Light-Front Holography: The AdS/CFT Correspondence and Light-Front QCD

Stan Brodsky, SLAC/IPPP

LIGHT CONE 2008, Mulhouse, July 8, 2008
Swiss German mathematician, astronomer, physicist, and philosopher who provided the first rigorous proof that $\pi$ (the ratio of a circle’s circumference to its diameter) is irrational, meaning that it cannot be expressed as the quotient of two integers.

born August 26, 1728, Mülhausen, Alsace
died September 25, 1777, Berlin, Prussia [Germany]
The World of Quarks and Gluons:

- Quarks and Gluons: Fundamental constituents of hadrons and nuclei
- Remarkable and novel properties of Quantum Chromodynamics (QCD)
- New Insights from higher space-time dimensions: Holography: AdS/CFT
- Hadron Dynamics at the Amplitude Level
- Light-Front wavefunctions analogous to Schrodinger wavefunctions of atomic physics
Dirac’s Amazing Idea: The Front Form

Evolve in ordinary time

\[ \sigma = ct - z \]

Evolve in light-front time!

\[ \tau = t + \frac{z}{c} \]

Instant Form

Front Form

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

LC 2008
July 8, 2008

Light-Front Holography

Stan Brodsky
SLAC & IPPP
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + \frac{z}{c} \)

\[ \sum_i^n x_i = 1 \]

\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Process Independent

Direct Link to QCD Lagrangian!

Invariant under boosts! Independent of \( P^\mu \)

Light-Front Holography

Stan Brodsky
SLAC & IPPP

LC 2008
July 8, 2008
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\Psi_n(x_i, \vec{k}_i, \lambda_i)$$

$x_i = \frac{k_i^+}{P^+}$

Invariant under boosts. Independent of $P^\mu$

$$H_{QCD}^{LF} |\psi> = M^2 |\psi>$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Each element of flash photograph illuminated at same LF time

\[ \tau = t + \frac{z}{c} \]

Boundary conditions set at fixed

\[ \tau = \tau_0 \]
Calculation of Form Factors in Equal-Time Theory

Instant Form:

\[ \sum \quad \]

Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form:

\[ \sum \quad \]

Absent for \( q^+ = 0 \) zero !!
Calculation of Hadron Form Factors
Instant Form

- Current matrix elements of hadron include interactions with vacuum-induced currents arising from infinitely-complex vacuum.

- Pair creation from vacuum occurs at any time before probe acts -- acausal.

- Knowledge of hadron wavefunction insufficient to compute current matrix elements.

- Requires dynamical boost of hadron wavefunction -- unknown except at weak binding.

- Complex vacuum even for QED.

- None of these complications occur for quantization at fixed LF time (front form).
\[
\frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2k_\perp] \sum_j e_j \frac{1}{2} \times 
\left[ -\frac{1}{q_L} \psi^\dagger_a(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) + \frac{1}{q_R} \psi^\dagger_a(x_i, k'_{\perp i}, \lambda_i) \psi_a(x_i, k_{\perp i}, \lambda_i) \right]
\]

\[ k'_{\perp i} = k_{\perp i} - x_i q_\perp \]

\[ k'_{\perp j} = k_{\perp j} + (1 - x_j) q_\perp \]

\[ q^2 = - q_\perp^2 \]

\[ q^+ = 0 \]

\[ q_{R,L} = q^x \pm iq^y \]

Must have \( \Delta \ell_z = \pm 1 \) to have nonzero \( F_2(q^2) \)

\[ \text{Checked to } O(\alpha^3) \text{ in QED} \]

Stan Brodsky
SLAC & IPPP

Light-Front Holography

LC 2008
July 8, 2008

Pinsky, Suaya, sjb
Anomalous gravitomagnetic moment $B(0)$

Okun, Kobzarev, Teryaev: $B(0)$ Must vanish because of Equivalence Theorem

$B(0) = 0$

Each Fock State

Hwang, Ma, Schmidt, sjb; Holstein et al

Light-Front Holography

Stan Brodsky
SLAC & IPPP

LC 2008
July 8, 2008
Angular Momentum on the Light-Front

**A⁺=0 gauge:**

\[ J^z = \sum_{i=1}^{n} s^z_i + \sum_{j=1}^{n-1} l^z_j. \]

No unphysical degrees of freedom

Conserved LF Fock state by Fock State

**Angular Momentum on the Light-Front**

**A⁺=0 gauge:**

\[ l^z_j = -i \left( k_1^j \frac{\partial}{\partial k_2^j} - k_2^j \frac{\partial}{\partial k_1^j} \right) \]

n-1 orbital angular momenta

Nonzero Anomalous Moment requires Nonzero orbital angular momentum.
\[ |p, S_z > = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; \vec{k}_{\perp i}, \lambda_i > \]

**sum over states with n=3, 4, ...constituents**

The Light Front Fock State Wavefunctions
\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]
are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction
\[ x_i = \frac{k^+_i}{p^+} = \frac{k^0_i + k^z_i}{P^0 + P^z} \]
are boost invariant.

\[ \sum_i^n k^+_i = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}. \]

**Intrinsic heavy quarks,**
\[ \bar{s}(x) \neq s(x) \]
\[ \bar{u}(x) \neq \bar{d}(x) \]

**Fixed LF time**

**LC 2008**
**July 8, 2008**

**Light-Front Holography**

Stan Brodsky
SLAC & IPPP
E866/NuSea (Drell-Yan)

$$\bar{d}(x) \neq \bar{u}(x)$$

$$s(x) \neq \bar{s}(x)$$

Intrinsic glue, sea, heavy quarks
Compare protons versus anti-proton in $s$ current quark fragmentation

$$D_{s\rightarrow p}(z) \neq D_{s\rightarrow \bar{p}}(z)$$

Tag $s$ quark via high $x_F$ $\Lambda$ production in proton fragmentation region.

$$A_s^{pp}(z) = \frac{D_{s\rightarrow p}(z) - D_{s\rightarrow \bar{p}}(z)}{D_{s\rightarrow p}(z) + D_{s\rightarrow \bar{p}}(z)}$$

Consequence of $s_p(x) \neq \bar{s}_p(x)$

$$|uudss\bar{s}| \simeq |K^+\Lambda|$$

B.Q. Ma and sjb
Hidden Color of Deuteron

Evolution of 5 color-singlet Fock states

\[ \Psi_n^d(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ \sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp} \]

\[ \sum_i^n x_i = 1 \]

\[ \Phi_n(x_i, Q) = \int k_{\perp i}^2 < Q^2 \prod' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j}) \]

Ji, Lepage, sjb

5 x 5 Matrix Evolution Equation for deuteron distribution amplitude
A Unified Description of Hadron Structure

\[ \Psi_n(x_i, \vec{k}_\perp, \lambda_i) \]

\text{LFWFs}

Elastic form factors

B-Decays

Real Compton scattering at high t

GPDs

Parton momentum distributions \text{TMDs}

Hadronization at the amplitude level

Deeply Virtual Compton Scattering

Deeply Virtual Meson production

Distribution Amplitudes

LC 2008
July 8, 2008

\textbf{Light-Front Holography}

Stan Brodsky
SLAC & IPPP
QCD Lagrangian

Yang-Mills Gauge Principle:
Invariance under Color Rotation and Phase Change at Every Point of Space and Time

\[ L_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr}(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i \bar{\psi}_f D^\mu \gamma^\mu \psi_f + \sum_{f=1}^{n_f} m_f \bar{\psi}_f \psi_f \]

Dimensionless Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

\[ L_{\text{QCD}} \rightarrow H_{\text{QCD}}^{LF} \rightarrow \psi_{n/H}^{LF}(x_i, \vec{k}_\perp i, \lambda_i) \]

LC 2008
July 8, 2008

Light-Front Holography

Stan Brodsky
SLAC & IPPP
\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_i [\frac{m^2 + k_\perp^2}{x}]_i + H^{int}_{LF} \]

\( H^{int}_{LF} \): Matrix in Fock Space

\[ H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h> \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
LIGHT-FRONT SCHRODINGER EQUATION

\[
\left( M_\pi^2 - \sum_i \frac{k_i^2}{x_i} \right) \begin{bmatrix}
\psi_{q\bar{q}/\pi} \\
\psi_{q\bar{q}g/\pi} \\
\vdots
\end{bmatrix} = \begin{bmatrix}
\langle q\bar{q} | V | q\bar{q} \rangle \\
\langle q\bar{q}g | V | q\bar{q} \rangle \\
\langle q\bar{q}g | V | q\bar{g}g \rangle \\
\vdots
\end{bmatrix} \begin{bmatrix}
\psi_{q\bar{q}/\pi} \\
\psi_{q\bar{q}g/\pi} \\
\vdots
\end{bmatrix}
\]

\[A^+ = 0\]
### Light-Front QCD

#### Heisenberg Matrix Formulation

<table>
<thead>
<tr>
<th>n Sector</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>gg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>gg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>gg</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>q̅q̅</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

### Discretized Light-Cone Quantization

### Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

**DLCQ:** Frame-independent, No fermion doubling; Minkowski Space

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrodinger Theory for Atomic Physics

- AdS/QCD Light-Front Holography

- Hadronic Spectra and Light-Front Wavefunctions
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu \nu}, P^\mu, D, K^\mu, \]

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5
Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

isomorphism

$x^\mu \rightarrow \lambda x^\mu, \ z \rightarrow \lambda z,$ maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \ z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

LC 2008
July 8, 2008

Light-Front Holography

Stan Brodsky
SLAC & IPPP
We will consider both holographic models

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

**We will consider both holographic models**
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $\text{AdS}_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal**: however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window**: $\alpha_s(Q^2) \simeq \text{const}$ at small $Q^2$

- **Use mathematical mapping of the conformal group** $\text{SO}(4,2)$ to $\text{AdS}_5$ space
Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

\[ \Gamma_{p^-n, Bj} (Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s g_1 (Q^2)}{\pi} \right] \]

IR conformal window

\[ \alpha_s(Q)/\pi \]

- JLab CLAS
- JLab PLB 650 4 244
- \( \alpha_{s,g_1}/\pi \) world data
- \( \alpha_{s,F3}/\pi \)
- GDH limit
- pQCD evol. eq.
- \( \alpha_{s,\pi}/\pi \) OPAL

Light-Front Holography

Stan Brodsky
SLAC & IPPP
Deur, Korsch, et al.

\[ \frac{\alpha_{s,gl}/\pi}{JLab} \]

- GDH limit
- Fit
- pQCD evol. eq.
- Cornwall
- Burkert-Ioffe
- Bloch et al.
- Godfrey-Isgur
- Bhagwat et al.
- Maris-Tandy
- Fischer et al.

DSE gluon couplings

\[ Q (GeV) \]

LC 2008
July 8, 2008

Light-Front Holography

Stan Brodsky
SLAC & IPPP
**IR Conformal Window for QCD?**

- **Dyson-Schwinger Analysis**: QCD gluon coupling has IR Fixed Point

- **Evidence from Lattice Gauge Theory**

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small** $Q^2$  
  
  \[ \Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 << 4m^2 \]

- **Justifies application of AdS/CFT in strong-coupling conformal window**

  Shrock, de Teramond, sjb

  Serber-Uehling

  \[ \ell^+ \quad \ell^- \]

**Light-Front Holography**

Stan Brodsky
SLAC & IPPP
Lesson from QED:

**Lamb Shift in Hydrogen**

\[ \Delta E \sim \alpha (Z\alpha)^4 \ln (Z\alpha)^2 m_e \]

\[ \lambda < \frac{1}{Z\alpha m_e} \]

\[ k > Z\alpha m_e \]

**Infrared divergence of free electron propagator removed because of atomic binding**

Maximum wavelength of bound electron

\[ \gamma^* \]

Bethe Log
Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{QCD}} \]

\[ \lambda < \Lambda_{QCD} \]

B-Meson

Shrock, sjb

gluon and quark propagators cutoff in IR because of color confinement
Lesson from QED and Lamb Shift: Consequences of Maximum Quark and Gluon Wavelength

- Infrared integrations regulated by confinement
- Infrared fixed point of QCD coupling
  \[ \alpha_s(Q^2) \text{ finite, } \beta \rightarrow 0 \text{ at small } Q^2 \]
- Bound state quark and gluon Dyson-Schwinger Equation
- Quark and Gluon Condensates exist within hadrons
• Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

• **Goal**: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances

• de Teramond, sjb: **AdS/QCD Holographic Model**: Initial “semi-classical” approximation to QCD. Predict light-quark hadron spectroscopy, form factors.

• Karch, Katz, Son, Stephanov: **Soft-Wall Model** -- **Linear Confinement**

• Mapping of AdS amplitudes to $3+1$ Light-Front equations, wavefunctions

• Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H_{\text{LF}}^{\text{QCD}}$; variational methods
AdS/CFT

• Use mapping of conformal group SO(4,2) to AdS$_5$

• Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension
  \[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

• Match solutions at small $z$ to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

• Hard wall model: Confinement at large distances and conformal symmetry in interior

• Truncated space simulates “bag” boundary conditions
  \[ 0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
- Physical AdS modes $\Phi_P(x, z) \sim e^{-iP \cdot x} \Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum $P^\mu$ and hadronic invariant mass states $P_\mu P^\mu = M^2$.

- For small-$z$ $\Phi(z) \sim z^\Delta$. The scaling dimension $\Delta$ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0\rangle \neq 0$.

$$\Delta = 2 + L$$

Twist dimension of meson

Identify hadron by its interpolating operator at $z \rightarrow 0$
Bosonic Solutions: Hard Wall Model

- Conformal metric: \( ds^2 = g_{\ell m} dx^\ell dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m} \).

- Action for massive scalar modes on AdS\(_{d+1}\):
  \[
  S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g_{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to \left(R/z\right)^{d+1}. 
  \]

- Equation of motion
  \[
  \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g_{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0. 
  \]

- Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = M^2 \):
  \[
  [z^2 \partial_z^2 - (d - 1) z \partial_z + z^2 M^2 - (\mu R)^2] \Phi(z) = 0. 
  \]

- Solution: \( \Phi(z) \to z^\Delta \) as \( z \to 0 \),
  \[
  \Phi(z) = C z^{d/2} J_{\Delta - d/2}(zM) \quad \Delta = \frac{1}{2} \left(d + \sqrt{d^2 + 4\mu^2 R^2} \right). 
  \]

\( \Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4 \)
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrodinger Equation for bound state of two scalar constituents:**

\[
\left[ -\frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
V(z) = -\frac{1-4L^2}{4z^2}
\]

Derived from variation of Action in AdS$_5$

**Hard wall model: truncated space**

\[
\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.
\]
**Match fall-off at small \( z \) to conformal twist-dimension at short distances**

- Pseudoscalar mesons: \( \mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\ell_1} \ldots D_{\ell_m} \psi \) (\( \Phi_\mu = 0 \) gauge). \( \Delta = 2 + L \)

- 4-\( d \) mass spectrum from boundary conditions on the normalizable string modes at \( z = z_0 \), \( \Phi(x, z_0) = 0 \), given by the zeros of Bessel functions \( \beta_{\alpha, k} \): \( M_{\alpha, k} = \beta_{\alpha, k} \Lambda_{QCD} \)

- Normalizable AdS modes \( \Phi(z) \)

\[ S = 0 \quad \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Light-Front Holography

Stan Brodsky
SLAC & IPPP

LC 2008
July 8, 2008

Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrödinger Equation for bound state of two scalar constituents:**

\[
\left[ -\frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)
\]

**Hard wall model: truncated space**

\[ V(z) = -\frac{1-4L^2}{4z^2} \quad \phi(z = z_0 = 1/\Lambda_0) = 0 \]

**Soft wall model: Harmonic oscillator confinement**

\[ V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2 \]

**Derived from variation of Action in AdS$_5$**
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
- \frac{d^2}{dz^2} + \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \phi_S(z) = M^2 \phi_S(z)
\]

with eigenvalues \(M^2 = 2\kappa^2 (2n + 2L + S)\).

- Compare with Nambu string result (rotating flux tube):

\[
M_n^2(L) = 2\pi \sigma (n + L + 1/2)
\]

- Compare with Nambu string result (rotating flux tube):

\[
M_n^2(L) = 2\pi \sigma (n + L + 1/2)
\]

Vector mesons orbital (a) and radial (b) spectrum for \(\kappa = 0.54\) GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories
Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = z Q K_1(z Q) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau - 1} \]

where \( \tau = \Delta_n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).

Polchinski, Strassler de Teramond, sjb

Dimensional Quark Counting Rule

General result from AdS/CFT
Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \ x^\ell = (x^\mu, z)$

$$ig_5 \int d^4x \ dz \sqrt{g} \ A^\ell(x, z) \Phi^{*}_P(x, z) \overleftrightarrow{\partial} \Phi_P(x, z).$$

- Electromagnetic probe polarized along Minkowski coordinates ($Q^2 = -q^2 > 0$)

$$A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q, z), \ A_z = 0.$$ 

- Propagation of external current inside AdS space described by the AdS wave equation

$$[z^2 \partial^2_z - z \partial_z - z^2 Q^2] \ J(Q, z) = 0,$$

subject to boundary conditions $J(Q = 0, z) = J(Q, z = 0) = 1.$

- Solution

$$J(Q, z) = zQ K_1(zQ).$$

- Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

$$\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \ \Phi(z) \rightarrow z^\Delta, \ z \rightarrow 0.$$
Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[
\left[ z^2 \partial_z^2 - z \left( 1 + 2\kappa^2 z^2 \right) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.
\]

- Solution bulk-to-boundary propagator

\[
J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right),
\]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[
\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.
\]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
\]

- For large \( Q^2 \gg 4\kappa^2 \)

\[
J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),
\]

the external current decouples from the dilaton field.
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement
Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

de Teramond, sjb
See also: Radyushkin
Stan Brodsky
SLAC & IPPP

Light-Front Holography

LC 2008
July 8, 2008
Constituent Counting Rules

\[
\frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s^{[n_{tot} - 2]}} \quad s = E_{cm}^2
\]

\[
F_{H}(Q^2) \sim [\frac{1}{Q^2}]^{n_{H} - 1}
\]

\[n_{tot} = n_A + n_B + n_C + n_D\]

Fixed \(t/s\) or \(\cos \theta_{cm}\)

Farrar & sjb; Matveev, Muradyan, Tavkhelidze

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests

LC 2008
July 8, 2008

Light-Front Holography

Stan Brodsky
SLAC & IPPP
Phenomenological success of dimensional scaling laws for exclusive processes

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^{n-2}}, \quad n = n_A + n_B + n_C + n_D, \]

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev et al. (1973).

Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space (hard behavior instead of soft behavior characteristic of strings) Polchinski and Strassler (2001).
Conformal Invariance: 

\[ \frac{d\sigma}{dt}(\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7} \]
Test of PQCD Scaling

Constituent counting rules

\[ s^7 \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) \sim \text{const fixed } \theta_{CM} \] scaling

PQCD and AdS/CFT:

\[ s^{n_{tot}} \frac{d\sigma}{dt}(A + B \rightarrow C + D) = F_{A+B\rightarrow C+D}(\theta_{CM}) \]

\[ s^{7} \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = F(\theta_{CM}) \]

\[ n_{tot} = 1 + 3 + 2 + 3 = 9 \]

No sign of running coupling

Conformal invariance
Quark-Counting

\[ \frac{d\sigma}{dt}(K^+p \rightarrow K^+p) = \frac{F(\theta_{CM})}{s^8} \]

\[ n = 2 \times 3 + 2 \times 2 - 2 = 8 \]
Quark-Counting: \[ \frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}} \]

\[ n = 4 \times 3 - 2 = 10 \]

Best Fit
\[ n = 9.7 \pm 0.5 \]

Reflects underlying conformal scale-free interactions

Angular distribution -- quark interchange

Light-Front Holography

Stan Brodsky
SLAC & IPPP