Light-Front Holography and Novel Effects in QCD

Stan Brodsky, SLAC

XIII Mexican School of Particles and Fields
San Carlos, Sonora, Mexico 2-11 October, 2008
• **Quarks and Gluons:**
  Fundamental constituents of hadrons and nuclei

• **Quantum Chromodynamics (QCD)**

• New Insights from higher space-time dimensions: *AdS/QCD*

• **Light-Front Holography:**
  First Approximation to QCD

• **Hadronization at the Amplitude Level**

• **Light Front Wavefunctions:**
  Analogous to Schrödinger wavefunctions of atomic physics
  \[ \Psi_n (x_i, \vec{k}_\perp i, \lambda_i) \]

---

San Carlos, Sonora  
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky  
SLAC & IPPP
\( \psi_M(x, k^2_\perp) \)

“Soft Wall” model

de Teramond, sjb

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

Fixed \( \tau = t + z/c \)

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Process Independent

Direct Link to QCD Lagrangian!

Invariant under boosts! Independent of \( P^\mu \)
Each element of flash photograph illuminated at same LF time

\[ \tau = t + z/c \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

Eigenstate -- independent of \( \tau \)
Light-Front Wavefunctions

Dirac’s Front Form: Fixed \( \tau = t + z/c \)

\[
\Psi_n(x_i, \vec{k}_\perp i, \lambda_i)
\]

\( x_i = \frac{k_i^+}{P^+} \)

Invariant under boosts. Independent of \( P^\mu \)

\[
H^{QCD}_{LF} |\psi> = M^2 |\psi>
\]

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Deep Inelastic Electron-Proton Scattering

Nonperturbative wavefunction
color confinement
spin, momenta, orbital angular
momentum ....

Gluonic Bremmstrahlung

DGLAP Evolution

Light-Front Quantization:
Rigorous realization of IMF

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
A Unified Description of Hadron Structure

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

- Elastic form factors
- B-Decays
- Real Compton scattering at high
- GPDs
- Deeply Virtual Compton Scattering
- Deeply Virtual Meson production
- Distribution Amplitudes
- Hadronization at the amplitude level
- Parton momentum distributions TMDs

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
Dirac’s Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!

\[ \sigma = ct - z \]

\[ \tau = t + z/c \]

Instant Form

Front Form

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

9

Stan Brodsky
SLAC & IPPP
Calculation of lepton g-2 in TOPTH (Instant form)

Only diagram in LFPTH ($q^+ = 0$)

$n!$ diagrams at order $e^n$

energy denominators:
frame-dependent and non-analytic

$$\sqrt{\left(\vec{p} + \vec{q} - \vec{k}\right)^2 + m^2}$$
**Light-Front Holography and Novel QCD**

**Instant Form:**

\[
\begin{align*}
\text{Calculation of Form Factors in Equal-Time Theory} \\
\text{Instant Form} \\
\text{Need vacuum-induced currents}
\end{align*}
\]

**Front Form:**

\[
\begin{align*}
\text{Calculation of Form Factors in Light-Front Theory} \\
\text{Front Form} \\
\text{Absent for } q^+ = 0 \quad \text{zero} !!
\end{align*}
\]
Calculation of Hadron Form Factors
Instant Form

• Current matrix elements of hadron include interactions with vacuum-induced currents arising from infinitely-complex vacuum

• Pair creation from vacuum occurs at any time before probe acts -- acausal

• Knowledge of hadron wavefunction insufficient to compute current matrix elements

• Requires dynamical boost of hadron wavefunction -- unknown except at weak binding

• Complex vacuum even for QED

• None of these complications occur for quantization at fixed LF time (front form)
\[ \frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_{\perp}] \sum_j e_j \frac{1}{2} \times \]

\[ \left[ -\frac{1}{q_L} \psi_{a}^{\dagger}(x_i, k_{\perp i}, \lambda_i) \psi_{a}(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q_R} \psi_{a}^{\dagger}(x_i, k'_{\perp i}, \lambda_i) \psi_{a}(x_i, k_{\perp i}, \lambda_i) \right] \]

\[ k'_{\perp i} = k_{\perp i} - x_i q_{\perp} \]

\[ k'_{\perp j} = k_{\perp j} + (1 - x_j) q_{\perp} \]

\[ q_R,L = q^x \pm iq^y \]

\[ q^2 = -q_{\perp}^2 \]

\[ q^+ = 0 \]

\[ p, S_z = -1/2 \]

\[ p + q, S_z = 1/2 \]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)

**Checked to \( O(\alpha^3) \) in QED**

Roskies, Suaya, sjb
Anomalous gravitomagnetic moment $B(0)$

Okun, Kobzarev, Teryaev: $B(0)$ Must vanish because of Equivalence Theorem

$$q^2 = -q_{\perp}^2$$

Hwang, Ma, Schmidt, sjb; Holstein et al

$B(0) = 0$

Each Fock State

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
‘Seagull’ contribution to real and virtual Compton scattering

Local coupling of photons to fundamental carriers of the em current

\[ M = -2 \sum_{q/p} e_q^2 F_q^+(t) \vec{\epsilon} \cdot \vec{\epsilon}' \]

Independent of \( s, q^2 \) at fixed \( t \)

\[ F_q^+(t) = \langle \frac{1}{x} \rangle \]
Instantaneous fermion exchange contribution to real and virtual Compton scattering

\[ M = -2 \sum_{q/p} e_q^2 F_q^+(t) \bar{\epsilon} \cdot \epsilon' \]

Local coupling of photons to fundamental carriers of the em current

Independent of \( s, q^2 \) at fixed \( t \)

\[ F_q^+(t) = \langle \frac{1}{x} \rangle \]
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Process Independent
Direct Link to QCD Lagrangian!

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
Angular Momentum on the Light-Front

\( A^+ = 0 \) gauge:

\[
J_z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.
\]

No unphysical degrees of freedom

Conserved

LF Fock state by Fock State

\[
l_j^z = -i \left( k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1} \right)
\]

n-1 orbital angular momenta

Nonzero Anomalous Moment requires

Nonzero orbital angular momentum.
\[ |p, S_z > = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp i}, \lambda_i > \]

**sum over states with \( n=3, 4, \ldots \) constituents**

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]

are boost invariant.

\[ \sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}. \]

**Intrinsic heavy quarks,**

\[ \bar{s}(x) \neq s(x) \]
\[ \bar{u}(x) \neq \bar{d}(x) \]
E866/NuSea (Drell-Yan)

\[ \bar{d}(x) \neq \bar{u}(x) \]

\[ s(x) \neq \bar{s}(x) \]

Intrinsic glue, sea, heavy quarks

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
Compare protons versus anti-proton in $\bar{s}$ current quark fragmentation

$$D_{s\rightarrow p}(z) \neq D_{s\rightarrow \bar{p}}(z)$$

Tag $s$ quark via high $x_F$ $\Lambda$ production in proton fragmentation region.

$$A_s^{p\bar{p}}(z) = \frac{D_{s\rightarrow p}(z) - D_{s\rightarrow \bar{p}}(z)}{D_{s\rightarrow p}(z) + D_{s\rightarrow \bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$

$$|uuds\bar{s}\bar{s}| \gtrsim |K^+\Lambda|$$

B.Q. Ma and sjb
Hadron Distribution Amplitudes

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[
\phi_{M}(x, Q) = \int d^{2}\vec{k} \ \psi_{q\bar{q}}(x, \vec{k}_{\perp})
\]

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman
Braun, Gardi

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
Hidden Color of Deuteron

Evolution of 5 color-singlet Fock states

\[ \Psi_n^d(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ \sum_i \vec{k}_{\perp i} = \vec{0}_{\perp} \]
\[ \sum_i x_i = 1 \]

\[ \Phi_n(x_i, Q) = \int k_{\perp i}^2 < Q^2 \prod' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j}) \]

Ji, Lepage, sjb

5 X 5 Matrix Evolution Equation for deuteron distribution amplitude
Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme
Deep Inelastic Electron-Proton Scattering

Conventional wisdom:
Final-state interactions of struck quark can be neglected
Single-spin asymmetries

Pseudo- $T$-Odd

$\mathbf{i} \mathbf{S}_p \cdot \mathbf{q} \times \mathbf{p}_q$

Leadina Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt
Ji, Yuan

QCD S- and P- Coulomb Phases
--Wilson Line

Moreno

Light-Front Holography and Novel QCD

San Carlos, Sonora
October 10, 2008

Light-Front Wavefunction
$S$ and $P$-Waves

Stan Brodsky
SLAC & IPPP
\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2k_\perp] \sum_j e_j \frac{1}{2} \times \\
\left[ -\frac{1}{q_L} \psi_{a}^{\dag}(x_i, k_{\perp i}, \lambda_i) \psi_{a}(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q_R} \psi_{a}^{\dag}(x_i, k_{\perp i}, \lambda_i) \psi_{a}(x_i, k_{\perp i}, \lambda_i) \right] \\
k'_{\perp i} = k_{\perp i} - x_i q_\perp \\
k'_{\perp j} = k_{\perp j} + (1 - x_j) q_\perp
\]

Drell, sjb

\[q_{R,L} = q^x \pm iq^y\]

Must have \(\Delta \ell_z = \pm 1\) to have nonzero \(F_2(q^2)\)

Same matrix elements appear in Sivers effect
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

- Leading-Twist Bjorken Scaling!

- Requires nonzero orbital angular momentum of quark

- Arises from the interference of Final-State QCD Coulomb phases in S- and P- waves; Wilson line effect; gauge independent

- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases

- QCD phase at soft scale: IR Fixed Point?

- New window to QCD coupling and running gluon mass in the IR

- QED S and P Coulomb phases infinite -- difference of phases finite
Remarkable observation at HERA

10% to 15% of DIS events are diffractive!

Fraction $r$ of events with a large rapidity gap, $\eta_{\text{max}} < 1.5$, as a function of $Q^2_{\text{DA}}$ for two ranges of $x_{\text{DA}}$. No acceptance corrections have been applied.

Final State Interactions in QCD

\[ \gamma^* \rightarrow q \rightarrow k_1, k_2 \]

Feynman Gauge

Light-Cone Gauge

Result is Gauge Independent
QCD Mechanism for Rapidity Gaps

Wilson Line: \( \bar{\psi}(y) \int_0^y dx \, e^{iA(x) \cdot dx} \psi(0) \)

Origin of Diffractive DIS
Reproduces lab-frame color dipole approach

San Carlos, Sonora
October 10, 2008
Integration over on-shell domain produces phase \( i \)

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

**Physics of FSI not in Wavefunction of Target**
Universal Frame-Independent Light Front virtual photon and proton Wavefunctions

\[ P^+ \]
\[ x_a \vec{l}_\perp + \vec{k}_\perp a \]

\[ \Psi^*_\gamma(x_a, \vec{k}_\perp a) \]

\[ \Psi_B(x_b, \vec{k}_\perp b) \]
DY $\cos 2\phi$ correlation at leading twist from double ISI
Double Initial-State Interactions
generate anomalous $\cos 2\phi$

Drell-Yan planar correlations

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} \propto \left( 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right)$$

PQCD Factorization (Lam Tung):

$$1 - \lambda - 2\nu = 0$$

$$\frac{\nu}{2} \propto h_1^+(\pi) h_1^+(N)$$

$$\pi N \rightarrow \mu^+ \mu^- X \quad \text{NA10}$$

Violates Lam-Tung relation!

Model: Boer, Hwang, sjb
Anomalous effect from Double ISI in Massive Lepton Production

- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization

\[ \cos 2\phi \text{ correlation} \]
Physics of Rescattering

- Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions!
- T-odd single-spin asymmetries,
- Nuclear Shadowing, Non-Universal Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon
Problem for factorization when both ISI and FSI occur
Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions


Implications for QCD at the LHC

The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.
### Static vs. Dynamic Structure Functions

#### Static

- Square of Target LFWFs
- No Wilson Line
- Probability Distributions
- Process-Independent
- T-even Observables
- No Shadowing, Anti-Shadowing
- Sum Rules: Momentum and $J^z$
- DGLAP Evolution; mod. at large $x$
- No Diffractive DIS

#### Dynamic

- Modified by Rescattering: ISI & FSI
- Contains Wilson Line, Phases
- No Probabilistic Interpretation
- Process-Dependent - From Collision
- T-Odd (Sivers, Boer-Mulders, etc.)
- Shadowing, Anti-Shadowing, Saturation
- Not Proven
- DGLAP Evolution
- Hard Pomeron and Odderon: DDIS
Light-Front Wave Functions in QCD

• Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n \psi_{n/h} |n\rangle$: the LF Hamiltonian $H_{LF} = P^2 = P^+ P^- - P_{\perp}^2$, $H_{LF} |P\rangle = M^2 |P\rangle$, at fixed LF time $\tau = t + z/c$ (Dirac ’49; Pauli and Pinsky, sjb Phys. Rept. 1988). 

**Process Independent**

**Direct Link to QCD Lagrangian!**

• Fock components

$$\psi_{n/h}(x_i, k_{\perp i}) = \langle n; x_i, k_{\perp i} | \psi_h(P^+, P_{\perp}) \rangle,$$

frame independent and encode hadron properties in high momentum-transfer collisions.

• Momentum fraction $x_i = k^+_i / P^+$ and $k_{\perp i}$ are the relative coordinates of parton $i$ in Fock-state $n$

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n k_{\perp i} = 0.$$

• Define transverse position coordinates $x_i r_{\perp i} = x_i R_{\perp} + b_{\perp i}$

$$\sum_{i=1}^n b_{\perp i} = 0, \quad \sum_{i=1}^n x_i r_{\perp i} = R_{\perp}.$$
**QCD Lagrangian**

\[ L_{QCD} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\psi}_f D_\mu \gamma^\mu \psi_f + \sum_{f=1}^{n_f} m_f \bar{\psi}_f \psi_f \]

**Yang-Mills Gauge Principle:**
- Invariance under Color Rotation and Phase Change at Every Point of Space and Time

**Dimensionless Coupling**
- Renormalizable
- Asymptotic Freedom

**Color Confinement**

\[ L_{QCD} \rightarrow H_{QCD}^{LF} \rightarrow \psi_{n/H}^{LF}(x_i, \vec{k}_\perp i, \lambda_i) \]

---

*San Carlos, Sonora October 10, 2008*

*Light-Front Holography and Novel QCD*

*Stan Brodsky*
*SLAC & IPPP*
Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

$$L_{QCD} \rightarrow H^QCD_{LF}$$

$$H^QCD_{LF} = \sum_i \left[ \frac{m^2 + k^2_{\perp}}{x} \right] i + H^int_{LF}$$

$H^int_{LF}$: Matrix in Fock Space

$$H^QCD_{LF} |\Psi_h> = M^2_h |\Psi_h>$$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in $x^-$. Discrete $k^+$; frame-independent truncation
\[ \hat{H}_{LF}^{QCD} | \Psi_h \rangle = M_h^2 | \Psi_h \rangle \]

**Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions**

**DLCQ: Frame-independent, No fermion doubling; Minkowski Space**

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
\[ \phi(z) \]

\[ \psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

**Light-Front Holography**

**Light Front Wavefunctions:**

Schrödinger Wavefunctions of Hadron Physics

San Carlos, Sonora  
October 10, 2008
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

San Carlos, Sonora  October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

de Teramond, sjb

String Theory

Bottom-Up

Top-Down
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrodinger Theory for Atomic Physics

- \textit{AdS/QCD Light-Front Holography}

- \textit{Hadronic Spectra and Light-Front Wavefunctions}
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu \nu}, P^\mu, D, K^\mu, \]

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \]

$x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

\[ x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z. \]

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
We will consider both holographic models

- Truncated AdS/CFT (Hard-Wall) model: cut-off at \( z_0 = 1/\Lambda_{QCD} \) breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) \(^{\text{Polchinski and Strassler (2001).}}\)

- Smooth cutoff: introduction of a background dilaton field \( \varphi(z) \) – usual linear Regge dependence can be obtained (Soft-Wall Model) \(^{\text{Karch, Katz, Son and Stephanov (2006).}}\)

San Carlos, Sonora
October 10, 2008

Light-Front Holography and Novel QCD

Stan Brodsky
SLAC & IPPP
**AdS/CFT:** Anti-de Sitter Space / Conformal Field Theory

Maldacena:

**Map AdS$_5 \times S_5$ to conformal $N=4$ SUSY**

- **QCD is not conformal:** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** $\alpha_s(Q^2) \sim \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group $\text{SO}(4,2)$ to AdS$_5$ space**