Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

\[ \Gamma_{\mu n}^{p-bj}(Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s g_1(Q^2)}{\pi} \right] \]

\( Q (\text{GeV}) \)

\( \frac{\alpha_s g_1}{\pi} \text{ world data} \)

\( \alpha_s/g_1 \)

\( \alpha_s/g_3 \)

JLab CLAS

JLab PLB 650 4 244

\( \alpha_s g_1/\pi \text{ world data} \)

\( \alpha_s/F_3/\pi \)

GDH limit

\( pQCD \text{ evol. eq.} \)

\( \alpha_s/\pi \text{ OPAL} \)

IR conformal window
Deur, Korsch, et al.

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SLAC & IPPP
IR Conformal Window for QCD?

- **Dyson-Schwinger Analysis:** QCD gluon coupling has IR Fixed Point

- **Evidence from Lattice Gauge Theory**

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small** $Q^2$

  \[
  \Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2
  \]

- **Justifies application of AdS/CFT in strong-coupling conformal window**

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QED One-Loop Vacuum Polarization

\[ \Pi(Q^2) = -\frac{\alpha(0)}{3\pi} \left[ \frac{5}{3} - \frac{4m^2}{Q^2} \right] - \left( 1 - \frac{2m^2}{Q^2} \right) \sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{|1 - \sqrt{1 + \frac{4m^2}{Q^2}}|} \]

\[ \Pi(Q^2) = \frac{\alpha(0) \log Q^2}{3\pi m^2} \quad Q^2 \gg 4M^2 \]

\[ \beta = \frac{d\left(\frac{\alpha}{4\pi}\right)}{d\log Q^2} = \frac{4}{3} \left(\frac{\alpha}{4\pi}\right)^2 n_\ell > 0 \]

\[ \Pi(Q^2) = \frac{\alpha(0) Q^2}{15\pi m^2} \quad Q^2 << 4M^2 \quad \text{Serber-Uehling} \]

\[ \beta \propto \frac{Q^2}{m^2} \quad \text{vanishes at small momentum transfer} \]
Lesson from QED:

Lamb Shift in Hydrogen

$$\Delta E \sim \alpha (Z\alpha)^4 \ln (Z\alpha)^2 m_e$$

\[\lambda < \frac{1}{Z\alpha m_e}\]

\[k > Z\alpha m_e\]

Maximum wavelength of bound electron

Infrared divergence of free electron propagator removed because of atomic binding

Bethe Log
Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{\text{QCD}}} \]

\[ \lambda < \Lambda_{\text{QCD}} \]

B-Meson

Shrock, sjb

gluon and quark propagators cutoff in IR because of color confinement
Lesson from QED and Lamb Shift:

Consequences of Maximum Quark and Gluon Wavelength

• Infrared integrations regulated by confinement

• Infrared fixed point of QCD coupling

\[ \alpha_s(Q^2) \text{ finite, } \beta \rightarrow 0 \text{ at small } Q^2 \]

• Bound state quark and gluon Dyson-Schwinger Equation

• Quark and Gluon Condensates exist within hadrons

Shrock, sjb
Lesson from QED and Lamb Shift:
maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\lambda_{QCD}} \]

\[ \lambda < \lambda_{QCD} \]

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

\[ < \bar{b} | \bar{q}q | \bar{b} > \text{ not } < 0 | \bar{q}q | 0 > \]

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Quark and Gluon condensates reside within hadrons, not vacuum

- **Bound-State Dyson-Schwinger Equations**
- LF vacuum trivial up to $k^+ = 0$ zero modes
- Analogous to finite size superconductor
- Usual picture for $m_\pi \rightarrow 0$
- Implications for cosmological constant -- reduction by 45 orders of magnitude!
Determinations of the vacuum Gluon Condensate

\[ < 0|\frac{\alpha_s}{\pi} G^2|0 > [\text{GeV}^4]\]

\[ -0.005 \pm 0.003 \text{ from } \tau \text{ decay.} \quad \text{Davier et al.}\]

\[ +0.006 \pm 0.012 \text{ from } \tau \text{ decay.} \quad \text{Geshkenbein, Ioffe, Zyablyuk}\]

\[ +0.009 \pm 0.007 \text{ from charmonium sum rules} \quad \text{Ioffe, Zyablyuk}\]

Consistent with zero vacuum condensate
• Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

• **Goal**: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances


• Karch, Katz, Son, Stephanov: Soft-Wall Model — Linear Confinement

• Mapping of AdS amplitudes to 3+1 Light-Front equations, wavefunctions!

• Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H_{\text{LFQCD}}$; variational methods
AdS/CFT

• Use mapping of conformal group SO(4,2) to AdS5

• Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension
  
  $x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$

• Match solutions at small $z$ to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

• Hard wall model: Confinement at large distances and conformal symmetry in interior

• Truncated space simulates “bag” boundary conditions
  
  $0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$
Bosonic Solutions: Hard Wall Model

- Conformal metric: \( ds^2 = g_{\ell m} dx^\ell dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m} \).

- Action for massive scalar modes on \( \text{AdS}_{d+1} \):
  \[
  S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.
  \]

- Equation of motion
  \[
  \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.
  \]

- Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = \mathcal{M}^2 \):
  \[
  [z^2 \partial_z^2 - (d - 1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0.
  \]

- Solution: \( \Phi(z) \rightarrow z^\Delta \) as \( z \rightarrow 0 \),
  \[
  \Phi(z) = Cz^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).
  \]

\( \Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4 \)
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrödinger Equation for bound state of two scalar constituents:**

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = M^2 \phi(z)$$

$L$: orbital angular momentum

**Derived from variation of Action in AdS$_5$**

**Hard wall model: truncated space**

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
• Physical AdS modes $\Phi_P(x, z) \sim e^{-i P \cdot x} \Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum $P^\mu$ and hadronic invariant mass states $P_\mu P^\mu = M^2$.

• For small-$z$ $\Phi(z) \sim z^\Delta$. The scaling dimension $\Delta$ of a normalizable string mode, is the same dimension of the interpolating operator $O$ which creates a hadron out of the vacuum: $\langle P | O | 0 \rangle \neq 0$.

Identify hadron by its interpolating operator at $z \rightarrow 0$
**Match fall-off at small $z$ to conformal twist-dimension at short distances**

- Pseudoscalar mesons: $\mathcal{O}^{2+L} = \bar{\psi} \gamma_5 D_{\ell_1} \ldots D_{\ell_m} \psi$ (\(\Phi_\mu = 0\) gauge). \(\Delta = 2 + L\)

- 4-\(d\) mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[ S = 0 \quad \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV
AdS Schrodinger Equation for bound state of two scalar constituents:

\[
\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z)\right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action

Dilaton-Modified AdS\(_5\)

Karch, Katz, Son, Stephanov

de Teramond, sjb
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$m_q = 0$
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \right] \phi_S(z) = M^2 \phi_S(z)
\]

with eigenvalues \( M^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube):

\[ M_n^2(L) = 2\pi\sigma (n + L + 1/2) \]

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \) GeV.

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\( \alpha(t) \approx \frac{1}{2} + 0.9t \)

**AdS/QCD Soft Wall Model** -- Reproduces Linear Regge Trajectories
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQK_1(zQ) \]

\[ F(Q^2)_{I \to F} = \int \frac{dz}{z^3} \Phi_F(z)J(Q, z)\Phi_I(z) \]

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi^{(n)} \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \to \left[ \frac{1}{Q^2} \right]^\tau - 1, \]

where \( \tau = \Delta_n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).
Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \ x^\ell = (x^\mu, z)$

$$
ig_5 \int d^4x \ dz \sqrt{g} A^\ell(x, z) \Phi^*_P(x, z) \overrightarrow{\partial}_\ell \Phi_P(x, z).
$$

- Electromagnetic probe polarized along Minkowski coordinates ($Q^2 = -q^2 > 0$)

$$
A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q, z), \ A_z = 0.
$$

- Propagation of external current inside AdS space described by the AdS wave equation

$$
[z^2 \partial_z^2 - z \partial_z - z^2 Q^2] J(Q, z) = 0,
$$

subject to boundary conditions $J(Q = 0, z) = J(Q, z = 0) = 1$.

- Solution

$$
J(Q, z) = z Q K_1(z Q).
$$

- Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

$$
\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \ \Phi(z) \rightarrow z^\Delta, \ z \rightarrow 0.
$$
Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[
[ z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 ] J_\kappa(Q, z) = 0. 
\]

- Solution bulk-to-boundary propagator

\[
J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right), 
\]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[
\Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.
\]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
\]

- For large \( Q^2 \gg 4\kappa^2 \)

\[
J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),
\]

the external current decouples from the dilaton field.

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Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

Data Compilation
Baldini, Kloe and Volmer
de Teramond, sjb
See also: Radyushkin

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Spacelike pion form factor from AdS/CFT

$$F_\pi(q^2)$$

Data Compilation from Baldini, Kloe and Volmer

SW: Harmonic Oscillator Confinement

HW: Truncated Space Confinement

One parameter - set by pion decay constant.

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Light-Front Holography and Novel QCD

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- Analytical continuation to time-like region \( q^2 \rightarrow -q^2 \) \( M_\rho = 2\kappa = 750 \text{ MeV} \)
- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

Space and time-like pion form factor for \( \kappa = 0.375 \text{ GeV} \) in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension $\tau$, $\Phi_\tau$ in the SW model

$$F(Q^2) = \Gamma(\tau) \frac{\Gamma \left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma \left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$  

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) = \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, \quad N = 2,$$

$$F(Q^2) = \frac{2}{(1 + \frac{Q^2}{4\kappa^2})(2 + \frac{Q^2}{4\kappa^2})}, \quad N = 3,$$

$$\ldots$$

$$F(Q^2) = \frac{(N - 1)!}{(1 + \frac{Q^2}{4\kappa^2})(2 + \frac{Q^2}{4\kappa^2}) \ldots (N-1 + \frac{Q^2}{4\kappa^2})}, \quad N.$$  

- For large $Q^2$:

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$
Constituent Counting Rules

\[ \frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{\text{cm}})}{s[n_{\text{tot}} - 2]} \quad s = E_{\text{cm}}^2 \]

\[ F_H(Q^2) \sim [\frac{1}{Q^2}]^{n_H - 1} \]

\( n_{\text{tot}} = n_A + n_B + n_C + n_D \)

Fixed \( t/s \) or \( \cos \theta_{\text{cm}} \)

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests
Phenomenological success of dimensional scaling laws for exclusive processes

$$d\sigma/dt \sim 1/s^{n-2}, \quad n = n_A + n_B + n_C + n_D,$$

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev et al. (1973).

Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space

Conformal Invariance:

\[ \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = F(\theta_{cm}) \]

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Quark-Counting: \[
\frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}} \]

\[n = 4 \times 3 - 2 = 10\]

Best Fit
\[n = 9.7 \pm 0.5\]

Reflects underlying conformal scale-free interactions

Angular distribution -- quark interchange

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PQCD and AdS/CFT:

$$s^{n_{tot}-2} \frac{d\sigma}{dt} (A + B \rightarrow C + D) = F_{A+B\rightarrow C+D}(\theta_{CM})$$

$$s^{11} \frac{d\sigma}{dt} (\gamma d \rightarrow np) = F(\theta_{CM})$$

$$n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11$$

Reflects conformal invariance
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi^*_P(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( (b = |\vec{b}_\perp|) \):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} \left| \tilde{\psi}(x, b) \right|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \left| \tilde{\psi}(x, b) \right|^2, \]

Soper 90
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[ F(q^2) = 2\pi \int_0^1 dx \frac{1 - x}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1 - x}{x}} \right) \tilde{\rho}(x, \zeta), \]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[ \zeta = \sqrt{\frac{x}{1 - x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|. \]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[ \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1 - x}{x}} \right) = \zeta Q K_1(\zeta Q), \]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
• Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2, \]

where \( J(Q^2, z) = zQK_1(zQ). \)

• Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1 - x}{x}} \right) \]

• Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1 - x}{x}} \right) |\Phi_{\pi^+}(z)|^2 \]

• Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1 - x) \frac{\left|\Phi_{\pi}(\zeta)\right|^2}{\zeta^4} \]

with \( \zeta = z, \ 0 \leq \zeta \leq \Lambda_{\text{QCD}} \)
Light-Front Holography: Unique mapping derived from equality of LF and \( \text{AdS}_5 \) formula for current matrix elements
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation! Frame Independent

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x)b^2 \perp.
\]

\[
U(\zeta) = \kappa^4 \zeta^2
\]

soft wall confining potential:

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Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

\[ M^2 = \int_0^1 dx \int \frac{d^2 k_\perp}{16\pi^3} \frac{k_\perp^2}{x(1-x)} |\psi(x, \vec{k}_\perp)|^2 + \text{interactions} \]

\[ = \int_0^1 dx \frac{d^2 b_\perp}{x(1-x)} \psi^*(x, \vec{b}_\perp) \left(-\nabla^2_{\vec{b}_\perp}\right) \psi(x, \vec{b}_\perp) + \text{interactions}. \]

**Change variables**

\[ (\vec{\zeta}, \varphi), \quad \vec{\zeta} = \sqrt{x(1-x)}\vec{b}_\perp: \quad \nabla^2 = \frac{1}{\zeta d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2} \]

\[ M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \]

\[ + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

\[ = \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \]
Consider the $AdS_5$ metric:

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

$ds^2$ invariant if $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$,

Maps scale transformations to scale changes of the holographic coordinate $z$.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0^2 - dx_3^2 - dx_\perp^2 = dx^+ dx^- - dx_\perp^2$

and

$$ds^2 = -\frac{R^2}{z^2} (dx_\perp^2 + dz^2) \text{ for } x^+ = 0.$$  

- $ds^2$ is invariant if $dx_\perp^2 \rightarrow \lambda^2 dx_\perp^2$, and $z \rightarrow \lambda z$, at equal LF time.

- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate $z$.

- Holographic connection of $AdS_5$ to the light-front.

- The effective wave equation in the two-dim transverse LF plane has the Casimir representation $L^2$ corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L + N - 2)$ ].

\[ \text{Light-Front/AdS}_5 \text{ Duality} \]
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

\[
\left[ - \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x) b_{\perp}^2.
\]

\[
U(\zeta) = \kappa^4 \zeta^2
\]

soft wall confining potential:

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\[ \psi_M(x, k_\perp^2) = \frac{4\pi}{k\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2k^2x(1-x)}} \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

**Note coupling**

\[ k_\perp^2, x \]

**“Soft Wall” model**

\[ \kappa = 0.375 \text{ GeV} \]

massless quarks

**Connection of Confinement to TMDs**

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Hadron Distribution Amplitudes

\[ \phi_H(x_i, Q) \]

\[ \sum_i x_i = 1 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[ \phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp) \]

San Carlos, Sonora October 10, 2008 Light-Front Holography and Novel QCD Stan Brodsky SLAC & IPPP
Linear potential ($m=0.22 \text{ GeV}, \beta=0.3659 \text{ GeV}$)
HO potential ($m=0.25 \text{ GeV}, \beta=0.3194 \text{ GeV}$)

$\phi_{as}(x) \sim x(1-x)$
$\phi_{\text{AdS/CFT}}(x) \sim [x(1-x)]^{1/2}$

$\phi_{\text{asympt}} \sim x(1-x)$

**AdS/CFT:**

$\phi(x, Q_0) \propto \sqrt{x(1-x)}$

Increases PQCD leading twist prediction $F_\pi(Q^2)$ by factor $16/9$
\[ F_\pi(Q^2) = \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2} \]

**AdS/CFT:**

Increases PQCD leading twist prediction for \( F_\pi(Q^2) \) by factor \( \frac{16}{9} \)

\[ \phi(x, Q_0) \propto \sqrt{x(1-x)} \]

\[ \phi_{\text{asymptotic}} \propto x(1-x) \]

Normalized to \( f_\pi \)
Second Moment of Pion Distribution Amplitude

\[
< \xi^2 > = \int_{-1}^{1} d\xi \, \xi^2 \phi(\xi)
\]

\[
\xi = 1 - 2x
\]

\[
< \xi^2 >_\pi = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1 - x)
\]

\[
< \xi^2 >_\pi = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1 - x)}
\]

Lattice (I) \quad < \xi^2 >_\pi = 0.28 \pm 0.03

Lattice (II) \quad < \xi^2 >_\pi = 0.269 \pm 0.039

Donnellan et al.

Braun et al.
Diffractive Dissociation of Pion into Quark Jets

E791 Ashery et al.

Measure Light-Front Wavefunction of Pion

Minimal momentum transfer to nucleus
Nucleus left Intact!
Two-gluon exchange measures the second derivative of the pion light-front wavefunction

\[ M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp) \]
Key Ingredients in E791 Experiment

Small color-dipole moment pion not absorbed; interacts with each nucleon coherently

QCD COLOR Transparency

\[ M_A = A \ M_N \]

\[ \frac{d\sigma}{dt}(\pi A \rightarrow q\bar{q}A') = A^2 \ \frac{d\sigma}{dt}(\pi N \rightarrow q\bar{q}N') \ F_T^2(t) \]

Target left intact

Diffraction, Rapidity gap

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October 10, 2008
Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets
• Fully coherent interactions between pion and nucleons.

• Emerging Di-Jets do not interact with nucleus.

\[ \mathcal{M}(A) = A \cdot \mathcal{M}(N) \]

\[ \frac{d\sigma}{dq_f^2} \propto A^2 \quad q_f^2 \sim 0 \]

\[ \sigma \propto A^{4/3} \]

Nuclear coherence

\[ F_A^2(q^2_\perp) \sim e^{-\frac{1}{3}R_A^2q^2_\perp} \]
Measure pion LFWF in diffractive dijet production

Confirmation of color transparency

A-Dependence results: $\sigma \propto A^\alpha$

<table>
<thead>
<tr>
<th>$k_t$ range (GeV/c)</th>
<th>$\alpha$</th>
<th>$\alpha$ (CT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.25 &lt; k_t &lt; 1.5$</td>
<td>$1.64 \pm 0.06 ; -0.12$</td>
<td>$1.25$</td>
</tr>
<tr>
<td>$1.5 &lt; k_t &lt; 2.0$</td>
<td>$1.52 \pm 0.12$</td>
<td>$1.45$</td>
</tr>
<tr>
<td>$2.0 &lt; k_t &lt; 2.5$</td>
<td>$1.55 \pm 0.16$</td>
<td>$1.60$</td>
</tr>
</tbody>
</table>

$\alpha$ (Incoh.) = $0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out!

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Light-Front Holography and Novel QCD

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Factor of 7

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E791 Diffractive Di-Jet transverse momentum distribution

Two Components

High Transverse momentum dependence $k_T^{-6.5}$ consistent with PQCD, ERBL Evolution

Gaussian component similar to AdS/CFT HO LFWF

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Narrowing of $x$ distribution at higher jet transverse momentum

$x$ distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

Possibly two components:
- Nonperturbative (AdS/CFT) and Perturbative (ERBL)

Evolution to asymptotic distribution

$\phi(x) \propto \sqrt{x(1-x)}$

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Light-Front Holography and Novel QCD

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Possibly two components:

Perturbative (ERBL) + Nonperturbative (AdS/CFT)

\[ \phi(x) = A_{\text{pert}}(k^2_\perp) x (1 - x) + B_{\text{nonpert}}(k^2_\perp) \sqrt{x(1-x)} \]

Narrowing of x distribution at high jet transverse momentum
Gravitational Form Factor of Composite Hadrons

- Gravitational FF defined by matrix elements of the energy momentum tensor $\Theta^{++}(x)$

$$\langle P' | \Theta^{++}(0) | P \rangle = 2 (P^+)^2 A(Q^2)$$

- $\Theta^{\mu\nu}$ is computed for each constituent in the hadron from the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu}$$

- Symmetric and gauge invariant $\Theta^{\mu\nu}$ from variation of $\mathcal{S}_{\text{QCD}} = \int d^4x \sqrt{g} \mathcal{L}_{\text{QCD}}$ with respect to four-dim Minkowski metric $g_{\mu\nu}$, $\Theta^{\mu\nu}(x) = -\frac{2}{\sqrt{g}} \frac{\delta \mathcal{S}_{\text{QCD}}}{\delta g_{\mu\nu}(x)}$:

$$\Theta^{\mu\nu} = \frac{1}{2} \bar{\psi} i(\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - g^{\mu\nu} \bar{\psi} (i\not{D} - m) \psi - G^{a \mu \lambda} G^{a \nu \lambda} + \frac{1}{4} g^{\mu\nu} G^{a}_{\mu\nu} G^{a\mu\nu}$$

- Quark contribution in light front gauge ($A^+ = 0$, $g^{++} = 0$)

$$\Theta^{++}(x) = \frac{i}{2} \sum_f \bar{\psi}^f(x) \gamma^+ \not{D}^+ \psi^f(x)$$