

# Confinement and Higher Fock States in Light-Front Holography

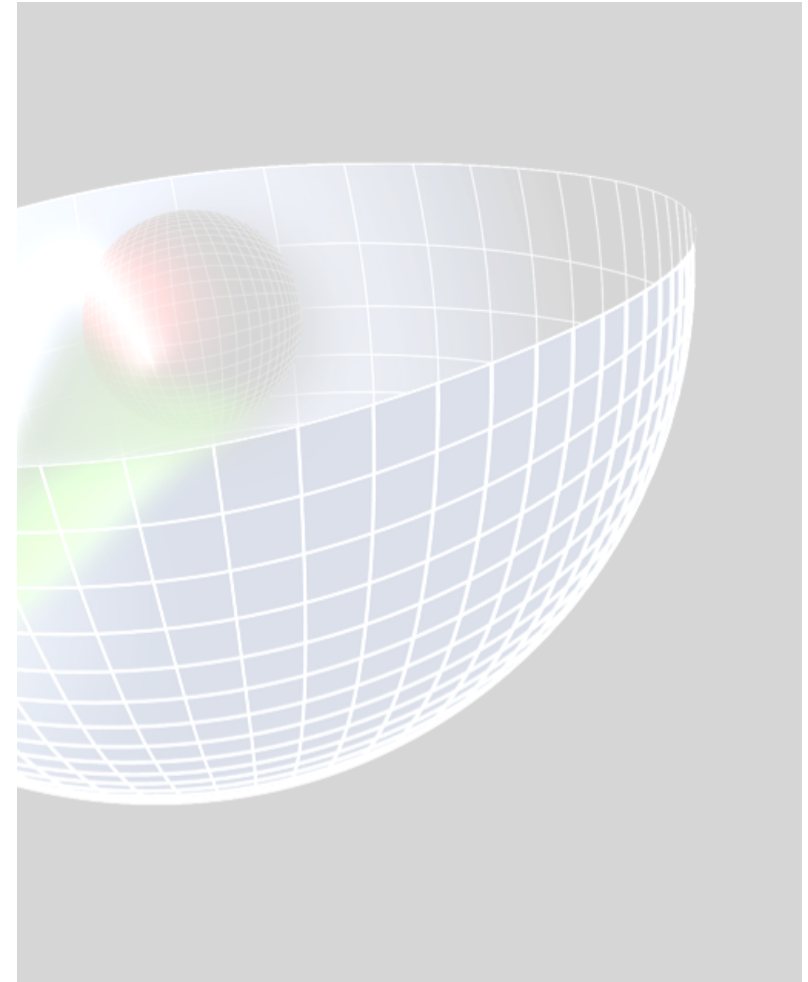
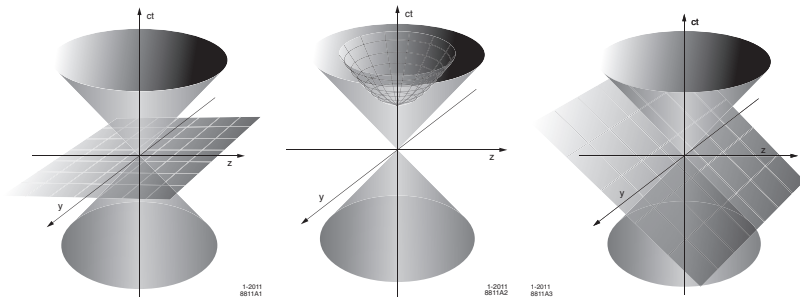
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## LIGHT CONE 2010

Applications of Light-Cone Coordinates  
to Highly Relativistic Systems

Dallas, May 23 - 27, 2011



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# 1 Introduction

## Gauge Gravity Correspondence and Light-Front QCD

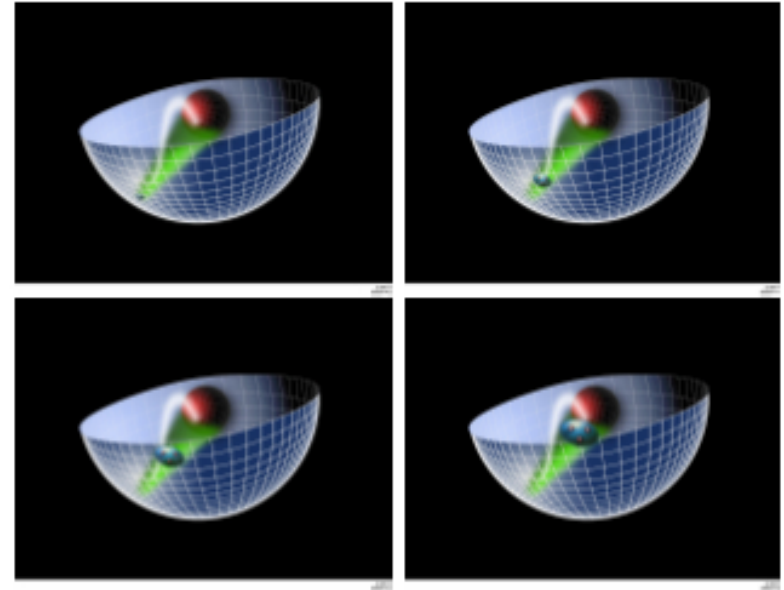
- Recent developments inspired by the AdS/CFT correspondence [Maldacena (1998)] between gravity in AdS space and conformal field theories in physical space-time have led to an analytical semiclassical approximation to light-front QCD, which provides physical insights into its non-perturbative dynamics
- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows to compute matrix elements diagonal in particle number
- Calculation of matrix elements  $\langle P + q | J | P \rangle$  requires boosting the hadronic bound state from  $|P\rangle$  to  $|P + q\rangle$ : boosts are trivial in LF
- Invariant Hamiltonian equation for bound states similar structure of AdS equations of motion: direct connection of QCD and AdS/CFT methods possible
- Isomorphism of  $SO(4, 2)$  group of conformal transformations with generators  $P^\mu, M^{\mu\nu}, K^\mu, D$ , with the group of isometries of  $AdS_5$ , a space of maximal symmetry, negative curvature and a four-dim boundary: Minkowski space (Dim isometry group of  $AdS_{d+1}$  is  $(d + 1)(d + 2)/2$ )

- AdS<sub>5</sub> metric:

$$\underbrace{ds^2}_{L_{\text{AdS}}} = \frac{R^2}{z^2} \left( \underbrace{\eta_{\mu\nu} dx^\mu dx^\nu}_{L_{\text{Minkowski}}} - dz^2 \right)$$

- A distance  $L_{\text{AdS}}$  shrinks by a warp factor  $z/R$  as observed in Minkowski space ( $dz = 0$ ):

$$L_{\text{Minkowski}} \sim \frac{z}{R} L_{\text{AdS}}$$



- Since the AdS metric is invariant under a dilatation of all coordinates  $x^\mu \rightarrow \lambda x^\mu$ ,  $z \rightarrow \lambda z$ , the variable  $z$  acts like a scaling variable in Minkowski space
- Short distances  $x_\mu x^\mu \rightarrow 0$  map to UV conformal AdS<sub>5</sub> boundary  $z \rightarrow 0$
- Large confinement dimensions  $x_\mu x^\mu \sim 1/\Lambda_{\text{QCD}}^2$  map to large IR region of AdS<sub>5</sub>,  $z \sim 1/\Lambda_{\text{QCD}}$ , thus there is a maximum separation of quarks and a maximum value of  $z$
- Use the isometries of AdS to map the local interpolating operators at the UV boundary of AdS into the modes propagating inside AdS

## 2 Light Front Dynamics

- Different possibilities to parametrize space-time [Dirac (1949)]
- Parametrizations differ by the hypersurface on which the initial conditions are specified. Each evolve with different “times” and has its own Hamiltonian, but should give the same physical results
- Forms of Relativistic Dynamics: dynamical vs. kinematical generators [Dirac (1949)]
- *Instant form*: hypersurface defined by  $t = 0$ , the familiar one

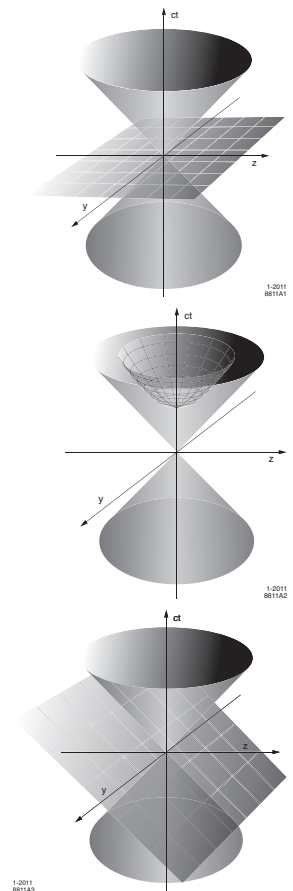
$$H, \mathbf{K} \text{ dynamical, } \mathbf{L}, \mathbf{P} \text{ kinematical}$$

- *Point form*: hypersurface is an hyperboloid

$$P^\mu \text{ dynamical, } M^{\mu\nu} \text{ kinematical}$$

- *Front form*: hypersurface is tangent to the light cone at  $\tau = t + z/c = 0$

$$P^-, L^x, L^y \text{ dynamical, } P^+, \mathbf{P}_\perp, L^z, \mathbf{K} \text{ kinematical}$$



## Semiclassical Approximation to QCD in the Light Front

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Compute  $\mathcal{M}^2$  from LF Lorentz invariant Hamiltonian equation for the relativistic bound state

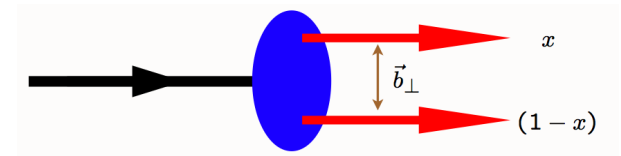
$$P_\mu P^\mu |\psi(P)\rangle = (P^- P^+ - \mathbf{P}_\perp^2) |\psi(P)\rangle = \mathcal{M}^2 |\psi(P)\rangle$$

- Relevant variable in the limit of zero quark masses (dual to the invariant mass)

$$\zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j} \right|$$

the  $x$ -weighted transverse impact coordinate of the spectator system ( $x$  active quark)

- For a two-parton system  $\zeta^2 = x(1-x) \mathbf{b}_\perp^2$



- To first approximation LF dynamics depend only on the invariant variable  $\zeta$ , and hadronic properties are encoded in the hadronic mode  $\phi(\zeta)$  from

$$\psi(x, \zeta, \varphi) = e^{iL^z \varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

factoring angular  $\varphi$ , longitudinal  $X(x)$  and transverse mode  $\phi(\zeta)$  ( $P^+$ ,  $\mathbf{P}_\perp$ ,  $J_z$  commute with  $P^-$ )

- Ultra relativistic limit  $m_q \rightarrow 0$  longitudinal modes  $X(x)$  decouple ( $L = |L^z|$ )

$$\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta)$$

where the confining forces from the interaction terms are summed up in the effective potential  $U(\zeta)$

- LF eigenvalue equation  $P_\mu P^\mu |\phi\rangle = \mathcal{M}^2 |\phi\rangle$  is a LF wave equation for  $\phi$

$$\left( \underbrace{-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2}}_{\text{kinetic energy of partons}} + \underbrace{U(\zeta)}_{\text{confinement}} \right) \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$



- Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable
- Eigenmodes  $\phi(\zeta)$  determine the hadronic mass spectrum and represent the probability amplitude to find  $n$ -massless partons at transverse impact separation  $\zeta$  within the hadron at equal light-front time
- Semiclassical approximation to light-front QCD does not account for particle creation and absorption

### 3 Light-Front Holographic Mapping of Wave Equations

#### Higher Spin Modes in AdS Space

- Description of higher spin modes in AdS space (Fronsdal, Fradkin, Vasiliev, Metsaev)
- Spin- $J$  in AdS represented by totally symmetric rank  $J$  tensor field  $\Phi_{M_1 \dots M_J}$
- Action for spin- $J$  field in  $\text{AdS}_{d+1}$  in presence of dilaton background  $\varphi(z)$  ( $x^M = (x^\mu, z)$ )

$$S = \frac{1}{2} \int d^d x dz \sqrt{g} e^{\varphi(z)} \left( g^{NN'} g^{M_1 M'_1} \dots g^{M_J M'_J} D_N \Phi_{M_1 \dots M_J} D_{N'} \Phi_{M'_1 \dots M'_J} - \mu^2 g^{M_1 M'_1} \dots g^{M_J M'_J} \Phi_{M_1 \dots M_J} \Phi_{M'_1 \dots M'_J} + \dots \right)$$

where  $D_M$  is the covariant derivative which includes parallel transport

- Physical hadron has plane-wave and polarization indices along  $3+1$  physical coordinates

$$\Phi_P(x, z)_{\mu_1 \dots \mu_J} = e^{-iP \cdot x} \Phi(z)_{\mu_1 \dots \mu_J}, \quad \Phi_{z\mu_2 \dots \mu_J} = \dots = \Phi_{\mu_1 \mu_2 \dots z} = 0$$

with four-momentum  $P_\mu$  and invariant hadronic mass  $P_\mu P^\mu = M^2$

- Construct effective action in terms of spin- $J$  modes  $\Phi_J$  with only physical degrees of freedom  
[Dosch, Brodsky and GdT (in preparation); Lyubovitskij *et al.*, operator construction (in preparation)]
- Introduce fields with tangent indices

$$\hat{\Phi}_{A_1 A_2 \dots A_J} = e_{A_1}^{M_1} e_{A_2}^{M_2} \dots e_{A_J}^{M_J} \Phi_{M_1 M_2 \dots M_J} = \left(\frac{z}{R}\right)^J \Phi_{A_1 A_2 \dots A_J}$$

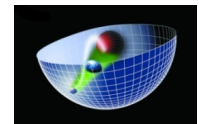
- Find effective action

$$S = \frac{1}{2} \int d^d x dz \sqrt{g} e^{\varphi(z)} \left( g^{NN'} \eta^{\mu_1 \mu'_1} \dots \eta^{\mu_J \mu'_J} \partial_N \hat{\Phi}_{\mu_1 \dots \mu_J} \partial_{N'} \hat{\Phi}_{\mu'_1 \dots \mu'_J} - \mu^2 \eta^{\mu_1 \mu'_1} \dots \eta^{\mu_J \mu'_J} \hat{\Phi}_{\mu_1 \dots \mu_J} \hat{\Phi}_{\mu'_1 \dots \mu'_J} \right)$$

upon  $\mu$ -rescaling

- Variation of the action gives AdS wave equation for spin- $J$  mode  $\Phi_J = \Phi_{\mu_1 \dots \mu_J}$

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$



with  $\hat{\Phi}_J(z) = (z/R)^J \Phi_J(z)$  and all indices along 3+1

## Dual QCD Light-Front Wave Equation

$$z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle$$

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Upon substitution  $z \rightarrow \zeta$  and  $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2} \Phi_J(\zeta)$  in AdS WE

$$\left[ -\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left( \frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z \right) + \left( \frac{\mu R}{z} \right)^2 \right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE ( $d = 4$ )

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$U(\zeta) = \frac{1}{2} \varphi''(z) + \frac{1}{4} \varphi'(z)^2 + \frac{2J - 3}{2z} \varphi'(z)$$

and  $(\mu R)^2 = -(2 - J)^2 + L^2$

- AdS Breitenlohner-Freedman bound  $(\mu R)^2 \geq -4$  equivalent to LF QM stability condition  $L^2 \geq 0$
- Scaling dimension  $\tau$  of AdS mode  $\hat{\Phi}_J$  is  $\tau = 2 + L$  in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

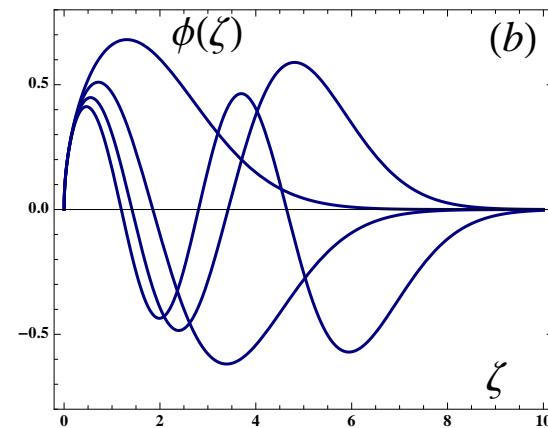
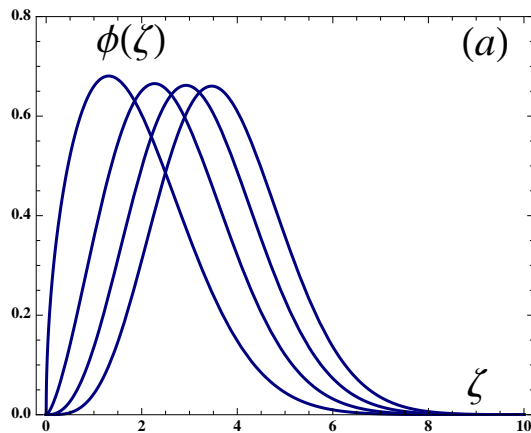
## Bosonic Modes and Meson Spectrum

Soft wall model: linear Regge trajectories [Karch, Katz, Son and Stephanov (2006)]

- Choose Dilaton  $\varphi(z) = +\kappa^2 z^2$  [ $\varphi(z) = -\kappa^2 z^2$  does not provide area law for the Wilson loop]
- Effective potential:  $U(z) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$
- Normalized eigenfunctions  $\langle \phi | \phi \rangle = \int d\zeta |\phi(z)|^2 = 1$

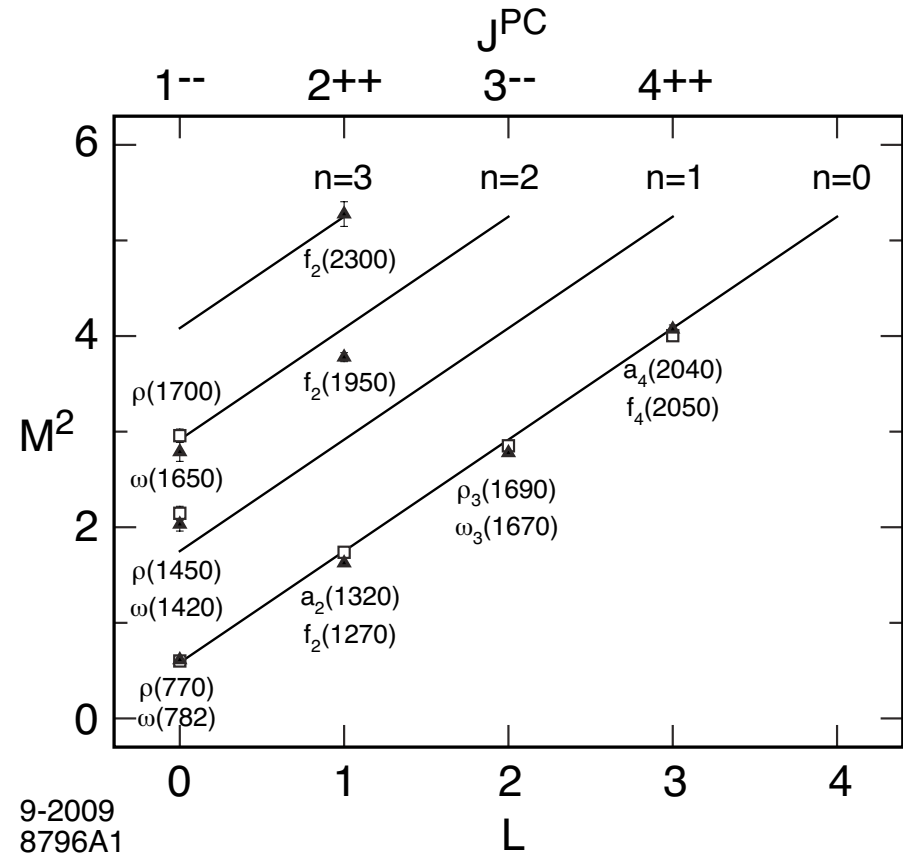
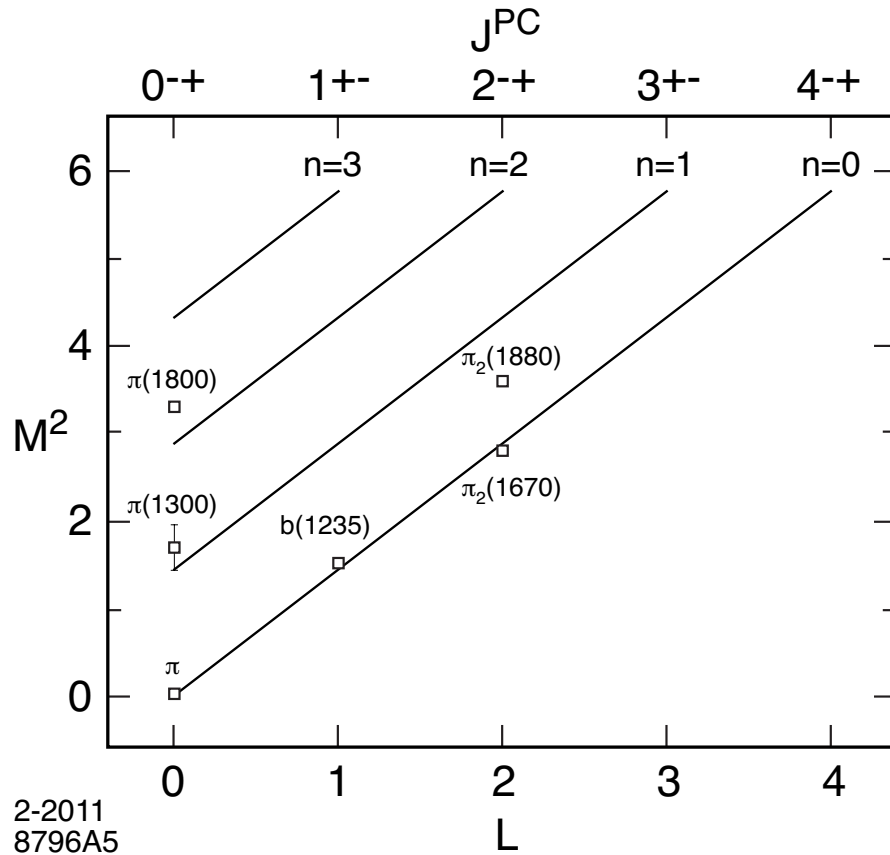
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues  $\mathcal{M}_{n,L,S}^2 = 4\kappa^2 (n + L + S/2)$



LFWFs  $\phi_{n,L}(\zeta)$  in physical space time for dilaton  $\exp(\kappa^2 z^2)$ : a) orbital modes and b) radial modes

$4\kappa^2$  for  $\Delta n = 1$   
 $4\kappa^2$  for  $\Delta L = 1$   
 $2\kappa^2$  for  $\Delta S = 1$

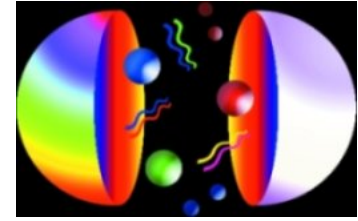


Regge trajectories for the  $\pi$  ( $\kappa = 0.6$  GeV) and the  $I = 1$   $\rho$ -meson and  $I = 0$   $\omega$ -meson families ( $\kappa = 0.54$  GeV)

## Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{3/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+1}(\kappa^2 \zeta^2)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L} \frac{1}{\sqrt{n+L+2}} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{5/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^{L+2}(\kappa^2 \zeta^2)$$

- Normalization

$$\int d\zeta \psi_+^2(\zeta) = \int d\zeta \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}_{n,L,S=1/2}^2 = 4\kappa^2 (n+L+1)$$

- “Chiral partners”

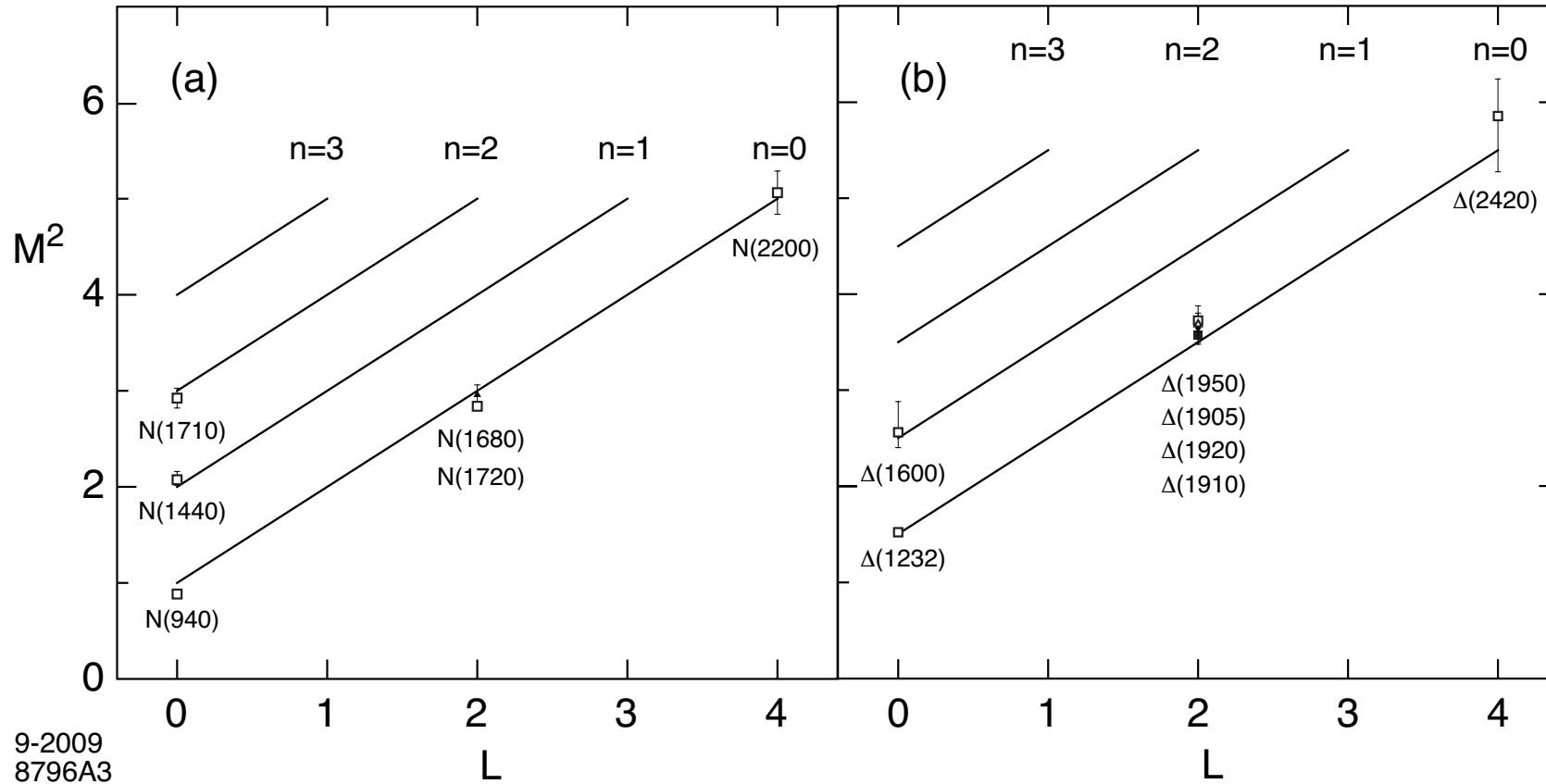
$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Same multiplicity of states for mesons and baryons!

$$4\kappa^2 \text{ for } \Delta n = 1$$

$$4\kappa^2 \text{ for } \Delta L = 1$$

$$2\kappa^2 \text{ for } \Delta S = 1$$



Regge trajectories for positive parity  $N$  and  $\Delta$  baryon families ( $\kappa = 0.5$  GeV)

## 4 Light-Front Holographic Mapping of Current Matrix Elements

[S. J. Brodsky and GdT, PRL **96**, 201601 (2006), PRD **77**, 056007 (2008)], PRD **78** 025032 (2008)]

- EM transition matrix element in QCD: local coupling to pointlike constituents

$$\langle \psi(P') | J^\mu | \psi(P) \rangle = (P + P')^\mu F(Q^2)$$

where  $Q = P' - P$  and  $J^\mu = e_q \bar{q} \gamma^\mu q$

- EM hadronic matrix element in AdS space from coupling of external EM field propagating in AdS with extended mode  $\Phi(x, z)$

$$\int d^4x dz \sqrt{g} e^{\varphi(z)} A^M(x, z) \Phi_{P'}^*(x, z) \overleftrightarrow{\partial}_M \Phi_P(x, z) \\ \sim (2\pi)^4 \delta^4(P' - P) \epsilon_\mu (P + P')^\mu F(Q^2)$$

- How to recover hard pointlike scattering at large  $Q$  out of soft collision of extended objects?

[Polchinski and Strassler (2002)]

- Mapping of  $J^+$  elements at fixed light-front time:  $\Phi_P(z) \Leftrightarrow |\psi(P)\rangle$

- QCD Drell-Yan-West electromagnetic FF in impact space [Soper (1977)]

$$F(q^2) = \sum_n \prod_{j=1}^{n-1} \int dx_j d^2\mathbf{b}_{\perp j} \sum_q e_q \exp\left(i\mathbf{q}_{\perp} \cdot \sum_{k=1}^{n-1} x_k \mathbf{b}_{\perp k}\right) |\psi_n(x_j, \mathbf{b}_{\perp j})|^2$$

- Consider a two-quark  $\pi^+$  Fock state  $|u\bar{d}\rangle$  with  $e_u = \frac{2}{3}$  and  $e_{\bar{d}} = \frac{1}{3}$

$$F_{\pi^+}(q^2) = \int_0^1 dx \int d^2\mathbf{b}_{\perp} e^{i\mathbf{q}_{\perp} \cdot \mathbf{b}_{\perp}(1-x)} \left| \psi_{u\bar{d}/\pi}(x, \mathbf{b}_{\perp}) \right|^2$$

with normalization  $F_{\pi^+}(q=0) = 1$

- Integrating over angle

$$F_{\pi^+}(q^2) = 2\pi \int_0^1 \frac{dx}{x(1-x)} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \left| \psi_{u\bar{d}/\pi}(x, \zeta) \right|^2$$

where  $\zeta^2 = x(1-x)\mathbf{b}_{\perp}^2$

- Compare with electromagnetic FF in AdS space [Polchinski and Strassler (2002)]

$$F(Q^2) = R^3 \int \frac{dz}{z^3} V(Q, z) \Phi_{\pi^+}^2(z)$$

where  $V(Q, z) = zQK_1(zQ)$

- Use the integral representation

$$V(Q, z) = \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right)$$

- Compare with electromagnetic FF in LF QCD for arbitrary  $Q$ . Expressions can be matched only if LFWF is factorized

$$\psi(x, \zeta, \varphi) = e^{iM\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Find

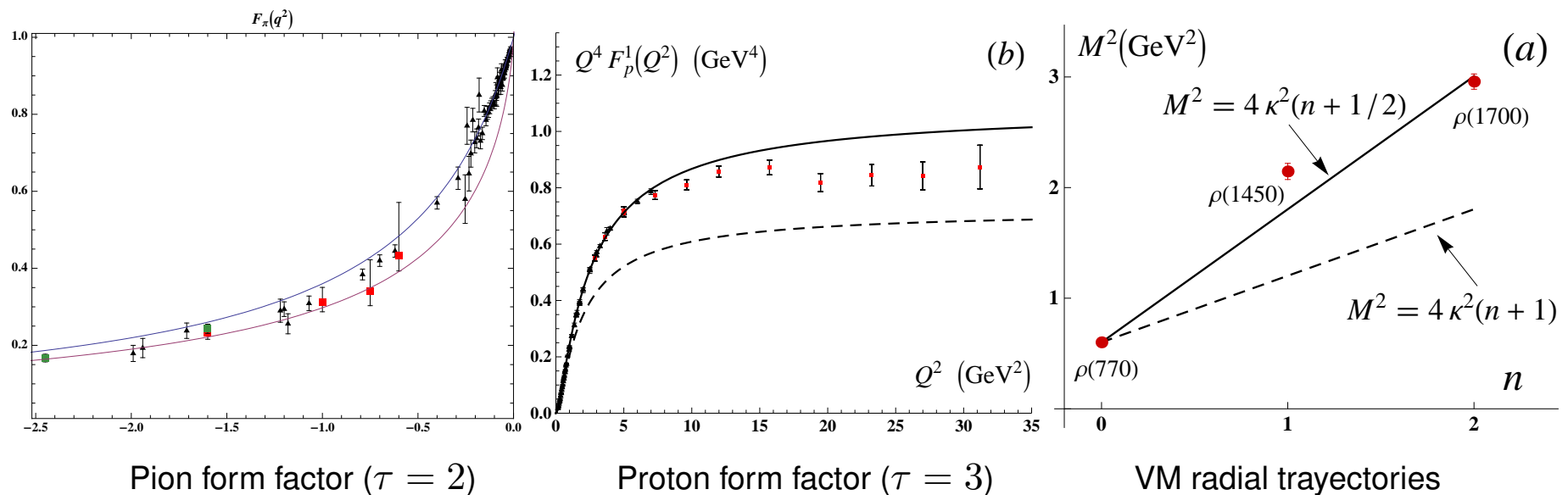
$$X(x) = \sqrt{x(1-x)}, \quad \phi(\zeta) = \left( \frac{\zeta}{R} \right)^{-3/2} \Phi(\zeta), \quad z \rightarrow \zeta$$

- Form factor in soft-wall model expressed as  $\tau - 1$  product of poles along vector radial trajectory (twist  $\tau = N + L$ ) [Brodsky and GdT, Phys.Rev. D77 (2008) 056007]

$$F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{\tau-2}}^2}\right)}$$

where  $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$  [negative SL (dashed line)  $\rightarrow$  positive TL dilaton (continuous)]

- Correct scaling incorporated in the model
- “Free current”  $V(Q, z) = zQK_1(zQ) \rightarrow$  infinite radius (mauve), no pole structure in time-like region
- “Dressed current” non-perturbative sum of an infinite number of terms  $\rightarrow$  finite radius (blue)



## Nucleon Form Factors

- Light Front Holographic Approach [Brodsky and GdT]
- EM hadronic matrix element in AdS space from non-local coupling of external EM field in AdS with fermionic mode  $\Psi_P(x, z)$

$$\int d^4x dz \sqrt{g} e^{\varphi(z)} \bar{\Psi}_P(x, z) e_A^M \Gamma^A A_M(x, z) \Psi_P(x, z)$$

$$\sim (2\pi)^4 \delta^4(P' - P) \epsilon_\mu \langle \psi(P'), \sigma' | J^\mu | \psi(P), \sigma \rangle$$

- Effective AdS/QCD model: additional term in the 5-dim action  
[Abidin and Carlson, Phys. Rev. D79, 115003 (2009)]

$$\eta \int d^4x dz \sqrt{g} e^{\varphi(z)} \bar{\Psi} e_A^M e_B^N [\Gamma^A, \Gamma^B] F_{MN} \Psi$$

Couplings  $\eta$  determined by static quantities

- Generalized Parton Distributions in gauge/gravity duals  
[Vega, Schmidt, Gutsche and Lyubovitskij, Phys.Rev. D83 (2011) 036001]  
[Nishio and Watari, arXiv:1105.290]

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q, z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}(\kappa^2 z^2) e^{-\kappa^2 z^2/2}$$

- Normalization ( $F_1^p(0) = 1$ ,  $V(Q=0, z) = 1$ )

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

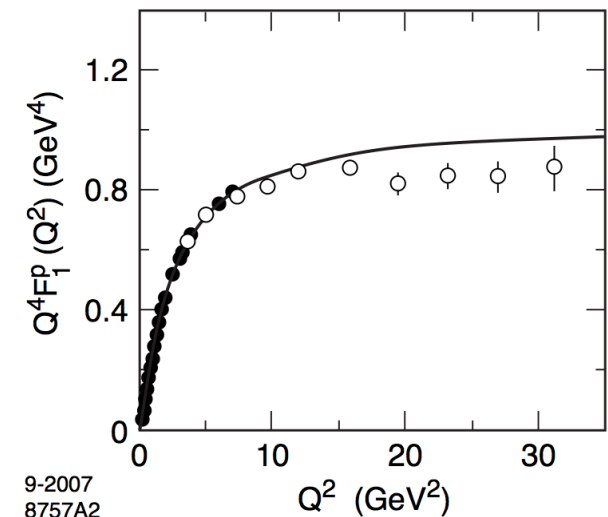
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with  $\mathcal{M}_{\rho n}^2 \rightarrow 4\kappa^2(n + 1/2)$



## Nucleon Transition Form Factors

- Compute spin non-flip EM transition  $N(940) \rightarrow N^*(1440)$ :  $\Psi_+^{n=0,L=0} \rightarrow \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1N \rightarrow N^*}^p(Q^2) = R^4 \int \frac{dz}{z^4} \Psi_+^{n=1,L=0}(z) V(Q, z) \Psi_+^{n=0,L=0}(z)$$

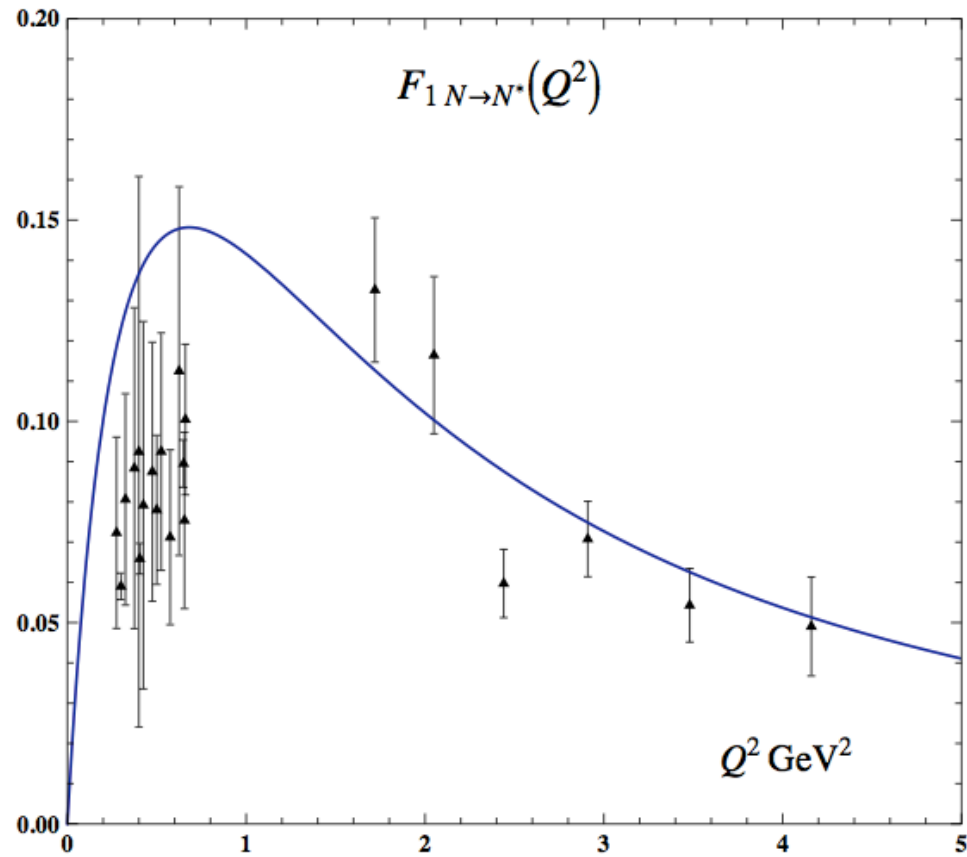
- Orthonormality of Laguerre functions  $(F_{1N \rightarrow N^*}^p(0) = 0, \quad V(Q=0, z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1N \rightarrow N^*}^p(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1 + \frac{Q^2}{\mathcal{M}_{\rho}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

with  $\mathcal{M}_{\rho_n}^2 \rightarrow 4\kappa^2(n + 1/2)$

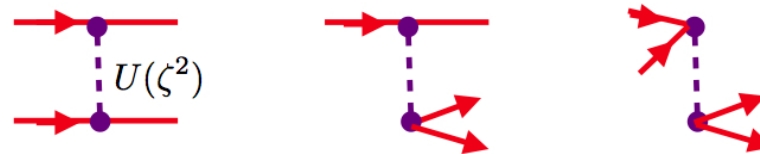


Data from I. Aznauryan, *et al.* CLAS (2009)

## 5 Confinement Interaction and Higher Fock States

[S. J. Brodsky and GdT (in progress)]

- Is the AdS/QCD confinement interaction responsible for quark pair creation?
- Only interaction in AdS/QCD is the confinement potential
- In QFT the resulting LF interaction is a 4-point effective interaction which leads to  $qq \rightarrow qq$ ,  $q \rightarrow qq\bar{q}$ ,  $q\bar{q} \rightarrow q\bar{q}$  and  $\bar{q} \rightarrow \bar{q}q\bar{q}$



- Create Fock states with extra quark-antiquark pairs.
- No mixing with  $q\bar{q}g$  Fock states (no dynamical gluons)
- Explain the dominance of quark interchange in large angle elastic scattering  
[C. White *et al.* Phys. Rev D **49**, 58 (1994)]
- Effective confining potential can be considered as an instantaneous four-point interaction in LF time, similar to the instantaneous gluon exchange in LC gauge  $A^+ = 0$ . For example

$$P_{\text{confinement}}^- \simeq \kappa^4 \int dx^- d^2 \vec{x}_\perp \frac{\bar{\psi} \gamma^+ T^a \psi}{P^+} \frac{1}{(\partial/\partial_\perp)^4} \frac{\bar{\psi} \gamma^+ T^a \psi}{P^+}$$

## Space and Time-Like Pion Form Factor

[GdT and S. J. Brodsky, arXiv:1010.1204]

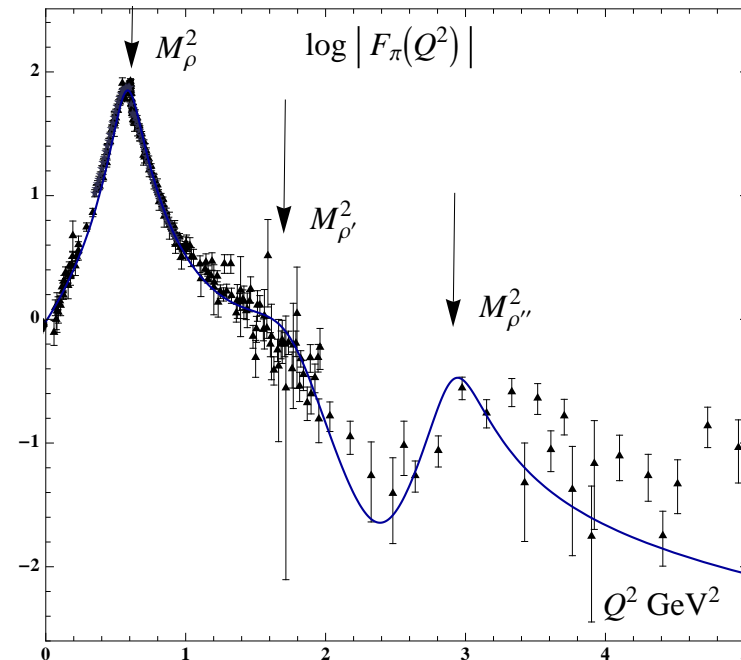
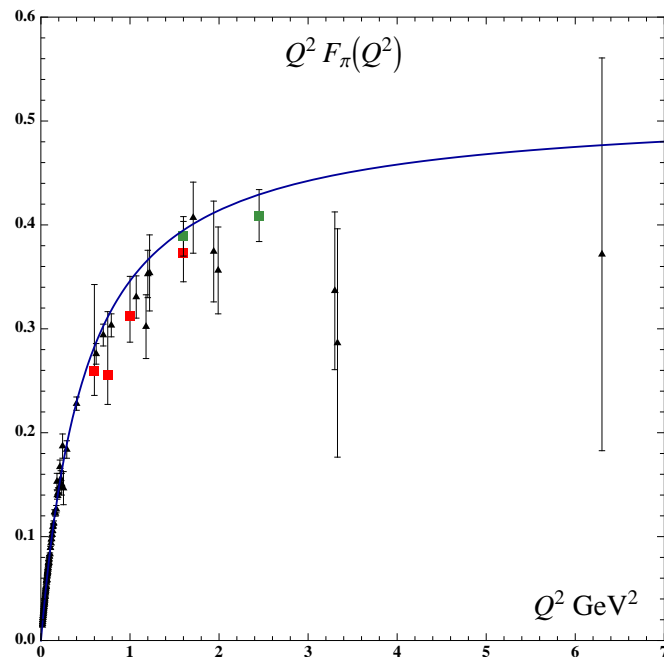
- Higher Fock components in pion LFWF

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}/\pi}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \dots$$

corresponding to interpolating operators  $\mathcal{O} = \bar{\psi}\gamma^+\gamma^5\psi$  and  $\mathcal{O} = \bar{\psi}\gamma^+\gamma^5\psi\psi\bar{\psi}$

- Expansion of LFWF up to twist 4

$$\kappa = 0.54 \text{ GeV}, \Gamma_{\rho} = 130, \Gamma_{\rho'} = 400, \Gamma_{\rho''} = 300 \text{ MeV}, P_{q\bar{q}q\bar{q}} = 13\%$$



## Meson Transition Form-Factors

[S. J. Brodsky, Fu-Guang Cao and GdT, arXiv:1005.39XX]

- Pion TFF from 5-dim Chern-Simons structure [Hill and Zachos (2005), Grigoryan and Radyushkin (2008)]

$$\int d^4x \int dz \epsilon^{LMNPQ} A_L \partial_M A_N \partial_P A_Q$$

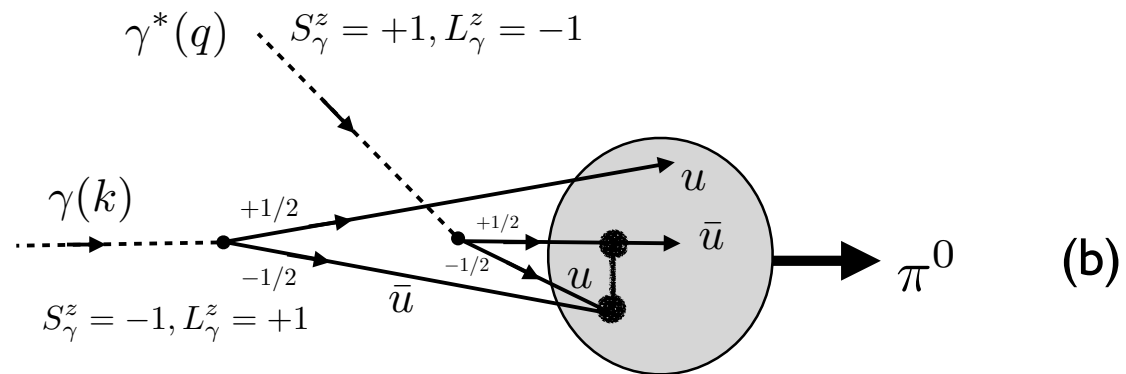
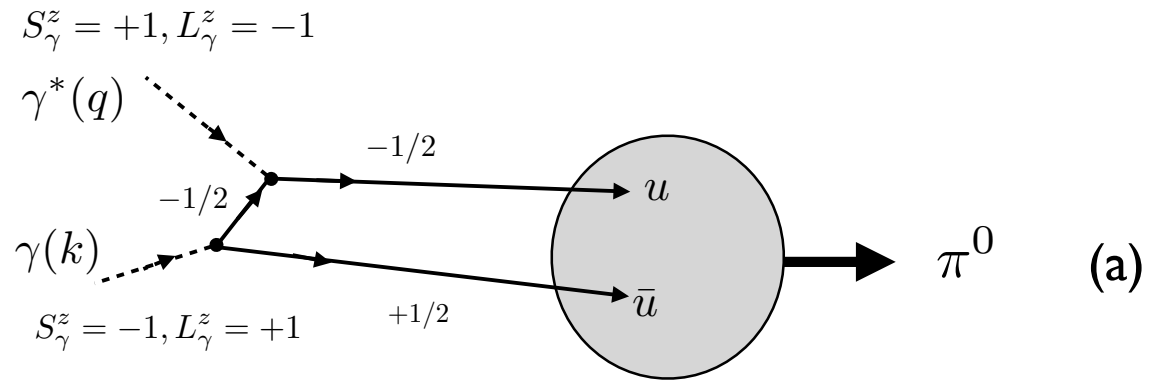
$$\sim (2\pi)^4 \delta^{(4)}(p_\pi + q - k) F_{\pi\gamma}(q^2) \epsilon^{\mu\nu\rho\sigma} \epsilon_\mu(q) (p_\pi)_\nu \epsilon_\rho(k) q_\sigma$$

- Take  $A_z \propto \Phi_\pi(z)/z$ ,  $\Phi_\pi(z) = \sqrt{2P_{q\bar{q}}} \kappa z^2 e^{-\kappa^2 z^2/2}$ ,  $\langle \Phi_\pi | \Phi_\pi \rangle = P_{q\bar{q}}$
- Find  $\phi(x) = \sqrt{3} f_\pi x(1-x)$ ,  $f_\pi = \sqrt{P_{q\bar{q}}} \kappa / \sqrt{2\pi}$

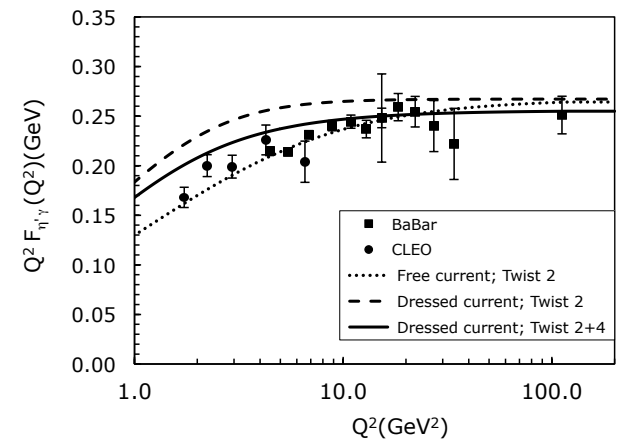
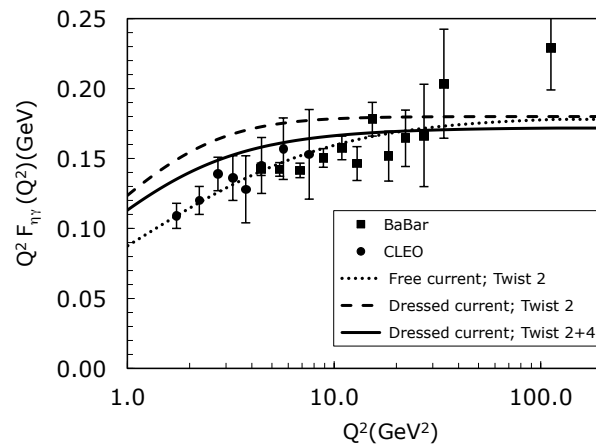
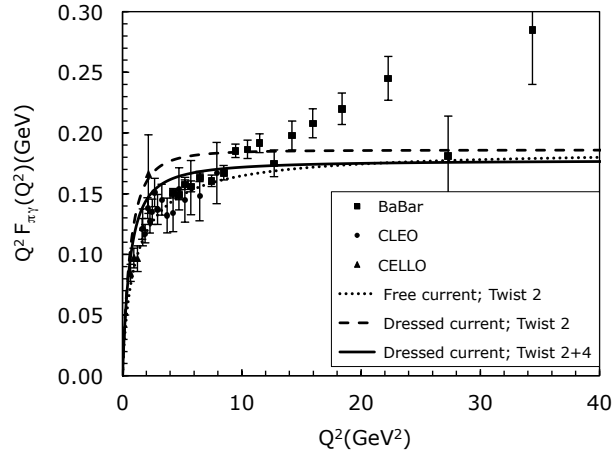
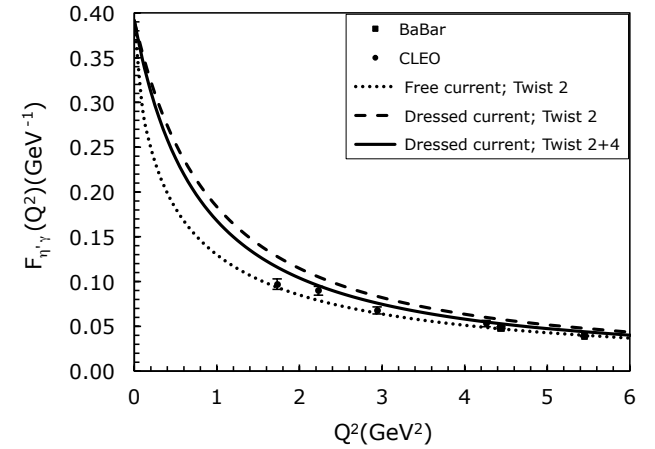
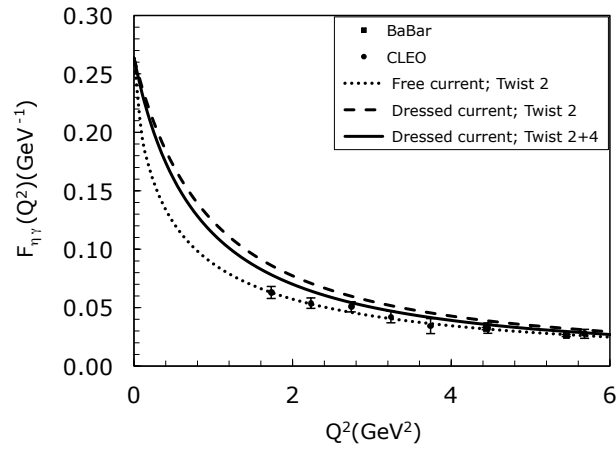
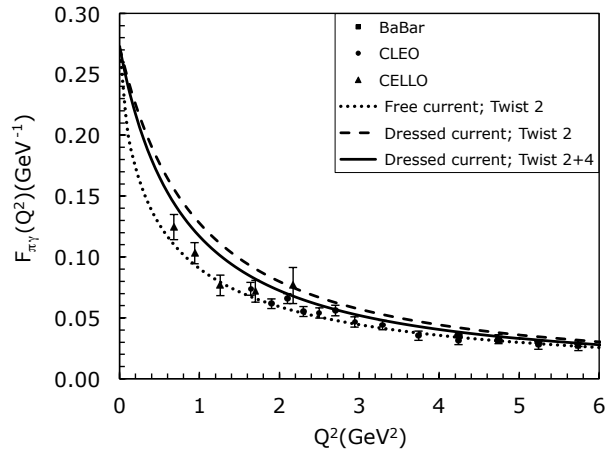
$$Q^2 F_{\pi\gamma}(Q^2) = \frac{4}{\sqrt{3}} \int_0^1 dx \frac{\phi(x)}{1-x} \left[ 1 - e^{-P_{q\bar{q}} Q^2 (1-x) / 4\pi^2 f_\pi^2 x} \right]$$

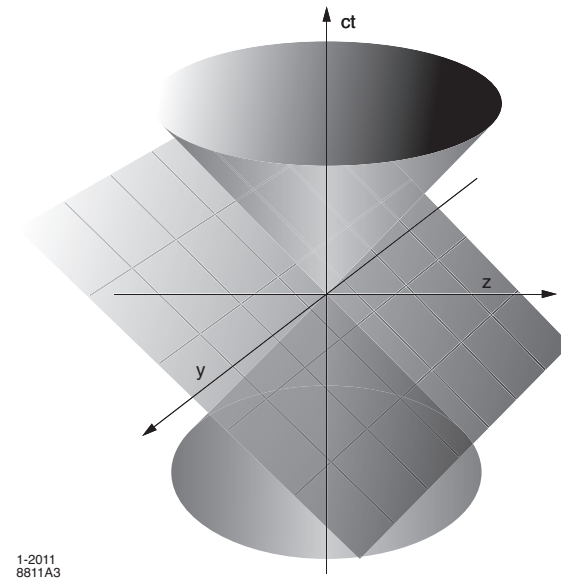
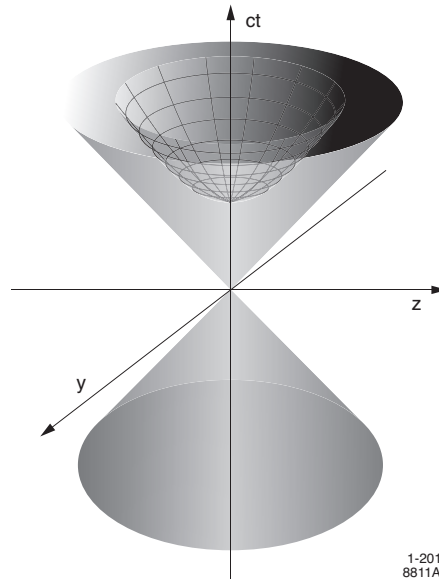
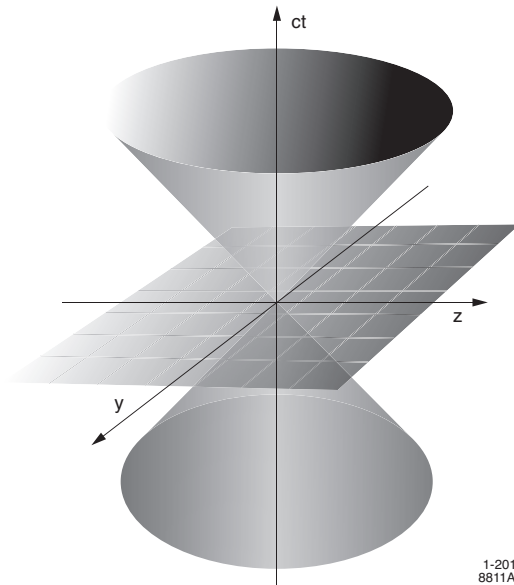
normalized to the asymptotic DA [ $P_{q\bar{q}} = 1 \rightarrow$  Musatov and Radyushkin (1997)]

- Large  $Q^2$  TFF is identical to first principles asymptotic QCD result  $Q^2 F_{\pi\gamma}(Q^2 \rightarrow \infty) = 2f_\pi$
- The CS form is local in AdS space and projects out only the asymptotic form of the pion DA



Twist-two (a) and twist-four contribution (b) to  $\gamma\gamma^* \rightarrow \pi^0$ .





*“ Working with a front is a process that is unfamiliar to physicists. But still I feel that the mathematical simplification that it introduces is all-important. I consider the method to be promising and have recently been making an extensive study of it. It offers new opportunities, while the familiar instant form seems to be played out ”*

P.A.M. Dirac (1977)