Production of a Double-Charm Baryon

SELEX high $x_F$  \[ < x_F > = 0.33 \]
Re-Analyzed Data
Restrict to Σ⁻–Beam
Peak wider than Resolution
Half decay to Ξ⁺_{cc}(3520)
Still working on Details
Intrinsic Charm Mechanism for Exclusive Diffraction Production

\[ p\ p \rightarrow J/\psi\ p\ p \]

\[ x_{J/\psi} = x_c + x_{\bar{c}} \]

Extrinsic Diffractive High-\(X_F\) Higgs Production

Intrinsic \(c\bar{c}\) pair formed in color octet \(8_C\) in proton wavefunction

Large Color Dipole

Collision produces color-singlet \(J/\psi\) through color exchange

RHIC Experiment

Kopeliovitch, Schmidt, Soffer, sjb

Stan Brodsky, SLAC
Intrinsic Charm Mechanism for Inclusive High-\(x_F\) Quarkonium Production

Quarkonia can have 80% of Proton Momentum!

Color-octet IC interacts at front surface of nucleus

IC can explains large excess of quarkonia at large \(x_F\), A-dependence

Goldhaber, Kopeliovich, Soffer, Schmidt, sjb
Intrinsic Charm Mechanism for Inclusive High-$X_F$ Higgs Production

$p p \rightarrow H X$

Also: intrinsic bottom, top

Higgs can have 80% of Proton Momentum!

New search strategy for Higgs

Goldhaber, Kopeliovich, Schmidt, sjb
Intrinsic Bottom Contribution to Inclusive Higgs Production

\[ \frac{d\sigma}{dx_F} (pp \to HX) [fb] \]

LHC: \( \sqrt{s} = 14\text{TeV} \)

Tevatron: \( \sqrt{s} = 2\text{TeV} \)

Goldhaber, Kopeliovich, Schmidt, sjb
Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization

IC Explains large excess of quarkonia at large $x_F$, $A$-dependence

$800 \text{ GeV } p-A \ (\text{FNAL}) \ \sigma_A = \sigma_p A^\alpha$

$PRL \ 84, \ 3256 \ (2000); \ PRL \ 72, \ 2542 \ (1994)$

Remarkably Strong Nuclear Dependence for Fast Charmonium

Violation of PQCD Factorization

IC Explains large excess of quarkonia at large $x_F$, $A$-dependence

$\frac{d\sigma}{dx_F} (pA \rightarrow J/\psi X)$

$M. \ Leitch$

$P. \ Hoyer, \ M. \ Vanttinen \ (\text{Helsinki U.}), \ U. \ Sukhatme \ (\text{Illinois U., Chicago})$. HU-TFT-90-14, May 1990. 7pp.

**J/ψ** nuclear dependence vs rapidity, $x_{Au}$, $xF$

PHENIX compared to lower energy measurements

**Huge “absorption” effect**

Violates PQCD factorization!

Hoyer, Sukhatme, Vanttinen

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**Klein, Vogt, PRL 91:142301, 2003**

**Kopeliovich, NP A696:669, 2001**


**NA3: ZP C20, 101 (1983)**

---

$\frac{d\sigma}{dx_F}(pA \rightarrow J/ψX)$
Scattering on front-face nucleon produces color-singlet $c\bar{c}$ pair

Octet-Octet IC Fock State

No absorption of small color-singlet

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^{2/3} \times \frac{d\sigma}{dx_F}(pN \rightarrow J/\psi X)$$
$A^{2/3}$ component

J. Badier et al, NA3

$$\frac{d\sigma}{dx_F}(pA \rightarrow J/\psi X) = A^1 \frac{d\sigma_1}{dx_F} + A^{2/3} \frac{d\sigma_{2/3}}{dx_F}$$

Excess beyond conventional PQCD subprocesses
• IC Explains Anomalous $\alpha(x_F)$ not $\alpha(x_2)$ dependence of $pA \rightarrow J/\psi X$
(Mueller, Gunion, Tang, SJB)

• Color Octet IC Explains $A^{2/3}$ behavior at high $x_F$ (NA3, Fermilab) (Kopeliovitch, Schmidt, Soffer, SJB)

• IC Explains $J/\psi \rightarrow \rho\pi$ puzzle
(Karliner, SJB)

• IC leads to new effects in $B$ decay
(Gardner, SJB)

Higgs production at $x_F = 0.8$
**Why is Intrinsic Charm Important for Flavor Physics?**

- New perspective on fundamental nonperturbative hadron structure
- Charm structure function at high $x$
- Dominates high $x_F$ charm and charmonium production
- Hadroproduction of new heavy quark states such as ccu, ccd at high $x_F$
- Intrinsic charm -- long distance contribution to penguin mechanisms for weak decay
- Novel Nuclear Effects from color structure of IC, Heavy Ion Collisions
- New mechanisms for high $x_F$ Higgs hadroproduction
- Dynamics of $b$ production: LHCb
- Fixed target program at LHC: produce bbb states
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

$x_i = \frac{k_i^+}{P^+}$

Invariant under boosts. Independent of $P^+$

$$H_{LF}^{QCD} |\psi> = M^2 |\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
QCD and the LF Hadron Wavefunctions

AdS/QCD
Light-Front Holography
LF Schrödinger Eqn

Initial and Final State
Rescattering
DDIS, DDIS, T-Odd

Non-Universal
Antishadowing

Baryon Excitations

Gluonic properties
DGLAP

Orbital Angular Momentum
Spin, Chiral Properties
Crewther Relation

Hard Exclusive Amplitudes
Form Factors
Counting Rules

Distribution amplitude
ERBL Evolution

\[ \phi_p(x_1, x_2, Q^2) \]

Nuclear Modifications
Baryon Anomaly
Color Transparency

Hadronization at
Amplitude Level

J=0 Fixed Pole
DVCS, GPDs, TMDs
LF Overlap, incl ERBL

Coordinate space
representation

Quark & Flavor Structure

Heavy Quark Fock States
Intrinsic Charm

Note: The image contains a flowchart diagram with various nodes and arrows connecting different topics related to QCD and the light-front hadron wavefunctions. The text highlights key concepts such as AdS/QCD, initial and final state rescattering, baryon excitation, gluonic properties, orbital angular momentum, hard exclusive amplitudes, nuclear modifications, and distribution amplitudes. The diagram uses mathematical symbols and terminology to represent these concepts.
Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable

de Teramond, Deur, Shrock, Roberts, Tandy
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond
\[ \phi(z) \]

- **Light-Front Holography**

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

- **Light Front Wavefunctions:**
  Schrödinger Wavefunctions of Hadron Physics
**String Theory**

**AdS/CFT**
- Mapping of Poincare' and Conformal $SO(4,2)$ symmetries of 3+1 space to AdS5 space
- Conformal behavior at short distances + Confinement at large distance

**AdS/QCD**
- Counting rules for Hard Exclusive Scattering
- Regge Trajectories
- QCD at the Amplitude Level

**Semi-Classical QCD / Wave Equations**
- Holography

**Boost Invariant 3+1 Light-Front Wave Equations**
- Integrable!

**Hadron Spectra, Wavefunctions, Dynamics**
- J = 0, 1, 1/2, 3/2 plus L

**Goal: First Approximant to QCD**
\[ \psi(x, \vec{b}_\perp) \quad \leftrightarrow \quad \phi(z) \]

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \leftrightarrow \quad z \]

\[ \psi(x, \zeta) = \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta) \]

**Light-Front Holography:** Unique mapping derived from equality of LF and AdS formula for current matrix elements.
**Light-Front Holography: Map AdS/CFT to 3+1 LF Theory**

**Relativistic LF radial equation**

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta^2 = x(1 - x)b_\perp^2.\]

**Frame Independent**

\[\left\{-\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right\} \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

**G. de Teramond, sjb**

\[U(\zeta) = \kappa^4 \zeta^2\]

**soft wall confining potential**
\( \mathbf{H}_{QED} \)

\[(H_0 + H_{int}) \left| \Psi \right> = E \left| \Psi \right>\]

\[\left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})\]

\[\left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell + 1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r)\]

\( V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r} \)

\[\text{Semiclassical first approximation to QED}\]

\( \text{QED atoms: positronium and muonium} \)

\( \text{Coupled Fock states} \)

\( \text{Effective two-particle equation} \)

\( \text{Includes Lamb Shift, quantum corrections} \)

\( \text{Spherical Basis } r, \theta, \phi \)

\( \text{Coulomb potential} \)

\( \text{Bohr Spectrum} \)
\[ H_{QCD}^{LF} \]

\[
(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle
\]

\[
\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)
\]

\[
-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)
\]

\[ U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \]

**QCD Meson Spectrum**

**Coupled Fock states**

**Effective two-particle equation**

\[ \zeta^2 = x(1-x)b_{\perp}^2 \]

**Azimuthal Basis** \[ \zeta, \phi \]

**Confining AdS/QCD potential**

**Semiclassical first approximation to QCD**
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6 \text{ GeV}$.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6 \text{ GeV}$. 

**Soft Wall Model**
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \right] \phi_S(z) = M^2 \phi_S(z)
\]

with eigenvalues \( M^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube): \( M^2_n(L) = 2\pi\sigma (n + L + 1/2) \).

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \text{ GeV} \).

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
**Spacelike pion form factor from AdS/CFT**

\[ F_\pi(q^2) \]

\( q^2(\text{GeV}^2) \)

Data Compilation
Baldini, Kloe and Volmer
de Teramond, sjb
See also: Radyushkin

**Soft Wall: Harmonic Oscillator Confinement**

**Hard Wall: Truncated Space Confinement**

*One parameter - set by pion decay constant.*
Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

• We write the Dirac equation

\[(\alpha \Pi(\zeta) - M) \psi(\zeta) = 0,\]

in terms of the matrix-valued operator \(\Pi\)

\[\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2} \gamma_5}{\zeta} - \kappa^2 \zeta \gamma_5 \right),\]

and its adjoint \(\Pi_\nu^\dagger\), with commutation relations

\[\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.\]

• Solutions to the Dirac equation

\[\psi_+(\zeta) \sim z^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_\nu^\nu(\kappa^2 \zeta^2),\]

\[\psi_-(\zeta) \sim z^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_{\nu + 1}^\nu(\kappa^2 \zeta^2).\]

• Eigenvalues

\[M^2 = 4\kappa^2 (n + \nu + 1).\]
• Baryon: twist-dimension $3 + L \quad (\nu = L + 1)$

\[
O_{3+L} = \psi D_{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m} \psi, \quad L = \sum_{i=1}^{m} \ell_i.
\]

\[
\mathcal{M}^2 = 4\kappa^2(n + L + 1).
\]
\[ \frac{\omega B}{\omega M} = \frac{5}{8} \]

\[ 4\kappa^2 \text{ for } \Delta n = 1 \]
\[ 4\kappa^2 \text{ for } \Delta L = 1 \]
\[ 2\kappa^2 \text{ for } \Delta S = 1 \]

Parent and daughter Regge trajectories for the \( N \) and \( \Delta \) baryon families for \( \kappa = 0.5 \text{ GeV} \).
E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

Other Applications of Light-Front Holography

- Light baryon spectrum
- Light meson spectrum
- Nucleon form-factors: space-like region
- Pion form-factors: space and time-like regions
- Gravitational form factors of composite hadrons
- $n$-parton holographic mapping
- Heavy flavor mesons

hep-th/0501022
hep-ph/0602252
arXiv:0707.3859
arXiv:0802.0514
arXiv:0804.0452
Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

\[ F_+(Q^2) = g_+ \int d\zeta \, J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_-(Q^2) = g_- \int d\zeta \, J(Q, \zeta) |\psi_-(\zeta)|^2, \]

where the effective charges \( g_+ \) and \( g_- \) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \( S^z = +1/2 \). The two AdS solutions \( \psi_+(\zeta) \) and \( \psi_-(\zeta) \) correspond to nucleons with \( J^z = +1/2 \) and \(-1/2\).

- For \( SU(6) \) spin-flavor symmetry

\[ F_1^p(Q^2) = \int d\zeta \, J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \]

where \( F_1^p(0) = 1, \ F_1^n(0) = 0. \)
• Scaling behavior for large $Q^2$: $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$  

Proton $\tau = 3$

 Scaling behavior for large $Q^2$: $Q^4 F_1^n(Q^2) \rightarrow \text{constant}$  

Neutron $\tau = 3$

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

$$F_2^p(Q^2)$$

Harmonic Oscillator
Confinement
Normalized to anomalous moment

$$\kappa = 0.49 \text{ GeV}$$

G. de Teramond, sjb

LBNL November 18, 2010

Novel QCD Phenomena

Stan Brodsky, SLAC
Prediction from AdS/CFT: Meson LFWF

\[ \phi_M(x, Q^2) \propto \sqrt{x(1-x)} \]

Increases PQCD prediction for \( F_\pi(Q^2) \) by \( 16/9 \)
Second Moment of Pion Distribution Amplitude

\[ < \xi^2 > = \int_{-1}^{1} d\xi \xi^2 \phi(\xi) \]

\[ \xi = 1 - 2x \]

\[ < \xi^2 >_{\pi} = 1/5 = 0.20 \]

\[ < \xi^2 >_{\pi} = 1/4 = 0.25 \]

\[ \phi_{asympt} \propto x(1 - x) \]

\[ \phi_{AdS/QCD} \propto \sqrt{x(1 - x)} \]

Lattice (I) \[ < \xi^2 >_{\pi} = 0.28 \pm 0.03 \]

Lattice (II) \[ < \xi^2 >_{\pi} = 0.269 \pm 0.039 \]

Donnellan et al.

Braun et al.
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite Nc = 3: Baryons built on 3 quarks -- Large Nc limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

\[ \frac{\alpha_s(Q)}{\pi} = e^{-Q^2/4\kappa^2} \]

\( \kappa = 0.54 \text{ GeV} \)

Deur, de Teramond, sjb
\[ \beta_{AdS}(Q^2) = \frac{d}{d\log Q^2} \alpha_{s\,AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2} \]
\[
\left[-\frac{d^2}{d\zeta^2} + V(\zeta)\right] \phi(\zeta) = M^2\phi(\zeta)
\]

Holographic Variable

\[
\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}
\]

LF Kinetic Energy in momentum space

Assume LFWF is a dynamical function of the quark-antiquark invariant mass squared

\[
-\frac{d}{d\zeta^2} \rightarrow -\frac{d}{d\zeta^2} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \equiv \frac{k_\perp^2 + m_1^2}{x} + \frac{k_\perp^2 + m_2^2}{1-x}
\]
Result: Soft-Wall LFWF for massive constituents

\[ \psi(x, k_\perp) = \frac{4\pi c}{\kappa \sqrt{x(1-x)}} e^{-\frac{1}{2\kappa^2} \left( \frac{k_\perp^2}{x(1-x)} + \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right)} \]

LFWF in impact space: soft-wall model with massive quarks

\[ \psi(x, b_\perp) = \frac{c \kappa}{\sqrt{\pi}} \sqrt{x(1-x)} e^{-\frac{1}{2} \kappa^2 x(1-x)b_\perp^2 - \frac{1}{2\kappa^2} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right]} \]

\[ z \rightarrow \zeta \rightarrow \chi \]

\[ \chi^2 = b^2 x(1-x) + \frac{1}{\kappa^4} \left[ \frac{m_1^2}{x} + \frac{m_2^2}{1-x} \right] \]
LFWF peaks at

\[ x_i = \sum_{j} \frac{m_{\perp i}}{m_{\perp j}} \]

where

\[ m_{\perp i} = \sqrt{m^2 + k_{\perp}^2} \]

minimum of LF energy denominator

\[ \kappa = 0.375 \text{ GeV} \]

\[ m_a = m_b = 1.25 \text{ GeV} \]
$| \pi^+ \rangle = | u \bar{d} \rangle$

$m_u = 2 \text{ MeV}$
$m_d = 5 \text{ MeV}$

$| K^+ \rangle = | u \bar{s} \rangle$

$m_s = 95 \text{ MeV}$

$| D^+ \rangle = | c \bar{d} \rangle$

$m_c = 1.25 \text{ GeV}$

$| \eta_c \rangle = | c \bar{c} \rangle$

$| B^+ \rangle = | u \bar{b} \rangle$

$m_b = 4.2 \text{ GeV}$

$| \eta_b \rangle = | b \bar{b} \rangle$

$\kappa = 375 \text{ MeV}$
Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb

\[ \bar{H}(\bar{p}e^+) \]

Coalescence of off-shell co-moving positron and antiproton

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level

LBNL November 18, 2010

Novel QCD Phenomena

Stan Brodsky, SLAC
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

\[ e^+ + e^- \rightarrow \gamma^* \rightarrow g \bar{q}q \gamma^* \rightarrow \bar{q}q \gamma^* \rightarrow \psi(x, \vec{k}_\perp, \lambda_i) \]

**AdS/QCD**
**Hard Wall**
**Confinement:**

Capture if \( \zeta^2 = x(1 - x)b_\perp^2 > \frac{1}{\Lambda_{QCD}^2} \)

i.e.,

\[ M^2 = \frac{k_\perp^2}{x(1-x)} < \Lambda_{QCD}^2 \]
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Features of LF T-Matrix Formalism  
"Event Amplitude Generator"

- Same principle as antihydrogen production: off-shell coalescence
- Coalescence to hadron favored at equal rapidity, small transverse momenta
- Leading heavy hadron production: D and B mesons produced at large $z$
- Hadron helicity conservation if hadron LFWF has $L_z = 0$
- Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp \]

\[ P^+ = P^0 + P_z \]
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Higher Fock State Coalescence \( |uud\bar{s}\bar{s}> \)

Asymmetric Hadronization! \( D_s \rightarrow p(z) \neq D_s \rightarrow \bar{p}(z) \)

B-Q Ma, sjb

Stan Brodsky, SLAC
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit required
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Chiral Features of Soft-Wall AdS/QCD Model

- Boost Invariant
- Trivial LF vacuum.
- Massless Pion
- Hadron Eigenstates have LF Fock components of different $L^z$
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$
  $J^z = +1/2 : < L^z >= 1/2, < S_q^z = 0 >$
- Self-Dual Massive Eigenstates: Proton is its own chiral partner.
- Label State by minimum L as in Atomic Physics
- Minimum L dominates at short distances
- **AdS/QCD Dictionary:** Match to Interpolating Operator Twist at $z=0$. 

\[ J^z = +1/2 : < L^z > = 1/2, < S_q^z = 0 > \]
An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- As Simple as Schrödinger Theory in Atomic Physics
- LF radial variable $\zeta$ conjugate to invariant mass squared
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales: Essential for Gauge Link phenomena
- Hadron Spectroscopy and Dynamics from one parameter $\kappa$
- Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates: Zero cosmological constant!
- Systematically improvable with DLCQ Methods
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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\[ (\Omega_\Lambda)_{QCD} \sim 10^{45} \]
\[ (\Omega_\Lambda)_{EW} \sim 10^{56} \]
\[ \Omega_\Lambda = 0.76 \text{(expt)} \]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb

“Condensates in Quantum Chromodynamics and the Cosmological Constant”
Chiral magnetism (or magneto-hadron-chronics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.\(^1\) Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.\(^2\) A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.\(^3\)
We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.
Simple physical argument for “in-hadron” condensate

Use Dyson-Schwinger Equation for bound-state quark propagator:
find confined condensate

\[ < B|\bar{q}q|B > \text{ not } < 0|\bar{q}q|0 > \]
**Bethe-Salpeter Analysis**

\[
f_H P^\mu = Z_2 \int^\Lambda d^4q \frac{d^4q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 \gamma^\mu S(\frac{1}{2}P + q)) \Gamma_H(q; P)S(\frac{1}{2}P - q)]
\]

\[f_H\] Meson Decay Constant
\[T_H\] flavor projection operator,
\[Z_2(\Lambda), Z_4(\Lambda)\] renormalization constants
\[S(p)\] dressed quark propagator
\[\Gamma_H(q; P) = F.T.\langle H|\psi(x_a)\bar{\psi}(x_b)|0\rangle\]
Bethe-Salpeter bound-state vertex amplitude.

\[
i \rho^H_\zeta \equiv \frac{-\langle qq\rangle^H_\zeta}{f_H} = Z_4 \int^\Lambda d^4q \frac{d^4q}{(2\pi)^4} \frac{1}{2} [T_H \gamma_5 S(\frac{1}{2}P + q)) \Gamma_H(q; P)S(\frac{1}{2}P - q)]
\]

**In-Hadron Condensate!**

\[
f_H m_H^2 = -\rho^H_\zeta \mathcal{M}_H \quad \mathcal{M}_H = \sum_{q \in H} m_q
\]

\[m^2_\pi \propto (m_q + m_{\bar{q}})/f_\pi \quad \text{GMOR}\]
Higher Light-Front Fock State of Pion Simulates DCSB

$$f_\pi P^+ = <0|\bar{q}\gamma^5\gamma^+q|\pi>$$

Instantaneous quark propagator contribution to $\rho_\pi$ derived from higher Fock state

$$i\rho_\pi = <0|\bar{q}\gamma^5q|\pi>$$

Higher Fock state acts like mass insertion

Roberts, Tandy, Shrock, sjb159
Gell-Mann - Oakes - Renner Relation (1968)

\[ f_\pi^2 m_\pi^2 = -2 m(\zeta) \langle \bar{q}q \rangle_0^\zeta \]

- Pion's leptonic decay constant, mass-dimensioned observable which describes rate of process \( \pi^+ \rightarrow \mu^+\nu \)
- *Vacuum quark condensate*

How is this expression modified and interpreted in a theory with confinement?
Pseudoscalar projection of pion’s Bethe-Salpeter wave-function onto the origin in configuration space – or the pseudoscalar pion-to-vacuum matrix element

\[ i \rho_\pi = -\langle 0 | \bar{q} i \gamma_5 q | \pi \rangle \]

\[ = Z_4(\zeta, \Lambda) \text{tr}_{\text{CD}} \int_0^\Lambda \frac{d^4 q}{(2\pi)^4} \gamma_5 S(q_+) \Gamma_\pi(q; P) S(q_-) \]

Rigorously defined in QCD – gauge-independent, cutoff-independent, etc.

- For arbitrary current-quark masses
- For any pseudoscalar meson
\[ f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^\zeta \]

**In-meson condensate**

Maris & Roberts

[nucl-th/9708029](https://arxiv.org/abs/nucl-th/9708029)

- Define
  \[
  -\langle \bar{q}q \rangle_\zeta^\pi = -f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m}; \zeta).
  \]

- Then, owing to the pion Goldberger-Treiman relations one derives, in the chiral limit
  
  **Chiral limit**
  
  \[
  \kappa_\pi(0; \zeta) = -\langle \bar{q}q \rangle^0_\zeta
  \]

- Namely, the so-called **vacuum quark condensate** is the **chiral-limit value** of the **in-pion condensate**
Paradigm shift: In-Hadron Condensates

Resolution

- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.

- So-called vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

- GMOR
cf.

\[ f_\pi^2 m_\pi^2 = -2 m(\zeta) \langle \bar{q}q \rangle_0^c \]

\[ f_\pi m_\pi^2 = 2 m(\zeta) \rho_\pi^c \]

QCD

Pion mass and decay constant.

Pi- and K meson Bethe-Salpeter amplitudes.

Concerning the quark condensate.

“**In-Meson Condensate**”

\[ - \langle \bar{q}q \rangle_\pi \zeta = f_\pi \langle 0|\bar{q}\gamma_5q|\pi \rangle. \]

Valid even for \( m_q \to 0 \)

\( f_\pi \) nonzero
Paradigm shift: In-Hadron Condensates

Resolution
- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.
- *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

\[ i f_\pi P_\mu = \langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle \]
\[ = Z_2(\zeta, \Lambda) \text{tr}_{CD} \int d^4q \frac{1}{(2\pi)^4} i \gamma_5 \gamma_\mu S(q_+ \Gamma_\pi(q; P)) S(q_-), \]

(5)

\[ i \rho_\pi = -\langle 0 | \bar{q} i \gamma_5 q | \pi \rangle \]
\[ = Z_4(\zeta, \Lambda) \text{tr}_{CD} \int d^4q \frac{1}{(2\pi)^4} \gamma_5 S(q_+ \Gamma_\pi(q; P)) S(q_-). \]
Paradigm shift: In-Hadron Condensates

“Void that is truly empty
solves dark energy puzzle”
Rachel Courtland, New Scientist 4th Sept. 2010

“EMPTY space may really be empty. Though quantum theory suggests that a vacuum
should be fizzing with particle activity, it turns out that this paradoxical picture of
nothingness may not be needed. A calmer view of the vacuum would also help resolve a
nagging inconsistency with dark energy, the elusive force thought to be speeding up the
expansion of the universe.”

Cosmological Constant:
✓ Putting QCD condensates back into hadrons reduces the
mismatch between experiment and theory by a factor of $10^{46}$
✓ Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model
Determinations of the vacuum Gluon Condensate

\[ \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \] [GeV\(^4\)]

\(-0.005 \pm 0.003\) from \(\tau\) decay.

\(+0.006 \pm 0.012\) from \(\tau\) decay.

\(+0.009 \pm 0.007\) from charmonium sum rules

Davier et al.

Geshkenbein, Ioffe, Zyablyuk

Ioffe, Zyablyuk

Consistent with zero vacuum condensate
Quark and Gluon condensates reside within hadrons, not vacuum

- Bound-State Dyson Schwinger Equations
- AdS/QCD
- Analogous to finite size superconductor
- Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict

R. Shrock, sjb
ArXiv:0905.1151
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
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\[(\Omega_\Lambda)_{QCD} \sim 10^{45}\]
\[(\Omega_\Lambda)_{EW} \sim 10^{56}\]
\[\Omega_\Lambda = 0.76 (expt)\]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb
"Condensates in Quantum Chromodynamics and the Cosmological Constant"
Quark and Gluon condensates reside within hadrons, not LF vacuum

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!
- Implications for cosmological constant

“Confined QCD Condensates”

LBNL November 18, 2010

Novel QCD Phenomena

Stan Brodsky, SLAC
- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Conformal Template (BLM, CSR, BFKL scale)
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict with Standard Model

Roberts, Shrock, Tandy, and sjb
Pervasive Myth in PQCD

- Renormalization Scale is Arbitrary
Measurement of the strong coupling $\alpha_S$ from the four-jet rate in $e^+e^-$ annihilation using JADE data

J.Schieck$^{1, a}$, S. Bethke$^1$, O. Biebel$^2$, S. Kluth$^1$, P.A.M. Fernández$^3$, C. Pahl$^1$, The JADE Collaboration$^b$


The theoretical uncertainty, associated with missing higher order terms in the theoretical prediction, is assessed by varying the renormalization scale factor $x_\mu$. The predictions of a complete QCD calculation would be independent of $x_\mu$, but a finite-order calculation such as that used here retains some dependence on $x_\mu$. The renormalization scale factor $x_\mu$ is set to 0.5 and two. The larger deviation from the default value of $\alpha_S$ is taken as systematic uncertainty.

$\alpha_S (M_{Z0})$ and the $\chi^2/d.o.f.$ of the fit to the four-jet rate as a function of the renormalization scale $x_\mu$ for $\sqrt{s} = 14$ GeV to 43.8 GeV. The arrows indicate the variation of the renormalization scale factor used for the determination of the systematic uncertainties.
Conventional wisdom concerning scale setting

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess
  \[ \mu_R = Q \]
  with an arbitrary range
  \[ \frac{Q}{2} < \mu_R < 2Q \]
- Factorization scale should be taken equal to renormalization scale
  \[ \mu_F = \mu_R \]

These assumptions are untrue in QED and thus they cannot be true for QCD!
**Electron-Electron Scattering in QED**

\[ M_{ee \rightarrow ee}(++;++) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

\[ \alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)} \]

**Gell Mann-Low Effective Charge**
QED Effective Charge

\[ \alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)} \]

All-orders lepton loop corrections to dressed photon propagator

\[ \alpha(t) = \alpha(t_0) \frac{1 - \Pi(t,t_0)}{1 - \Pi(t_0)} \]

Initial scale \( t_0 \) is arbitrary -- Variation gives RGE Equations

Physical renormalization scale \( t \) not arbitrary
Another Example in QED: Muonic Atoms

Another Example in QED: Muonic Atoms

\[ V(q^2) = -\frac{Z\alpha_{QED}(q^2)}{q^2} \]

\[ \mu_R^2 \equiv q^2 \]

\[ \alpha_{QED}(q^2) = \frac{\alpha_{QED}(0)}{1 - \Pi(q^2)} \]

Scale is unique: Tested to ppm

Gyulassy: Higher Order VP verified to 0.1% precision in \( \mu \) Pb
Must recover QED result using \( \alpha_{\overline{MS}}(\mu^2) \)

\[
\alpha(q^2) = \alpha(q_0^2) \frac{(1-\Pi(q_0^2))}{(1-\Pi(q^2))} \quad \text{where } \Pi(q^2 = 0) = 0
\]

\[
\Pi(q^2) = \ldots
\]

Identical QED result if

\[
\ln\left(-\frac{\mu^2}{m^2}\right) = 6 \int_0^1 d\alpha [\alpha(1-\alpha)] \ln(1 - \frac{q_0^2 \alpha(1-\alpha)}{m^2})
\]

\[
\mu^2 = q_0^2 e^{-5/3} \quad \text{at large } q_0^2
\]

q\(_0^2\): Normalization point
Electron-Positron Scattering in QED

\[ M_{e^+e^-\to e^+e^-}(s,t) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi t}{s} \alpha(s) \]

Gell Mann-Low Running Charge
sums all vacuum polarization insertions

\[ \alpha(t) = \frac{\alpha(0)}{1 - \Pi(t)} \]

Running Coupling is Complex for Timelike Argument
Electron-Electron Scattering in QED

- No renormalization scale ambiguity!

\[ \mathcal{M}_{ee \to ee}(++; +) = \frac{8\pi s}{t} \alpha(t) + \frac{8\pi s}{u} \alpha(u) \]

- Two separate physical scales: \( t, u \) = photon virtuality

- Gauge Invariant. Dressed photon propagator

- Sums all vacuum polarization, non-zero beta terms into running coupling.

- If one chooses a different scale, one can sum an infinite number of graphs -- but always recover same result! Scheme independent.

- Number of active leptons correctly set

- Analytic: reproduces correct behavior at lepton mass thresholds

- No renormalization scale ambiguity!

- Two separate physical scales.

- Gauge Invariant. Dressed photon propagator

- Sums all vacuum polarization, non-zero beta terms into running coupling.

- If one chooses a different scale, one must sum an infinite number of graphs -- but then recover same result!
\[
\lim_{N_C \to 0} \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F/C_F
\]

QCD $\rightarrow$ Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

Scale-Setting procedure for QCD must be applicable to QED
Renormalization Scale-Setting Not Ambiguous

\[ \alpha_{MS}(e^{-5/3 t}) \]
Example of Multiple BLM Scales

Angular distributions of massive quarks and leptons close to threshold.

\[ F_1 + F_2 = 1 + \frac{\alpha(s \beta^2) \pi}{4 \beta} - 2 \frac{\alpha(s e^{3/4}/4)}{\pi} \]

\[ \approx \left( 1 - 2 \frac{\alpha(s e^{3/4}/4)}{\pi} \right) \left( 1 + \frac{\alpha(s \beta^2) \pi}{4 \beta} \right) \]
On the elimination of scale ambiguities in perturbative quantum chromodynamics

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We present a new method for resolving the scheme-scale ambiguity that has plagued perturbative analyses in quantum chromodynamics (QCD) and other gauge theories. For Abelian theories the method reduces to the standard criterion that only vacuum-polarization insertions contribute to the effective coupling constant. Given a scheme, our procedure automatically determines the coupling-constant scale appropriate to a particular process. This leads to a new criterion for the convergence of perturbative expansions in QCD. We examine a number of well known reactions in QCD, and find that perturbation theory converges well for all processes other than the gluonic width of the $\gamma$. Our analysis calls into question recent determinations of the QCD coupling constant based upon $\gamma$ decay.

BLM: Choose $\mu_R$ in $\alpha_s$ to absorb all $\beta$ terms
**BLM Scale Setting**

\[ \beta_0 = 11 - \frac{2}{3} n_f \]

\[ \rho = C_0 \alpha_{\overline{MS}}(Q) \left[ 1 + \frac{\alpha_{\overline{MS}}(Q)}{\pi} \left( -\frac{3}{2} \beta_0 A_{VP} + \frac{33}{2} A_{VP} + B \right) + \cdots \right] \]

by

\[ \rho = C_0 \alpha_{\overline{MS}}(Q^*) \left[ 1 + \frac{\alpha_{\overline{MS}}(Q^*)}{\pi} C_1^* + \cdots \right], \]

where

\[ Q^* = Q \exp(3A_{VP}) , \]

\[ C_1^* = \frac{33}{2} A_{VP} + B . \]

The term \( 33A_{VP} / 2 \) in \( C_1^* \) serves to remove that part of the constant \( B \) which renormalizes the leading-order coupling. The ratio of these gluonic corrections to the light-quark corrections is fixed by \( \beta_0 = 11 - \frac{2}{3} n_f \).

**Use skeleton expansion:**

Gardi, Grunberg, Rathsman, sjb
Features of BLM Scale Setting

- All terms associated with nonzero beta function summed into running coupling
- BLM Scale $Q^*$ sets the number of active flavors
- Only $n_f$ dependence required to determine renormalization scale at NLO
- Result is scheme independent: $Q^*$ has exactly the correct dependence to compensate for change of scheme
- Result independent of starting scale
- Correct Abelian limit
- Resulting series identical to conformal series!
- Renormalon $n!$ growth of PQCD coefficients from beta function eliminated!
- In general, BLM scale depends on all invariants
Three-Jet rate in electron-positron annihilation

The scale $\mu/\sqrt{s}$ according to the BLM (dashed-dotted), PMS (dashed), FAC (full), and $\sqrt{y}$ (dotted) procedures for the three-jet rate in $e^+e^-$ annihilation, as computed by Kramer and Lampe [10]. Notice the strikingly different behavior of the BLM scale from the PMS and FAC scales at low $y$. In particular, the latter two methods predict increasing values of $\mu$ as the jet invariant mass $M < \sqrt{(ys)}$ decreases.

Other Jet Observables: Rathsman
Heavy Quark Hadroproduction

\[ p_T \neq 0 \]

\[ Q \]

\[ \bar{Q} \]

3-gluon coupling depends on 3 physical scales

\[ p_1 \]

\[ \mu_1 \]

\[ p_2 \]

\[ \mu_2 \]

\[ p_3 \]

\[ \mu_3 \]

\[ p_T \neq 0 \]

\[ \text{crossed} \]

\[ = \]

\[ + \]

\[ + \text{crossed} \]
The Renormalization Scale Problem

\[ \rho(Q^2) = C_0 + C_1 \alpha_s(\mu_R) + C_2 \alpha_s^2(\mu_R) + \cdots \]

\[ \mu_R^2 = CQ^2 \]

Is there a way to set the renormalization scale \( \mu_R \)?

What happens if there are multiple physical scales?
General Structure of the Three-Gluon Vertex

\[ \hat{\Gamma}_{\mu_1\mu_2\mu_3} = \text{Full analytic calculation, general masses, spin Pinch Scheme} \]

3 index tensor \( \hat{\Gamma}_{\mu_1\mu_2\mu_3} \) built out of \( g_{\mu\nu} \) and \( p_1, p_2, p_3 \)

with \( p_1 + p_2 + p_3 = 0 \)

14 basis tensors and form factors

Form factors of the gauge-invariant three-gluon vertex

Michael Binger* and Stanley J. Brodsky†

PHYSICAL REVIEW D 74, 054016 (2006)
Multi-scale Renormalization of the Three-Gluon Vertex

\[ \tilde{g}(p_1^2, p_2^2, p_3^2) \]

\[ g(p_1^2) \]

\[ g(p_2^2) \]

\[ g(p_3^2) \]

gauge-invariant subset of rad. cor.

coupling at each vertex absorb the rad. cor.
\[ \hat{\Gamma}_{\mu_1 \mu_2 \mu_3} = \]

\[ \mu^2 \frac{2}{R} \sim \frac{p^2_{\text{min}} p^2_{\text{med}}}{p^2_{\text{max}}} \]

H. J. Lu
Properties of the Effective Scale

\[ Q_{\text{eff}}^2(a, b, c) = Q_{\text{eff}}^2(-a, -b, -c) \]

\[ Q_{\text{eff}}^2(\lambda a, \lambda b, \lambda c) = \lambda |Q_{\text{eff}}^2(a, b, c)| \]

\[ Q_{\text{eff}}^2(a, a, a) = |a| \]

\[ Q_{\text{eff}}^2(a, -a, -a) \approx 5.54 |a| \]

\[ Q_{\text{eff}}^2(a, a, c) \approx 3.08 |c| \quad \text{for} \quad |a| >> |c| \]

\[ Q_{\text{eff}}^2(a, -a, c) \approx 22.8 |c| \quad \text{for} \quad |a| >> |c| \]

\[ Q_{\text{eff}}^2(a, b, c) \approx 22.8 \frac{|bc|}{|a|} \quad \text{for} \quad |a| >> |b|, |c| \]

Surprising dependence on Invariants
Heavy Quark Hadro-production

- Preliminary calculation using (massless) results for tree level form factor
- Very low effective scale much larger cross section than $\overline{MS}$ with scale $\mu_R = M_{Q\bar{Q}}$ or $M_Q$
- Future: repeat analysis using the full mass-dependent results and include all form factors

Expect that this approach accounts for most of the one-loop corrections
Relate Observables to Each Other

• Eliminate intermediate scheme
• No scale ambiguity
• Transitive!
• Commensurate Scale Relations
• Example: Generalized Crewther Relation
Relate Observables to Each Other

- Eliminate intermediate scheme
- No scale ambiguity
- Transitive!
- Commensurate Scale Relations
- Conformal Template
- Example: Generalized Crewther Relation

\[
R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right].
\]

\[
\int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha g_1(Q)}{\pi} \right]
\]
\[
\frac{\alpha_R(Q)}{\pi} = \frac{\alpha_{MS}(Q)}{\pi} + \left(\frac{\alpha_{MS}(Q)}{\pi}\right)^2 \left[\left(\frac{41}{8} - \frac{11}{3} \zeta_3\right) C_A - \frac{1}{8} C_F + \left(-\frac{11}{12} + \frac{2}{3} \zeta_3\right) f \right]
+ \left(\frac{\alpha_{MS}(Q)}{\pi}\right)^3 \left\{\left(\frac{90445}{2592} - \frac{2737}{108} \zeta_3 - \frac{55}{18} \zeta_5 - \frac{121}{432} \pi^2\right) C_A^2 + \left(-\frac{127}{48} - \frac{143}{12} \zeta_3 + \frac{55}{3} \zeta_5\right) C_A C_F - \frac{23}{32} C_F^2 \right. \\
\left. + \left[\left(\frac{970}{81} + \frac{224}{27} \zeta_3 + \frac{5}{9} \zeta_5 + \frac{11}{108} \pi^2\right) C_A + \left(-\frac{29}{96} + \frac{19}{6} \zeta_3 - \frac{10}{3} \zeta_5\right) C_F \right] f \\
+ \left(\frac{151}{162} - \frac{19}{27} \zeta_3 - \frac{1}{108} \pi^2\right) f^2 + \left(\frac{11}{144} - \frac{1}{6} \zeta_3\right) \frac{d^{abc} d^{abc}}{C_F d(R)} \left(\frac{\sum f Q_f}{\sum f Q_f^2}\right)^2 \right\}.
\]

\[
\frac{\alpha_{g_1}(Q)}{\pi} = \frac{\alpha_{MS}(Q)}{\pi} + \left(\frac{\alpha_{MS}(Q)}{\pi}\right)^2 \left[\frac{23}{12} C_A - \frac{7}{8} C_F - \frac{1}{3} f \right]
+ \left(\frac{\alpha_{MS}(Q)}{\pi}\right)^3 \left\{\left(\frac{5437}{648} - \frac{55}{18} \zeta_5\right) C_A^2 + \left(-\frac{1241}{432} + \frac{11}{9} \zeta_3\right) C_A C_F + \frac{1}{32} C_F^2 \right. \\
\left. + \left[\left(-\frac{3535}{1296} - \frac{1}{2} \zeta_3 + \frac{5}{9} \zeta_5\right) C_A + \left(\frac{133}{864} + \frac{5}{18} \zeta_3\right) C_F \right] f + \frac{115}{648} f^2 \right\}.
\]
\[ R_{e^+e^-}(Q^2) \equiv 3 \sum_{\text{flavors}} e_q^2 \left[ 1 + \frac{\alpha_R(Q)}{\pi} \right]. \]

\[ \int_0^1 dx \left[ g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] \equiv \frac{1}{3} \left| \frac{g_A}{g_V} \right| \left[ 1 - \frac{\alpha_g(Q)}{\pi} \right] \]

\[ \frac{\alpha_g(Q)}{\pi} = \frac{\alpha_R(Q^*)}{\pi} - \left( \frac{\alpha_R(Q^{**})}{\pi} \right)^2 + \left( \frac{\alpha_R(Q^{***})}{\pi} \right)^3 \]

**Geometric Series in Conformal QCD**

**Generalized Crewther Relation**

Lu, Kataev, Gabadadze, Sjb
Generalized Crewther Relation

\[
\left[ 1 + \frac{\alpha_R(s^*)}{\pi} \right] \left[ 1 - \frac{\alpha g_1(q^2)}{\pi} \right] = 1
\]

\[\sqrt{s^*} \simeq 0.52Q\]

Conformal relation true to all orders in perturbation theory

No radiative corrections to axial anomaly

Nonconformal terms set relative scales (BLM)

Analytic matching at quark thresholds

No renormalization scale ambiguity!
Transitivity Property of Renormalization Group

Relation of observables independent of intermediate scheme $C$
Conventional wisdom concerning scale setting

- Renormalization scale “unphysical”: No optimal physical scale
- Can ignore possibility of multiple physical scales
- Accuracy of PQCD prediction can be judged by taking arbitrary guess
  \[ \mu_R = Q \]
- with an arbitrary range
  \[ Q/2 < \mu_R < 2Q \]
- Factorization scale should be taken equal to renormalization scale
  \[ \mu_F = \mu_R \]

These assumptions are untrue in QED and thus they cannot be true for QCD!

Worse: result is scheme dependent!
QCD Myths

- Anti-Shadowing is Universal
- ISI and FSI are higher twist effects and universal
- High transverse momentum hadrons arise only from jet fragmentation -- baryon anomaly!
- Heavy quarks arise only from gluon splitting
- Renormalization scale cannot be fixed
- QCD condensates are vacuum effects
- Infrared Slavery
- Nuclei are composites of nucleons only
- Real part of DVCS arbitrary