• Fully coherent interactions between pion and nucleons.

• Emerging Di-Jets do not interact with nucleus.

\[ \mathcal{M}(A) = A \cdot \mathcal{M}(N) \]

\[ \frac{d\sigma}{dq_{t}^{2}} \propto A^{2} \quad q_{t}^{2} \sim 0 \]

\[ \sigma \propto A^{4/3} \]
**Measure pion LFWF in diffractive dijet production**

**Confirmation of color transparency**

<table>
<thead>
<tr>
<th>$k_t$ range (GeV/c)</th>
<th>$\alpha$</th>
<th>$\alpha$ (CT)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.25 &lt; k_t &lt; 1.5$</td>
<td>$1.64 \pm 0.06$</td>
<td>1.25</td>
</tr>
<tr>
<td>$1.5 &lt; k_t &lt; 2.0$</td>
<td>$1.52 \pm 0.12$</td>
<td>1.45</td>
</tr>
<tr>
<td>$2.0 &lt; k_t &lt; 2.5$</td>
<td>$1.55 \pm 0.16$</td>
<td>1.60</td>
</tr>
</tbody>
</table>

$\alpha$ (Incoh.) = $0.70 \pm 0.1$

*Conventional Glauber Theory Ruled Out!*  

*Factor of 7*
E791 Diffractive Di-Jet transverse momentum distribution

Two Components

High Transverse momentum dependence consistent with $k_T^{-6.5}$ PQCD, ERBL Evolution

Gaussian component similar to AdS/CFT HO LFWF
Narrowing of $x$ distribution at higher jet transverse momentum

$x$: distribution of diffractive dijets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ distribution amplitudes. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.

**Possibly two components: Nonperturbative (AdS/CFT) and Perturbative (ERBL)**

$$\phi(x) \propto \sqrt{x(1-x)}$$

Evolution to asymptotic distribution

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**Novel Hadron Physics**  
Stan Brodsky, SLAC & CP3
Possibly two components:
Perturbative (ERBL) + Nonperturbative (AdS/CFT)

$$\phi(x) = A_{\text{pert}}(k_\perp^2)x(1 - x) + B_{\text{nonpert}}(k_\perp^2)\sqrt{x(1 - x)}$$

Narrowing of $x$ distribution at high jet transverse momentum
Light-Front formalism links dynamics to spectroscopy

\[ L_{QCD} \rightarrow H_{LF}^{QCD} \]

Heisenberg Matrix Formulation

\[ H_{LF}^{QCD} = \sum_i \left[ \frac{m^2 + k^2}{x} \right]_i + H_{LF}^{int} \]

\( H_{LF}^{int} \): Matrix in Fock Space

\[ H_{LF}^{QCD} | \Psi_h \rangle = \mathcal{M}_h^2 | \Psi_h \rangle \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Physical gauge: \( A^+ = 0 \)
Light-Front QCD

Heisenberg Matrix Formulation

\[
H_{QCD}^{LF} |\Psi_h \rangle = M^2_h |\Psi_h \rangle
\]

D.L.C.Q: Frame-independent, No fermion doubling; Minkowski Space

D.L.C.Q: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation

**Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions**
\[
\left( M^2 - \sum_i \frac{\not{k}_i^2 + m_i^2}{x_i} \right) \begin{bmatrix}
\psi_{q\bar{q}/\pi} \\
\psi_{q\bar{g}/\pi} \\
\vdots
\end{bmatrix} = \begin{bmatrix}
\langle q\bar{q}|V|q\bar{q} \rangle \\
\langle q\bar{g}|V|q\bar{q} \rangle \\
\langle q\bar{g}|V|q\bar{g} \rangle \\
\vdots
\end{bmatrix} \begin{bmatrix}
\psi_{q\bar{q}/\pi} \\
\psi_{q\bar{g}/\pi} \\
\vdots
\end{bmatrix}
\]

\[A^+ = 0\]
Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable
  \textit{de Teramond, Deur, Shrock, Roberts, Tandy}
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi >= M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Goal:

Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions, Running coupling in IR

$\Psi_n(x_i, k_{\perp i}, \lambda_i)$

in collaboration with Guy de Teramond and Alexandre Deur

Central problem for strongly-coupled gauge theories
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension $z$

in collaboration with Guy de Teramond

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Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu\nu}, P^\mu, D, K^\mu, \]

the generators of SO(4,2)

SO(4,2) has a mathematical representation on AdS5
\textbf{AdS/CFT:} Anti-de Sitter Space / Conformal Field Theory

Maldacena:

\textbf{Map AdS}_5 \times S_5 \textbf{ to conformal } N=4 \textbf{ SUSY}

- **QCD is not conformal:** however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window:** \( \alpha_s(Q^2) \simeq \text{const at small } Q^2 \)

- **Use mathematical mapping of the conformal group \( \text{SO}(4,2) \) to AdS}_5 \text{ space}
Deur, Korsch, et al.

\[ \frac{\alpha_{s,gl}}{\pi \ JLab} \ \text{GDH limit} \]

- \text{Fit}
- \text{pQCD evol. eq.}

\[ \alpha_{s,g1} \]

- Cornwall
- Godfrey-Isgur
- Burkert-Ioffe
- Bloch et al.

\[ Q (\text{GeV}) \]

DSE gluon couplings

Bhagwat et al.

Maris-Tandy

Fischer et al.

Lattice QCD

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Nearly conformal QCD?

Define $\Gamma_{p-n}^n$ from Björkén sum,

$$\Gamma_{p-n}^n \equiv \int_0^1 dx \left( g_1^p(x, Q^2) - g_1^n(x, Q^2) \right) = \frac{1}{6} g_A \left( 1 - \frac{\alpha_s g_1}{\pi} \right)$$

$g_1$ = spin dependent structure function

Recent JLab data from EG1(2008), CLAS, and Hall A

$s$ runs only modestly at small $Q^2$

Gribov

Fig. from 0803.4119, Duer et al.
Confinement: maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{QCD}} \]

\[ \lambda < \Lambda_{QCD} \]

B-Meson

Glueon and quark propagators cutoff in IR because of color confinement

R. Shrock, sjb
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain \((x - y)^2 < \Lambda_{QCD}^{-2}\)
- All perturbative calculations regulated in IR
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation

Quark and Gluon vacuum polarization insertions decouple; IR fixed Point

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).

J. D. Bjorken, SLAC-PUB 1053
Cargese Lectures 1989

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Scale Transformations

- Isomorphism of $SO(4, 2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu$, $z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.
• Truncated AdS/CFT (Hard-Wall) model: cut-off at \( z_0 = 1/\Lambda_{QCD} \) breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) \textit{Polchinski and Strassler (2001)}.

• Smooth cutoff: introduction of a background dilaton field \( \varphi(z) \) – usual linear Regge dependence can be obtained (Soft-Wall Model) \textit{Karch, Katz, Son and Stephanov (2006)}.
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

\[
\left[ - \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta^2 = x(1 - x)b_\perp^2.\]

\[U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\]

G. de Teramond, sjb

soft wall confining potential:

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Stan Brodsky, SLAC & CP³
\( \alpha_{g_1}/\pi \) (pQCD)
\( \alpha_{g_1}/\pi \) world data
GDH limit \( \times \) \( \alpha_{F_3}/\pi \)
\( \alpha/\pi \) OPAL
\( \alpha_{g_1}/\pi \) JLab CLAS
\( \alpha_{g_1}/\pi \) Hall A/CLAS

**Legend:**
- **Dashed line:** Modified AdS
- **Solid line:** AdS
- **Light blue:** \( \alpha_{g_1}/\pi \) (pQCD)
- **Red square:** \( \alpha_{g_1}/\pi \) world data
- **Green diamond:** GDH limit \( \times \) \( \alpha_{F_3}/\pi \)
- **Brown triangle:** \( \alpha/\pi \) OPAL
- **Blue diamond:** \( \alpha_{g_1}/\pi \) JLab CLAS
- **Blue double triangle:** \( \alpha_{g_1}/\pi \) Hall A/CLAS
- **Red circle:** Lattice QCD (2004) \( \nabla \) (2007)

\begin{align*}
\alpha_{g_1}/\pi \text{ (pQCD)} & \quad \alpha_{g_1}/\pi \text{ world data} \\
\text{GDH limit} \times \alpha_{F_3}/\pi & \quad \alpha/\pi \text{ OPAL} \\
\alpha_{g_1}/\pi \text{ JLab CLAS} & \quad \alpha_{g_1}/\pi \text{ Hall A/CLAS} \\
\text{Lattice QCD (2004)} \nabla (2007) & \\
\end{align*}
Bosonic Solutions: Hard Wall Model

• Conformal metric: \( ds^2 = g_{\ell m} dx^\ell dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m} \).

• Action for massive scalar modes on AdS\(_{d+1}\):

\[
S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g_{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.
\]

• Equation of motion

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g_{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.
\]

• Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = M^2 \):

\[
\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 M^2 - (\mu R)^2 \right] \Phi(z) = 0.
\]

• Solution: \( \Phi(z) \rightarrow z^\Delta \) as \( z \rightarrow 0 \),

\[
\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z M) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4 \mu^2 R^2} \right).
\]

\[
\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4
\]
AdS Schrodinger Equation for bound state of two scalar constituents:

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} \right] \phi(z) = M^2 \phi(z)$$

$L$: light-front orbital angular momentum

Derived from variation of Action in AdS$_5$

Hard wall model: truncated space

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
Match fall-off at small $z$ to conformal twist-dimension at short distances

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi} \gamma_5 D_{\ell_1 \cdots D_{\ell_m}} \psi$ (\(\Phi_\mu = 0\) gauge). \(\Delta = 2 + L\)

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, \(\Phi(x, z_0) = 0\), given by the zeros of Bessel functions \(\beta_{\alpha, k}\): \(\mathcal{M}_{\alpha, k} = \beta_{\alpha, k} \Lambda_{QCD}\)

- Normalizable AdS modes $\Phi(z)$

\[
\begin{align*}
\mathcal{S} &= 0 \quad \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.}
\end{align*}
\]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
\textbf{Soft-Wall Model}

\[ S = \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2 \]

\textit{Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field}

\[ [z^2 \partial^2_z - (3 \mp 2\kappa^2 z^2) \, z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \, \Phi(z) = 0 \]

with \((\mu R)^2 \geq -4\).

- Equation of motion for scalar field \( \mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2) \)

- LH holography requires ‘plus dilaton’ \( \varphi = +\kappa^2 z^2 \). Lowest possible state \((\mu R)^2 = -4\)

\[ \mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2} \]

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

\textit{Massless pion}
AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action

Dilaton-Modified AdS$_5$

\[ e^{\Phi(z)} = e^{+\kappa^2 z^2} \]

Positive-sign dilaton
Quark separation increases with $L$

Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

$\Phi(z)$

Soft Wall Model

Pion mass automatically zero!

$m_q = 0$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

HCAA June 7, 2010  Novel Hadron Physics  Stan Brodsky, SLAC & CP$^3$
Higher-Spin Hadrons

• Obtain spin-$J$ mode $\Phi_{\mu_1\cdots\mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left( \frac{z}{R} \right)^{-J} \Phi(z)$$

• Substituting in the AdS scalar wave equation for $\Phi$

$$\left[ z^2 \partial_z^2 - \left( 3 - 2J - 2\kappa^2 z^2 \right) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

• Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1\cdots\mu_J} = \mathcal{M}^2 \phi_{\mu_1\cdots\mu_J}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$
Quark separation increases with $L$.

$$
\Phi(z) = \begin{cases} 
L = 3 \\
L = 2 \\
L = 1 \\
L = 0 
\end{cases}
$$

$$
\Phi(z) = \begin{cases} 
n = 0 \\
n = 1 \\
n = 2 \\
n = 3 
\end{cases}
$$

(a) $S = 1$

$$
\begin{align*}
\omega (782) & \quad \omega_3 (1670) \\
a_2 (1320) & \quad \rho (770) \\
f_2 (1270) & \quad a_4 (2040) \\
f_4 (2050) & \\
\end{align*}
$$

(b) $S = 1$

$$
\begin{align*}
\rho (1700) & \\
\rho (1450) & \\
\rho (770) & \\
\end{align*}
$$
Parent and daughter Regge trajectories for the $I = 1$ $\rho$-meson family (red) and the $I = 0$ $\omega$-meson family (black) for $\kappa = 0.54$ GeV.
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQ K_1(zQ) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \(|n\rangle\). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1} \]

where \( \tau = \Delta_n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).

High \( Q^2 \) from small \( z \sim 1/Q \)

Polchinski, Strassler

de Teramond, sjb

Dimensional Quark Counting Rules:
General result from AdS/CFT and Conformal Invariance
**Spacelike pion form factor from AdS/CFT**

\[ F_\pi(q^2) \]

**One parameter - set by pion decay constant**

- **Soft Wall: Harmonic Oscillator Confinement**
- **Hard Wall: Truncated Space Confinement**

Data Compilation
Baldini, Kloe and Volmer

de Teramond, sjb
See also: Radyushkin

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Novel Hadron Physics

Stan Brodsky, SLAC & CP³
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi^*_P(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( b = |\vec{b}_\perp| \):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, |\tilde{\psi}(x, b)|^2, \]

\[ \tilde{q}_\perp^2 = Q^2 = -q^2 \]
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[ F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0(\zeta q \sqrt{\frac{1-x}{x}}) \tilde{\rho}(x, \zeta), \]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[ \zeta = \sqrt{x(1-x)b_{\perp}^2} \]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[ \int_0^1 dx J_0(\zeta Q \sqrt{\frac{1-x}{x}}) = \zeta Q K_1(\zeta Q), \]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

\[ A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2, \]

where \( H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ) \)

- Use integral representation for \( H(Q^2, z) \)

\[ H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0(zQ \sqrt{\frac{1-x}{x}}) \]

- Write the AdS gravitational form-factor as

\[ A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0(zQ \sqrt{\frac{1-x}{x}}) |\Phi_\pi(z)|^2 \]

- Compare with gravitational form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{qq/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}, \]

Identical to LF Holography obtained from electromagnetic current

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Stan Brodsky, SLAC & CP3
Holography: Unique mapping derived from equality of LF and \( \text{AdS} \) formula for current matrix elements

\[
\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)
\]
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

\[ \left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta) \]

\[ \zeta^2 = x(1 - x)b_\perp^2. \]

\[ U(z) = \kappa^4 z^2 + 2\kappa^2(L + S - 1) \]

G. de Teramond, sjb

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Novel Hadron Physics

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Current Matrix Elements in AdS Space (SW)

• Propagation of external current inside AdS space described by the AdS wave equation

\[ \left[ z^2 \partial_z^2 - z \left( 1 + 2\kappa^2 z^2 \right) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0. \]

• Solution bulk-to-boundary propagator

\[ J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right) U \left( \frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2 \right), \]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[ \Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt. \]

• Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[ F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z). \]

• For large \( Q^2 \gg 4\kappa^2 \)

\[ J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z), \]

the external current decouples from the dilaton field.
Dressed soft-wall current bring in higher Fock states and more vector meson poles.
Form Factors in AdS/QCD

\[ F(Q^2) = \frac{1}{1 + \frac{Q^2}{\mathcal{M}_\rho^2}}, \quad N = 2, \]

\[ F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots}, \quad N = 3, \]

\[ F(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right) \left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right) \cdots \left(1 + \frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}, \quad N, \]

Positive Dilaton Background \( \exp\left(\kappa^2 z^2\right) \quad \mathcal{M}_n^2 = 4\kappa^2 \left(n + \frac{1}{2}\right) \)

\[ F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)} \quad Q^2 \rightarrow \infty \]

Constituent Counting
AdS/CFT now extensive field---apologies for all omitted references
Original 1997 Maldacena paper has 6016 citations

Calculations of form factors: “fancy”
Start from string theory, develop QCD analogs on lower dimensional branes

“Bottom-up”
Anticipate what 5D Lagrangian must be (guess), directly involving desired rho, pi, a1, ... fields and connect to matching QCD structures

EM form factors in “bottom-up” approach

Gravitational form factors in bottom-up approach

Soft-wall

Sakai & Sugimoto
Erlich et al.
Da Rold & Pomarol
Brodsky & de Teramond
Radyushkin & Grigoryan
Zainul Abidin & me
Karch, Katz, Son, and Stephanov
Batell, Gherghetta, and Sword
\[ \psi(x, \vec{b}_\perp) \ \leftrightarrow \ \phi(\zeta) \]

\[ \zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \]

\[ \psi(x, \vec{b}_\perp) = \sqrt{\frac{x(1-x)}{2\pi\zeta}} \phi(\zeta) \]

**Light-Front Holography:** Unique mapping derived from equality of LF and AdS formula for current matrix elements.
Light-Front Holography:
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = M^2 \phi(\zeta)
\]

Frame Independent

\[
\zeta^2 = x(1 - x)b^2_\perp.
\]

G. de Teramond, sbj

soft wall confining potential:

\[
U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)
\]
Derivation of the Light-Front Radial Schrödinger Equation
directly from LF QCD

\[ M^2 = \int_0^1 dx \int \frac{d^2 k_\perp}{16 \pi^3} \frac{k_\perp^2}{x(1-x)} |\psi(x, k_\perp)|^2 + \text{interactions} \]

\[ = \int_0^1 dx \int d^2 b_\perp \psi^*(x, b_\perp) \left(-\nabla_{b_\perp}^2\right) \psi(x, b_\perp) + \text{interactions.} \]

**Change variables**

\( (\zeta, \varphi), \quad \zeta = \sqrt{x(1-x)b_\perp} : \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2} \)

\[ M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \]

\[ + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

\[ = \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \]
\( H_{QED} \)

\[
(H_0 + H_{int}) |\Psi\rangle \geq E |\Psi\rangle
\]

\[
\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})
\]

\[
\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell + 1)}{r^2} + V_{\text{eff}}(r, S, \ell)\right] \psi(r) = E \psi(r)
\]

\( V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r} \)

**QED atoms: positronium and muonium**

**Coupled Fock states**

**Effective two-particle equation**

**Includes Lamb Shift, quantum corrections**

**Spherical Basis** \( r, \theta, \phi \)

**Coulomb potential**

**Bohr Spectrum**

**Semiclassical first approximation to QED**
\[
H_{QCD}^{LF} \\
\]

\[
(H_{LF}^0 + H_{LF}^I)|\Psi| \geq M^2|\Psi| \\
\]

\[
\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1 - x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp) \\
\]

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta) \\
\]

\[
U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \\
\]

**QCD Meson Spectrum**

**Coupled Fock states**

**Effective two-particle equation**

\[
\zeta^2 = x(1 - x)b_\perp^2 \\
\]

**Azimuthal Basis** \( \zeta, \phi \)

**Confining AdS/QCD potential**

**Semiclassical first approximation to QCD**
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k^2) \]

de Teramond, sjb

"Soft Wall" model

\[ \kappa = 0.375 \text{ GeV} \]

massless quarks

Note coupling \( k^2 \), \( x \)

\[ \psi_M(x, k^2) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k^2}{2\kappa^2 x(1-x)}} \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

Connection of Confinement to TMDs

HCAA June 7, 2010

Novel Hadron Physics

Stan Brodsky, SLAC & CP3
Second Moment of Pion Distribution Amplitude

\[ \langle \xi^2 \rangle = \int_{-1}^{1} d\xi \, \xi^2 \phi(\xi) \]

\[ \xi = 1 - 2x \]

\[ \langle \xi^2 \rangle_\pi = 1/5 = 0.20 \]

\[ \langle \xi^2 \rangle_\pi = 1/4 = 0.25 \]

\[ \phi_{asympt} \propto x(1 - x) \]

\[ \phi_{AdS/QCD} \propto \sqrt{x(1 - x)} \]

Lattice (I) \[ \langle \xi^2 \rangle_\pi = 0.28 \pm 0.03 \]

Lattice (II) \[ \langle \xi^2 \rangle_\pi = 0.269 \pm 0.039 \]

Donnellan et al.

Braun et al.
Baryons in Ads/CFT

• Action for massive fermionic modes on AdS$_5$:

\[
S[\bar{\Psi}, \Psi] = \int d^4x \, dz \, \sqrt{g} \, \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z)
\]

• Equation of motion:

\[
\left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0
\]

\[
\left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu_R \right] \Psi(x^\ell) = 0
\]

• Solution ($\mu_R = \nu + 1/2$)

\[
\Psi(z) = C z^{5/2} \left[ J_\nu(z\mathcal{M}) u_+ + J_{\nu+1}(z\mathcal{M}) u_- \right]
\]

• Hadron mass spectrum determined from IR boundary conditions $\psi_\pm (z = 1/\Lambda_{\text{QCD}}) = 0$

\[
\mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}}
\]

with scale independent mass ratio

• Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}, J > 1/2$, with all indices along 3+1 from $\Psi$ by shifting dimensions
Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_{\mu}(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha \Pi(\zeta) \psi(\zeta) = M \psi(\zeta),$$

where $H_{LF} = \alpha \Pi$ and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$
• Note: in the Weyl representation ($i\alpha = \gamma_5 \beta$)

\[
i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.
\]

• Baryon: twist-dimension $3 + L$ ($\nu = L + 1$)

\[
O_{3+L} = \psi D_{\ell_1} \cdots D_{\ell_q} \psi D_{\ell_{q+1}} \cdots D_{\ell_m} \psi, \quad L = \sum_{i=1}^{m} \ell_i.
\]

• Solution to Dirac eigenvalue equation with UV matching boundary conditions

\[
\psi(\zeta) = C \sqrt{\zeta} \left[ J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right].
\]

Baryonic modes propagating in AdS space have two components: orbital $L$ and $L + 1$.

• Hadronic mass spectrum determined from IR boundary conditions

\[
\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,
\]

given by

\[
\mathcal{M}_{\nu,k}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}_{\nu,k}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}},
\]

with a scale independent mass ratio.
Fig: Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The 56 trajectory corresponds to $L$ even $P = +$ states, and the 70 to $L$ odd $P = -$ states.
Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

• We write the Dirac equation

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0,\]

in terms of the matrix-valued operator \(\Pi\)

\[\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),\]

and its adjoint \(\Pi^\dagger\), with commutation relations

\[\left[ \Pi_\nu(\zeta), \Pi^\dagger_\mu(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.\]

• Solutions to the Dirac equation

\[\psi_+(\zeta) \sim z^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^{\nu}(\kappa^2 \zeta^2),\]

\[\psi_-(\zeta) \sim z^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_{n+1}^{\nu+1}(\kappa^2 \zeta^2).\]

• Eigenvalues

\[\mathcal{M}^2 = 4\kappa^2 (n + \nu + 1).\]

\[
\frac{\omega_B}{\omega_M}^2 = \frac{5}{8} \quad 4\kappa^2 \text{ for } \Delta n = 1
\]
\[
4\kappa^2 \text{ for } \Delta L = 1
\]
\[
2\kappa^2 \text{ for } \Delta S = 1
\]

\[
M^2
\]

\[
L
\]

Parent and daughter 56 Regge trajectories for the \( N \) and \( \Delta \) baryon families for \( \kappa = 0.5 \text{ GeV} \)
E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

\[
F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta)|\psi_+(\zeta)|^2,
\]
\[
F_-(Q^2) = g_- \int d\zeta J(Q, \zeta)|\psi_-(\zeta)|^2,
\]

where the effective charges \(g_+\) and \(g_-\) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \(S^z = +1/2\). The two AdS solutions \(\psi_+(\zeta)\) and \(\psi_-(\zeta)\) correspond to nucleons with \(J^z = +1/2\) and \(-1/2\).

- For \(SU(6)\) spin-flavor symmetry

\[
F_1^p(Q^2) = \int d\zeta J(Q, \zeta)|\psi_+(\zeta)|^2,
\]
\[
F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2\right],
\]

where \(F_1^p(0) = 1, \ F_1^n(0) = 0\).
• Scaling behavior for large $Q^2$: $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$  \hspace{1cm} \text{Proton } \tau = 3

 Scaling behavior for large $Q^2$: \( Q^4 F_1^m(Q^2) \rightarrow \text{constant} \)

Neutron $\tau = 3$

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

$F_2^p(Q^2)$

$\kappa = 0.49$ GeV

G. de Teramond, sjb

Harmonic Oscillator
Confinement
Normalized to anomalous moment

HCAA June 7, 2010

Novel Hadron Physics

Stan Brodsky, SLAC & CP3
String Theory

AdS/CFT

AdS/QCD

Semi-Classical QCD / Wave Equations

Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering
Regge Trajectories
QCD at the Amplitude Level

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space
Conformal behavior at short distances
+ Confinement at large distance

Boost Invariant 3+1 Light-Front Wave Equations

J = 0, 1, 1/2, 3/2 plus L
Integrable!

Holography

Hadron Spectra, Wavefunctions, Dynamics

HCAA June 7, 2010
Novel Hadron Physics
Stan Brodsky, SLAC & CP3
Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$

where the coupling $\alpha_s^{AdS}$ incorporates the non-conformal dynamics of confinement
Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point

\[ \alpha_s(Q) / \pi = e^{-Q^2/4\kappa^2} \]

\( \kappa = 0.54 \text{ GeV} \)

Deur, de Teramond, sjb
\[ \beta_{AdS}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{AdS}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2} \]
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion \( (m_q = 0) \)
- Regge Trajectories: universal slope in \( n \) and \( L \)
- Valid for all integer \( J \) & \( S \). Spectrum is independent of \( S \)
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large \( N_c \) limit
- Add quark masses to LF kinetic energy

- Systematically improvable -- diagonalize \( H_{LF} \) on AdS basis

Novel Hadron Physics

HCAA June 7, 2010

Stan Brodsky, SLAC & CP3
Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

- Hamiltonian light-front field theory within an AdS/QCD basis.

J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,

Pauli, Hornbostel, Hiller, McCartor, sjb
Use AdS/QCD basis functions

HCAA June 7, 2010  Novel Hadron Physics  168  Stan Brodsky, SLAC & CP3

### Heisenberg Equation

\[ H_{QCD}^{LC} \Psi_h = M_h^2 \Psi_h \]

### Light-Front QCD

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### Diagrams

- Diagram (a)
- Diagram (b)
- Diagram (c)
Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Collision can produce 3 collinear quarks

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

n_{active} = 6

\phi_p(x_1, x_2, x_3) \propto \Lambda^2_{QCD}

n_{eff} = 2n_{active} - 4

qq \rightarrow B\bar{q}

n_{eff} = 8

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Novel Hadron Physics

Stan Brodsky, SLAC & CP3
\[ \pi N \rightarrow \mu^+ \mu^- X \text{ at high } x_F \]

In the limit where \((1-x_F)Q^2\) is fixed as \(Q^2 \rightarrow \infty\)

Light-Front Wavefunctions from AdS/CFT

Entire pion wf contributes to hard process

Virtual photon is longitudinally polarized

"Direct" Subprocess

Berger, sjb
Khoze, Brandenburg, Muller, sjb
Hoyer, Vanttinen

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Novel Hadron Physics

Stan Brodsky, SLAC & CP3
**π⁻N → μ⁺μ⁻X at 80 GeV/c**

\[
\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2\theta + \rho \sin2\theta \cos\phi + \omega \sin^2\theta \cos2\phi.
\]

\[
\frac{d^2\sigma}{dx_\pi d\cos\theta} \propto x_\pi \left[(1-x_\pi)^2(1+\cos^2\theta) + \frac{4}{9} \frac{\langle k_F^2 \rangle}{M^2} \sin^2\theta\right]
\]

\[
\langle k_F^2 \rangle = 0.62 \pm 0.16 \text{ GeV}^2/c^2
\]

\[
Q^2 = M^2
\]

**Dramatic change in angular distribution at large x_F**

**Example of a higher-twist direct subprocess**

---

**Chicago-Princeton Collaboration**

*Phys.Rev.Lett.55:2649,1985*

Stan Brodsky, SLAC & CP³
Crucial Test of Leading-Twist QCD: Scaling at fixed $x_T$

$$x_T = \frac{2p_T}{\sqrt{s}}$$

$$E \frac{d\sigma}{d^3p} (pN \to \pi X) = \frac{F(x_T, \theta_{CM})}{p_T^{n_{eff}}}$$

Parton model: $n_{eff} = 4$

As fundamental as Bjorken scaling in DIS

Conformal scaling: $n_{eff} = 2n_{active} - 4$
$pp \rightarrow \gamma X$

$E \frac{d\sigma}{d^3p} (pp \rightarrow \gamma X) = \frac{F(\theta_{cm}, x_T)}{p_T^4}$

$gu \rightarrow \gamma u$

$n_{active} = 4$

$n_{eff} = 2n_{active} - 4$

$n_{eff} = 4$
\[ \sqrt{s^n} E \frac{d\sigma}{d^3p} (pp \to \gamma X) \] at fixed \( x_T \)

\( x_T \)-scaling of direct photon production is consistent with PQCD
Leading-Twist Contribution to Hadron Production

\[ \frac{d\sigma}{d^3p/E} = \alpha_s^2 \frac{F(x_\perp, y)}{p_{\perp}^4} \]

Parton model and Conformal Scaling:

\[ G_{q/p}(x_1, p_{\perp}^2) \quad G_{q/p}(x_2, p_{\perp}^2) \]

\[ D_{\pi/q}(z, p_{\perp}^2) \]

HCAA June 7, 2010

Novel Hadron Physics

Stan Brodsky, SLAC & CP3
QCD prediction: Modification of power fall-off due to DGLAP evolution and the Running Coupling

\[ \frac{d\sigma}{d^3p/E} = \frac{F(x_\perp, y)}{p_\perp^n(x_\perp)} \]

Key test of PQCD: power-law fall-off at fixed \( x_T \)

\[ 5 < p_\perp < 20 \text{ GeV} \]

\[ 70 \text{ GeV} < \sqrt{s} < 4 \text{ TeV} \]
\[ [\sqrt{s}]^n \frac{d\sigma}{d^3p/E} (pp \rightarrow \pi^0 X) \text{ at fixed } x_T = \frac{2p_T}{\sqrt{s}} \]
$$E \frac{d\sigma}{d^3p}(pp \to HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_T^{n_{eff}}}$$

\begin{align*}
\sqrt{s}=38.8/31.6 \text{ GeV E706} \\
\sqrt{s}=62.4/22.4 \text{ GeV PHENIX/FNAL} \\
\sqrt{s}=62.8/52.7 \text{ GeV R806} \\
\sqrt{s}=52.7/30.6 \text{ GeV R806} \\
\sqrt{s}=200/62.4 \text{ GeV PHENIX} \\
\sqrt{s}=500/200 \text{ GeV UA1} \\
\sqrt{s}=900/200 \text{ GeV UA1} \\
\sqrt{s}=1800/630 \text{ GeV CDF} \\
\end{align*}
Significant increase of the hadron $n^{\exp}$ with $x_{\perp}$

- $n^{\exp} \sim 8$ at large $x_{\perp}$

Huge contrast with photons and jets!

- $n^{\exp}$ constant and slight above 4 at all $x_{\perp}$
Direct Contribution to Hadron Production

\[ \phi_\pi(x, p_{\perp}^2) \propto f_\pi \]

\[
\frac{d\sigma}{d^3p/E} = \alpha_s^3 f_\pi^2 \frac{F(x_{\perp}, y)}{p_{\perp}^6} \]

No Fragmentation Function
Direct Proton Production

$E \frac{d\sigma}{d^3p}(p\ p \rightarrow p\ X) \sim \frac{F(x_\perp, y_{cm})}{p_\perp^8}$

$n_{active} = 6$

Explains “Baryon anomaly” at RHIC

Sickles, sjb
Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Collision can produce 3 collinear quarks

Bjorken Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive
Sickles; sjb

Small color-singlet
Color Transparent
Minimal same-side energy

Baryon anomaly

HCAA June 7, 2010

Novel Hadron Physics

Stan Brodsky, SLAC & CP3
Chiral Symmetry Breaking in AdS/QCD

- Chiral symmetry breaking effect in AdS/QCD depends on weighted \( z^2 \) distribution, not constant condensate

\[
\delta M^2 = -2m_q < \bar{\psi} \psi > \times \int dz \ \phi^2 (z) z^2
\]

- \( z^2 \) weighting consistent with higher Fock states at periphery of hadron wavefunction

- AdS/QCD: confined condensate

- Suggests “In-Hadron” Condensates

Erlich et al.
de Teramond, Shrock, sjb
Chiral magnetism (or magneto-hadrochironics)

Aharon Casher and Leonard Susskind

Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.\textsuperscript{1} Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.\textsuperscript{2} A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.\textsuperscript{3}
Simple physical argument for “in-hadron” condensate

Use Dyson-Schwinger Equation for bound-state quark propagator: find confined condensate

\[ < B|\bar{q}q|B > \text{ not } < 0|\bar{q}q|0 > \]
Lesson from QED and Lamb Shift:

maximum wavelength of bound quarks and gluons

\[ k > \frac{1}{\Lambda_{QCD}} \]

\[ \lambda < \Lambda_{QCD} \]

Use Dyson-Schwinger Equation for bound-state quark propagator:
find confined condensate

\[ < \bar{b} | \bar{q} q | \bar{b} > \text{ not } < 0 | \bar{q} q | 0 > \]

B-Meson

Shrock and sjb

Roberts et al.
Higher Light-Front Fock State of Pion Simulates DCSB

\[ f_\pi P^+ = < 0 | \bar{q} \gamma^5 \gamma^+ q | \pi > \]

Instantaneous quark propagator contribution to \( \pi \) derived from higher Fock state

\[ i \rho_\pi = < 0 | \bar{q} \gamma^5 q | \pi > \]

Higher Fock state acts like mass insertion

Roberts, Tandy, Shrock, sjb
We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
Quark and Gluon condensates reside within hadrons, not vacuum

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Analogous to finite size superconductor**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

R. Shrock, sjb
PNAS
ArXiv:0905.1151

Casher and Susskind   Maris, Roberts, Tandy   Shrock and sjb
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

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\[
\left( \Omega_\Lambda \right)_{QCD} \sim 10^{45} \quad \Omega_\Lambda = 0.76 (\text{expt})
\]

\[
\left( \Omega_\Lambda \right)_{EW} \sim 10^{56}
\]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not vacuum!

R. Shrock, sjb
“Condensates in Quantum Chromodynamics and the Cosmological Constant.”
Quark and Gluon condensates reside within hadrons, not LF vacuum

- **Bound-State Dyson-Schwinger Equations**

- **Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs**

- **Finite size phase transition - infinite # Fock constituents**

- **AdS/QCD Description -- CSB is in-hadron Effect**

- **Analogous to finite-size superconductor!**

- **Phase change observed at RHIC within a single-nucleus-nucleus collisions-- quark gluon plasma!**

- **Implications for cosmological constant**

“Confined QCD Condensates”

HCAA June 7, 2010

Novel Hadron Physics

Stan Brodsky, SLAC & CP³

Maris, Roberts, Tandy

Casher Susskind

Shrock, sjb
Determinations of the vacuum Gluon Condensate

\[ < 0 \left| \frac{\alpha_s}{\pi} G^2 \right| 0 > [\text{GeV}^4] \]

\[-0.005 \pm 0.003 \text{ from } \tau \text{ decay.} \]
\[+0.006 \pm 0.012 \text{ from } \tau \text{ decay.} \]
\[+0.009 \pm 0.007 \text{ from charmonium sum rules} \]

Davier et al.
Geshkenbein, Ioffe, Zyablyuk
Ioffe, Zyablyuk

Consistent with zero vacuum condensate
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite $N_c = 3$: Baryons built on 3 quarks -- Large $N_c$ limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD at all scales
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
\begin{align*}
H_{\text{QCD}}^{LF} & \\
(H_{LF}^0 + H_{LF}^I)|\Psi| & \geq M^2|\Psi| \\
\left[\begin{array}{c}
\frac{\vec{k}_\perp^2 + m^2}{x(1 - x)} + V_{\text{eff}}^{LF} \\
\end{array}\right] \psi_{LF}(x, \vec{k}_\perp) & = M^2 \psi_{LF}(x, \vec{k}_\perp) \\
\left[\begin{array}{c}
-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \\
\end{array}\right] \psi_{LF}(\zeta) & = M^2 \psi_{LF}(\zeta) \\
U(\zeta, S, L) & = \kappa^2 \zeta^2 + \kappa^2(L + S - 1/2) \\
\end{align*}

QCD Meson Spectrum

Coupled Fock states

Effective two-particle equation

\[ \zeta^2 = x(1 - x)b_\perp^2 \]

Azimuthal Basis \( \zeta, \phi \)

Confining AdS/QCD potential

Semiclassical first approximation to QCD
An analytic first approximation to QCD
AdS/QCD + Light-Front Holography

• As Simple as Schrödinger Theory in Atomic Physics
• LF radial variable $\xi$ conjugate to invariant mass squared
• Relativistic, Frame-Independent, Color-Confining
• QCD Coupling at all scales: Essential for Gauge Link phenomena
• Hadron Spectroscopy and Dynamics from one parameter $\kappa$
• Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules
• Insight into QCD Condensates: Zero cosmological constant!
• Systematically improvable with DLCQ Methods
QCD and the LF Hadron Wavefunctions

- AdS/QCD: Light-Front Holography, LF Schrödinger Eqn
- Heavy Quark Fock States: Intrinsic Charm
- Coordinate space representation
- Quark & Flavor Structure
- J=0 Fixed Pole: DVCS, GPDs, TMDs, LF Overlap, incl ERBL
- Initial and Final State Rescattering: DDIS, DDIS, T-Odd
- Non-Universal Antishadowing
- Baryon Excitations
- Gluonic properties: DGLAP
- Orbital Angular Momentum Spin, Chiral Properties Crewther Relation
- Hard Exclusive Amplitudes: Form Factors Counting Rules
- Nuclear Modifications: Baryon Anomaly Color Transparency
- Distribution amplitude: ERBL Evolution
  \[ \phi_p(x_1, x_2, Q^2) \]
- Baryon Decay
- Hadronization at Amplitude Level
Single-spin asymmetries

$\mathbf{Pseudo-T-Odd}$

$\mathbf{Leading~Twist}$

$\mathbf{Sivers~Effect}$

$\mathbf{Hwang, Schmidt, sjb}$

$\mathbf{Collins, Burkardt}$

$\mathbf{Ji, Yuan}$

$\mathbf{QCD S- and P-Coulomb Phases}$

$\mathbf{--Wilson~Line}$

$\mathbf{Leading-Twist}$

$\mathbf{Rescattering}$

$\mathbf{Violates~pQCD}$

$\mathbf{Factorization!}$

$\mathbf{HCAA June 7, 2010}$

$\mathbf{Novel Hadron Physics}$

$\mathbf{Stan Brodsky, SLAC & CP3}$
• Color Confinement: Maximum Wavelength of Quark and Gluons
• Conformal symmetry of QCD coupling in IR
• Conformal Template (BLM, CSR, BFKL scale)
• Motivation for AdS/QCD
• QCD Condensates inside of hadronic LFWFs
• Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
• Simple physical solution to cosmological constant conflict with Standard Model

Robert, Shrock, Tandy, and sjb
A Theory of Everything Takes Place

String theorists have broken an impasse and may be on their way to converting this mathematical structure -- physicists’ best hope for unifying gravity and quantum theory -- into a single coherent theory.

I thought I had discovered the Theory of Everything
But everything canceled out!
The Novel World of Quarks and Gluons

Thanks to Francesco Sannino, CP³-Origins, SDU, and the HCAA for hosting my visit!

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Particle Physics & Origin of Mass