with 500 GeV/c pions on Pt and C.

Fit to $\sigma = \sigma_0 A^\alpha$

$\alpha = 0.76$ from pion-nucleus total cross-section.

Aitala et al., PRL 86 4773 (2001)

Two-gluon exchange measures the second derivative of the pion light-front wavefunction.

\[ M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp) \]
Two Components

High Transverse momentum component consistent with PQCD, ERBL Evolution

Gaussian component similar to AdS/CFT HO LFWF

Shuryak: Transition reflects domain walls
Prediction from AdS/CFT: Meson LFWF

\[ \phi_M(x, Q_0) \propto \sqrt{x(1 - x)} \]

de Teramond, sjb

Harmonic oscillator model

AdS/QCD

Stan Brodsky, SLAC/IPPP
Narrowing of $x$ distribution at higher jet transverse momentum

$x$: distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/$c$ (left) and for $1.5 \leq k_t \leq 2.5$ GeV/$c$ (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
Nonperturbative (AdS/CFT) and Perturbative (ERBL)
Evolution to asymptotic distribution

$\phi(x) \propto \sqrt{x(1-x)}$
Color Transparency

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets
Measurement of Nuclear Transparency for the $A(e, e'\pi^+)$ Reaction

$$eA \rightarrow e'\pi^+ X$$

B. Clasie, et al, Jlab

PRL 99, 242502 (2007)

Transparency vs. $Q^2 (\text{GeV/c})^2$

- $^2\text{H}$
- $^{12}\text{C}$
- $^{63}\text{Cu}$
- $^{197}\text{Au}$

$^{137}$

Color Transparency Prediction

Glauber
GPDs & Deeply Virtual Exclusive Processes  
- New Insight into Nucleon Structure

Deeply Virtual Compton Scattering (DVCS)

\[ H(x, \xi, t), E(x, \xi, t), \ldots \]

"Generalized Parton Distributions"

Quark angular momentum (Ji sum rule)

\[ J^q = \frac{1}{2} - J^G = \frac{1}{2} \int x dx \left[ H^q(x, \xi, 0) + E^q(x, \xi, 0) \right] \]


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AdS/QCD

138  
Stan Brodsky, SLAC/IPP
The position of the struck quark differs by $x^-$ in the two wave functions.

**Measure $x$-distribution from DVCS:**

Take Fourier transform of skewness, the longitudinal momentum transfer.

$$\zeta = \frac{Q^2}{2p \cdot q}$$
AdS/CFT Holographic Model

\[ \psi(\sigma, b_\perp) \]

\[ |b_\perp| (\text{GeV}^{-1}) \]

\[ \sigma = x^+ = ct - x^3 \]

\[ x^+ = ct + x^3 \]

The front form

3-dimensional photograph: meson LFWF at fixed LF Time

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Hadron Optics

$$A(\sigma, b_\perp) = \frac{1}{2\pi} \int d\zeta e^{i\sigma\zeta} \tilde{A}(b_\perp, \zeta)$$

$$\sigma = \frac{1}{2} x^- P^+ \quad \zeta = \frac{Q^2}{2p \cdot q}$$

The Fourier Spectrum of the DVCS amplitude in $\sigma$ space for different fixed values of $|b_\perp|$.

$\Lambda_{QCD} = 0.32$

DVCS Amplitude using holographic QCD meson LFWF
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances
Quark Interchange
(Spin exchange in atom-atom scattering)

\[ \frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2} \]

\[ M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2} \]

Gluon Exchange
(Van der Waal -- Landshoff)

\[ M(s, t)_{\text{gluon exchange}} \propto sF(t) \]

MIT Bag Model (de Tar), large \( N_c \), (‘t Hooft), AdS/CFT all predict dominance of quark interchange:
AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions.

\[ M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2} \]

Non-linear Regge behavior:

\[ \alpha_R(t) \to -1 \]