# Match fall-off at small z to Conformal Dimension of State at short distances

- Pseudoscalar mesons: $\mathcal{O}_{3+L} = \overline{\psi}\gamma_5 D_{\{\ell_1} \dots D_{\ell_m\}}\psi$ ($\Phi_\mu = 0$ gauge).
- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k}\Lambda_{QCD}$
- Normalizable AdS modes $\Phi(z)$

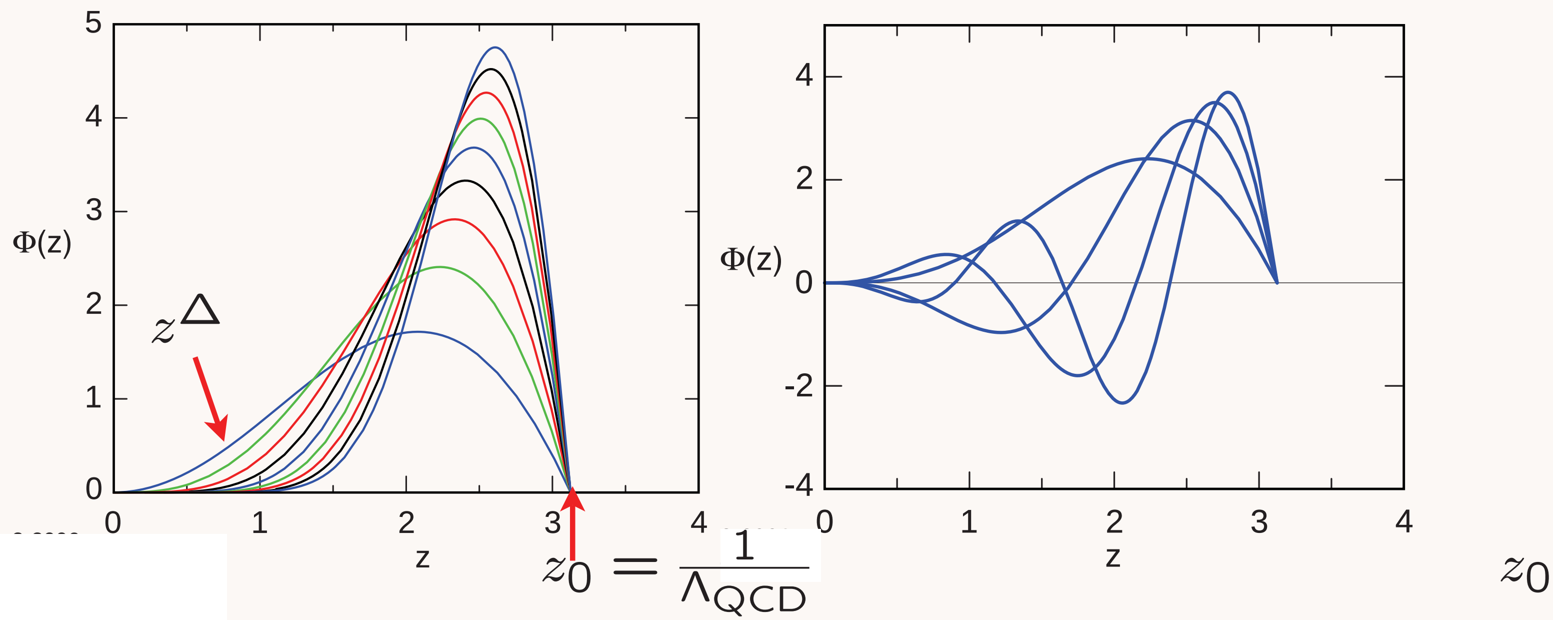



Fig: Meson orbital and radial AdS modes for $\Lambda_{QCD} = 0.32$ GeV.

# Meson Spectrum

- Vector meson interpolating operator with twist-dimension minus spin-two, and conformal dimension $\Delta = 3 + L$

$$\mathcal{O}^{\mu}_{3+L} = \overline{\psi}\gamma^{\mu} D_{\{\ell_1} \ldots D_{\ell_m\}}\psi.$$

$z \to 0$

$x^2 \to 0$

- AdS wave equation with effective 5-dim mass $\mu$. Solution is a vector field $\Phi_\mu$ with polarization along Poincaré coordinates:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2 + d - 1\right] f_\mu(z) = 0,$$

with $\Phi_\mu(x,z) = e^{-iP\cdot x}\, f_\mu(z)$ and $P_\mu P^\mu = \mathcal{M}^2$ ($\Phi_z = 0$ gauge).

$d = 4$

- Normalizable AdS vector mode:

$$\Phi_\mu(x,z) = Ce^{-iP\cdot x} z^{\frac{d}{2}} J_{\Delta-\frac{d}{2}}(z\mathcal{M})\,\epsilon_\mu.$$

$\sim z^{\Delta}$ at $z \to 0$

with $\Delta = d - 1 + L$ and $(\mu R)^2 = L(L + d - 2)$.

(Casimir )

# Introduction of Twist (Spin 0 and 1 AdS Modes)

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^{n} \sigma_i.$
- For a two quark state $\Delta \to \Delta - 1$. Change compensated in $\mu$ by the shift $L \to L - 1$.
- Lowest state corresponds to $(\mu R)^2 = -1$. Thus $-1 \leq (\mu R)^2$: Breitenlohner-Freedman stability bound for a 1-form.
- Two-quark vector meson described by wave equation (d=4)

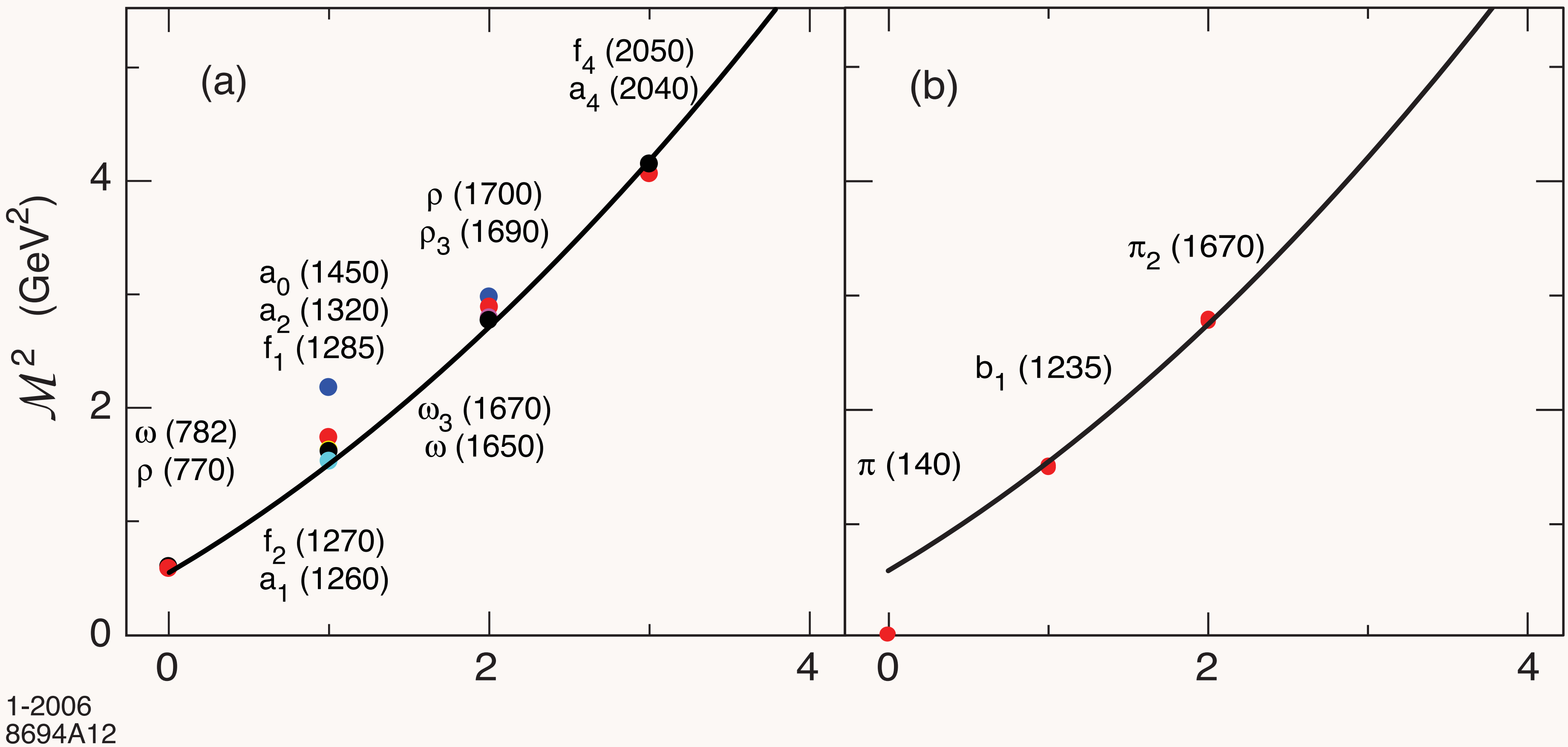
$$\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - L^2 + 4\right] f_\mu(z) = 0}$$

with solution

$$\Phi_\mu(x, z) = C e^{-iP\cdot x} z^2 J_L(z\mathcal{M})\, \epsilon_\mu.$$

- Same equation for $\Delta = 4$, $\tau = 2$ glueball 0-form with $-4 \leq (\mu R)^2$ and solution

$$\Phi(x, z) = C e^{-iP\cdot x} z^2 J_L(z\mathcal{M}).$$




Light meson orbital spectrum $\Lambda_{QCD} = 0.32 \text{ GeV}$

Guy de Teramond
SJB

# Baryon Spectrum

- Baryon: twist-three, dimension $\Delta = \frac{9}{2} + L$

$$\mathcal{O}_{\frac{9}{2}+L} = \psi D_{\{\ell_1} \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m\}} \psi, \quad L = \sum_{i=1}^{m} \ell_i.$$

- Solve full 10-dim Dirac Eq., $\not{D} \hat{\Psi} = 0$, since baryons are charged under $SU(4) \sim SO(6)$. Baryon number conservation?

- $\hat{\Psi}$ is expanded in terms of eigenfunctions $\eta_\kappa(y)$ of the Dirac operator on compact space $X$ with eigenvalues $\lambda_\kappa$:

$$\hat{\Psi}(x, z, y) = \sum_\kappa \Psi_\kappa(x, z) \eta_\kappa(y).$$

- From the 10-dim Dirac equation, $\not{D} \hat{\Psi} = 0$:

$$\left[ z^2 \, \partial_z^2 - d \, z \, \partial_z + z^2 \mathcal{M}^2 - (\lambda_\kappa + \mu)^2 R^2 + \frac{d}{2} \left( \frac{d}{2} + 1 \right) + (\lambda_\kappa + \mu) R \, \hat{\Gamma} \right] f(z) = 0,$$

$$i \not{D}_X \eta(y) = \lambda \, \eta(y),$$

where $\Psi(x, z) = e^{-iP \cdot x} \, f(z)$, $P_\mu P^\mu = \mathcal{M}^2$ and $\hat{\Gamma} u_\pm = \pm u_\pm$.

See: Muck and Viswanathan, hep-ph/9805945.

- Normalizable AdS baryon mode:

$$\Psi(x,z) = Ce^{-iP\cdot x} z^{\frac{d+1}{2}} \left[ J_{(\mu+\lambda_\kappa)R-\frac{1}{2}}(z\mathcal{M})\, u_+(P) + J_{(\mu+\lambda_\kappa)R+\frac{1}{2}}(z\mathcal{M})\, u_-(P) \right].$$

with $\Delta = \frac{d}{2} + |(\mu + \lambda_\kappa)R|$.

- For $d = 4$, $\hat{\Gamma} = \gamma_5$ and spinors $u_\pm(P)$ are defined along 4-dim coordinates.

- $\mu$ determined asymptotically by spectral comparison with orbital excitations in the boundary: $\mu = L/R$ and $\lambda_\kappa$ are the eigenvalues of the Dirac equation on $S^{d+1}$:

$$\lambda_\kappa R = \pm \left( \kappa + \frac{d}{2} + \frac{1}{2} \right), \quad \kappa = 0, 1, 2...$$

See: Camporesi and Higuchi: gr-gc/9505009.

- Spin-$\frac{3}{2}$ Rarita-Schwinger eq. in AdS similar to spin-$\frac{1}{2}$ in the $\Psi_z = 0$ gauge for polarization along Minkowski coordinates, $\Psi_\mu$. See: Volovich, hep-th/9809009.

## Introduction of Twist (Spin $\frac{1}{2}$ and $\frac{3}{2}$ AdS Modes)

- For spin-carrying constituents: $\Delta \to \tau = \Delta - \sigma, \;\; \sigma = \sum_{i=1}^{n} \sigma_i$.
- For a three quark state $\Delta \to \Delta - 3/2$. Change compensated in $\mu$ by the shift $L \to L - 1$ and $\Psi(z) \to z^{-\frac{1}{2}} \Psi(z)$.
- Three-quark baryon described by wave equation ($d = 4$, $\kappa = 0$)

$$\boxed{\left[z^2\, \partial_z^2 - 3z\, \partial_z + z^2 \mathcal{M}^2 - \mathcal{L}_\pm^2 + 4\right] f_\pm(z) = 0}$$

with $\mathcal{L}_+ = L + 1$, $\mathcal{L}_- = L + 2$, and solution

$$\Psi(x,z) = C e^{-iP\cdot x} z^2 \Big[J_{1+L}(z\mathcal{M})\, u_+(P) + J_{2+L}(z\mathcal{M})\, u_-(P)\Big].$$

- 4-$d$ mass spectrum $\Psi(x, z_o)^{\pm} = 0 \quad \Longrightarrow \quad$ parallel Regge trajectories for baryons !

$$\mathcal{M}^+_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}, \quad \mathcal{M}^-_{\alpha,k} = \beta_{\alpha+1,k} \Lambda_{QCD}.$$

- Ratio of eigenvalues determined by the ratio of zeros of Bessel functions !

Predictions of AdS/CFT

Only one parameter!

Entire light quark baryon spectrum

## PARITY DOUBLING

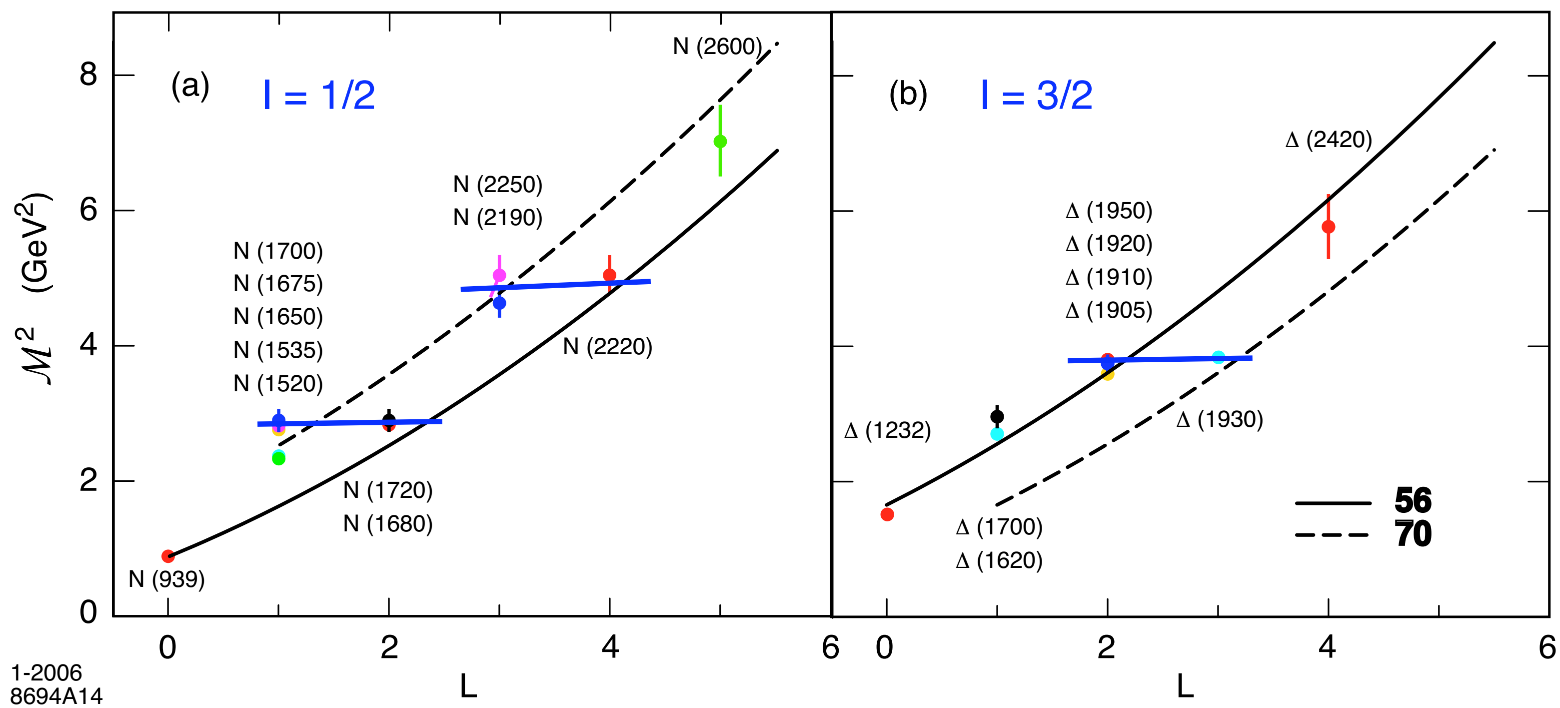



Fig: Predictions for the light baryon orbital spectrum for $\Lambda_{QCD}$ = 0.25 GeV. The $\mathbf{56}$ trajectory corresponds to $L$ even $P = +$ states, and the $\mathbf{70}$ to $L$ odd $P = -$ states.

Guy de Teramond
SJB

- $SU(6)$ multiplet structure for $N$ and $\Delta$ orbital states, including internal spin $S$ and $L$.

| $SU(6)$ | $S$ | $L$ | Baryon State |
|---|---|---|---|
| **56** | $\frac{1}{2}$ | 0 | $N\frac{1}{2}^+(939)$ |
| | $\frac{3}{2}$ | 0 | $\Delta\frac{3}{2}^+(1232)$ |
| **70** | $\frac{1}{2}$ | 1 | $N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$ |
| | $\frac{3}{2}$ | 1 | $N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$ |
| | $\frac{1}{2}$ | 1 | $\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$ |
| **56** | $\frac{1}{2}$ | 2 | $N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$ |
| | $\frac{3}{2}$ | 2 | $\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$ |
| **70** | $\frac{1}{2}$ | 3 | $N\frac{5}{2}^-$ $N\frac{7}{2}^-$ |
| | $\frac{3}{2}$ | 3 | $N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$ |
| | $\frac{1}{2}$ | 3 | $\Delta\frac{5}{2}^-(1930)$ $\Delta\frac{7}{2}^-$ |
| **56** | $\frac{1}{2}$ | 4 | $N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$ |
| | $\frac{3}{2}$ | 4 | $\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$ |
| **70** | $\frac{1}{2}$ | 5 | $N\frac{9}{2}^-$ $N\frac{11}{2}^-$ |
| | $\frac{3}{2}$ | 5 | $N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$ |

# Baryons

- Coupling of the extended AdS mode with an external gauge field $A^\mu(x,z)$

$$ig_5 \int d^4x\, dz\, \sqrt{g}\, A_\mu(x,z)\, \overline{\Psi}(x,z)\gamma^\mu \Psi(x,z),$$

where

$$\Psi(x,z) = e^{-iP\cdot x}\left[\psi_+(z)u_+(P) + \psi_-(z)u_-(P)\right],$$

$$\psi_+(z) = Cz^2 J_1(zM), \quad \psi_-(z) = Cz^2 J_2(zM),$$

and

$$u(P)_\pm = \frac{1 \pm \gamma_5}{2}\, u(P).$$

- In the large $P^+$ limit

$$\psi_+(z) \equiv \psi^\uparrow(z), \quad \psi_-(z) \equiv \psi^\downarrow(z),$$

the LC $\pm$ spin projection along $\hat{z}$.

- Constant $C$ determined by charge normalization:

$$C = \frac{\sqrt{2}\Lambda_{\rm QCD}}{R^{3/2}\left[-J_0(\beta_{1,1})J_2(\beta_{1,1})\right]^{1/2}}.$$

# Glueball Spectrum

- AdS wave function with effective mass $\mu$:

$$\left[z^2\,\partial_z^2 - (d-1)z\,\partial_z + z^2\,\mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

where $\Phi(x,z) = e^{-iP\cdot x}\,f(z)$ and $P_\mu P^\mu = \mathcal{M}^2$.

- Glueball interpolating operator with twist -dimension minus spin- two, and conformal dimension $\Delta = 4 + L$

$$\mathcal{O}_{4+L} = F D_{\{\ell_1} \dots D_{\ell_m\}} F,$$

where $L = \sum_{i=1}^m \ell_i$ is the total internal space-time orbital momentum.

- Normalizable scalar AdS mode ( d = 4):

$$\Phi_{\alpha,k}(x,z) = C_{\alpha,k} e^{-iP\cdot x} z^2 J_\alpha\left(z\,\beta_{\alpha,a}\Lambda_{QCD}\right)$$

with $\alpha = 2 + L$ and scaling dimension $\Delta = 4 + L$.

# Glueball Regge trajectories from gauge/string duality and the Pomeron

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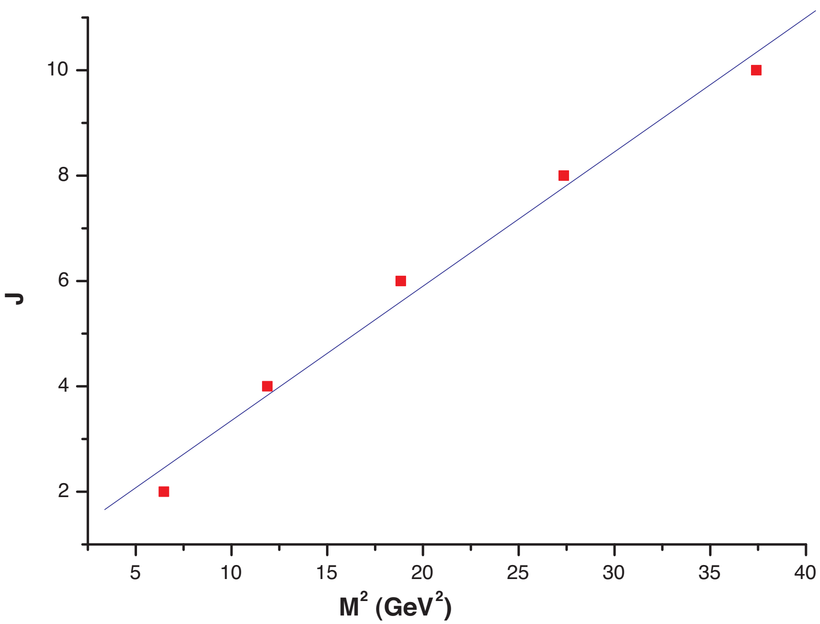
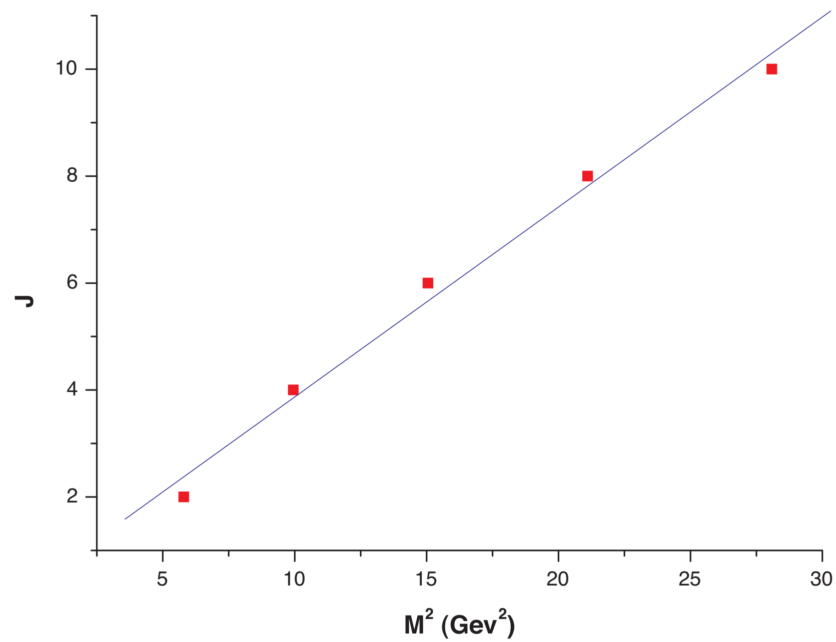



Neumann Boundary Conditions




Dirichlet Boundary Conditions

# Features of HolographicModel

de Teramond, sjb

- Ratio of proton to Delta trajectories= ratio of zeroes of Bessel functions.
- One scale $\Lambda_{QCD}$ determines hadron spectrum (different for mesons and baryons)
- Only quark-antiquark, qqq, and g g hadrons appear at classical level
- Covariant version of bag model: confinement+conformal symmetry

# Hadronic Form Factor in Space and Time-Like Regions

- The form factor in AdS/QCD is the overlap of the normalizable modes dual to the incoming and outgoing hadron $\Phi_I$ and $\Phi_F$ and the non-normalizable mode $J$, dual to the external source (hadron spin $\sigma$):

$$
\begin{aligned}
F(Q^2)_{I\to F} &= R^{3+2\sigma}\int_0^\infty \frac{dz}{z^{3+2\sigma}}\, e^{(3+2\sigma)A(z)}\Phi_F(z)\, J(Q,z)\,\Phi_I(z) \\
&\simeq R^{3+2\sigma}\int_0^{z_o} \frac{dz}{z^{3+2\sigma}}\, \Phi_F(z)\, J(Q,z)\,\Phi_I(z),
\end{aligned}
$$

- $J(Q,z)$ has the limiting value 1 at zero momentum transfer, $F(0)=1$, and has as boundary limit the external current, $A^\mu = \epsilon^\mu e^{iQ\cdot x} J(Q,z)$. Thus:

$$\lim_{Q\to 0} J(Q,z) = \lim_{z\to 0} J(Q,z) = 1.$$

- Solution to the AdS Wave equation with boundary conditions at $Q=0$ and $z\to 0$:

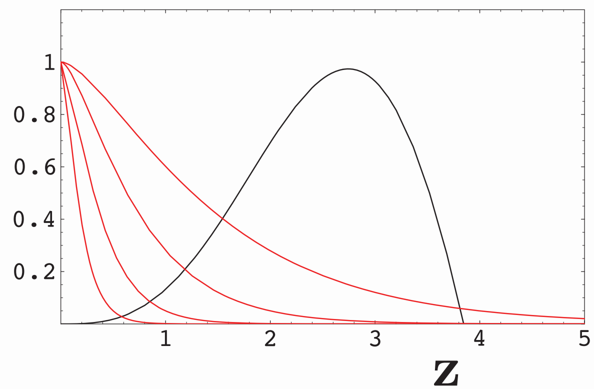
$$J(Q,z) = zQK_1(zQ).$$

Polchinski and Strassler, hep-th/0209211; Hong, Yong and Strassler, hep-th/0409118.

- Propagation of external perturbation suppressed inside AdS.

- At large enough $Q \sim r/R^2$, the interaction occurs in the large-$r$ conformal region. Important contribution to the FF integral from the boundary near $z \sim 1/Q$.

$\mathbf{J}(\mathbf{Q}, \mathbf{z}), \quad \mathbf{\Phi}(\mathbf{z})$




- Consider a specific AdS mode $\Phi^{(n)}$ dual to an $n$ partonic Fock state $|n\rangle$. At small $z$, $\Phi^{(n)}$ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

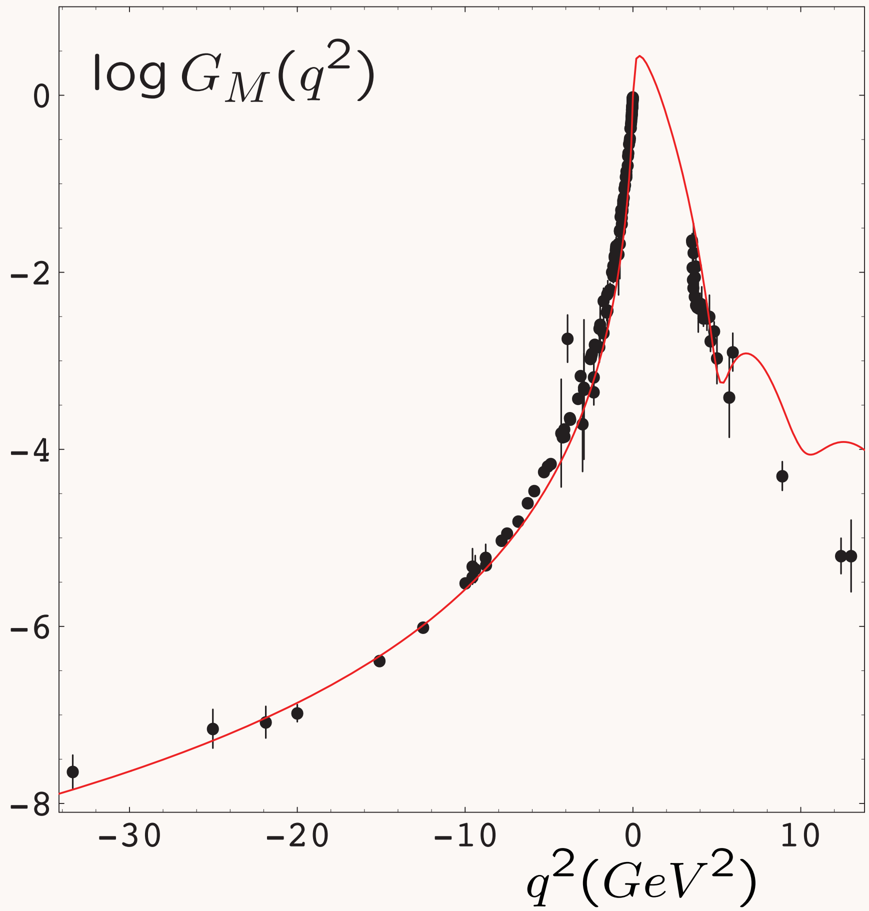
General result from AdS/CFT

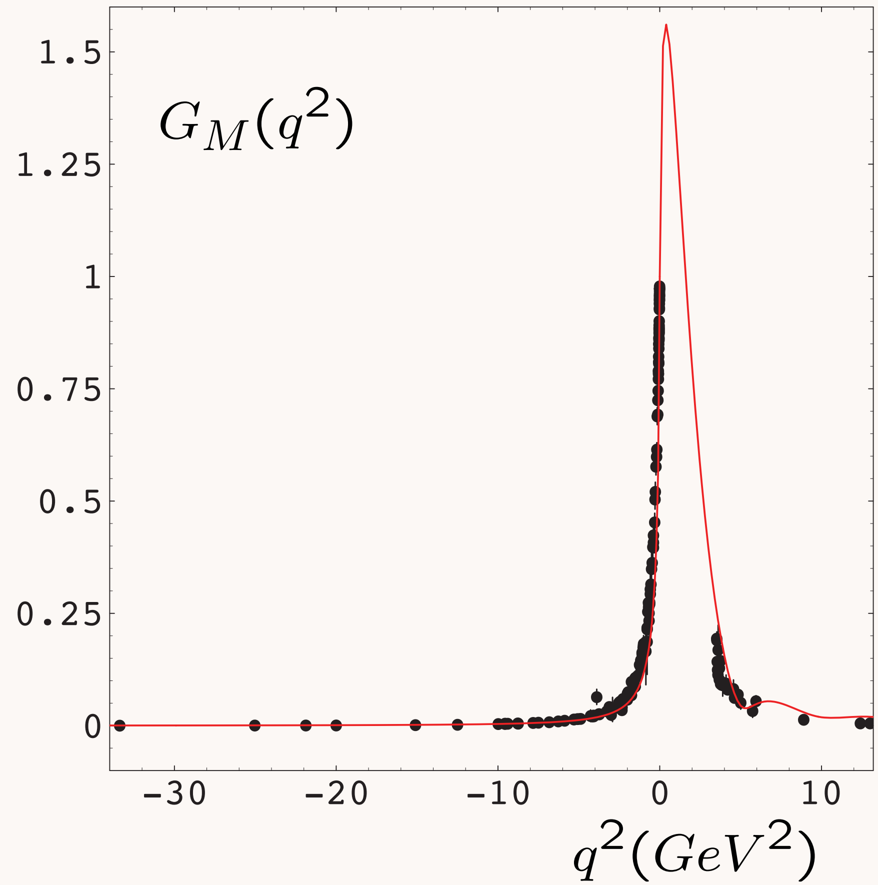
where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n} \sigma_i$. The twist is equal to the number of partons, $\tau = n$.

## Prediction of AdS/CFT Holographic Model

One parameter: $\Lambda_{QCD} = 0.15$ GeV.

Guy F. de Téramond and sjb




Space-like and time-like structure of the proton magnetic form factor in AdS/QCD for $\Lambda_{QCD} = 0.15$ GeV. The data are from the compilation given by Baldini et al.

$$J(Q, z) = zQK_1(zQ)$$

- Consider the spin non-flip form factors in the infinite wall approximation

$$F_+(Q^2) = g_+ R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_+(z)|^2,$$
$$F_-(Q^2) = g_- R^3 \int \frac{dz}{z^3} J(Q,z) \, |\psi_-(z)|^2,$$

where the effective charges $g_+$ and $g_-$ are determined from the spin-flavor structure of the theory.

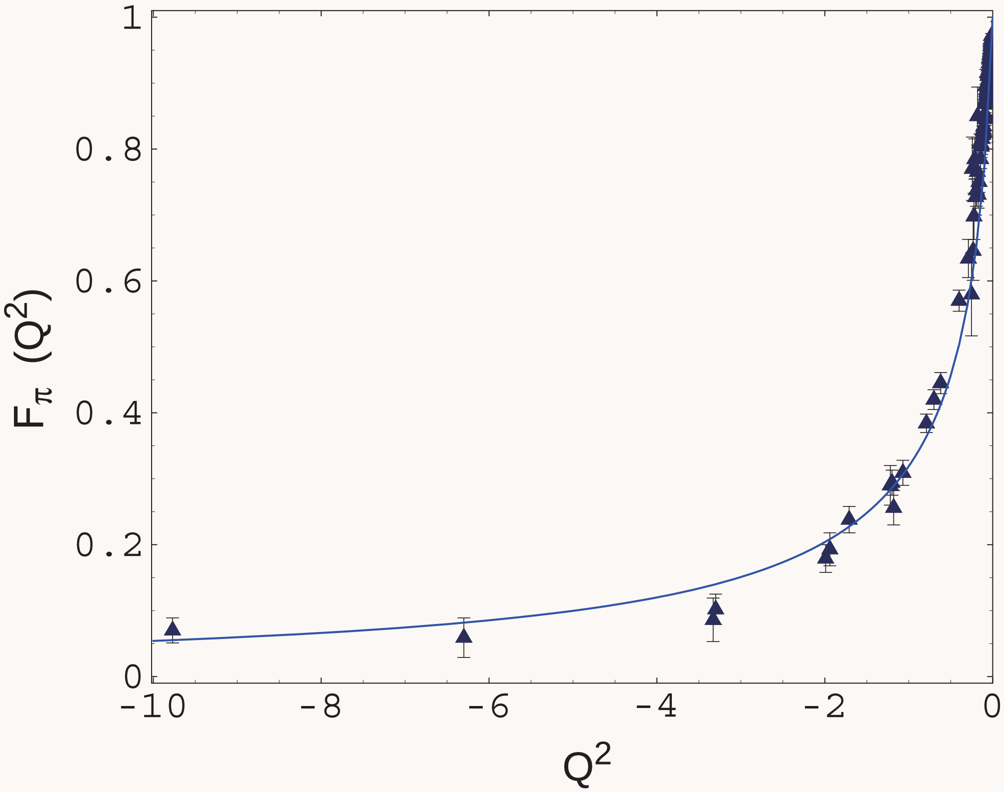
- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(z)$ and $\psi_-(z)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = R^3 \int \frac{dz}{z^3} J(Q,z) |\psi_+(z)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} R^3 \int \frac{dz}{z^3} J(Q,z) \left[ |\psi_+(z)|^2 - |\psi_-(z)|^2 \right],$$

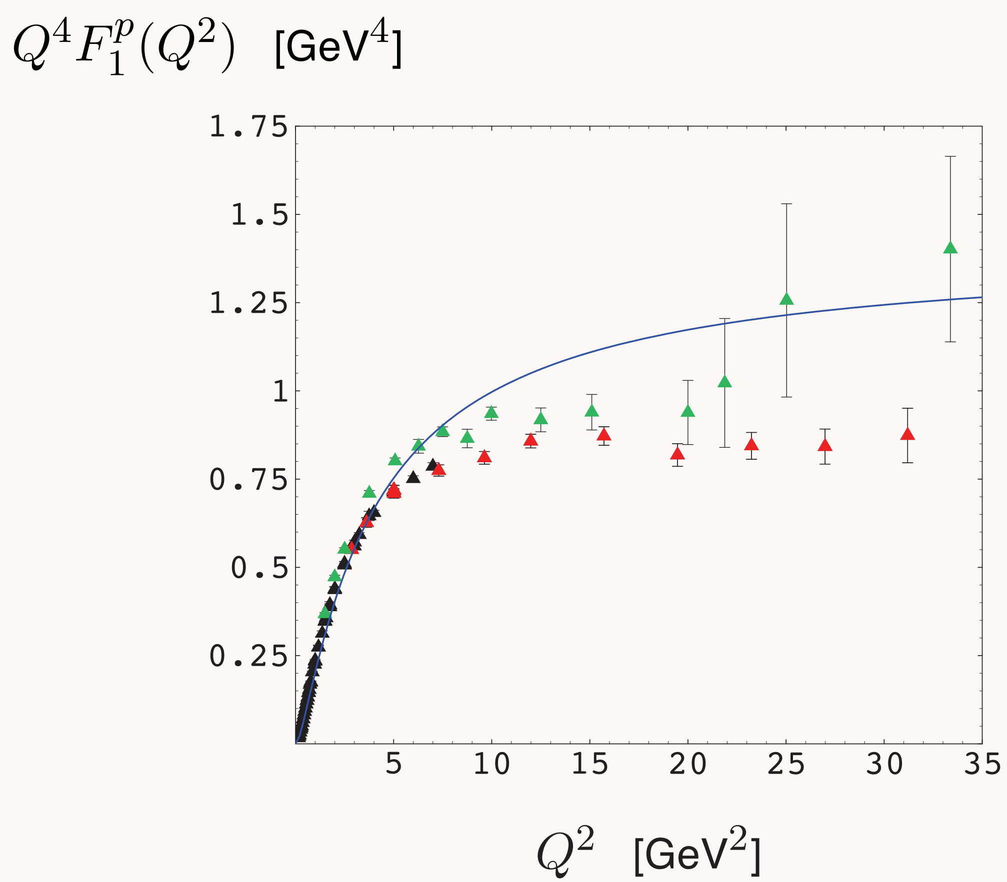
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Large $Q$ power scaling: $F_1(Q^2) \to \left[1/Q^2\right]^2$.



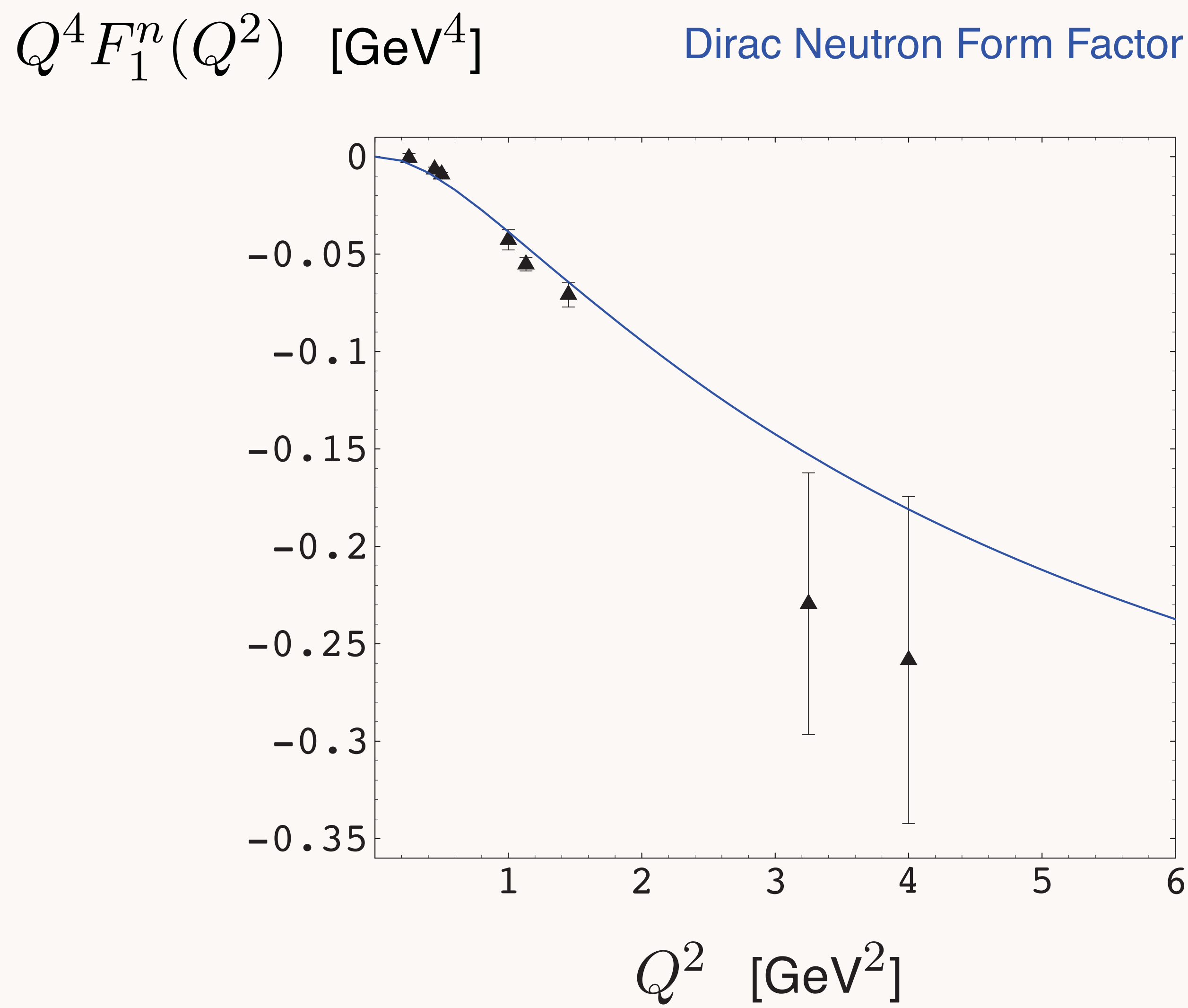

Space-like pion form factor in holographic model for $\Lambda_{QCD} = 0.2$ GeV.

## Dirac Proton Form Factor $F_1^p$




Prediction for $Q^4 F_1^p(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the infinite wall approximation from Kirk (superimposed green points assuming $G_E^p = G_M^p$): P. N. Kirk *et al.*, Phys. Rev. D **8** (1973) 63.

# Dirac Neutron Form Factor $F_1^n$




Prediction for $Q^4 F_1^n(Q^2)$ for $\Lambda_{\rm QCD} = 0.21$ GeV in the infinite wall approximation.

# Exact Representation of Form Factors using LFWFs

Hadron form factors can be expressed as a sum of overlap integrals of light-front wave functions:

Drell Yan, West, Drell, SJB

$$F(q^2) = \sum_n \int \left[dx_i\right] \left[d^2\vec{k}_{\perp i}\right] \sum_j e_j \psi_n^*(x_i, \vec{k}'_{\perp i}, \lambda_i) \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i), \qquad (1)$$

where the variables of the light-cone Fock components in the final-state are given by

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\ \vec{q}_\perp, \qquad (2)$$

for a struck constituent quark and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\ \vec{q}_\perp, \qquad (3)$$

for each spectator. The momentum transfer is $q^2 = -\vec{q}_\perp^{\,2} = -2P \cdot q = -Q^2$. The measure of the phase-space integration is

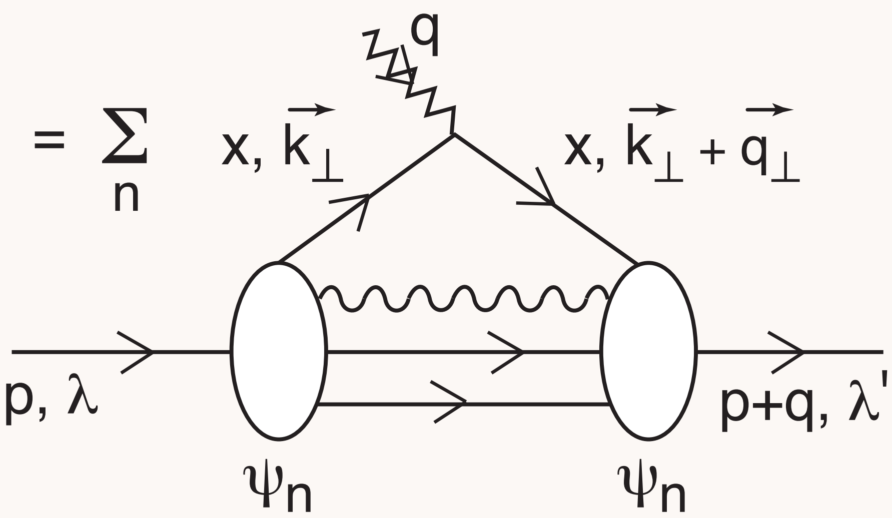
$$\left[dx_i\right] = \prod_{i=1}^{n} \frac{dx_i}{\sqrt{x_i}}\, \delta\left(1 - \sum_{j=1}^{n} x_j\right), \qquad (4)$$

$$\left[d^2\vec{k}_{\perp i}\right] = (16\pi^3) \prod_{i=1}^{n} \frac{d^2\vec{k}_{\perp i}}{16\pi^3} \delta^{(2)}\left(\sum_{\ell=1}^{n} \vec{k}_{\perp \ell}\right). \qquad (5)$$

# Light-Front Calculation of Form Factors

Form Factors $\ell p \to \ell' p'$ $\langle p' \lambda' | J^+(0) | p\lambda \rangle$

$$F_{\lambda\lambda'}(Q^2) = \sum_n$$




$$\psi(x, \vec{k}_\perp) \qquad \psi(x, \vec{k}_\perp + (1-x)\vec{q}_\perp)$$

# Drell-Yan West formula for Form Factor of meson

$$F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2\vec{k}_\perp}{16\pi^3} \; \psi^*_{P'}(x, \vec{k}_\perp + (1-x)\vec{q}_\perp) \; \psi_P(x, \vec{k}_\perp).$$

## In Impact Space:

$$F(q^2) = 4\pi \int_0^1 dx \int \; d^2\vec{b}_\perp \; e^{i\vec{b}_\perp \cdot \vec{q}_\perp} \left|\widetilde{\psi}(x,b)\right|^2$$

$$= 8\pi^2 \int_0^1 dx \int_0^\infty b \; db \; J_0\left(bqx\right) \; \left|\widetilde{\psi}(x,b)\right|^2,$$

For a two-parton state, the light-front wave function is effectively cutoff at

$$\vec{b}_\perp^2 \simeq \frac{1}{x(1-x)\Lambda^2_{\rm QCD}},$$

## Change Integration variable to: $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F_{IS}(Q^2) = 8\pi^2 \int_0^1 \frac{dx}{x(1-x)} \int_0^{\Lambda^{-1}_{\rm QCD}} \zeta \; d\zeta \; J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \; \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Change the integration variable $\zeta = |\vec{b}_\perp|\sqrt{x(1-x)}$

$$F(Q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int_0^{\zeta_{max}=\Lambda^{-1}_{\rm QCD}} \zeta \, d\zeta \, J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) \left|\widetilde{\psi}(x,\zeta)\right|^2,$$

- Compare with AdS form factor for arbitrary $Q$. Find:

$$J(Q,\zeta) = \int_0^1 dx J_0\left(\frac{\zeta Q x}{\sqrt{x(1-x)}}\right) = \zeta Q K_1(\zeta Q),$$

Recover AdS/CFT formula !

the solution for the electromagnetic potential in AdS space, and

$$\widetilde{\psi}(x,\vec{b}_\perp) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{0,1}\Lambda_{QCD}\right) \theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda^{-2}_{\rm QCD}}{x(1-x)}\right)$$

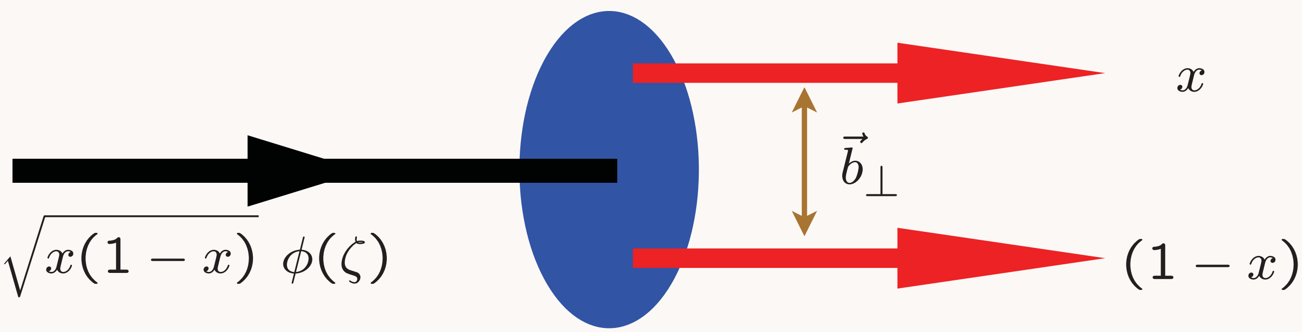
the holographic LFWF for the valence Fock state of the pion $\psi_{\overline{q}q/\pi}$.

- The variable $\zeta$, $0 \le \zeta \le \Lambda^{-1}_{QCD}$, represents the scale of the invariant separation between quarks and is also the holographic coordinate $\zeta = z$ !

Similar in form to Radyushkin model

# Mapping between LF(3+1) and AdS$_5$

| LF(3+1) | | AdS$_5$ |
| --- | --- | --- |
| $\psi(x, \vec{b}_\perp)$ | $\Leftrightarrow$ | $\phi(z)$ |
| $\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$ | $\Leftrightarrow$ | $z$ |




$$\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)}\ \phi(\zeta)$$

# AdS/CFT and Light-Front Wavefunctions

- Light-Front Wavefunctions can be determined by matching functional dependence in fifth dimension to scaling in impact space.

$$\left[z^2\, \partial_z^2 - (d-1) z\, \partial_z + z^2\, \mathcal{M}^2 - (\mu R)^2\right] f(z) = 0,$$

$$z \to \zeta = b\sqrt{x(1-x)}$$

de Teramond and sjb

- High transverse momentum behavior matches (mod logs) PQCD LFWF with orbital: Belitsky, Ji, Yuan

- Perfect match of LF and AdS/CFT formulae for form factors

G. de Teramond and sjb

# Map AdS/CFT to 3+1 LF Theory

Effective radial equation:

$$\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right]\phi(\zeta) = \mathcal{M}^2\phi(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Effective conformal potential:

$$V(\zeta) = -\frac{1-4L^2}{4\zeta^2}.$$

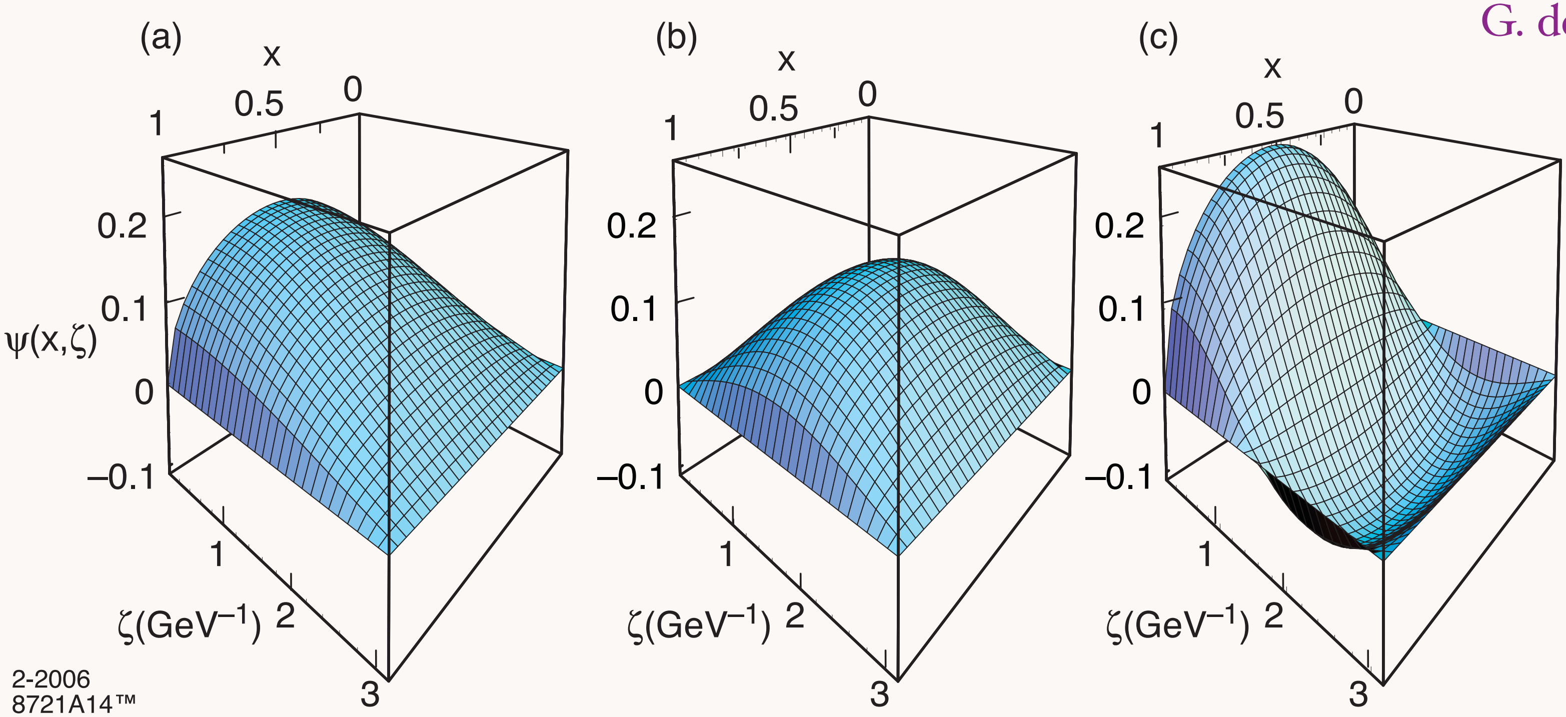
General solution:

$$\widetilde{\psi}_{L,k}(x,\vec{b}_\perp) = B_{L,k}\sqrt{x(1-x)}$$

$$\times \quad J_L\left(\sqrt{x(1-x)}|\vec{b}_\perp|\beta_{L,k}\Lambda_{QCD}\right)\theta\left(\vec{b}_\perp^{\,2} \le \frac{\Lambda_{QCD}^{-2}}{x(1-x)}\right),$$

# AdS/CFT Prediction for Meson LFWF

G. de Teramond
SJB

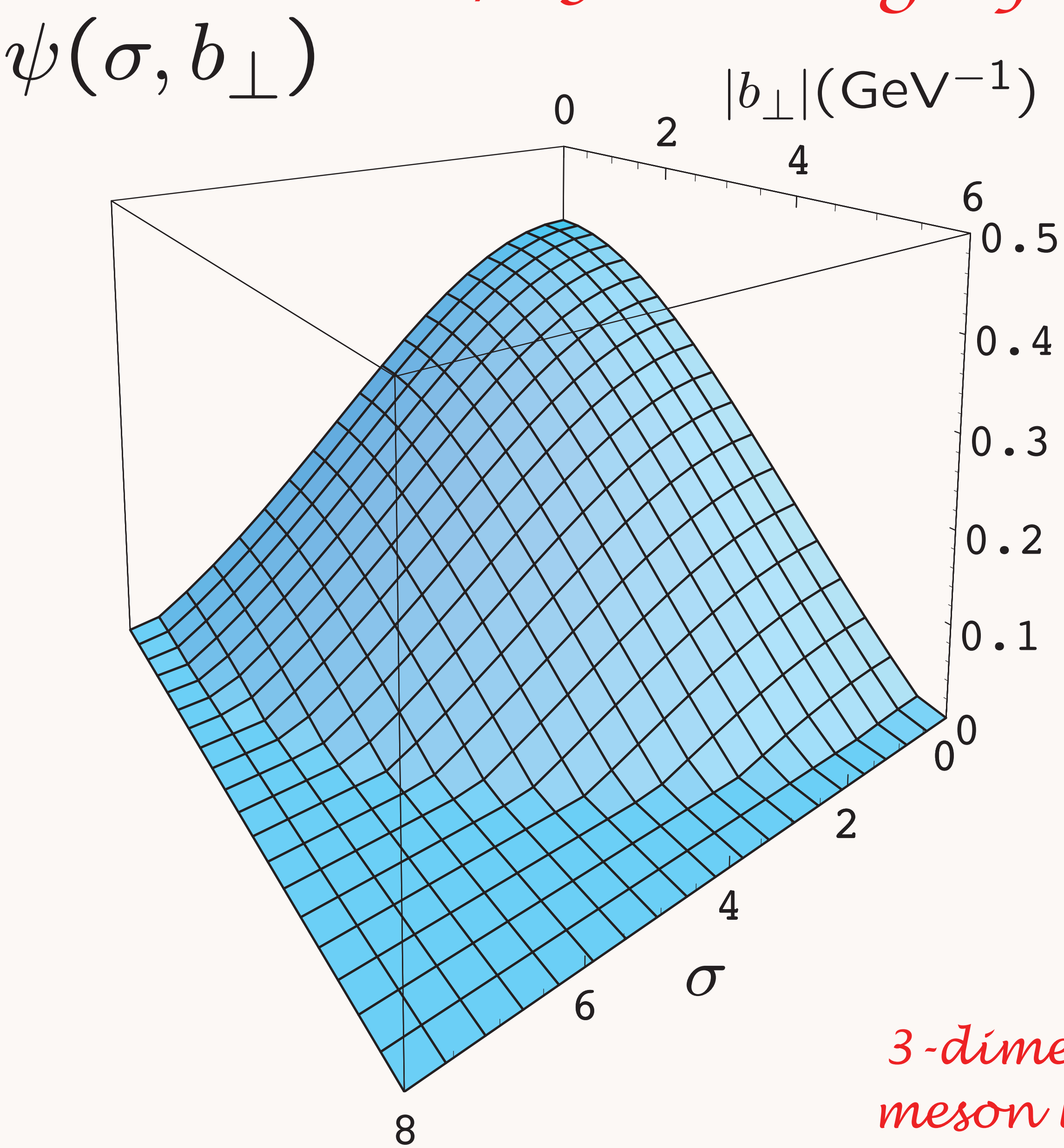


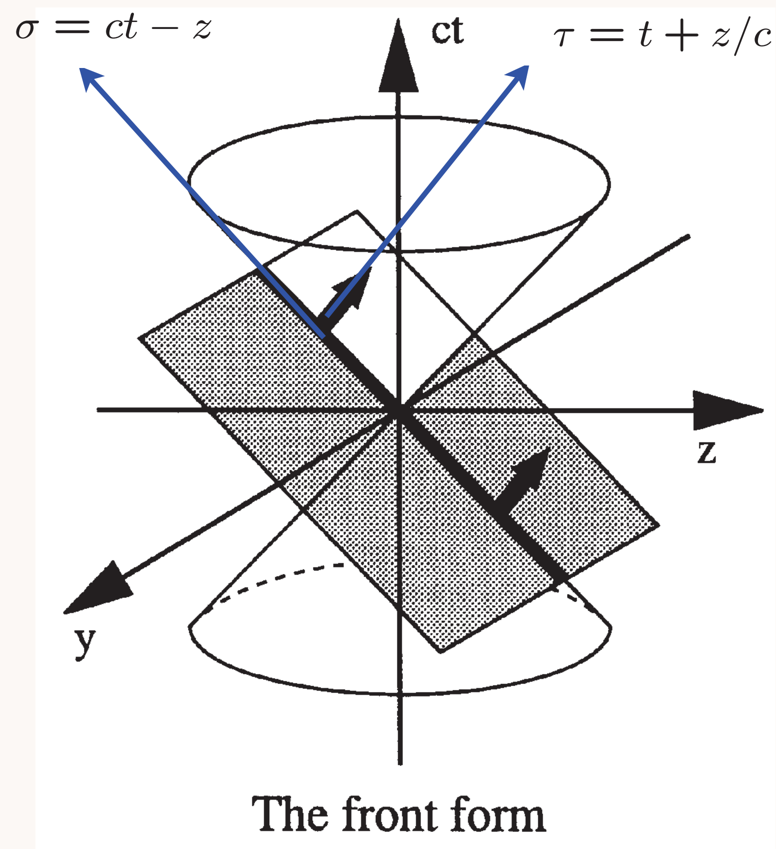

Two-parton holographic LFWF in impact space $\widetilde{\psi}(x,\zeta)$ for $\Lambda_{QCD} = 0.32$ GeV: (a) ground state $L = 0,\ k = 1$; (b) first orbital exited state $L = 1,\ k = 1$; (c) first radial exited state $L = 0,\ k = 2$. The variable $\zeta$ is the holographic variable $z = \zeta = |b_\perp|\sqrt{x(1-x)}$.

$$\widetilde{\psi}(x,\zeta) = \frac{\Lambda_{\rm QCD}}{\sqrt{\pi} J_1(\beta_{0,1})} \sqrt{x(1-x)} J_0\left(\zeta \beta_{0,1} \Lambda_{QCD}\right) \theta\left(z \le \Lambda_{\rm QCD}^{-1}\right)$$

# AdS/CFT Holographic Model

G. de Teramond
SJB




*3-dimensional photograph: meson LFWF at fixed LF Time*

## Evaluation of QCD Matrix Elements: Example $f_\pi$

- Pion decay constant defined by the matrix element of EW current $J^+_W$:

$$\left\langle 0 \left| \overline{\psi}_u \gamma^+ (1-\gamma_5) \psi_d \right| \pi^- \right\rangle = i\sqrt{2} P^+ f_\pi ,$$

with

$$\left|\pi^-\right\rangle = |d\overline{u}\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left( b^\dagger_{c\, d\downarrow} d^\dagger_{c\, u\uparrow} - b^\dagger_{c\, d\uparrow} d^\dagger_{c\, u\downarrow} \right) |0\rangle .$$

- Use light-cone expression:

$$f_\pi = 2\sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \, \psi_{\overline{q}q/\pi}(x, k_\perp) .$$

Lepage and Brodsky, Phys. Rev. D **22**, 2157 (1980)

- Find:

$$f_\pi = \frac{\sqrt{3}\Lambda_{\rm QCD}}{8 J_1(\beta_{0,1})} = 83.4 \, {\rm Mev},$$

for $\Lambda_{\rm QCD} = 0.2$ GeV.

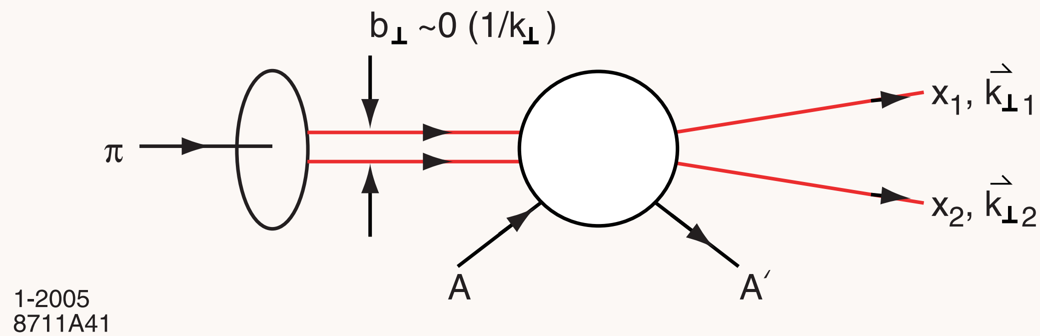
Experiment: $f_\pi = 92.4$ Mev.

# Use Diffraction to Resolve Hadron Substructure

- Measure Light-Front Wavefunctions
- Test AdS/CFT predictions
- Novel Aspects of Hadron Wavefunctions: Intrinsic Charm, Hidden Color, Color Transparency/Opaqueness
- Diffractive Di-Jet Production
- Nuclear Shadowing and Antishadowing
- New Mechanism for Higgs Production

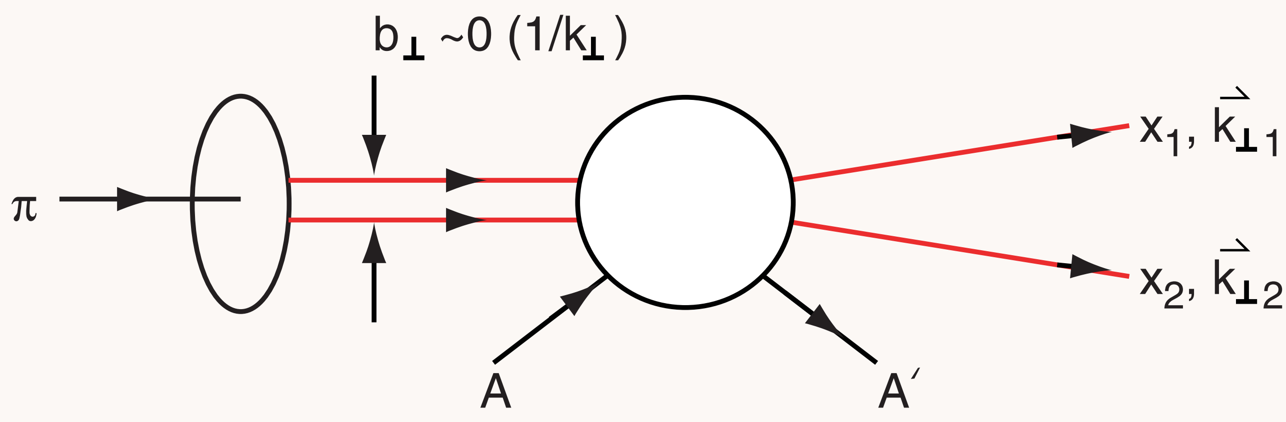
# Diffractive Dissociation of Pion

E791 Ashery et al.

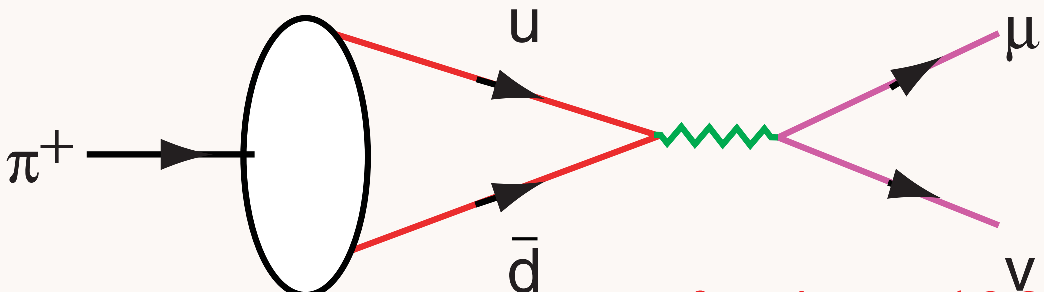



Measure Light-Front Wavefunction of Pion
Two-gluon Exchange
Minimal momentum transfer to nucleus
Nucleus left Intact

# Fluctuation of a Pion to a Compact Color Dipole State

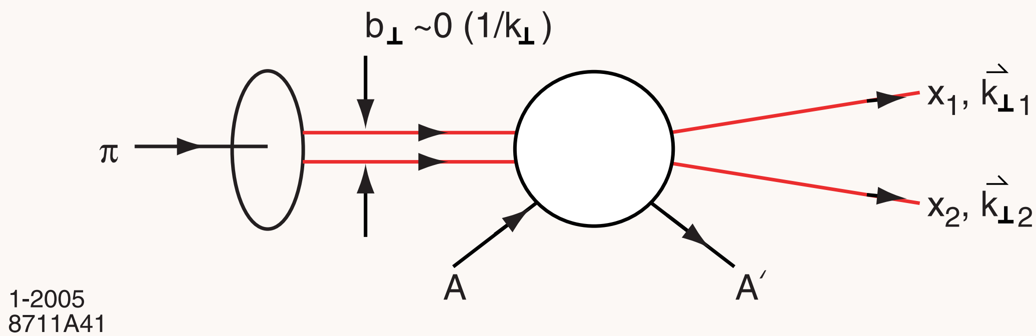



Color-Transparent Fock State For High Transverse Momentum Di-Jets




Same Fock State Determines Weak Decay

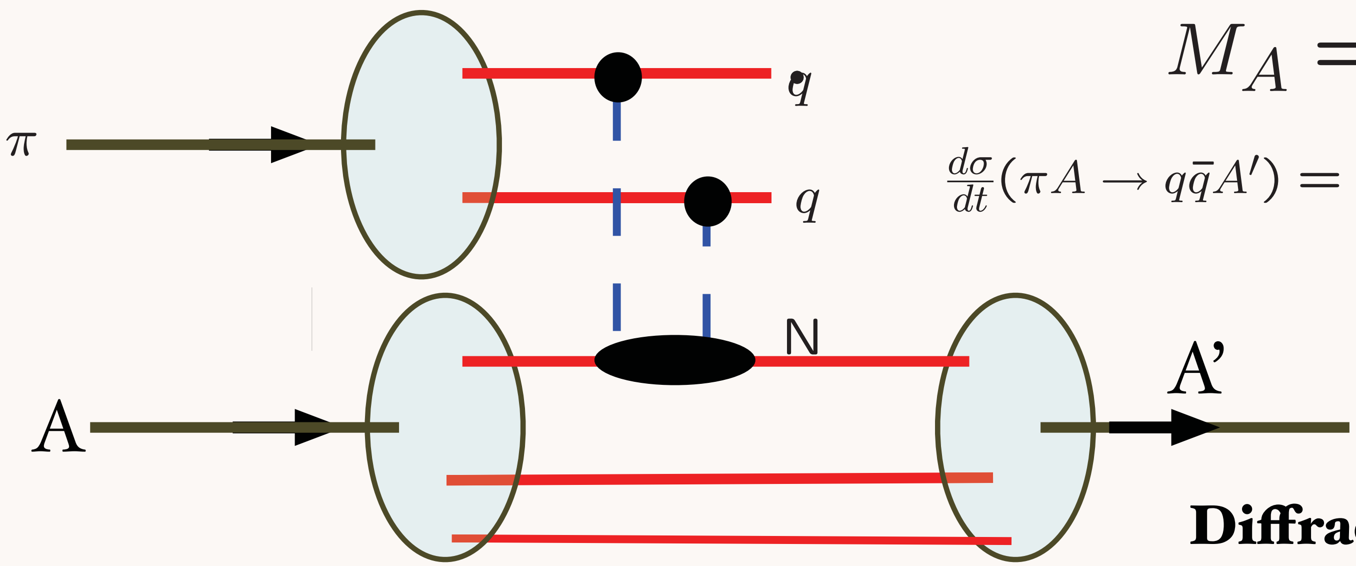
# Key Ingredients in Ashery Experiment




Brodsky Mueller
Frankfurt Miller
Strikman

*Small color-dipole moment pion not absorbed; interacts with each nucleon coherently*

*QCD COLOR Transparency*




$$M_A = A\ M_N$$

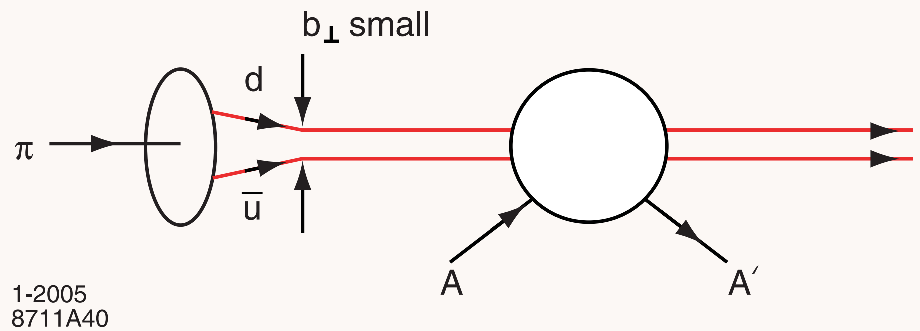
$$\frac{d\sigma}{dt}(\pi A \to q\bar{q}A') = A^2\ \frac{d\sigma}{dt}(\pi N \to q\bar{q}N')\ F_A^2(t)$$

**Target left intact**

**Diffraction, Rapidity gap**

# Fluctuation of a Pion to a Compact Color Dipole State

Small Size Pion Can Interact Coherently on Each Nucleon of Nucleus




Diffractive Dijet Cross Section Color Transparent

$$M(\pi A \to Jet Jet A') = A^1 M(\pi N \to Jet Jet N') F_A(t)$$

$$d\sigma/dt(\pi A \to Jet Jet A') =$$

$$A^2 d\sigma/dt(\pi N \to Jet Jet N') |F_A(t)|^2$$

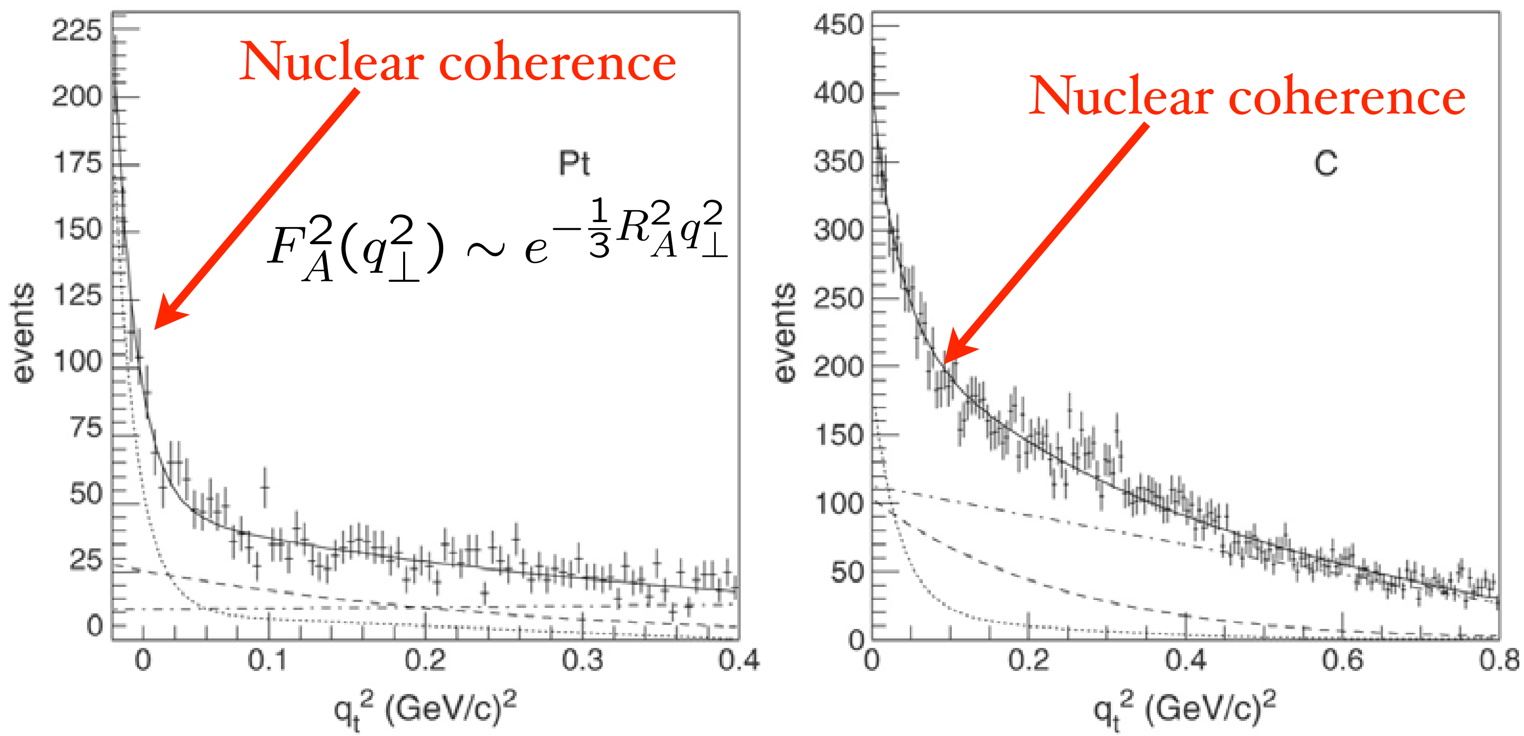
$$\sigma \propto \frac{A^2}{R_A^2} \sim A^{4/3}$$

- Fully coherent interactions between pion and nucleons.
- Emerging Di-Jets do not interact with nucleus.

$$\mathcal{M}(\mathcal{A}) = \mathcal{A} \cdot \mathcal{M}(\mathcal{N})$$

$$\frac{d\sigma}{dq_t^2} \propto A^2 \quad q_t^2 \sim 0$$

$$\sigma \propto A^{4/3}$$



# Ashery E791:
# Measure pion LFWF in diffractive dijet production
# Confirms color transparency !

Mueller, sjb; Bertsch et al; Frankfurt, Miller, Strikman

A-Dependence results: $\sigma \propto A^{\alpha}$

| $k_t$ range (GeV/c) | $\alpha$ | $\alpha$ (CT) |
|---|---|---|
| $1.25 < k_t < 1.5$ | 1.64 +0.06 -0.12 | 1.25 |
| $1.5 < k_t < 2.0$ | $1.52 \pm 0.12$ | 1.45 |
| $2.0 < k_t < 2.5$ | $1.55 \pm 0.16$ | 1.60 |

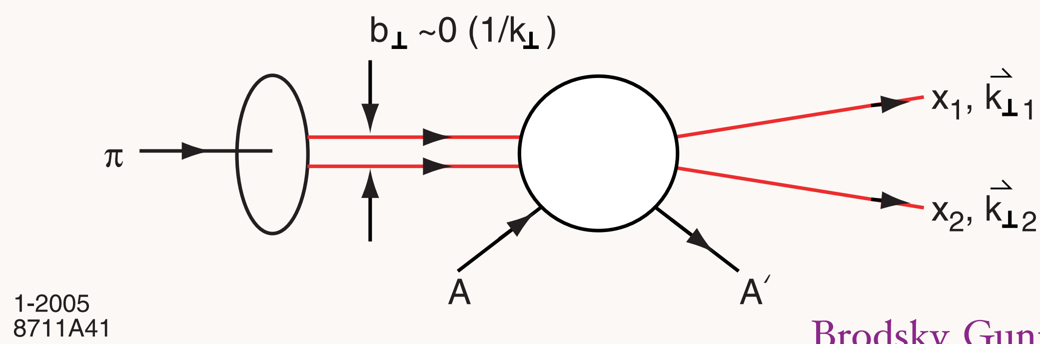
$\alpha$ (Incoh.) = $0.70 \pm 0.1$

Conventional Glauber Theory Ruled Out !

Factor of 7

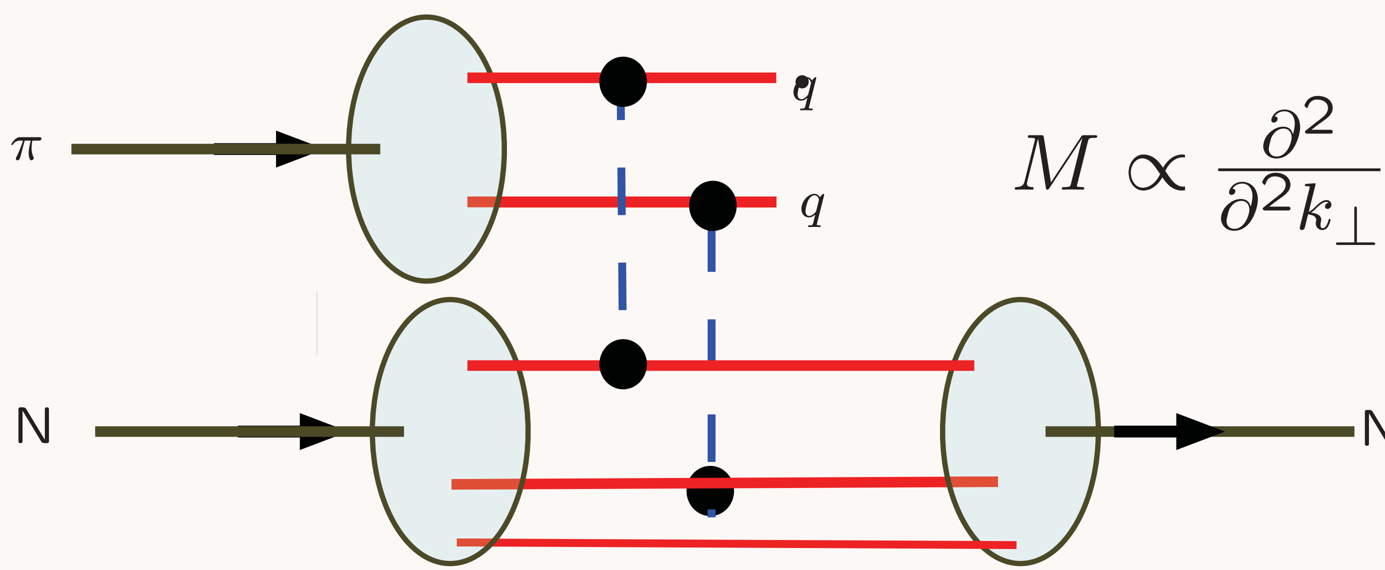
FermiLab E791
Ashery et al

# *Key Ingredients in Ashery Experiment*

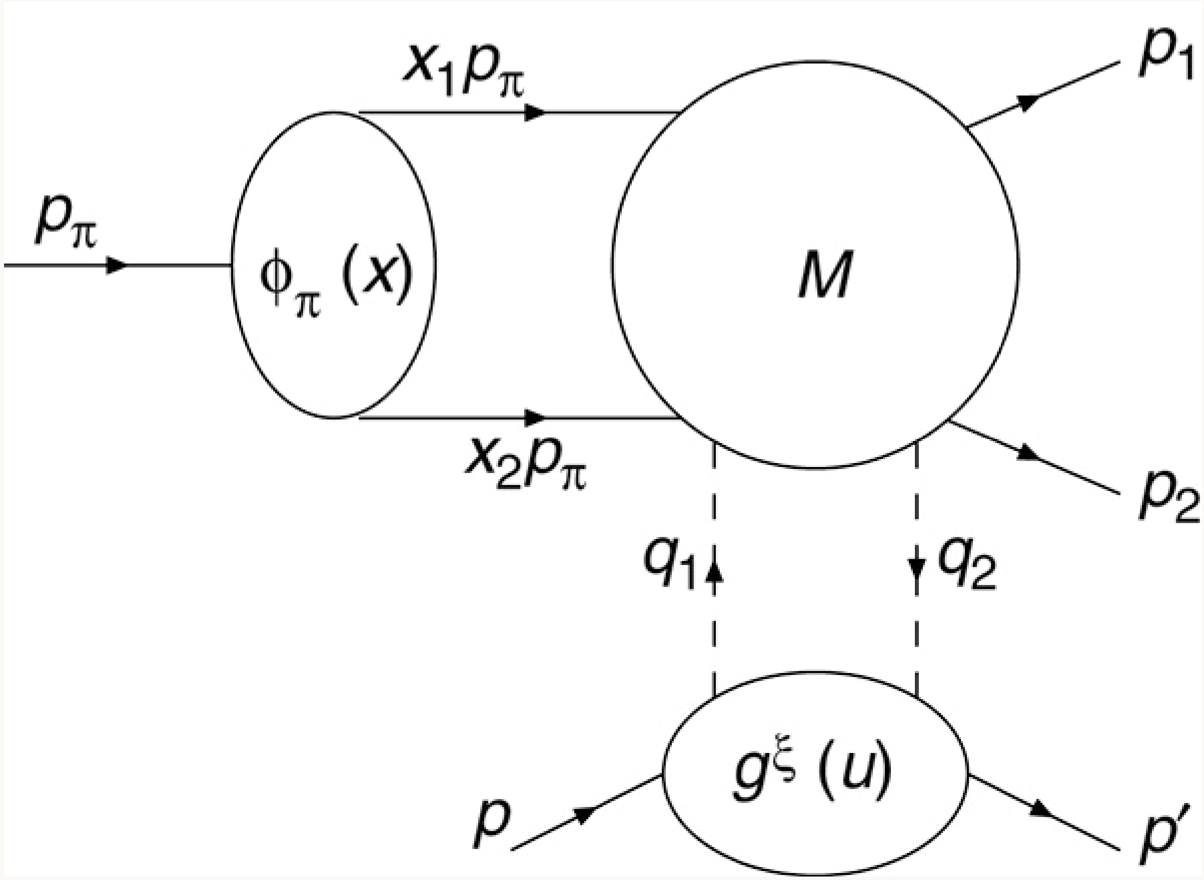



Brodsky, Gunion, Frankfurt, Mueller, Strikman
Frankfurt, Miller, Strikman

## *Two-gluon exchange measures the second derivative of the pion light-front wavefunction*




$$M \propto \frac{\partial^2}{\partial^2 k_\perp} \psi_\pi(x, k_\perp)$$




x

1-x

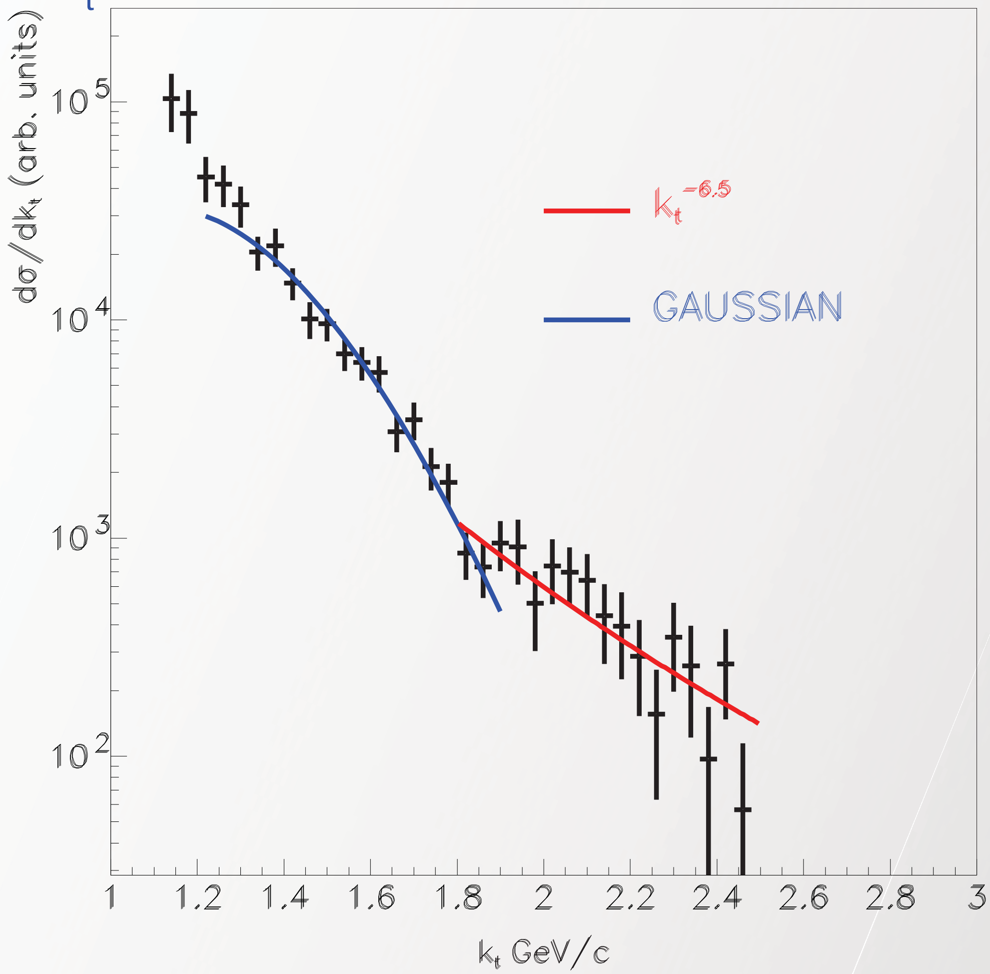
*gluons measure size of color dipole*

$$\frac{d\sigma}{dk_t^2} \propto |\alpha_s(k_t^2) x_N G(u, k_t^2)|^2 \left| \frac{\partial^2}{\partial k_t^2} \psi(x, k_t) \right|^2$$

# THE $k_t$ DEPENDENCE OF DI-JETS YIELD

$$\frac{d\sigma}{dk_t^2} \propto \left|\alpha_s(k_t^2)G(x,k_t^2)\right|^2 \left|\frac{\partial^2}{\partial k_t^2}\psi(u,k_t)\right|^2$$

**With** $\psi \sim \frac{\phi}{k_t^2}$**, weak** $\phi(k_t^2)$ **and** $\alpha_s(k_t^2)$ **dependences and** $G(x,k_t^2) \sim k_t^{1/2}$ : $\frac{d\sigma}{dk_t} \sim k_t^{-6}$




*High Transverse momentum dependence consistent with PQCD*

# Diffractive Dissociation of a Pion into Dijets

$$\pi A \rightarrow Jet\, Jet\, A'$$

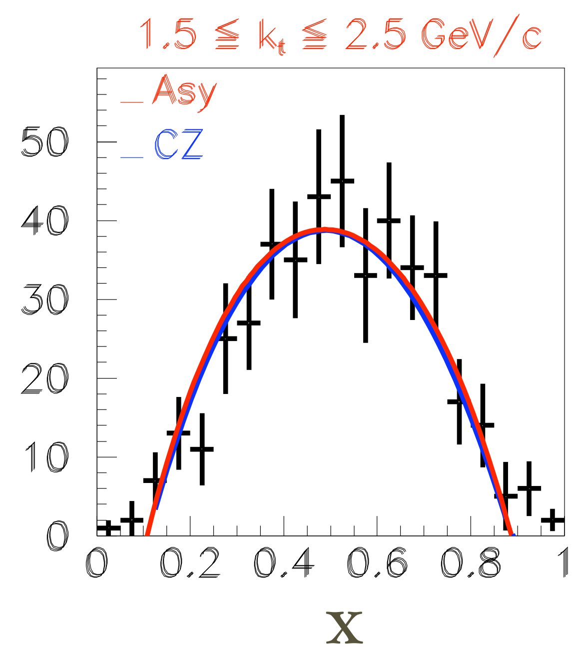
$$\psi^{\pi}_{q\bar{q}}(x, \vec{k}_{\perp})$$

- E789 Fermilab Experiment Ashery et al

- 500 GeV pions collide on nuclei keeping it intact

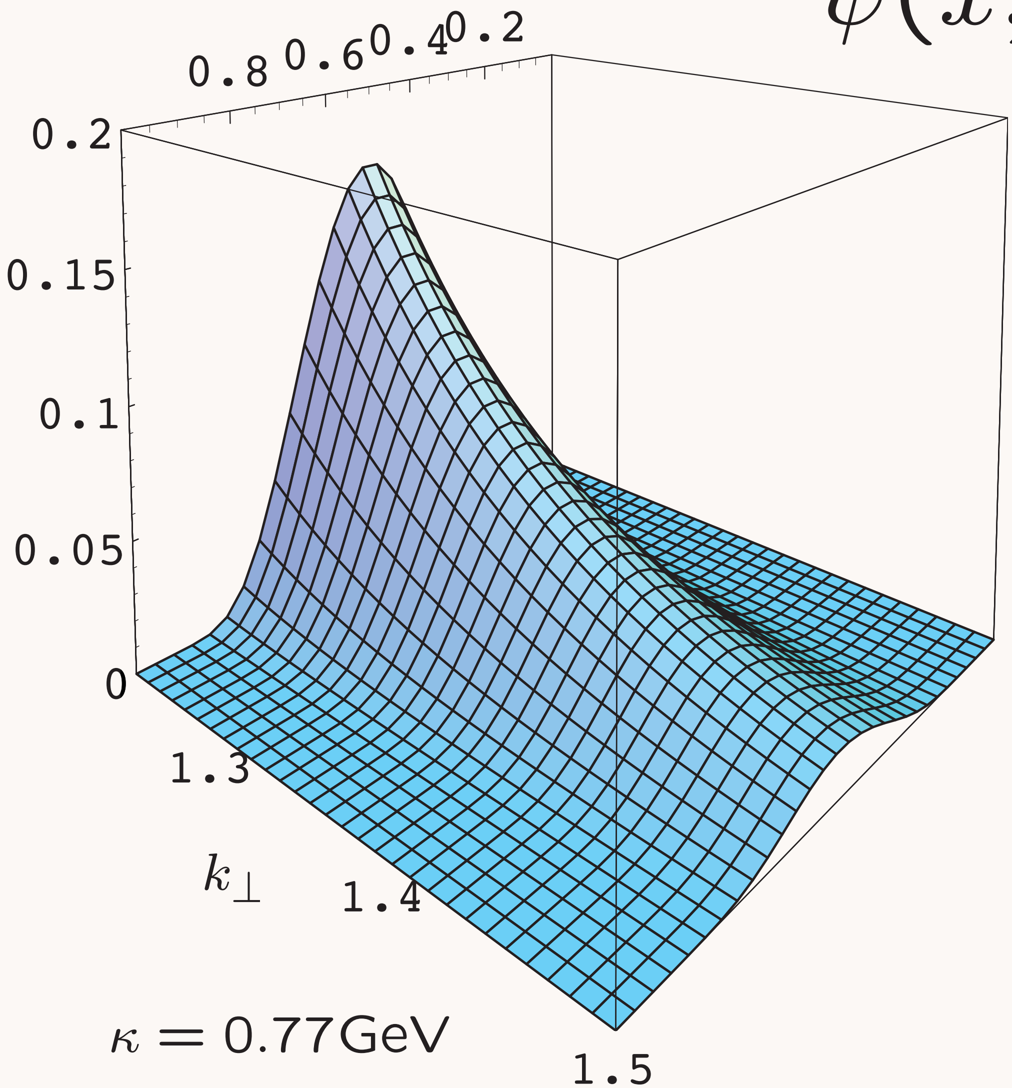
- Measure momentum of two jets

- Study momentum distributions of pion LF wavefunction

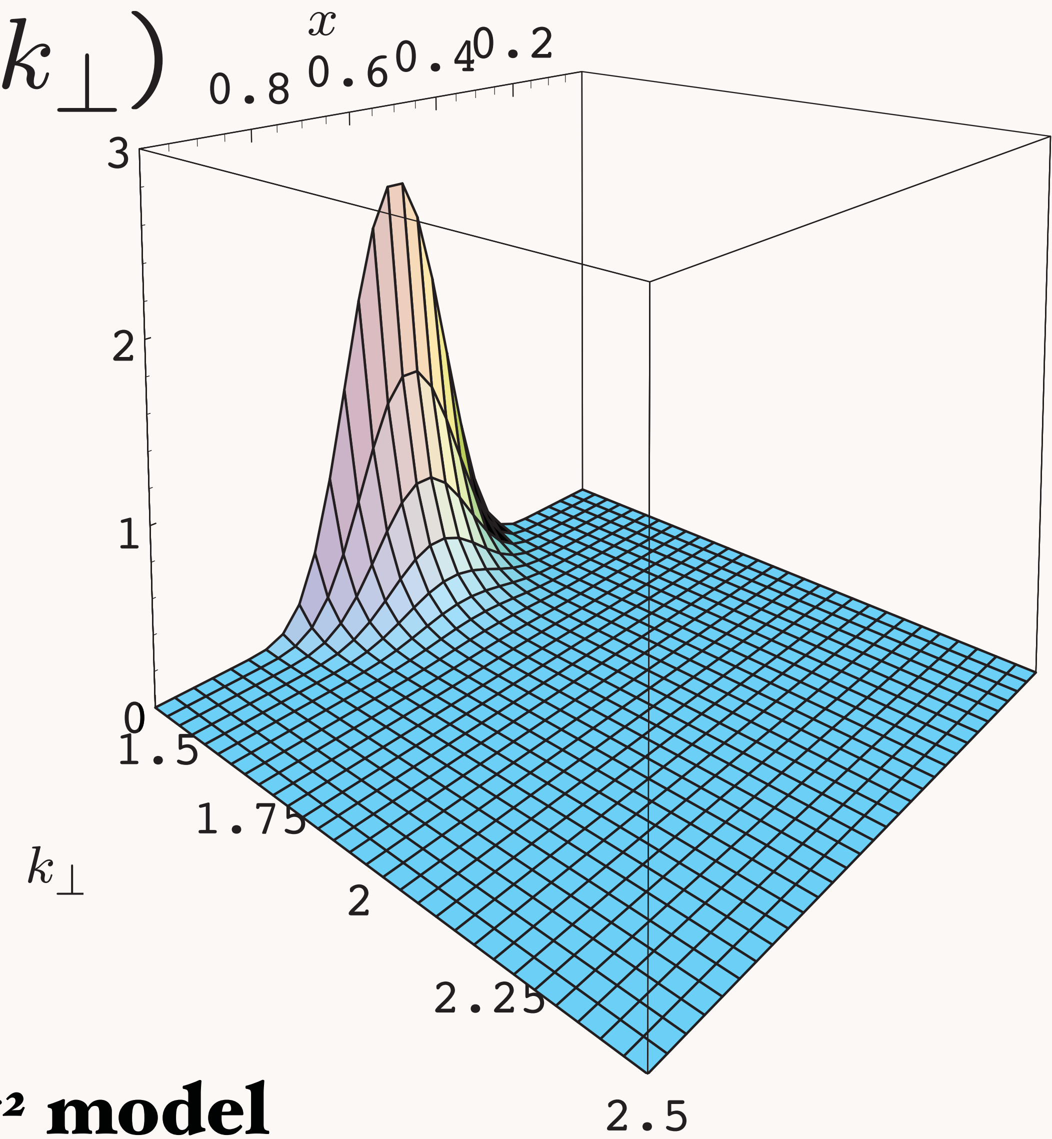


# Predictions from AdS/CFT

$\psi(x, k_\perp)$




$\kappa = 0.77\text{GeV}$




**$z^2$ model**

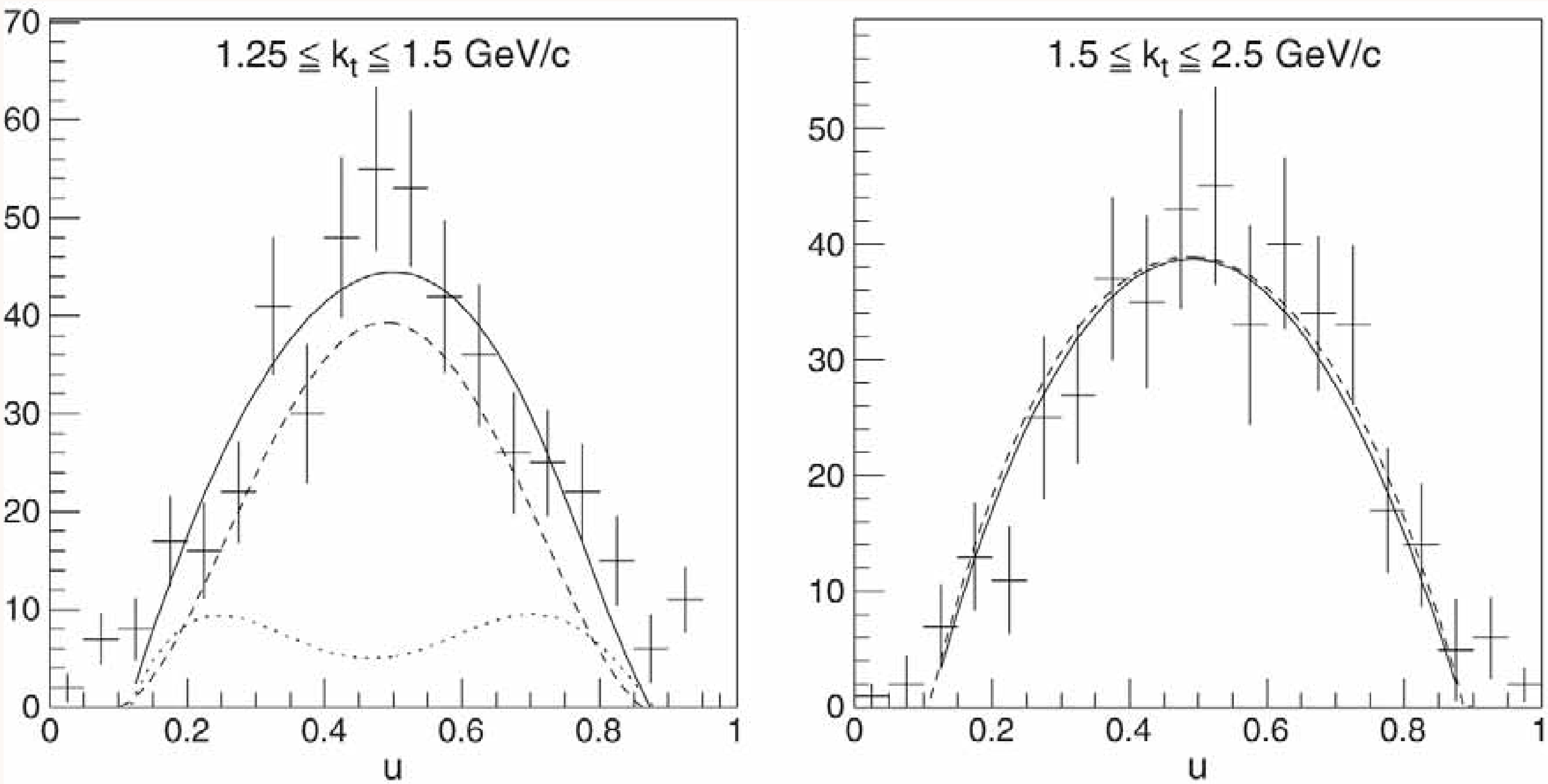
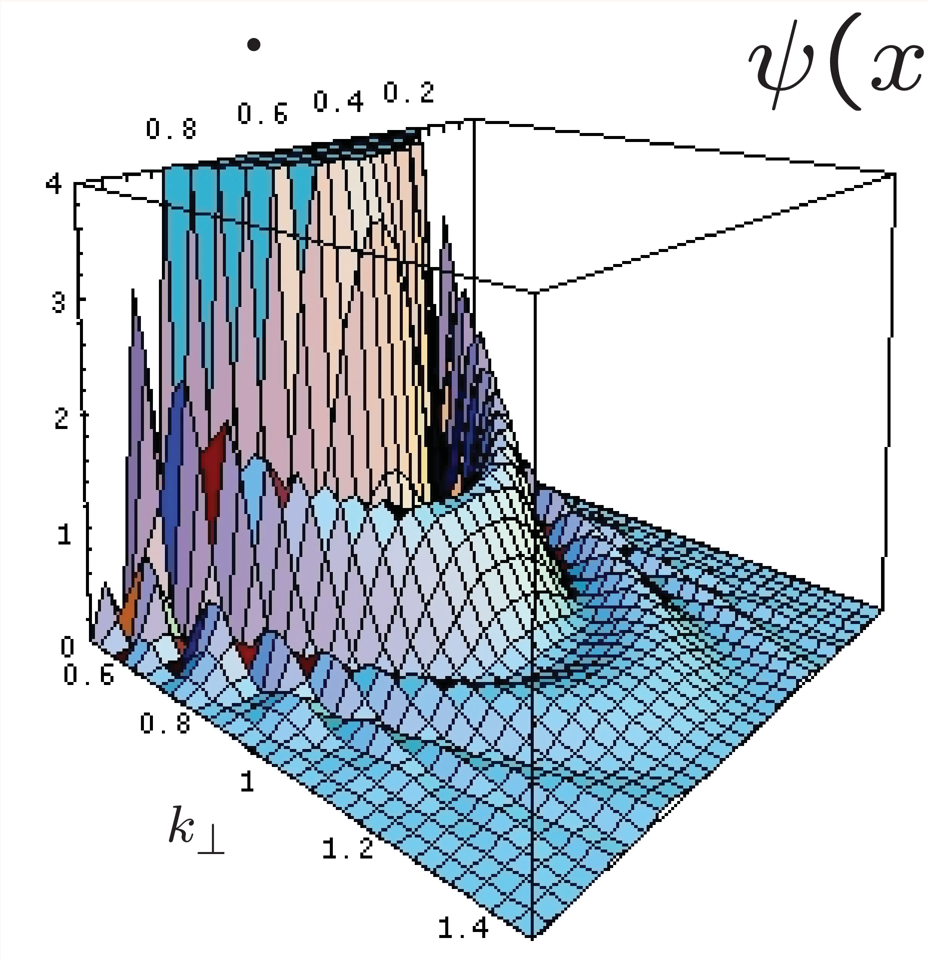
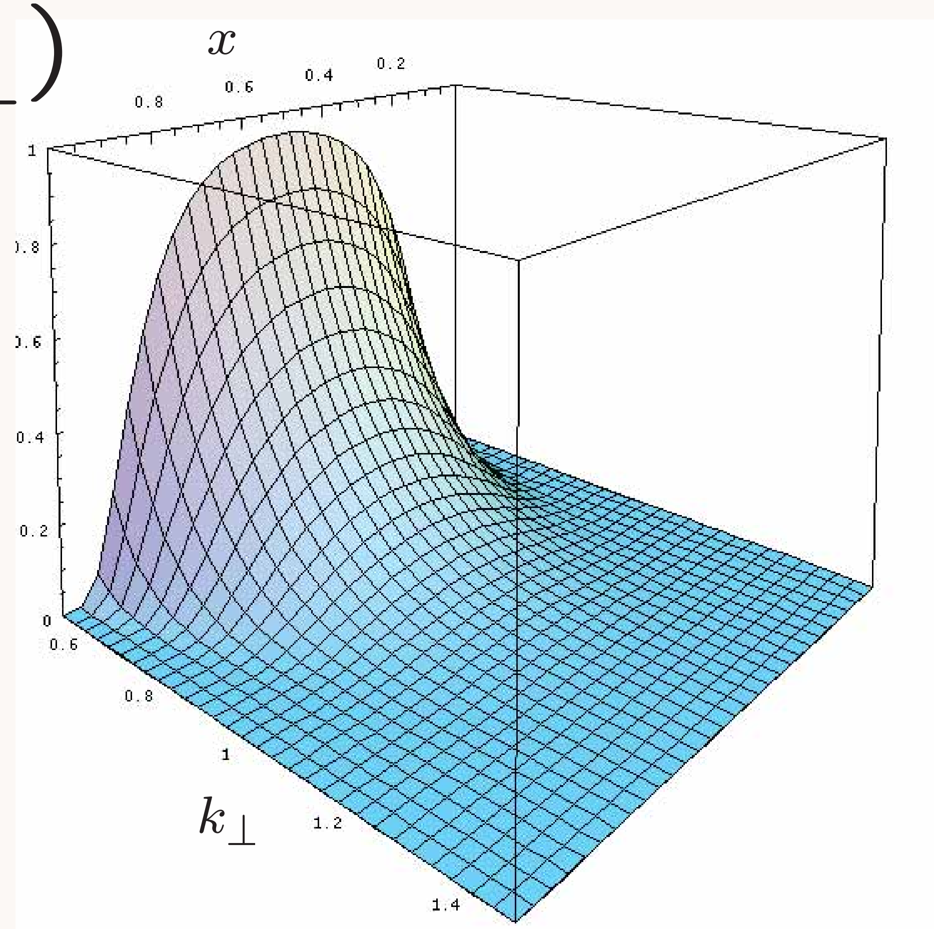



Fig. 22. The $u$ distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/$c$ (left) and for $1.5 \leq k_t \leq 2.5$ GeV/$c$ (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

# Predictions from AdS/CFT

$$\psi(x, k_\perp)$$

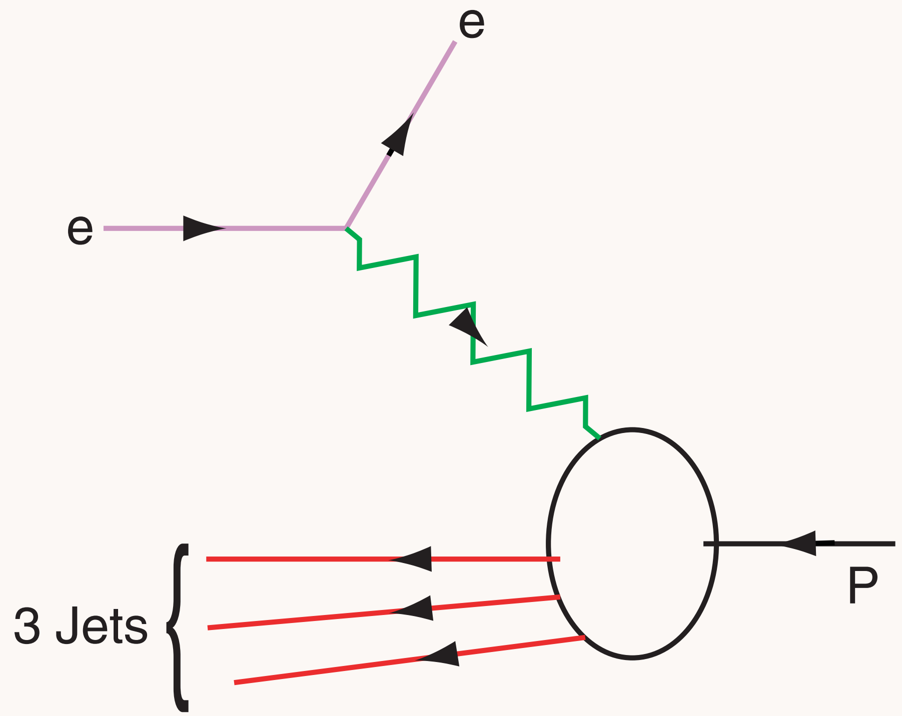



truncated space: need smoothing

"oscillator" $z^2$

# Coulomb-Dissociate Proton to Three Jets at HERA




Frankfurt
Strikman
Miller

Measure $\Psi_{qqq}(x_i, \vec{k}_{\perp i})$ valence wavefunction of proton

# Quantum Chromodynamic Predictions for the Deuteron Form Factor

$$F_d(Q^2) = \int_0^1 [dx][dy]\varphi_d^\dagger(y,Q) \times T_H^{6q+\gamma^*\to 6q}(x,y,Q)\varphi_d(x,Q), \quad (1)$$

where the hard-scattering amplitude

$$T_H^{6q+\gamma^*\to 6q} = [\alpha_s(Q^2)/Q^2]^5 t(x,y) \times [1+O(\alpha_s(Q^2))] \quad (2)$$

gives the probability amplitude for scattering six quarks collinear with the initial to the final deuteron momentum and

$$\varphi_d(x_i,Q) \propto \int^{k_{\perp i}<Q} [d^2k_\perp]\psi_{qqq\,qqq}(x_i,\vec{k}_{\perp i}) \quad (3)$$

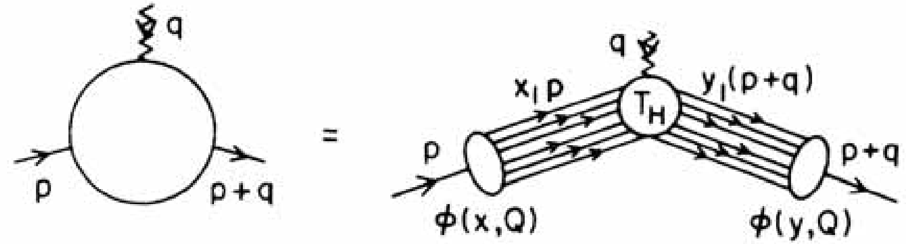
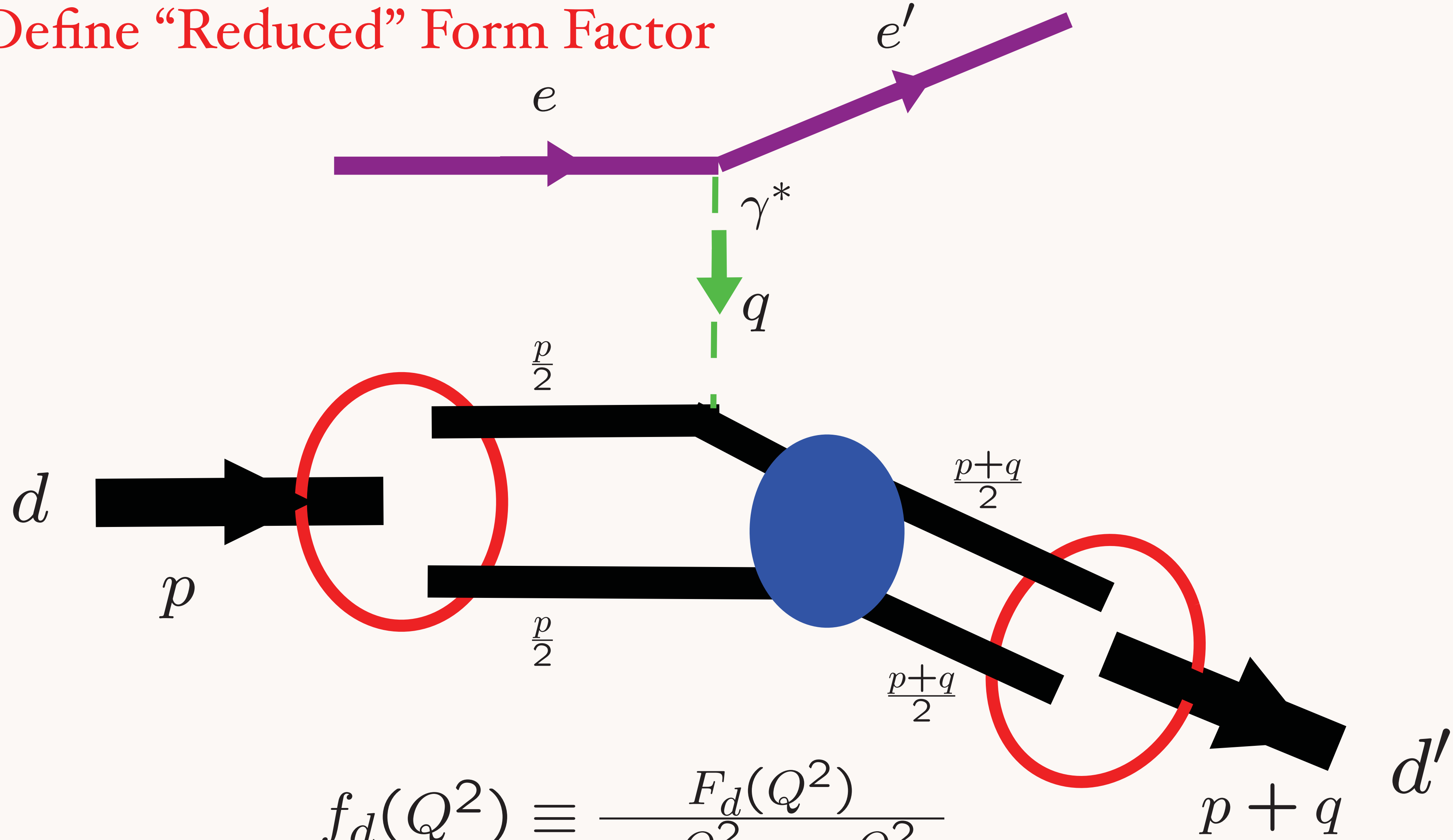



FIG. 1. The general structure of the deuteron form factor at large $Q^2$.

Ji, Lepage, sjb

# Define "Reduced" Form Factor




$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

*Elastic electron-deuteron scattering*

# QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Define "Reduced" Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N^{\,2}(Q^2/4)} .$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

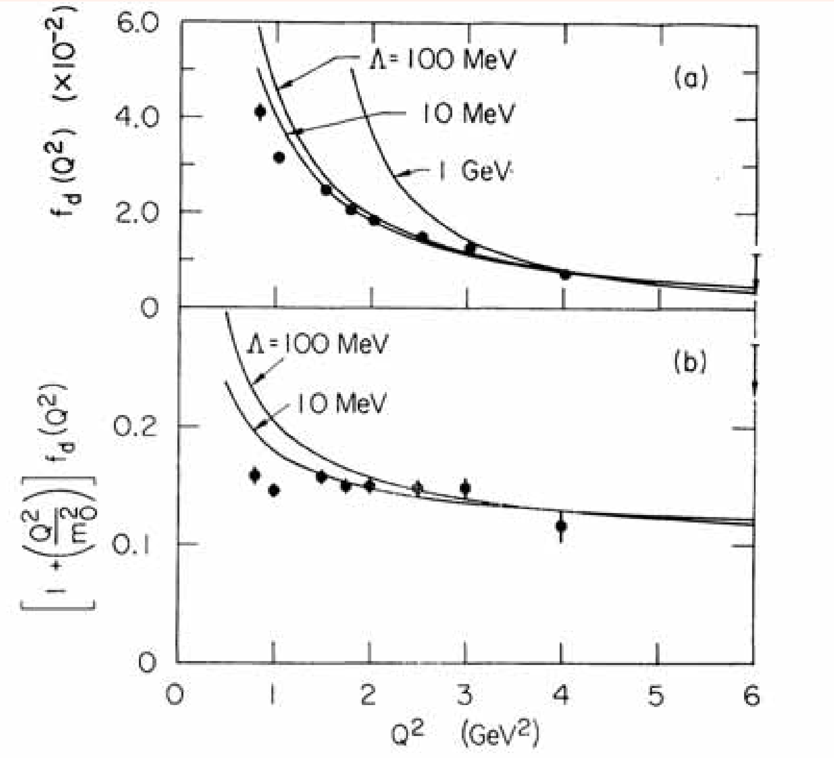
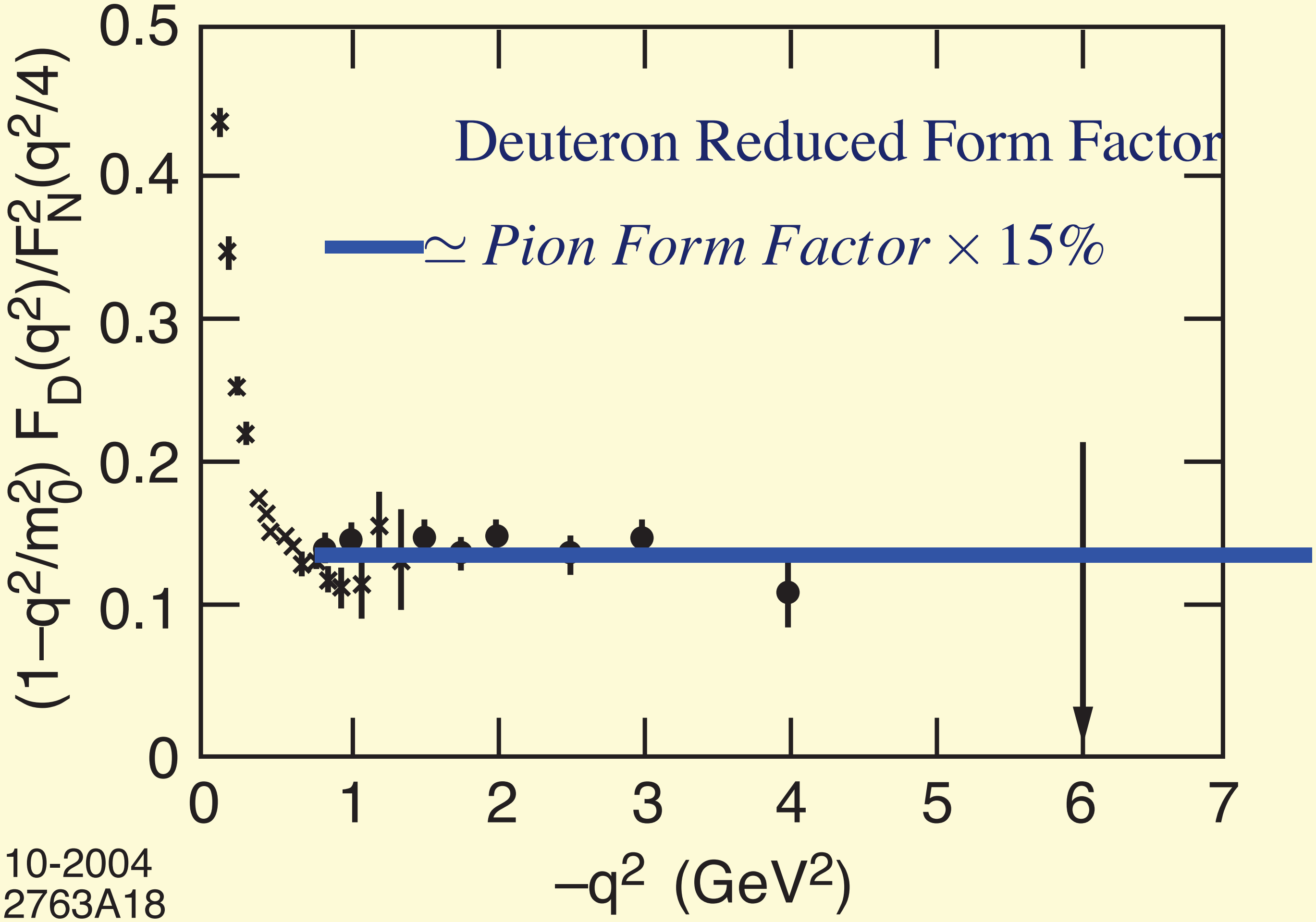



FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4$ GeV$^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0^2)] f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0^2 = 0.28$ GeV$^2$ is used (Ref. 8).

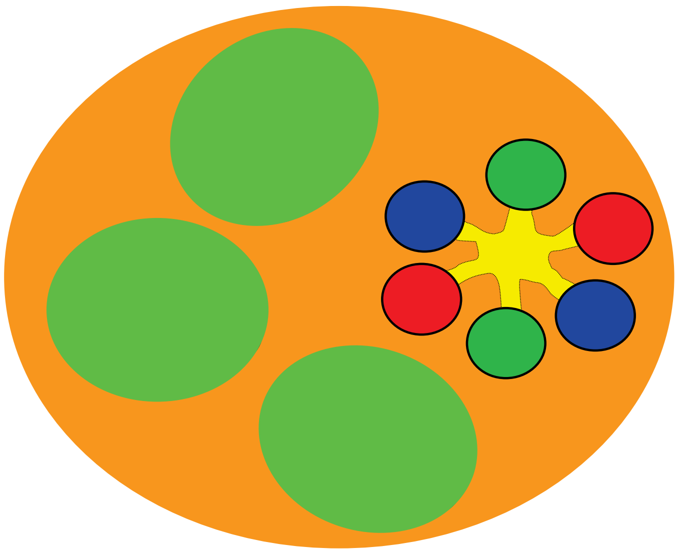



Magnitude of reduced form factor very large -- short range n-p correlation

- 15% ``Hidden Color” in the Deuteron

# Do multi-quark clusters exist in the nuclear wavefunction?

*Does the nucleus only consist of nucleons?*



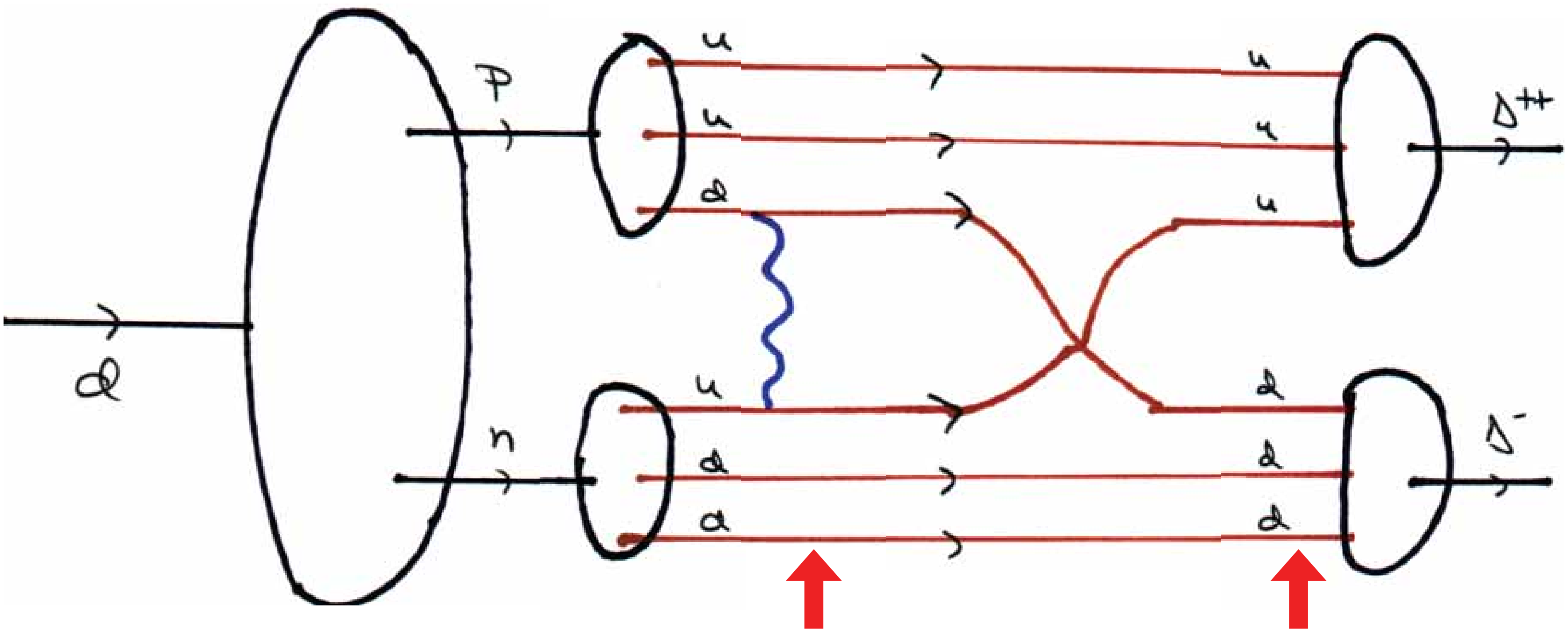
*Hidden Color!*

# Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six-quark wavefunction
- 5 color-singlet combinations of 6 color-triplets -- only one state is | n p>
- Short-range correlations (Frankfurt, Strikman)
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict $\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn)$ at high $Q^2$

# Structure of Deuteron in QCD



The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions $x_i$ $(i=1,2,\ldots,6)$ can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, $C_d$, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum $Q$, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy]=\delta(1-\sum_{i=1}^{6}y_i)\prod_{i=1}^{6}dy_i$, $C_F=(n_c^2-1)/2n_c=\frac{4}{3}$, $\beta=11-\frac{2}{3}n_f$, and $n_f$ is the effective number of flavors$\}$

$$\prod_{k=1}^{6} x_k\left[\frac{\partial}{\partial\xi}+\frac{3C_F}{\beta}\right]\tilde{\Phi}(x_i,Q)=-\frac{C_d}{\beta}\int_0^1[dy]V(x_i,y_i)\,\tilde{\Phi}(y_i,Q),$$

$$\xi(Q^2)=\frac{\beta}{4\pi}\int_{Q_0^2}^{Q^2}\frac{dk^2}{k^2}\alpha_s(k^2)\sim\ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i,y_i)=2\prod_{k=1}^{6}x_k\sum_{i\neq j}^{6}\theta(y_i-x_i)\prod_{l\neq i,j}^{6}\delta(x_l-y_l)\frac{y_j}{x_j}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i+x_j}+\frac{\Delta}{y_i-x_i}\right)$$

where $\delta_{h_i\bar{h}_j}=1$ (0) when the helicities of the constituents $\{i,j\}$ are antiparallel (parallel). The infrared singularity at $x_i=y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i,Q)=\tilde{\Phi}(y_i,Q)-\tilde{\Phi}(x_i,Q)$ since the deuteron is a color singlet.

# Test Hidden Color of Deuteron

- Diffractive, Coulomb Dissociation to $\Delta^{++}\Delta^{-}$
- Photodisintegration of Deuteron to $\Delta^{++}\Delta^{-}$
- Connection to EMC
- **Deuteron not simply n + p**
- **Connection to x > 1 physics (Pialetzky)**

# New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

# AdS/CFT and QCD

Bottom-Up Approach

- Nonperturbative derivation of dimensional counting rules of hard exclusive glueball scattering for gauge theories with mass gap dual to string theories in warped space:

  Polchinski and Strassler, hep-th/0109174.

- Deep inelastic structure functions at small $x$:

  Polchinski and Strassler, hep-th/0209211.

- Derivation of power falloff of hadronic light-front Fock wave functions, including orbital angular momentum, matching short distance behavior with string modes at AdS boundary:

  Brodsky and de Téramond, hep-th/0310227.

- Low lying hadron spectra, chiral symmetry breaking and hadron couplings in AdS/QCD:

  Boschi-Filho and Braga, hep-th/0212207; de Téramond and Brodsky, hep-th/0501022; Erlich, Katz, Son and Stephanov, hep-ph/0501128; Hong, Yong and Strassler, hep-th/0501197; Da Rold and Pomarol, hep-ph/0501218; Hirn and Sanz, hep-ph/0507049; Boschi-Filho, Braga and Carrion, arXiv:hep-th/0507063; Katz, Lewandowski and Schwartz, arXiv:hep-ph/0510388.

- Gluonium spectrum (top-bottom):

  Csaki, Ooguri, Oz and Terning, hep-th/9806021; de Mello Kock, Jevicki, Mihailescu and Nuñez, hep-th/9806125; Csaki, Oz, Russo and Terning, hep-th/9810186; Minahan, hep-th/9811156; Brower, Mathur and Tan, hep-th/0003115, Caceres and Nuñez, hep-th/0506051.

- D3/D7 branes (top-bottom):

  Karch and Katz, hep-th/0205236; Karch, Katz and Weiner, hep-th/0211107; Kruczenski, Mateos, Myers and Winters, hep-th/0311270; Sakai and Sonnenschein, hep-th/0305049; Babington, Erdmenger, Evans, Guralnik and Kirsch, hep-th/0312263; Nuñez, Paredes and Ramallo, hep-th/0311201; Hong, Yoon and Strassler, hep-th/0312071; hep-th/0409118; Kruczenski, Pando Zayas, Sonnenschein and Vaman, hep-th/0410035; Sakai and Sugimoto, hep-th/0412141; Paredes and Talavera, hep-th/0412260; Kirsh and Vaman, hep-th/0505164; Apreda, Erdmenger and Evans, hep-th/0509219; Casero, Paredes and Sonnenschein, hep-th/0510110.

- Other aspects of high energy scattering in warped spaces:

  Giddings, hep-th/0203004; Andreev and Siegel, hep-th/0410131; Siopsis, hep-th/0503245.

- Strongly coupled quark-gluon plasma ($\eta/s = 1/4\pi$):

  Policastro, Son and Starinets, hep-th/0104066; Kang and Nastase, hep-th/0410173 ...

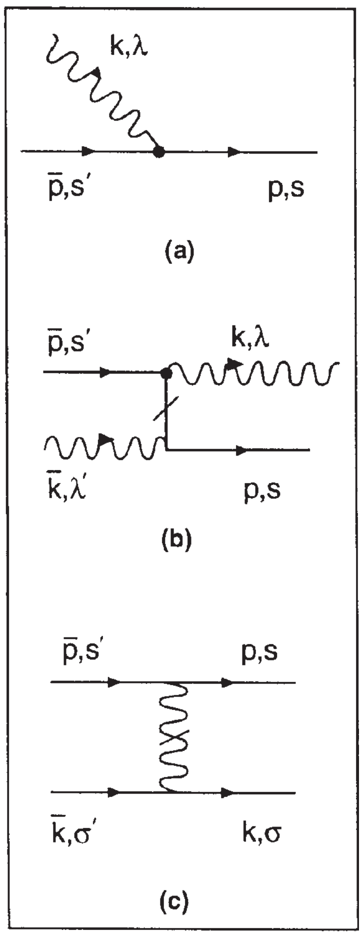
# Solving the LF Heisenberg Eqn.

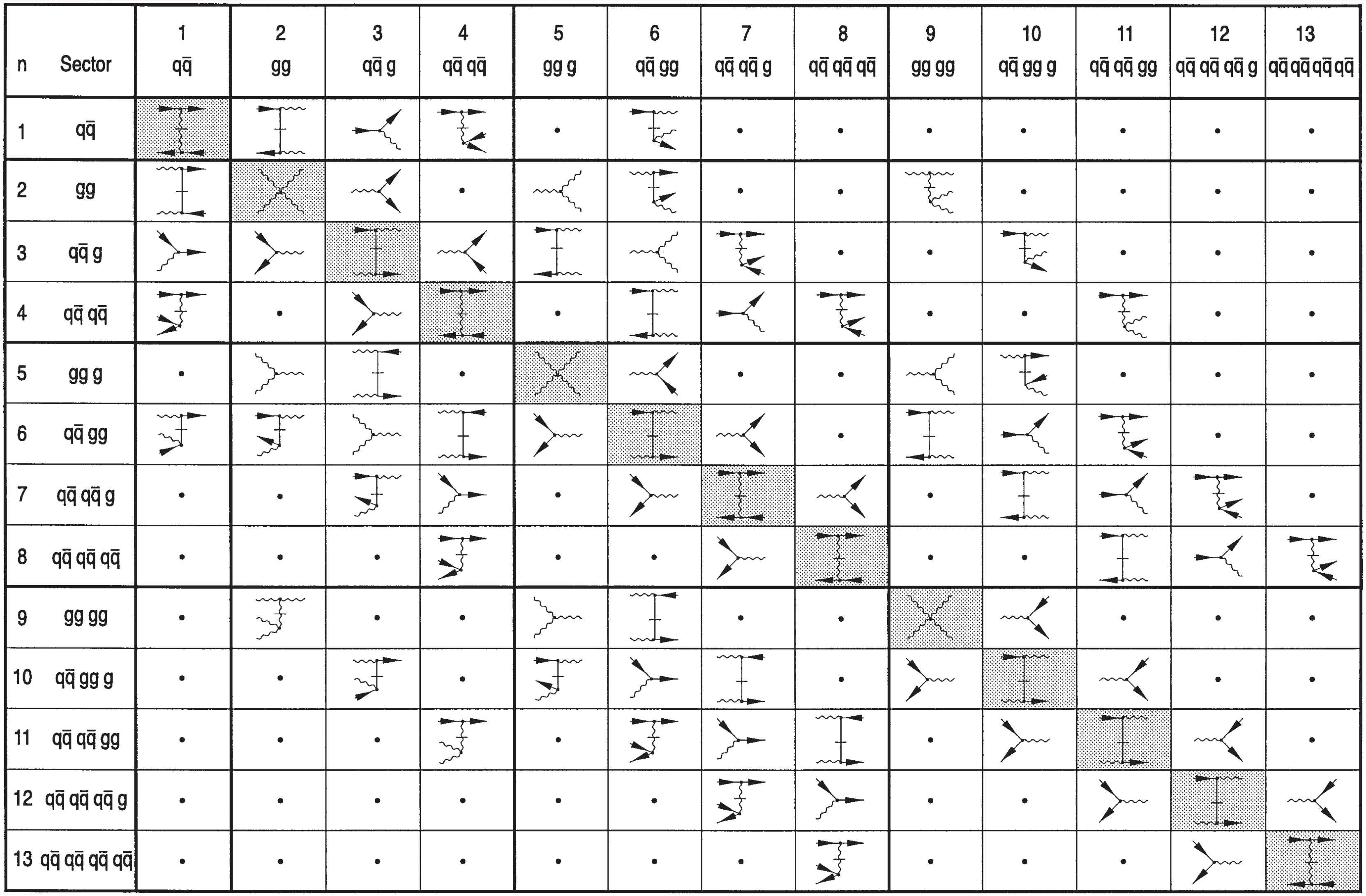
- Discretized Light-Cone Quantization (DLCQ) Pauli, sjb
  Minkowski space !
- Many 1+1 model field theories completely solved using DLCQ Hornbostel, Pauli, sjb; Klebanov
- UV Regularization: 3+ 1 Pauli Villars Hiller, McCartor, sjb
- Transverse Lattice Bardeen, Peterson,Rabinovici, Burkardt, Dalley
- Bethe-Salpeter/Dyson-Schwinger at fixed LF time
- Angular Structure of Solutions known Karmanov, Hwang, sjb
- **Use AdS/CFT model solutions and AdS/LF Equations as starting point**! Vary, de Teramond sjb

# Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

# DLCQ

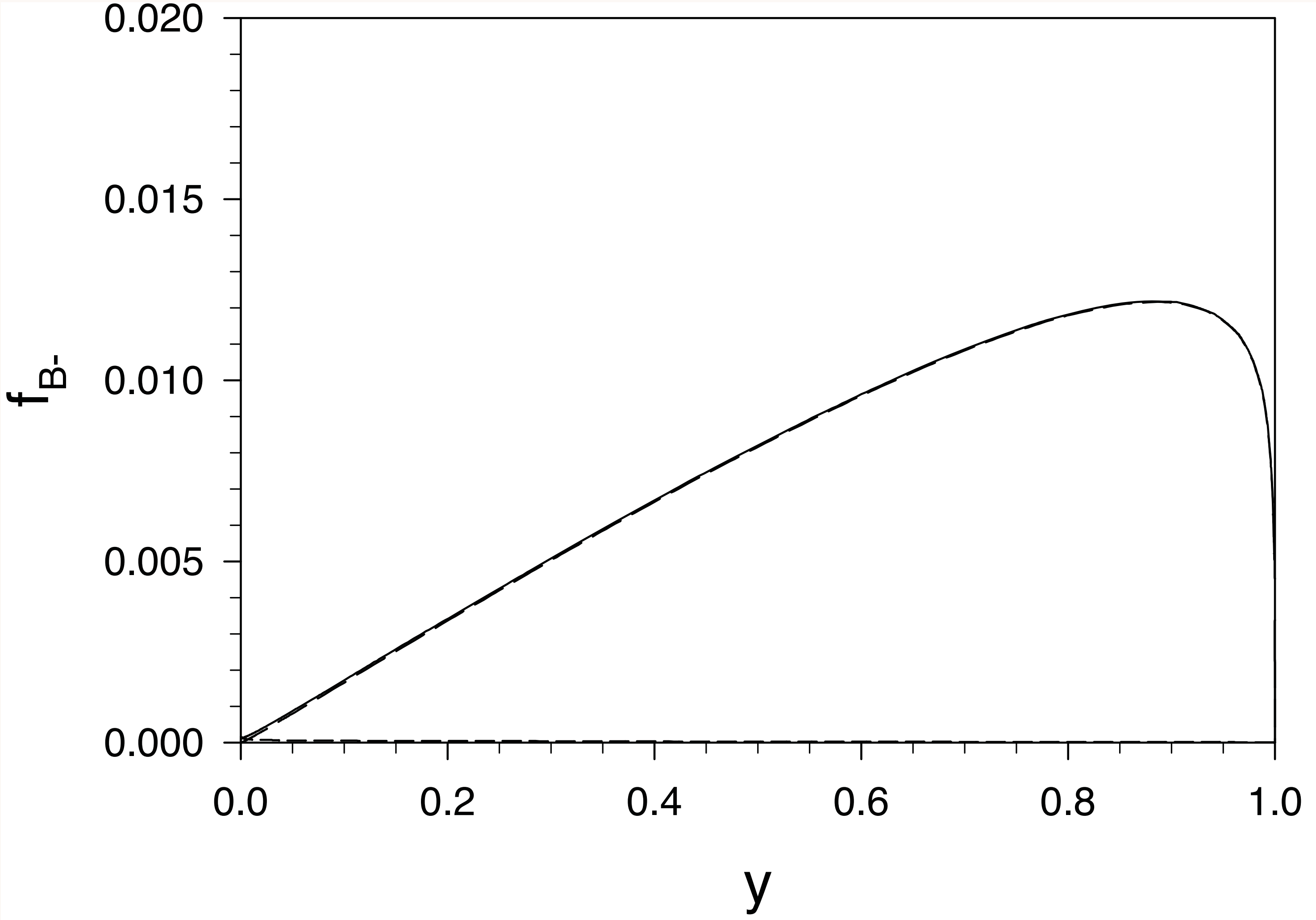



Pauli, Pinsky, sjb

# *Structure function of boson constituent in 3+1 Yukawa theory*

## Three-particle Fock state truncation




Pauli-Villars Regularization

Hiller, McCartor, sjb

# New Perspectives on QCD from AdS/CFT

- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- AdS/CFT predicts Light-front wavefunctions: Fundamental description of hadrons at amplitude level
- AdS/CFT: gluonium (gg) , meson ($q \bar{q}$), and baryon (qqq) spectra
- **Quark-interchange dominates scattering amplitudes**

# Why is quark-interchange dominant over gluon exchange?

Example: $M(K^+p \to K^+p) \propto \frac{1}{ut^2}$

Exchange of common $u$ quark

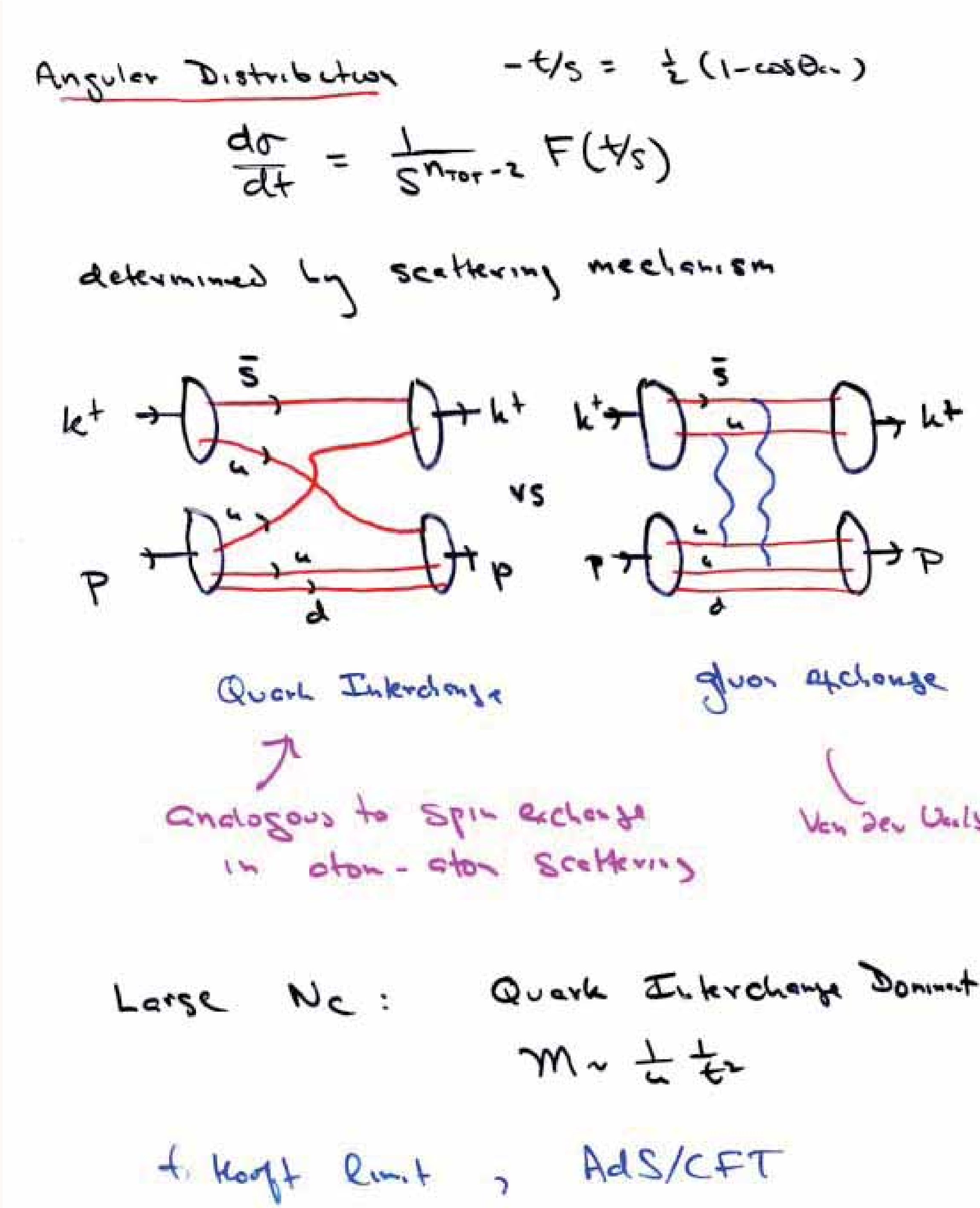
$M_{QIM} = \int d^2k_\perp dx\ \psi^\dagger_C \psi^\dagger_D \Delta \psi_A \psi_B$

Holographic model (Classical level):

Hadrons enter 5th dimension of $AdS_5$

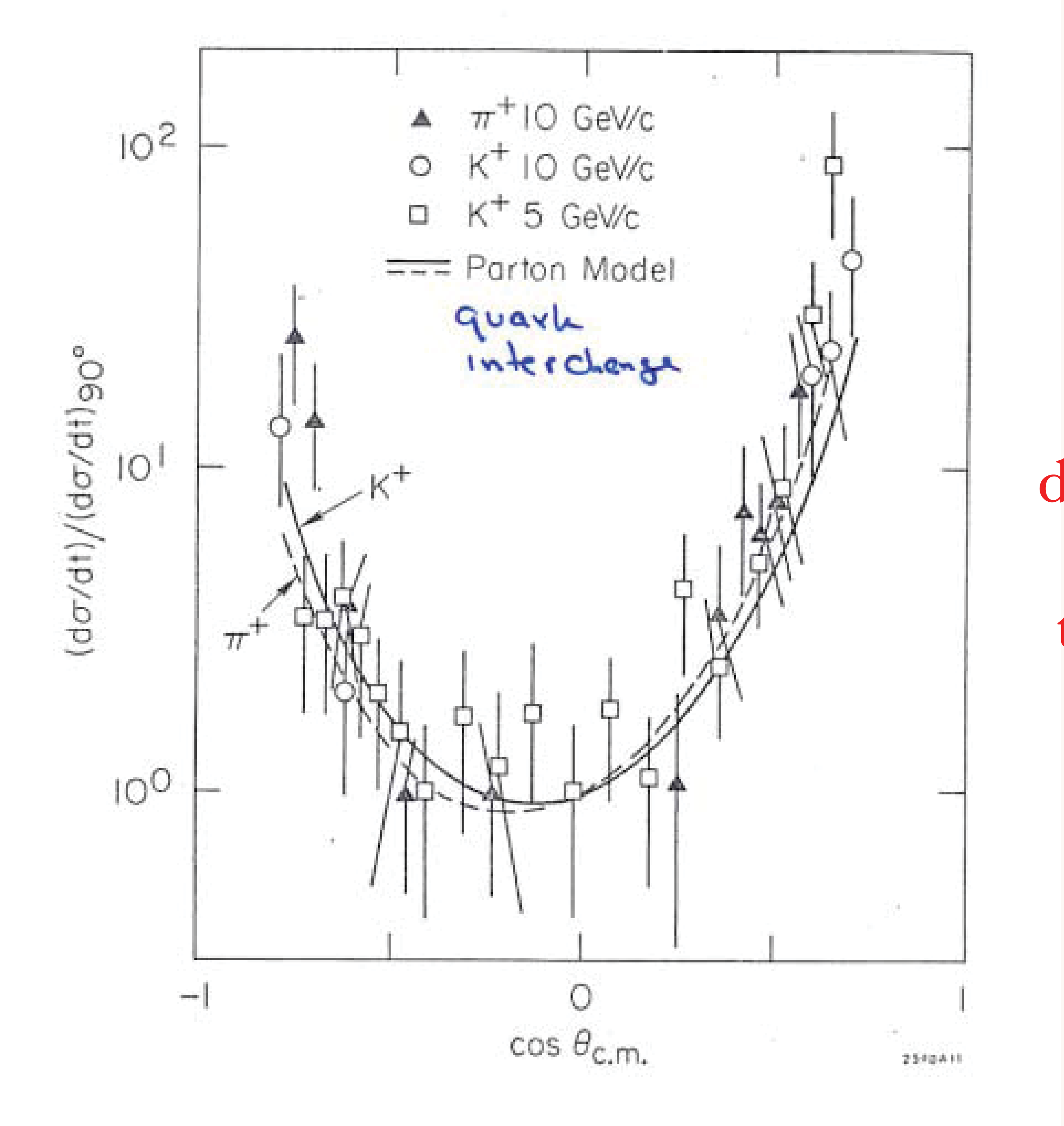
Quarks travel freely within cavity as long as separation $z < z_0 = \frac{1}{\Lambda_{QCD}}$

LFWFs obey conformal symmetry producing quark counting rules.




Blankenbecler, Gunion, sjb

# MIT Bag Model predicts dominance of quark interchange: deTar




AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

# Formula for quark interchange using LFWFs

Blankenbecler, Gunion, sjb; Sivers

$$M_{FI} = \langle \psi_F | E - K | \psi_I \rangle$$

$$\equiv \langle \psi_F | \Delta | \psi_I \rangle$$

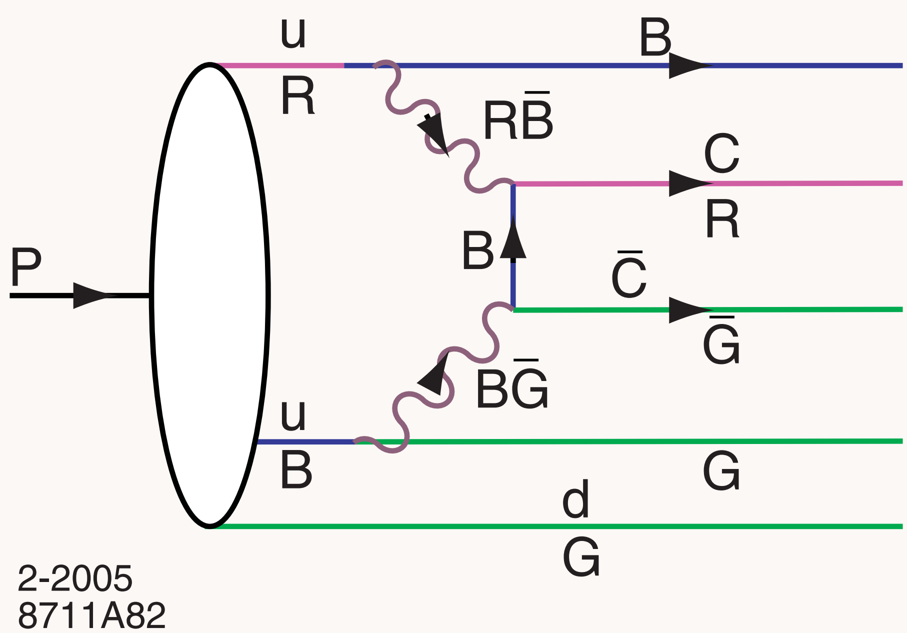
$$= \frac{1}{2(2\pi)^3} \int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2} \Delta \psi_C(\vec{k}_\perp - x\vec{r}_\perp, x) \psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) \psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x) \psi_B(\vec{k}_\perp, x)$$

where

$$\Delta = s - M_A{}^2 - M_B{}^2 - K_a - K_b - K_c - K_d$$

$$= M_A{}^2 + M_B{}^2 - S_A(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp, x) - S_B(\vec{k}_\perp, x)$$

$$= M_C{}^2 + M_D{}^2 - S_C(\vec{k}_\perp - x\vec{r}_\perp, x) - S_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x) .$$




$|uudc\bar{c}>$ Fluctuation in Proton

QCD: Probability $\frac{\sim\Lambda^2_{QCD}}{M^2_Q}$

$|e^+e^-\ell^+\ell^->$ Fluctuation in Positr

QED: Probability $\frac{\sim(m_e\alpha)^4}{M^4_\ell}$

OPE derivation - M.Polyakov et al.

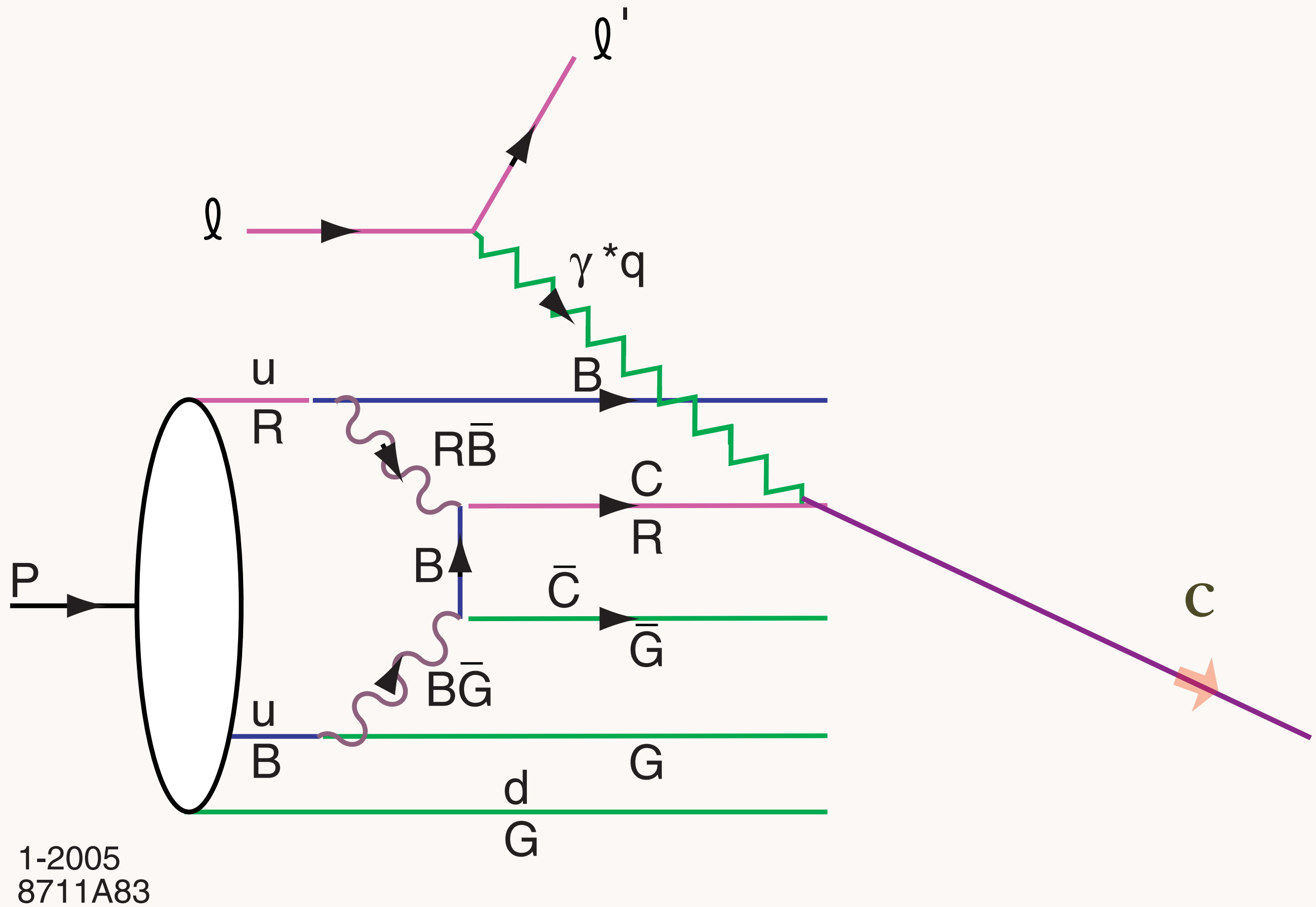
$c\bar{c}$ in Color Octet

# *High x charm*

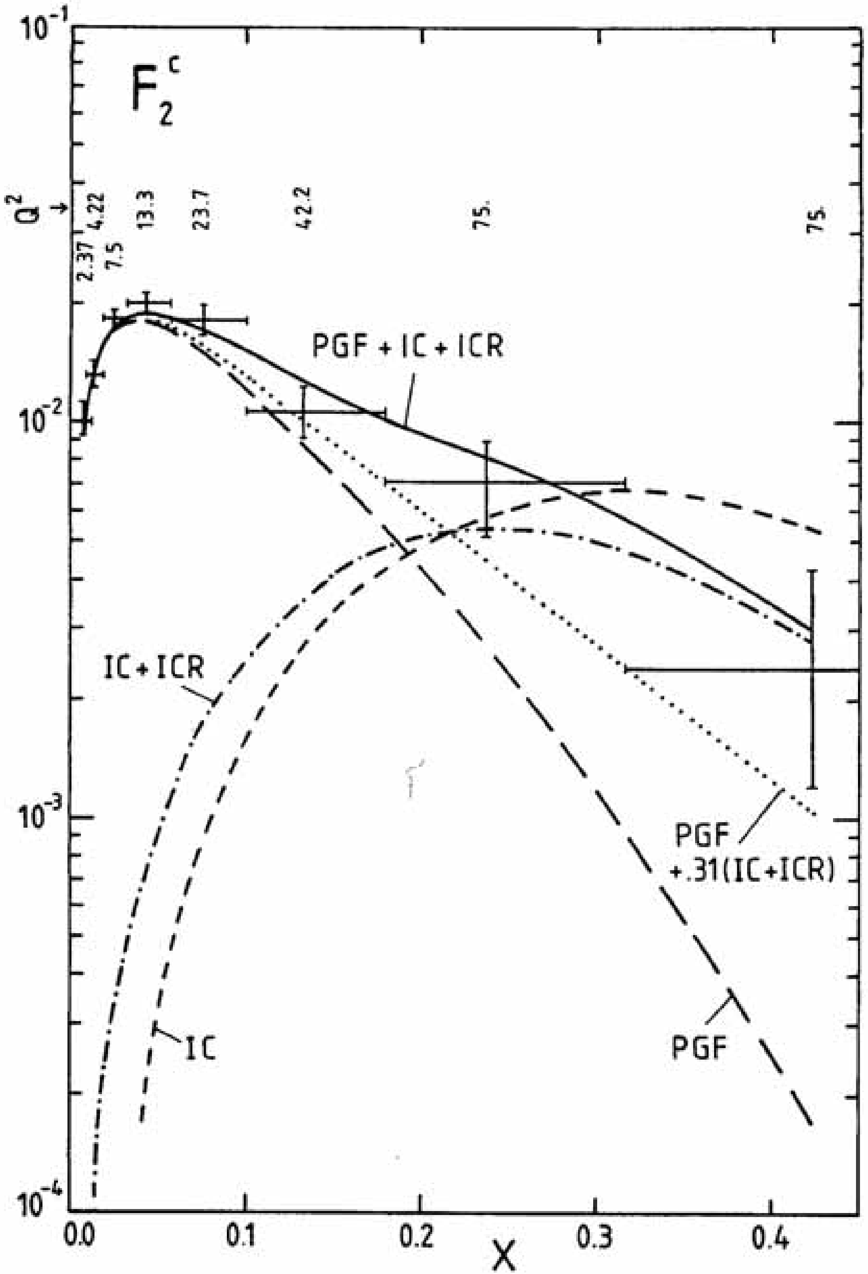
Distribution peaks at equal rapidity (velocity)

Therefore heavy particles carry the largest momentum fractions

# Measure c(x) in Deep Inelastic Lepton-Proton Scattering



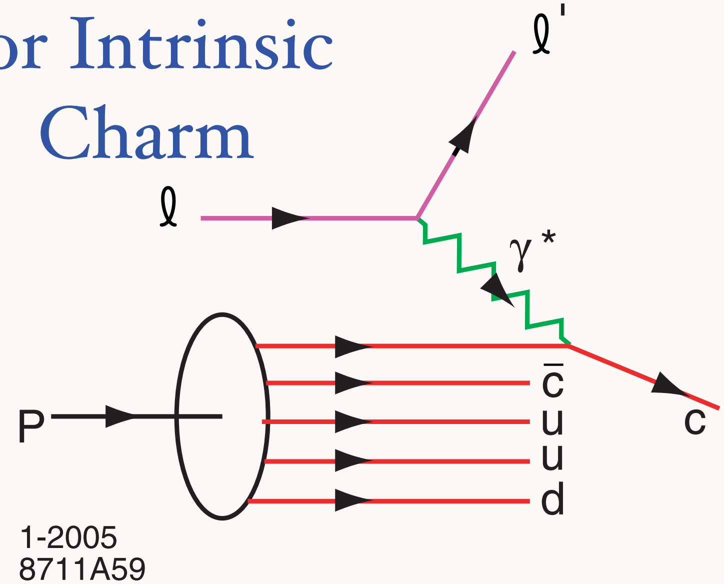

Hoyer, Peterson, SJB



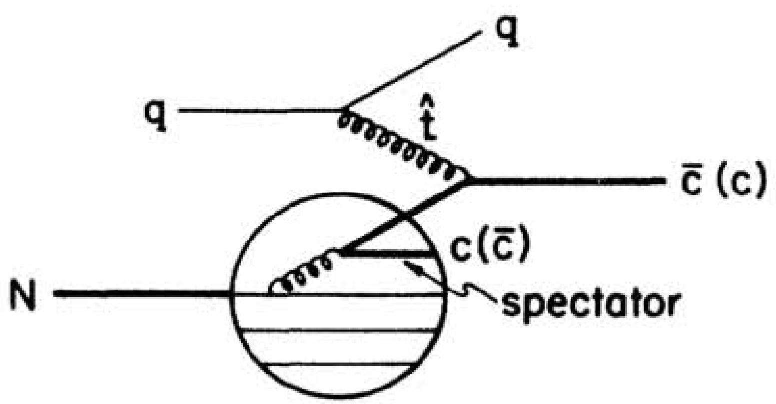

# Measurement of Charm Structure Function

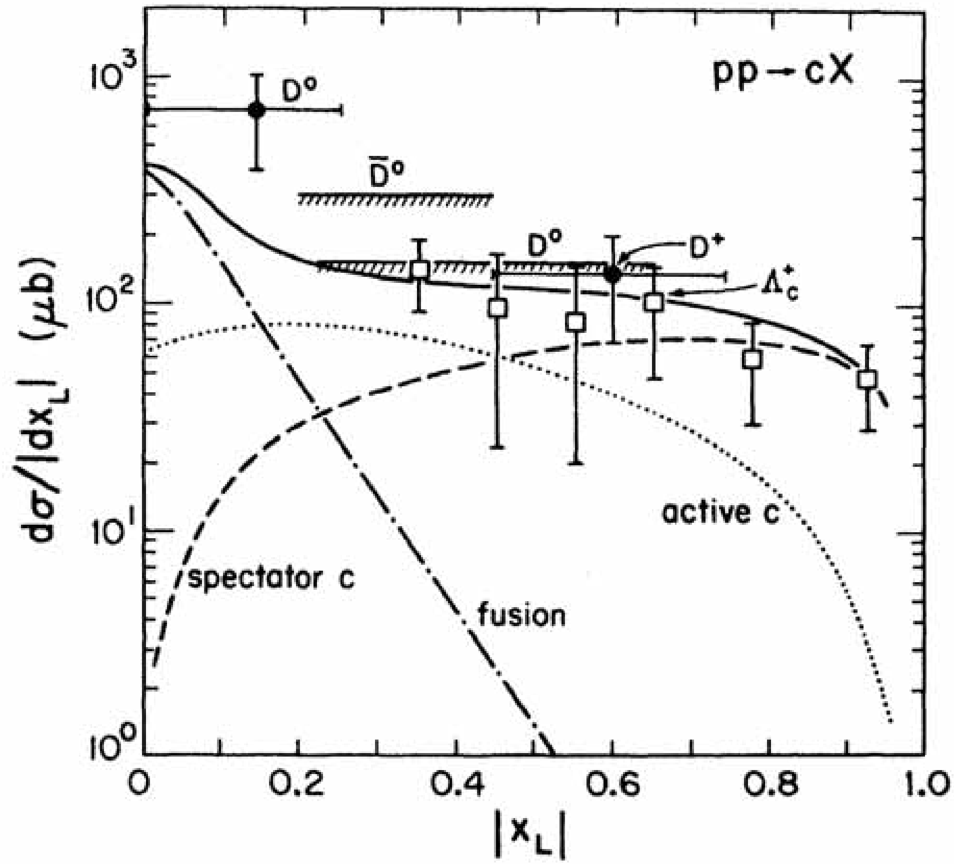
J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

## Evidence for Intrinsic Charm




## DGLAP / Photon-Gluon Fusion Factor of 30 too small




Predictions for Inclusive Charm ProductionDistributions at the ISR. Assumes active and spectator charm distribution in proton patterned on IC, plus coalescence of valence and charm quarks.

V. D. Barger, F. Halzen and W. Y. Keung,
"The Central And Diffractive Components Of Charm Production,"
Phys. Rev. D 25, 112 (1982).

- EMC data: $c(x,Q^2) > 30 \times$ DGLAP
  $Q^2 = 75$ GeV$^2$, $x = 0.42$

- High $x_F$ $pp \to J/\psi X$

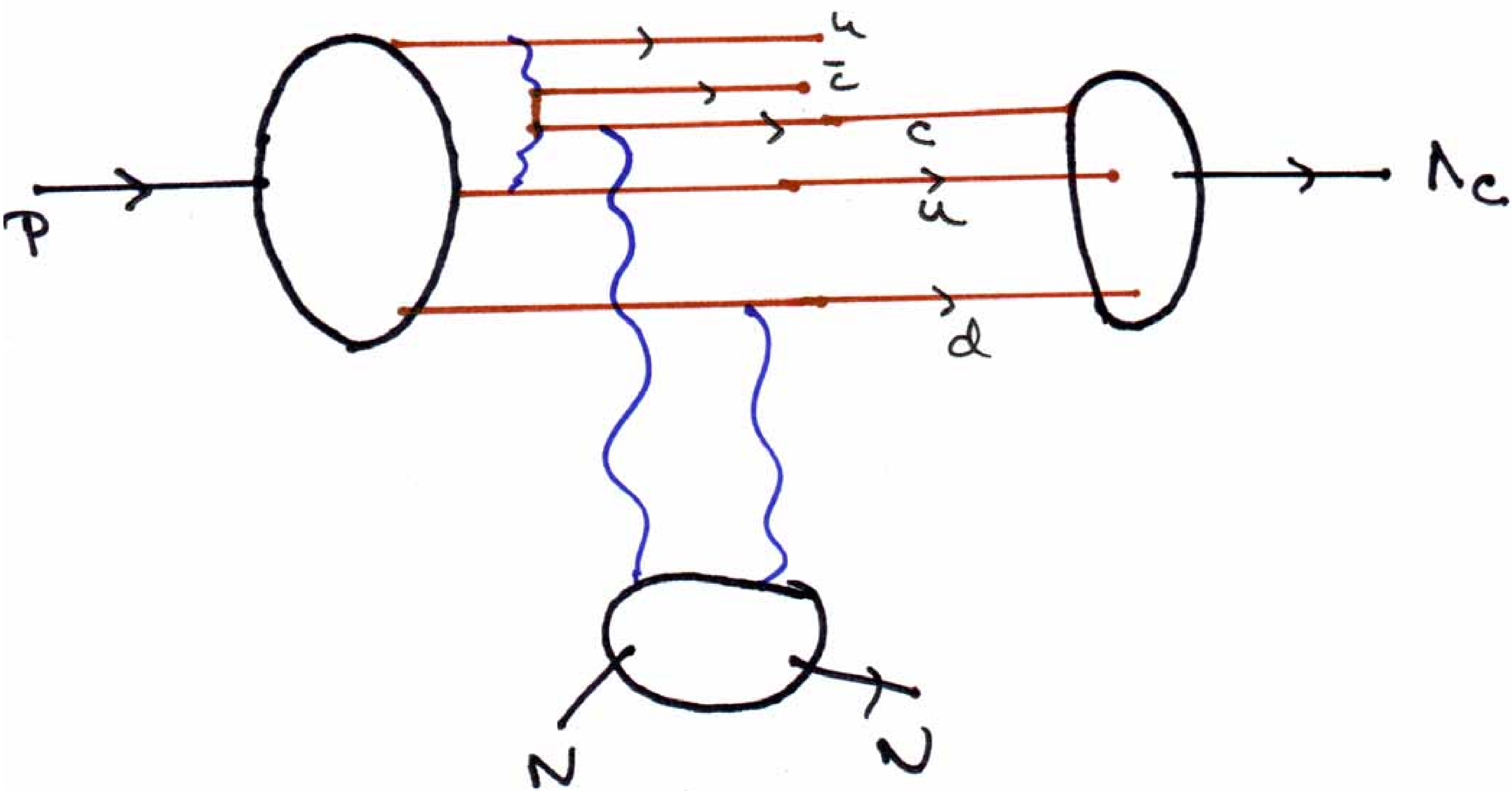
- High $x_F$ $pp \to J/\psi J/\psi X$

- High $x_F$ $pp \to \Lambda_c X$

- High $x_F$ $pp \to \Lambda_b X$

- High $x_F$ $pp \to \Xi(ccd)X$ (SELEX)

## $p\ p \to p\Lambda_c X$
## Diffractive Dissociation of Intrinsic Charm



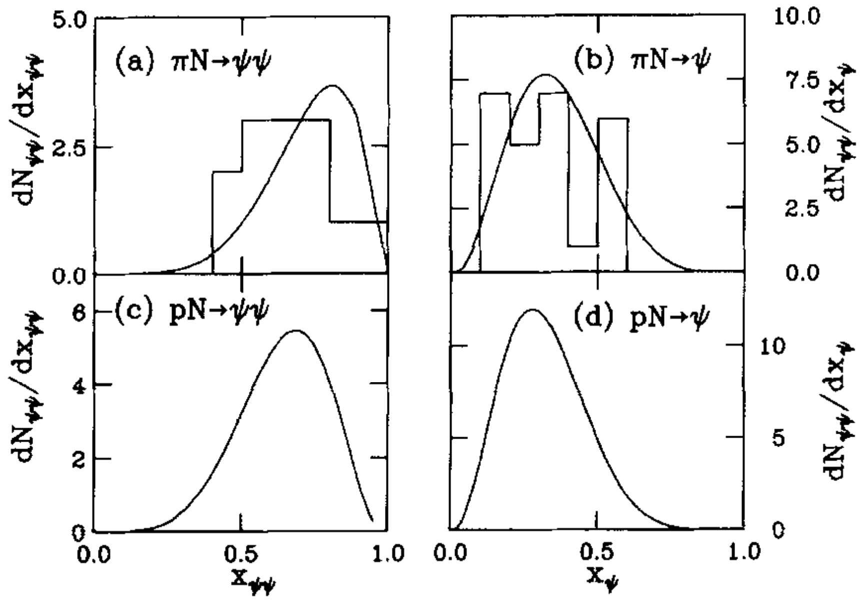



Fig. 3. The $\psi\psi$ pair distributions are shown in (a) and (c) for the pion and proton projectiles. Similarly, the distributions of $J/\psi$'s from the pairs are shown in (b) and (d). Our calculations are compared with the $\pi^- N$ data at 150 and 280 GeV/$c$ [1]. The $x_{\psi\psi}$ distributions are normalized to the number of pairs from both pion beams (a) and the number of pairs from the 400 GeV proton measurement (c). The number of single $J/\psi$'s is twice the number of pairs.

## NA3 Data

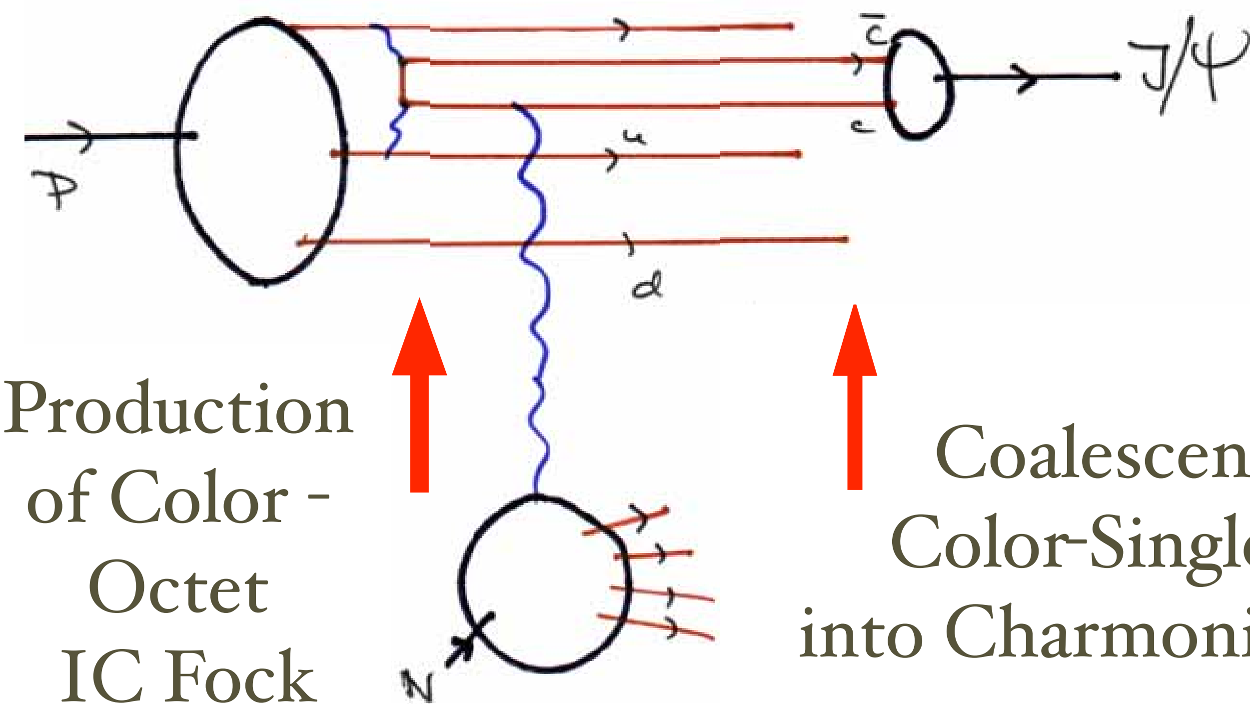
## $\pi A \to J/\psi J/\psi X$

### Intrinsic charm contribution to double quarkonium hadroproduction ★

R. Vogt [a], S.J. Brodsky [b]

The probability distribution for a general $n$-parti intrinsic $c\bar{c}$ Fock state as a function of $x$ and $\boldsymbol{k}_T$ written as

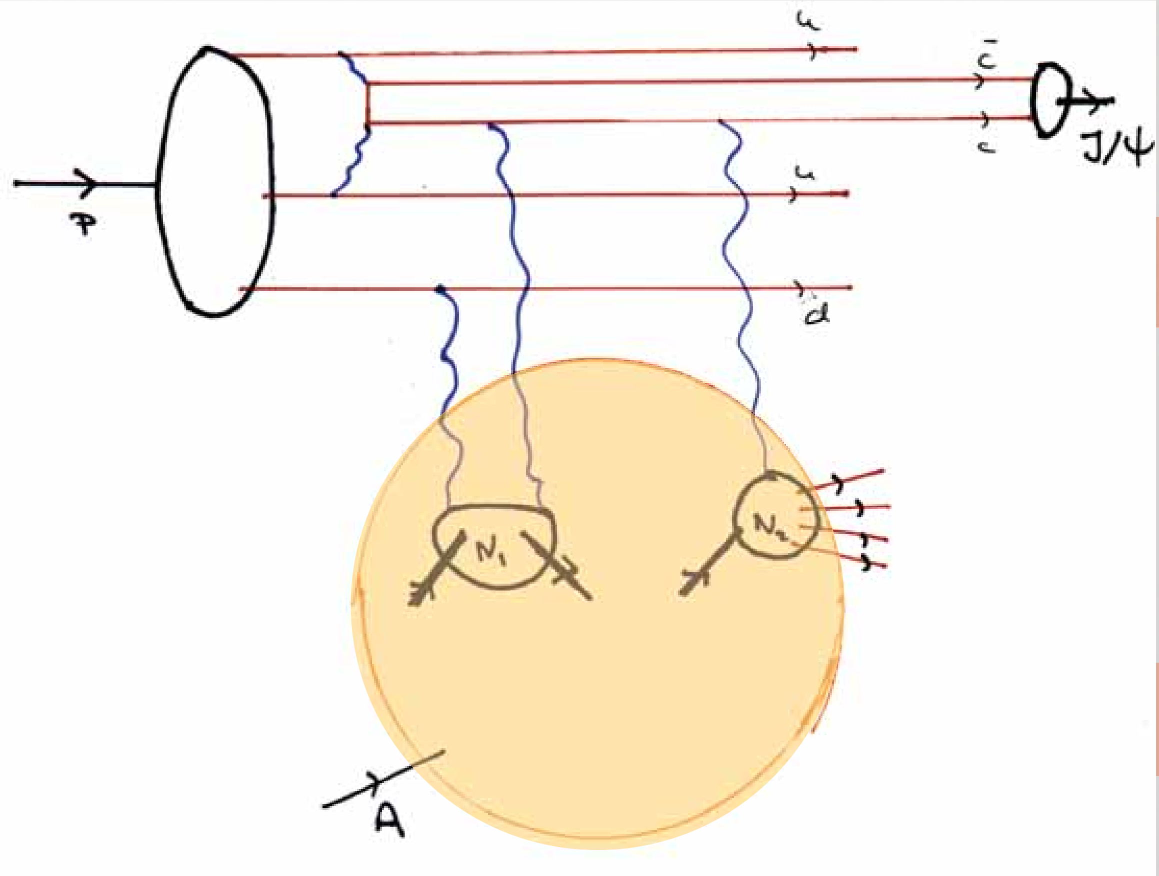
$$\frac{dP_{\rm ic}}{\prod_{i=1}^n dx_i d^2 k_{T,i}} = N_n \alpha_s^4(M_{c\bar{c}}) \frac{\delta(\sum_{i=1}^n \boldsymbol{k}_{T,i})\delta(1-\sum_{i=1}^n x_i)}{(m_h^2 - \sum_{i=1}^n (m_{T,i}^2/x_i))^2},$$




Production of Color - Octet IC Fock State

Coalescence of Color-Singlet Pair into Charmonium State

Scattering on Nucleon via one Gluon



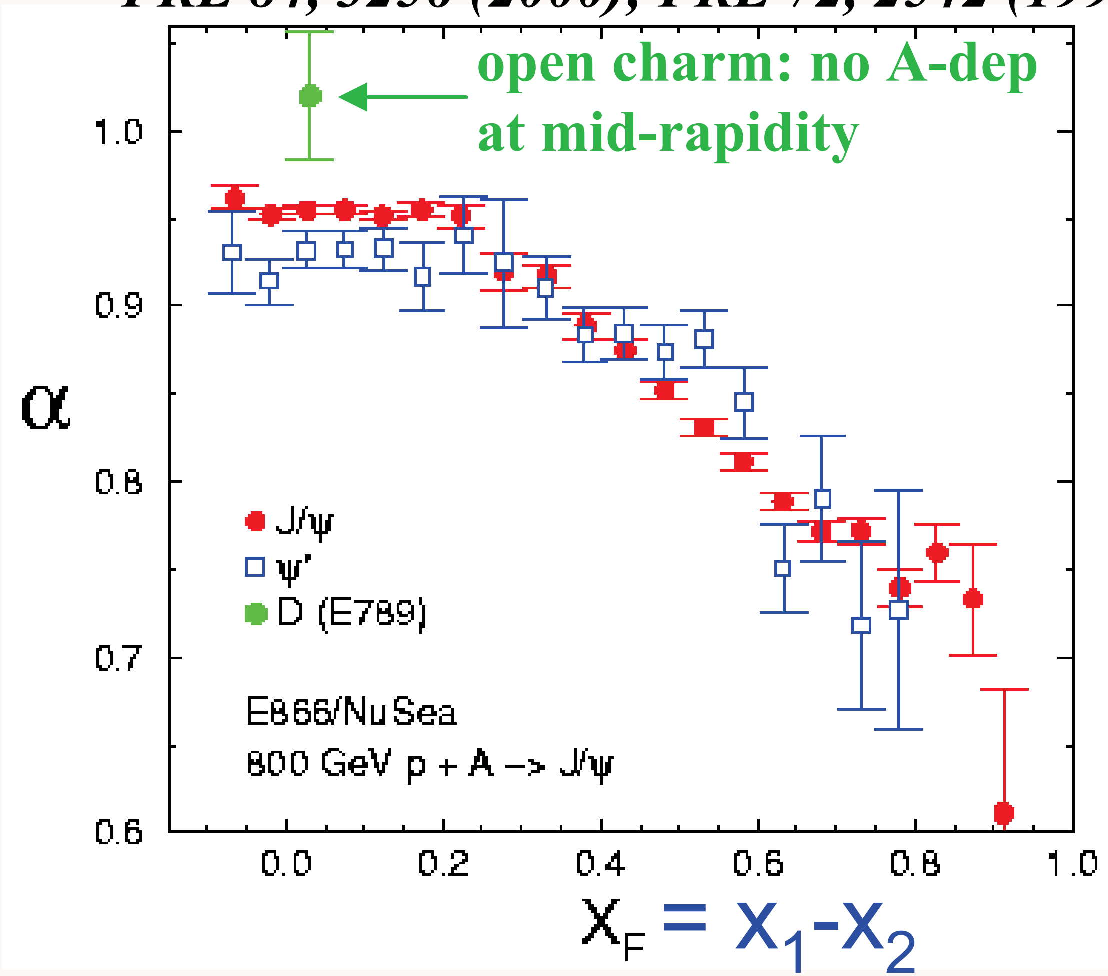

## Shadowing of $pA \rightarrow J/\Psi X$

$J/\Psi$ Production on Front Surface
No Absorption of Propagating $J/\Psi$
$\sigma(p + A \rightarrow J/\Psi + X) \propto A^{2/3}$

Elastic scattering of IC Fock state:
$|[uud]_{8_C}[c\bar{c}]_{8_C} > + N_1 \rightarrow |[uud]_{8_C}[c\bar{c}]_{8_C} > + N_1$
followed by:
$|[uud]_{8_C}[c\bar{c}]_{8_C} > + N_2 \rightarrow J/\Psi + X$
Depleted flux on downstream nucleons

## 800 GeV p-A (FNAL) $\sigma_A = \sigma_p * A^\alpha$

*PRL 84, 3256 (2000); PRL 72, 2542 (1994)*



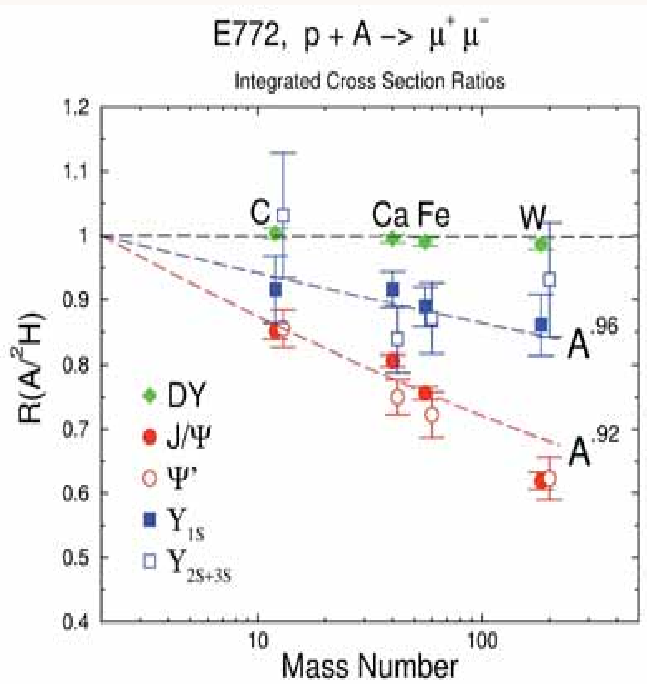

Remarkably Strong Nuclear Dependence for Fast Charmonium

M. Leitch

# Nuclear effects in Quarkonium production

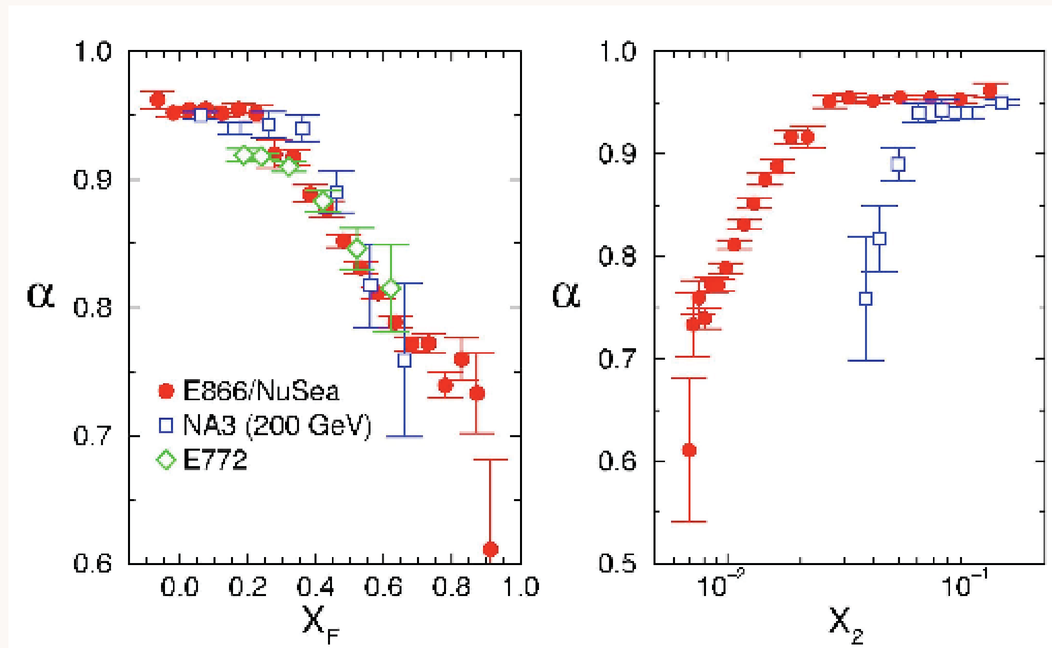
p + A at $s^{1/2}$ = 38.8 GeV

E772 data




$\sigma(p+A) = A^\alpha \, \sigma(p+N)$

Strong $x_F$ - dependence




Nuclear effects scale with $x_F$, not $x_2$ !!!

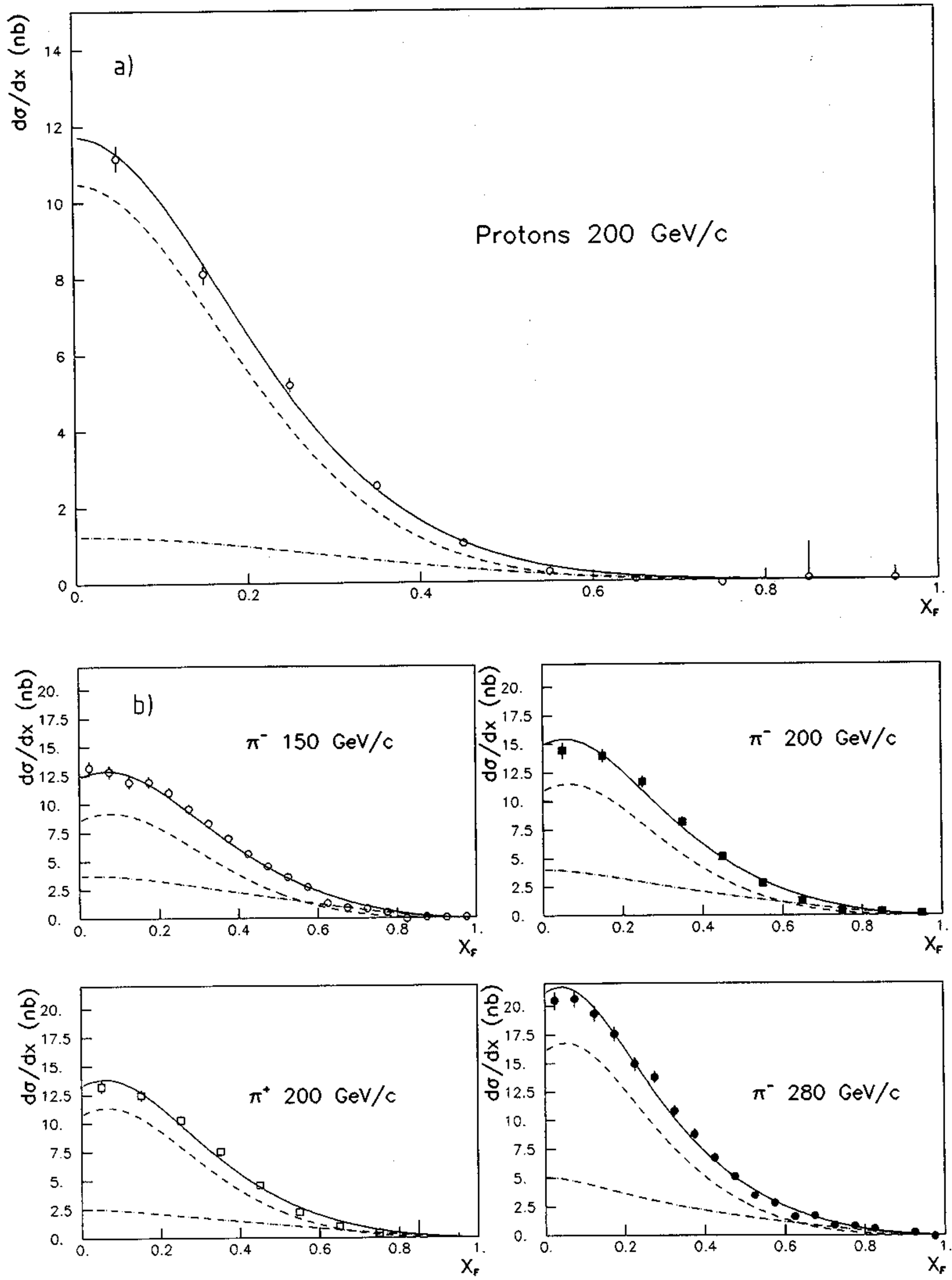
M.Leitch

# Nuclear Dependence of Quarkonium Production

NA3 data for $\frac{d\sigma}{dx_F}(p(\pi)A \to J/\psi X)$: hard $A^1$ and "diffractive" $A^{2/3}$ components

Diffractive contribution extends to large $x_F$

$A^{\alpha(x_F)}$ not $A^{\alpha(x_2)}$ : PQCD Factorization Violated!




Hard Component $\frac{d\sigma}{dx_F}(p(\pi)A \to J/\psi X)$

The fit: $gg$ fusion (dashed)

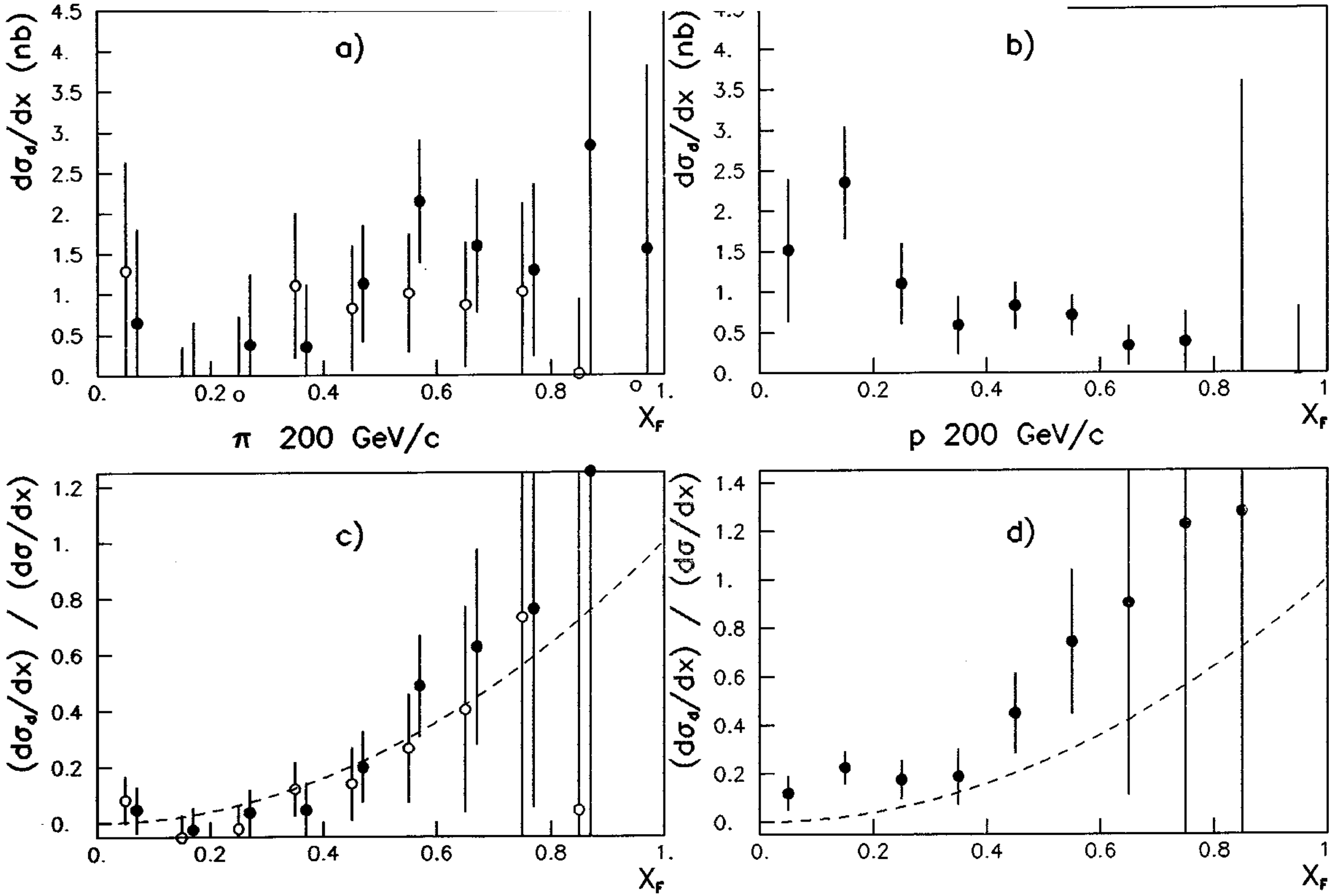
$q\bar{q}$ fusion (dashed-dot)

total (solid)

EXPERIMENTAL J/ψ HADRONIC PRODUCTION FROM 150 TO 280 GeV/c

NA3 COLLABORATION

CERN-EP/83-86
June 29th, 1983

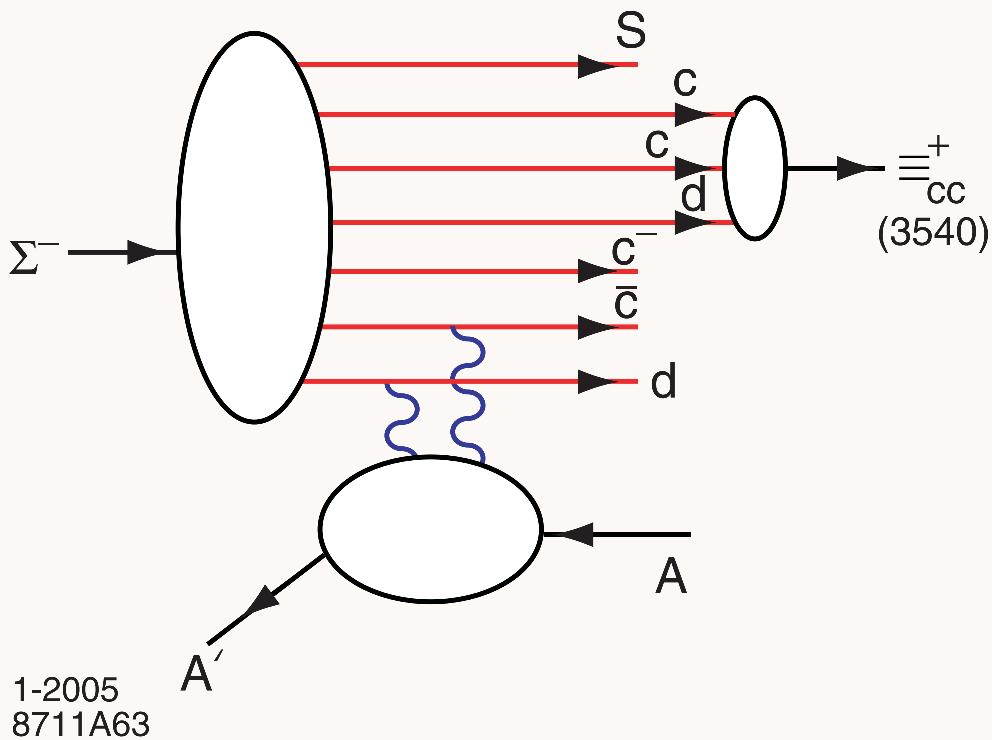


- IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \to J/\psi X$ (Mueller, Gunion, Tang, SJB)

- Color Octet IC Explains $A^{2/3}$ behavior at high $x_F$ (NA3, Fermilab) (Kopeliovitch, Schmidt, Soffer, SJB)

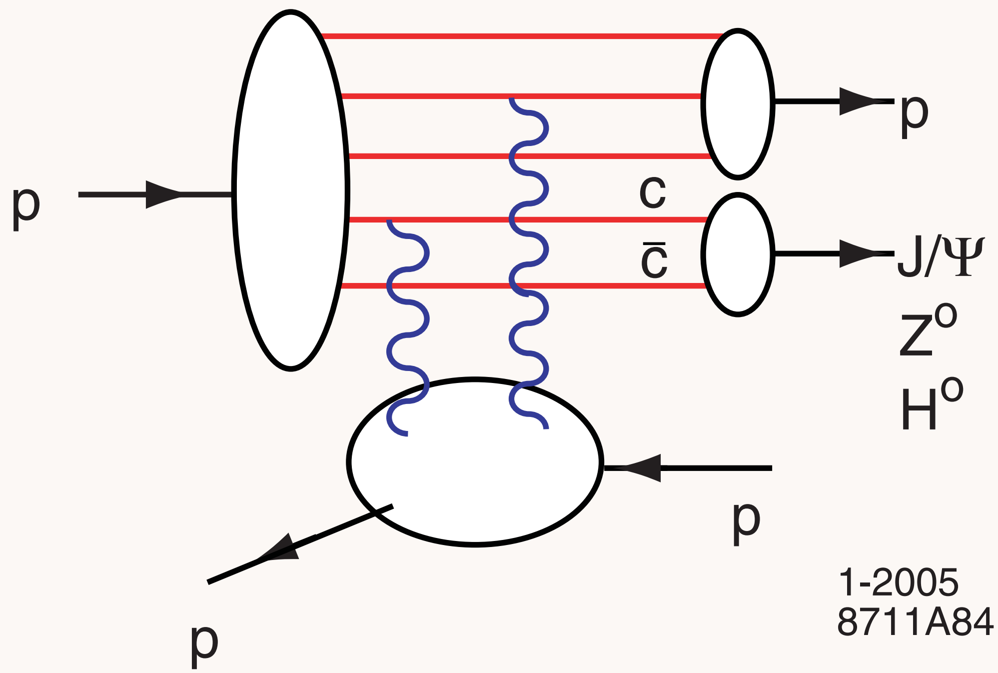
- IC Explains $J/\psi \to \rho\pi$ puzzle (Karliner, SJB)

- IC leads to new effects in $B$ decay (Gardner, SJB)




# Production of a Double-Charm Baryon

# Intrinsic Charm Mechanism for Exclusive Diffraction Production




$$p\,p \rightarrow J/\psi\; p\; p$$

$$x_{J/\psi} = x_c + x_{\bar{c}}$$

Exclusive Diffractive High-$X_F$ Higgs Production

Kopeliovitch, Schmidt, Soffer, sjb

Intrinsic $c\bar{c}$ pair formed in color octet $8_C$ in proton wavefunction    Large Color Dipole

Collision produces color-singlet $J/\psi$ through color exchange

RHIC Experiment

# *New Perspectives for QCD from AdS/CFT*

- LFWFs: Fundamental description of hadrons at amplitude level
- Holographic Model from AdS/CFT : Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances

# QCD at The Amplitude Level

- Light-Front Fock Expansions
- LFWFs boost invariant
- Direct connection to form factors, structure functions, distribution amplitudes, GPDs
- Higher Twist Correlations
- Orbital Angular Momentum
- Validated in QED, Bethe-Salpeter
- AdS/CFT Holographic Model

# *Anomalous QCD Effects*

- Hidden Color of Nuclear Wavefunction
- Odderon Trajectory: Charm jet asymmetry
- Anomalous Regge Behavior: J=0 Fixed Pole
- Proton-Proton Scattering:
  Color Transparency Breakdown and $A_{NN}$
- Non-Universality of Antishadowing
- Intrinsic Heavy Quarks at large x
- Anomalous scaling of single-particle inclusive at high $p_T$

# Light-Front QCD Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme

# Essential to test QCD

- GSI antiprotons
- 12 GeV Jlab
- J-PARC
- BaBar/Belle: ISR, two-gamma, timelike DVCS
- RHIC/LHC Nuclear Collisions; LHCb; forward heavy quarks, higgs
- photon-photon collider at the ILC
- electron-proton, electron-nucleus collisions

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# A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists' best hope for unifying gravity and quantum theory -- into a single coherent theory.