Light-Front Holographic QCD and the Bound State Structure of Hadrons

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1 Introduction

Gauge Gravity Correspondence and Light-Front QCD

- The AdS/CFT correspondence [Maldacena (1998)] between gravity on AdS space and conformal field theories in physical spacetime has led to a semiclassical approximation for strongly-coupled QCD, which provides physical insights into its non-perturbative dynamics.

- Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...

- Light-front holography provides a remarkable connection between the equations of motion in AdS and the bound-state LF Hamiltonian equation in QCD [GdT and S. J. Brodsky, PRL 102, 081601 (2009)].

- Isomorphism of $SO(4,2)$ group of conformal transformations with generators $P^\mu, M^{\mu\nu}, K^\mu, D$, with the group of isometries of AdS$_5$, a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space $\text{Dim isometry group of AdS}_{d+1} = (d + 1)(d + 2)/2$
\[ e^{iW_{QCD}[J=\Phi_0]} = Z_{\text{grav}}[\Phi] = \int \mathcal{D}\Phi e^{iS_{\text{eff}}[\Phi]} \]
• AdS$_5$ metric:

$$ds^2_{L_{AdS}} = \frac{R^2}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right)_{L_{Minkowski}}$$

• A distance $L_{AdS}$ shrinks by a warp factor $z/R$ as observed in Minkowski space ($dz = 0$):

$$L_{Minkowski} \sim \frac{z}{R} L_{AdS}$$

• Since the AdS metric is invariant under a dilatation of all coordinates $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, the variable $z$ acts like a scaling variable in Minkowski space

• Short distances $x_\mu x^\mu \rightarrow 0$ map to UV conformal AdS$_5$ boundary $z \rightarrow 0$

• Large confinement dimensions $x_\mu x^\mu \sim 1/\Lambda^2_{QCD}$ maps to large IR region of AdS$_5$, $z \sim 1/\Lambda_{QCD}$, thus there is a maximum separation of quarks and a maximum value of $z$

• Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS
2 Light Front Dynamics

• Different possibilities to parametrize space-time [Dirac (1949)]

• Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results

• *Instant form*: hypersurface defined by $t = 0$, the familiar one

• *Front form*: hypersurface is tangent to the light cone at $\tau = t + z/c = 0$

\[
\begin{align*}
x^+ &= x^0 + x^3 \quad \text{light-front time} \\
x^- &= x^0 - x^3 \quad \text{longitudinal space variable} \\
k^+ &= k^0 + k^3 \quad \text{longitudinal momentum} \quad (k^+ > 0) \\
k^- &= k^0 - k^3 \quad \text{light-front energy} \\

k \cdot x &= \frac{1}{2} (k^+ x^- + k^- x^+) - \mathbf{k}_\perp \cdot \mathbf{x}_\perp
\end{align*}
\]

On shell relation $k^2 = m^2$ leads to dispersion relation $k^- = \frac{k^2 + m^2}{k^+}$
• QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} \left( G^{\mu\nu} G_{\mu\nu} \right) + i \bar{\psi} D_\mu \gamma^\mu \psi + m \bar{\psi} \psi \]

• LF Momentum Generators \( P = (P^+, P^-, P_\perp) \) in terms of dynamical fields \( \psi, A_\perp \)

\[
P^- = \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi} \gamma^+ \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \psi + \text{interactions}
\]

\[
P^+ = \int dx^- d^2x_\perp \bar{\psi} \gamma^+ i\partial^+ \psi
\]

\[
P_\perp = \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi} \gamma^+ i\nabla_\perp \psi
\]

• LF Hamiltonian \( P^- \) generates LF time translations

\[ [\psi(x), P^-] = i \frac{\partial}{\partial x^+} \psi(x) \]

and the generators \( P^+ \) and \( P_\perp \) are kinematical
Light-Front Fock Representation

- Dirac field $\psi$, expanded in terms of ladder operators on the initial surface

$$P^- = \sum_\lambda \int \frac{dq^+ d^2q_\perp}{(2\pi)^3} \left( \frac{q_\perp^2 + m^2}{q^+} \right) b_\lambda^\dagger(q) b_\lambda(q) + \text{interactions}$$

- Construct LF Lorentz invariant Hamiltonian equation for the relativistic bound state

$$P_\mu P^\mu |\psi(P)\rangle = \left( P^- P^+ - P_\perp^2 \right) |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

- State $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$ of the free LF Hamiltonian

$$|\psi\rangle = \sum_n \psi_n |n\rangle, \quad |n\rangle = \{ |uud\rangle, |uudg\rangle, |uud\bar{q}q\rangle, \ldots \}$$

with $k_i^2 = m_i^2$, $k_i = (k_i^+, k_i^-, k_{\perp i})$, for each constituent $i$ in state $n$

- Fock components $\psi_n(x_i, k_{\perp i}, \lambda_i^z)$ independent of $P^+$ and $P_\perp$ and depend only on relative partonic coordinates: momentum fraction $x_i = k_i^+ / P^+$, transverse momentum $k_{\perp i}$ and spin $\lambda_i^z$

$$\sum_{i=1}^n x_i = 1, \quad \sum_{i=1}^n k_{\perp i} = 0.$$
Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL 102, 081601 (2009)]

- Compute $\mathcal{M}^2$ from hadronic matrix element

$$\langle \psi (P') | P_\mu P'^\mu | \psi (P) \rangle = \mathcal{M}^2 \langle \psi (P') | \psi (P) \rangle$$

- Find

$$\mathcal{M}^2 = \sum_n \int [dx_i] [d^2 k_{\perp i}] \sum_\ell \left( \frac{k_{\perp \ell}^2 + m_{\ell}^2}{x_q} \right) |\psi_n(x_i, k_{\perp i})|^2 + \text{interactions}$$

- LFWF $\psi_n$ represents a bound state which is off the LF energy shell $\mathcal{M}^2 - \mathcal{M}_n^2$

$$\mathcal{M}_n^2 = \left( \sum_{a=1}^n k_{\mu a}^2 \right)^2 = \sum_a \frac{k_{\perp a}^2 + m_{a}^2}{x_a}$$

with $k_{a}^2 = m_{a}^2$ for each constituent

- Invariant mass $M_n^2$ key variable which controls the bound state: LFWF peaks at the minimum $\mathcal{M}_n^2$

- Semiclassical approximation to QCD:

$$\psi_n(k_1, k_2, \ldots, k_n) \rightarrow \phi_n \left( \frac{(k_1 + k_2 + \cdots + k_n)^2}{\mathcal{M}_n^2} \right), \quad m_q \rightarrow 0$$
• In terms of $n-1$ independent transverse impact coordinates $b_{\perp j}, j = 1, 2, \ldots, n-1$,

$$M^2 = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2 b_{\perp j} \psi_n^*(x_i, b_{\perp i}) \sum_\ell \left( -\frac{\nabla_{\perp \ell}^2 + m_\ell^2}{x_q} \right) \psi_n(x_i, b_{\perp i}) + \text{interactions}$$

• Relevant variable conjugate to invariant mass in the limit of zero quark masses

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|$$

the $x$-weighted transverse impact coordinate of the spectator system ($x$ active quark)

• For a two-parton system $\zeta^2 = x(1-x)b_{\perp}^2$

• To first approximation LF dynamics depend only on the invariant variable $\zeta$, and hadronic properties are encoded in the hadronic mode $\phi(\zeta)$ from

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular $\varphi$, longitudinal $X(x)$ and transverse mode $\phi(\zeta)$
• Ultra relativistic limit $m_q \rightarrow 0$ longitudinal modes $X(x)$ decouple ($L = L^z$)

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential $U(\zeta)$

• LF eigenvalue equation $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$ is a LF wave equation for $\phi$

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

• Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

• Eigenmodes $\phi(\zeta)$ determine the hadronic mass spectrum and represent the probability amplitude to find $n$-massless partons at transverse impact separation $\zeta$ within the hadron at equal light-front time

• Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in LF Hamiltonian EOM by applying the L-S formalism or evolution equations
3 Light-Front Holographic Mapping

Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

- Action for spin-\(J\) field in AdS\(_{d+1}\) in presence of dilaton background \(\varphi(z)\) \((x^M = (x^\mu, z))\)

\[
S = \frac{1}{2} \int d^d x \, dz \, \sqrt{g} \, e^{\varphi(z)} \left( g^{NN'} g^{M_1 M_1'} \cdots g^{M_J M_J'} D_N \Phi_{M_1 \cdots M_J} D_{N'} \Phi_{M_1' \cdots M_J'} - \mu^2 g^{M_1 M_1'} \cdots g^{M_J M_J'} \Phi_{M_1 \cdots M_J} \Phi_{M_1' \cdots M_J'} + \cdots \right)
\]

where \(D_M\) is the covariant derivative which includes parallel transport

\[
[D_N, D_K] \Phi_{M_1 \cdots M_J} = - R^L_{M_1 N K} \Phi_{L \cdots M_J} - \cdots - R^L_{M_J N K} \Phi_{M_1 \cdots L}
\]

- Physical hadron has plane-wave and polarization indices along 3 + 1 physical coordinates

\[
\Phi_{P}(x, z)_{\mu_1 \cdots \mu_J} = e^{-i P \cdot x} \Phi_{(z)_{\mu_1 \cdots \mu_J}}, \quad \Phi_{z^{\mu_2 \cdots \mu_J}} = \cdots = \Phi_{\mu_1 \mu_2 \cdots z} = 0
\]

with four-momentum \(P_\mu\) and invariant hadronic mass \(P_\mu P^\mu = M^2\)
• Construct effective action in terms of spin-\textit{J} modes $\Phi_{J}$ with only physical degrees of freedom

H. G. Dosch, S. J. Brodsky, J. Erlich, and GdT (in progress)

• Introduce fields with tangent indices

$\hat{\Phi}_{A_{1}A_{2}...A_{J}} = e_{A_{1}}^{M_{1}} e_{A_{2}}^{M_{2}} ... e_{A_{J}}^{M_{J}} \Phi_{M_{1}M_{2}...M_{J}} = \left( \frac{z}{R} \right)^{J} \Phi_{A_{1}A_{2}...A_{J}}$

• Find effective action

$S = \frac{1}{2} \int d^{d}x \, dz \, \sqrt{g} \, e^{\varphi(z)} \left[ g^{N'N} \eta_{\mu_{1}}^{\mu_{1}'} ... \eta_{\mu_{J}}^{\mu_{J}'} \partial_{N} \hat{\Phi}_{\mu_{1}...\mu_{J}} \partial_{N'} \hat{\Phi}_{\mu_{1}'}...\mu_{J}'}

- \mu^{2} \eta_{\mu_{1}}^{\mu_{1}'} ... \eta_{\mu_{J}}^{\mu_{J}'} \hat{\Phi}_{\mu_{1}...\mu_{J}} \hat{\Phi}_{\mu_{1}'}...\mu_{J}'} \right]

upon \mu\text{-rescaling}

• Variation of the action gives AdS wave equation for spin-\textit{J} mode $\check{\Phi}_{J} = \Phi_{\mu_{1}...\mu_{J}}$

\[
\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_{z} \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_{z} \right) + \left( \frac{\mu R}{z} \right)^{2} \right] \check{\Phi}_{J}(z) = \mathcal{M}^{2} \Phi_{J}(z)
\]

with $\check{\Phi}_{J}(z) = \left( z/R \right)^{J} \Phi_{J}(z)$ and all indices along 3+1
Dual QCD Light-Front Wave Equation

\[ \Phi_P(z) \Leftrightarrow |\psi(P)\rangle \]

- LF Holographic mapping found originally matching expressions of EM and gravitational form factors of hadrons in AdS and LF QCD \[ [\text{Brodsky and GdT (2006, 2008)}] \]

- Upon substitution \( z \rightarrow \zeta \) and \( \phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta) \) in AdS WE

\[
\left[-z^{d-1-2J} \frac{e^{\varphi(z)}}{e^{\varphi(z)}} \partial_z \left( z^{d-1-2J} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = M^2 \Phi_J(z)
\]

find LFWE \((d = 4)\)

\[
\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)
\]

with

\[
U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)
\]

and \((\mu R)^2 = -(2 - J)^2 + L^2\)

- AdS Breitenlohner-Freedman bound \((\mu R)^2 \geq -4\) equivalent to LF QM stability condition \(L^2 \geq 0\)

- Scaling dimension \(\tau\) of AdS mode \(\hat{\Phi}_J\) is \(\tau = 2 + L\) in agreement with twist scaling dimension of a two parton bound state in QCD
Bosonic Modes and Meson Spectrum

- Positive dilaton background: $\varphi = \kappa^2 z^2$ : $U(z) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$
- Normalized eigenfunctions: $\langle \phi | \phi \rangle = \int d\zeta |\phi(z)^2| = 1$

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$M_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$$

LFWFs $\phi_{n,L}(\zeta)$ in physical spacetime for dilaton $\exp(\kappa^2 z^2)$: a) orbital modes and b) radial modes
$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Regge trajectories for the $\pi$ ($\kappa = 0.6$ GeV) and the $I = 1$ $\rho$-meson and $I = 0$ $\omega$-meson families ($\kappa = 0.54$ GeV)
For baryons LFWE equivalent to system of coupled linear equations \((\nu = L + 1)\)

\[
-\frac{d}{d\zeta} \psi_- - \frac{\nu + \frac{1}{2}}{\zeta} \psi_- - \kappa^2 \zeta \psi_- + 2i\kappa \psi_+ = \mathcal{M} \psi_+ \\
\frac{d}{d\zeta} \psi_+ - \frac{\nu + \frac{1}{2}}{\zeta} \psi_+ - \kappa^2 \zeta \psi_+ - 2i\kappa \psi_- = \mathcal{M} \psi_-
\]

with eigenfunctions

\[
\psi_+ (\zeta) \sim \zeta^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu} (\kappa^2 \zeta^2) \\
\psi_- (\zeta) \sim \zeta^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu+1} (\kappa^2 \zeta^2)
\]

and eigenvalues

\[
\mathcal{M}^2 = 4\kappa^2 (n + \nu)
\]

- Large \(N_C\): \(\mathcal{M}^2 = 4\kappa^2 (N_C + n + L - 2) \Rightarrow \mathcal{M} \sim \sqrt{N_C} \Lambda_{\text{QCD}}\)
Same multiplicity of states for mesons and baryons!

Regge trajectories for positive parity $N$ and $\Delta$ baryon families ($\kappa = 0.5$ GeV)
4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL 96, 201601 (2006)], PRD 77, 056007 (2008)]

• EM transition matrix element in QCD: local coupling to pointlike constituents

\[ \langle \psi(P')|J^\mu|\psi(P)\rangle = (P + P') F(Q^2) \]

where \( Q = P' - P \) and \( J^\mu = e_q\bar{q}\gamma^\mu q \)

• EM hadronic matrix element in AdS space from non-local coupling of external EM field propagating in AdS with extended mode \( \Phi(x, z) \)

\[ \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} A^\ell(x, z) \Phi^*_P(x, z) \longleftrightarrow \partial^\ell \Phi_P(x, z) \]

• Are the transition amplitudes related?

• How to recover hard pointlike scattering at large \( Q \) out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

• Mapping of \( J^+ \) elements at fixed light-front time: \( \Phi_P(z) \leftrightarrow |\psi(P)\rangle \)
• Electromagnetic probe polarized along Minkowski coordinates, \((Q^2 = -q^2 > 0)\)

\[
A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} V(Q, z), \quad A_z = 0
\]

• Propagation of external current inside AdS space described by the ‘free’ AdS wave equation

\[
\left[ z^2 \partial_z^2 - z \partial_z - z^2 Q^2 \right] V(Q, z) = 0
\]

• Solution \( V(Q, z) = zQ K_1(zQ) \)

• Substitute hadronic modes \( \Phi(x, z) \) in the AdS EM matrix element

\[
\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \quad \Phi(z) \rightarrow z^\tau, \quad z \rightarrow 0
\]

• Find form factor in AdS as overlap of normalizable modes dual to the in and out hadrons \( \Phi_P \) and \( \Phi_P' \), with the non-normalizable mode \( V(Q, z) \) dual to external source \[[Polchinski and Strassler (2002)]\].

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{\varphi(z)} V(Q, z) \Phi^2_J(z) \rightarrow \left( \frac{1}{Q^2} \right)^{\tau - 1}
\]

At large \( Q \) important contribution to the integral from \( z \sim 1/Q \) where \( \Phi \sim z^\tau \) and power-law point-like scaling is recovered \[[Polchinski and Susskind (2001)]\].
Electromagnetic Form-Factor

- Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

\[
F(q^2) = \sum_{n} \prod_{j=1}^{n-1} \int dx_j d^2 b_{\perp j} \sum_{e} e_q \exp \left( i \mathbf{q}_{\perp} \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k} \right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2
\]

- Consider a two-quark $\pi^+$ Fock state $|u\bar{d}\rangle$ with $e_u = \frac{2}{3}$ and $e_d = \frac{1}{3}$

\[
F_{\pi^+}(q^2) = \int_{0}^{1} dx \int d^2 b_{\perp} e^{i \mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp} (1-x)} |\psi_{ud/\pi}(x, \mathbf{b}_{\perp})|^2
\]

with normalization $F_{\pi^+}(q = 0) = 1$

- Integrating over angle

\[
F_{\pi^+}(q^2) = 2\pi \int_{0}^{1} \frac{dx}{x(1-x)} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) |\psi_{ud/\pi}(x, \zeta)|^2
\]

where $\zeta^2 = x(1-x)b_{\perp}^2$
• Compare with electromagnetic FF in AdS space

\[ F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi^2_{\pi^+}(z) \]

where \( V(Q, z) = zQK_1(zQ) \)

• Use the integral representation

\[ V(Q, z) = \int_0^1 dx J_0(zQ \sqrt{1-x/x}) \]

• Find

\[ F(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0(zQ \sqrt{1-x/x}) \Phi^2_{\pi^+}(z) \]

• Compare with electromagnetic FF in LF QCD for arbitrary \( Q \). Expressions can be matched only if LFWF is factorized

\[ \psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi \zeta}} \]

• Find

\[ X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = \left( \frac{\zeta}{R} \right)^{-3/2} e^{\varphi(z)/2} \Phi(\zeta), \quad z \to \zeta \]
- “Free current” \( V(Q, z) = zQK_1(zQ) \rightarrow \) infinite hadron radius (mauve)

- “Dressed current” non-perturbative sum of an infinite number of terms \( \rightarrow \) finite radius (blue)

- Form factor in soft-wall model expressed as \( N - 1 \) product of poles along vector radial trajectory

  \[ F(Q^2) = \left[ \left( 1 + \frac{Q^2}{M^2_\rho} \right) \left( 1 + \frac{Q^2}{M^2_\rho'} \right) \cdots \left( 1 + \frac{Q^2}{M^2_\rho^{N-2}} \right) \right]^{-1} \]

\[ (M^2_\rho \rightarrow 4\kappa^2(n + 1/2)) \]

Pion form factor (lowest mode) Proton form factor (lowest mode)
Gravitational or Energy-Momentum Form-Factor

[S. J. Brodsky and GdT, PRD 78, 025032 (2008)]

- Gravitational form factor of composite hadrons in QCD: local coupling to pointlike constituents

\[ \langle P' | \Theta_{\mu}^\nu | P \rangle = (P^\nu P'_\mu + P_\mu P'^\nu) A(Q^2) \]

where \( Q = P' - P \) and

\[ \Theta_{\mu\nu} = \frac{1}{2} \bar{q} i (\gamma_\mu D_\nu + \gamma_\nu D_\mu) q - g_{\mu\nu} \bar{q} (iD - m) q - G^a_{\mu\lambda} G^a_{\nu\lambda} + \frac{1}{4} g_{\mu\nu} G^a_{\lambda\sigma} G^a_{\lambda\sigma} \]

- Hadronic matrix element of energy-momentum tensor from perturbing the static AdS metric: non-local coupling of external graviton field propagating in AdS with extended mode \( \Phi(x, z) \)

\[ \int d^4x \, dz \sqrt{g} h_{\ell m} \left( \partial^\ell \Phi^*_P \partial^m \Phi_P + \partial^m \Phi^*_P \partial^\ell \Phi_P \right) \]

- Are the transition amplitudes related?

- Mapping of \( \Theta^{++} \) elements at fixed LF time: Identical mapping \( \Phi_P(z) \leftrightarrow |\psi(P)\rangle \) as EM FF
5 Beyond the Lowest Order Approximation

Higher Fock states


- Only interaction in LF holographic semiclassical approx is the confinement potential: create Fock states with extra quark-antiquark pairs, no dynamical gluons

- Explain the dominance of quark interchange in large angle elastic scattering

- Form factor in soft-wall model expressed as $N - 1$ product of poles along vector radial trajectory

\[ \left( \mathcal{M}_\rho^2 \rightarrow 4\kappa^2 (n + 1/2) \right) \]

\[ F(Q^2) = \left[ \left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right) \right]^{-1} \]

- Higher Fock components in pion LFWF

\[ |\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}qq/\pi}|q\bar{q}qq\rangle_{\tau=4} + \cdots \]

- Expansion of LFWF up to twist 4 (monopole + tripole)

$\kappa = 0.54 \text{ GeV}, \Gamma_\rho = 130, \Gamma_{\rho'} = 400, \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}qq} = 13\%$
Higher Loop Effects
S. J. Brodsky, F.-G. Cao and GdT (in preparation)

• Pion distribution amplitude

\[ \phi(x, Q) = \int \frac{dz^-}{2\pi} e^{i(2x-1)z^-/2} \left\langle 0 \left| \bar{\psi}(-z) \frac{\gamma^+ \gamma_5}{2\sqrt{2}} \Omega \psi(z) \right| \pi \right\rangle^{(Q)} \]

\[ = \int_{0}^{Q^2} \frac{dk^2}{16\pi^2} \psi(x, k_{\perp}) \]

• Normalization

\[ \int_{0}^{1} dx \phi(x, \mu_0) = \frac{f_\pi}{2\sqrt{3}} \]

• Evolution of pion DA given by the ERBL equation.

\[ \phi(x, Q^2) = x(1-x) \sum_{n=0,2,4,\ldots}^{\infty} a_n(Q^2) C_n^{3/2}(2x - 1) \]

• Meson transition form factor \((\bar{x} = 1 - x)\)

\[ Q^2 F_M^\gamma(Q^2) = c_M \frac{4}{\sqrt{3}} \int_{0}^{1} dx \frac{\phi(x, \bar{x}Q)}{\bar{x}} \left[ 1 - \exp \left( -\frac{\bar{x}Q^2}{2\kappa^2 x} \right) \right] \]
Asymptotic form: $\phi_{\text{asy}}(x, \mu_0) = \sqrt{3} f_\pi x (1 - x)$

AdS/QCD form: $\phi^{\text{AdS}}(x, \mu_0) = \frac{4}{\sqrt{3} \pi} f_\pi \sqrt{x (1 - x)}$

DA evolution $Q^2 = 0.5, 1, 10, 100, 1000 \text{ GeV}^2$
“Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out”

P.A.M. Dirac (1977)