Anomalous effect from Double ISI in Massive Lepton Production

- Leading Twist, valence quark dominated
- Violates Lam-Tung Relation!
- Not obtained from standard PQCD subprocess analysis
- Normalized to the square of the single spin asymmetry in semi-inclusive DIS
- No polarization required
- Challenge to standard picture of PQCD Factorization
Physics of Rescattering

• Diffractive DIS: New Insights into Final State Interactions in QCD

• Origin of Hard Pomeron

• Structure Functions not Probability Distributions!

• T-odd single-spin asymmetries,

• Nuclear Shadowing, Non-Universal Antishadowing

• Diffractive dijets/ trijets, doubly diffractive Higgs

• Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon
Problem for factorization when both ISI and FSI occur
Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions


**Implications for QCD at the LHC**

The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.
Light-Front Wave Functions in QCD

• Hadronic bound state expanded in n-particle Fock eigenstates $|\psi_h\rangle = \sum_n |\psi_{n/h} n\rangle$: the LF Hamiltonian $H_{LF} = P^2 = P^+ P^- - P^2_\perp$, $H_{LF} |P\rangle = M^2 |P\rangle$, at fixed LF time $\tau = t + z/c$ (Dirac '49; Pauli and Pinsky, sjb Phys. Rept. 1988).

• Fock components

$\psi_{n/h}(x_i, k_{\perp i}) = \langle n; x_i, k_{\perp i}, |\psi_h(P^+, P_\perp)\rangle$, frame independent and encode hadron properties in high momentum-transfer collisions.

• Momentum fraction $x_i = k_{i}^+ / P^+$ and $k_{\perp i}$ are the relative coordinates of parton $i$ in Fock-state $n$

$$\sum_{i=1}^{n} x_i = 1, \quad \sum_{i=1}^{n} k_{\perp i} = 0.$$

• Define transverse position coordinates $x_i r_{\perp i} = x_i R_\perp + b_{\perp i}$

$$\sum_{i=1}^{n} b_{\perp i} = 0, \quad \sum_{i=1}^{n} x_i r_{\perp i} = R_\perp.$$
Light-Front QCD

Heisenberg Matrix Formulation

Physical gauge: $A^+ = 0$

$L^{QCD} \rightarrow H^{QCD}_{LF}$

$H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k^2_\perp}{x} \right]_i + H^{int}_{LF}$

$H^{int}_{LF}$: Matrix in Fock Space

$H^{QCD}_{LF} |\Psi_h> = M^2_h |\Psi_h>$

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

DLCQ: Periodic BC in $x^-$. Discrete $k^+$; frame-independent truncation
LIGHT-FRONT SCHRODINGER EQUATION

\[
\left( M^2_\pi - \sum_i \frac{k^2_i}{x_i} \frac{m_i^2}{x_i} \right) \begin{pmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{g}/\pi} \\ \vdots \end{pmatrix} = \begin{pmatrix} \langle q\bar{q} | V | q\bar{q} \rangle \\ \langle q\bar{g} | V | q\bar{g} \rangle \\ \vdots \end{pmatrix} \begin{pmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{g}/\pi} \\ \vdots \end{pmatrix}
\]

\[
A^+ = 0
\]

G.P. Lepage, sjb
**Light-Front QCD**

**Heisenberg Matrix Formulation**

\[ H_{LF}^{QCD} \Psi_h \geq M_h^2 \Psi_h \]

**Discretized Light-Cone Quantization**

**Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions**

**DLCQ: Frame-independent, No-fermion doubling; Minkowski Space**

DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation
Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

in collaboration with Guy de Teramond

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AdS/QCD & Novel Phenomena

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Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrodinger Theory for Atomic Physics

- AdS/QCD Light-Front Holography

- Hadronic Spectra and Light-Front Wavefunctions
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu \nu}, P^\mu, D, K^\mu, \]

the generators of \( SO(4,2) \)

\( SO(4,2) \) has a mathematical representation on AdS5
Conformal symmetry: Template for QCD

- Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses
  
  Frishman, Lepage, Mackenzie, Sachrajda, sjb
  E. Gardi, V. Braun et al;

- Eigensolutions of ERBL evolution equation for distribution amplitudes

- Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation

- Fix Renormalization Scale (BLM, Effective Charges)
  Gardi, Grunberg, Rathsman, Gabadadze, Kataev, Lepage, Lu, Mackenzie, sjb

- Use AdS/CFT

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Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \]

\[ x^\mu \rightarrow \lambda x^\mu, \quad z \rightarrow \lambda z, \]

maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

\[ x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z. \]

\[ x^2 = x_\mu x^\mu: \text{invariant separation between quarks} \]

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
We will consider both holographic models

- Truncated AdS/CFT (Hard-Wall) model: cut-off at \( z_0 = 1/\Lambda_{QCD} \) breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field \( \varphi(z) \) – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
AdS/CFT: Anti-de Sitter Space / Conformal Field Theory

Maldacena:

Map $AdS_5 \times S_5$ to conformal $N=4$ SUSY

- **QCD is not conformal**: however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window**: $\alpha_s(Q^2) \simeq \text{const at small } Q^2$

- **Use mathematical mapping of the conformal group** $SO(4,2)$ to $AdS_5$ space
Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

\[ \Gamma_{b_j}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s g_1(Q^2)}{\pi} \right] \]

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**Figure:** Graph showing the effective charge \( \Gamma_{b_j}^{p-n}(Q^2) \) as a function of \( Q^2 \). The graph includes data points from experiments such as JLab, CLAS, and OPAL, along with theoretical predictions from pQCD evolution equations and the GDH limit. The IR conformal window is indicated on the graph.
Deur, Korsch, et al.

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\[ \frac{\alpha_s(g_1/\pi)}{\alpha_s(g_2/\pi)} \]

\[ \frac{\alpha_s}{\pi} \]

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**IR Conformal Window for QCD?**

- **Dyson-Schwinger Analysis**: QCD gluon coupling has an IR Fixed Point.

- **Evidence from Lattice Gauge Theory**

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**.

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small Q^2**.  
  
  $\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2}$  
  
  $Q^2 << 4m^2$  
  
  ![Diagram](image)

- **Justifies application of AdS/CFT in strong-coupling conformal window**

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**AdS/QCD & Novel Phenomena**

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QED One-Loop Vacuum Polarization

\[ \Pi(Q^2) = \frac{\alpha(0)}{3\pi} \left[ \frac{5}{3} - \frac{4m^2}{Q^2} - (1 - \frac{2m^2}{Q^2}) \sqrt{1 + \frac{4m^2}{Q^2}} \log \frac{1 + \sqrt{1 + \frac{4m^2}{Q^2}}}{1 - \sqrt{1 + \frac{4m^2}{Q^2}}} \right] \]

\[ \Pi(Q^2) = \frac{\alpha(0) \log Q^2}{3\pi m^2} \quad Q^2 >> 4M^2 \]

\[ \beta = \frac{d(\alpha/4\pi)}{d \log Q^2} = \frac{4}{3} (\frac{\alpha}{4\pi})^2 n_{\ell} > 0 \]

\[ \Pi(Q^2) = \frac{\alpha(0) Q^2}{15\pi m^2} \quad Q^2 << 4M^2 \]  

Serber-Uehling

\[ \beta \propto \frac{Q^2}{m^2} \quad \text{vanishes at small momentum transfer} \]
• Polchinski & Strassler: AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation

• **Goal**: Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances


• Karch, Katz, Son, Stephanov: Soft-Wall Model -- Linear Confinement

• Mapping of AdS amplitudes to 3+1 Light-Front equations, wavefunctions

• Use AdS/CFT wavefunctions as expansion basis for diagonalizing $H_{QCD}^{LF}$; variational methods
AdS/CFT

• Use mapping of conformal group SO(4,2) to AdS5

• Scale Transformations represented by wavefunction $\psi(z)$ in 5th dimension

\[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

• Match solutions at small $z$ to conformal dimension of hadron wavefunction at short distances $\psi(z) \sim z^\Delta$ at $z \rightarrow 0$

• Hard wall model: Confinement at large distances and conformal symmetry in interior

• Truncated space simulates “bag” boundary conditions

\[ 0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
- Physical AdS modes $\Phi_P(x, z) \sim e^{-iP\cdot x} \Phi(z)$ are plane waves along the Poincaré coordinates with four-momentum $P^\mu$ and hadronic invariant mass states $P_\mu P^\mu = M^2$.

- For small-$z$ $\Phi(z) \sim z^\Delta$. The scaling dimension $\Delta$ of a normalizable string mode, is the same dimension of the interpolating operator $\mathcal{O}$ which creates a hadron out of the vacuum: $\langle P|\mathcal{O}|0 \rangle \neq 0$. 

\[ \Delta = 2 + L \]

Confinement in the 5th dimension

Twist dimension of meson

Identify hadron by its interpolating operator at $z \rightarrow 0$
Bosonic Solutions: Hard Wall Model

- Conformal metric:  
  \[ ds^2 = g_{\ell m} dx^\ell dx^m . \quad x^\ell = (x^\mu, z) , \quad g_{\ell m} \to \left( \frac{R^2}{z^2} \right) \eta_{\ell m} . \]

- Action for massive scalar modes on AdS_{d+1}:
  \[
  S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g_{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right] , \quad \sqrt{g} \to (R/z)^{d+1} .
  \]

- Equation of motion
  \[
  \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g_{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0 .
  \]

- Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) , \quad P_\mu P^\mu = \mathcal{M}^2 :\)
  \[
  \left[ z^2 \partial_z^2 - (d - 1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0 .
  \]

- Solution: \( \Phi(z) \to z^\Delta \) as \( z \to 0 ,\)
  \[
  \Phi(z) = C z^{d/2} J_{\Delta - d/2} (z \mathcal{M}) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right) .
  \]

  \[\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4\]
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrödinger Equation for bound state of two scalar constituents:**

$$\left[ -\frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)$$

$$V(z) = -\frac{1 - 4L^2}{4z^2}$$

Derived from variation of Action in AdS$_5$

**Hard wall model: truncated space**

$$\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.$$
Match fall-off at small $z$ to conformal twist-dimension at short distances

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \bar{\psi} \gamma_5 D_{\ell_1} \ldots D_{\ell_m} \psi$ ($\Phi_\mu = 0$ gauge). $\Delta = 2 + L$

- 4-$d$ mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_0) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[
S = 0 \quad \text{Meson orbital and radial AdS modes for } \Lambda_{QCD} = 0.32 \text{ GeV.}
\]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
Let $\Phi(z) = z^{3/2} \phi(z)$

**AdS Schrodinger Equation for bound state of two scalar constituents:**

$$\left[ -\frac{d^2}{dz^2} + V(z) \right] \phi(z) = M^2 \phi(z)$$

**Hard wall model:** truncated space

$$V(z) = -\frac{1-4L^2}{4z^2} \quad \phi(z = z_0 = 1/\Lambda_0) = 0$$

**Soft wall model:** Harmonic oscillator confinement

$$V(z) = -\frac{1-4L^2}{4z^2} + \kappa^4 z^2$$

**Derived from variation of Action in AdS$_5$**
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[
\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1)\right] \phi_S(z) = M^2 \phi_S(z)
\]

with eigenvalues \( M^2 = 2\kappa^2(2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube):

\[
M_n^2(L) = 2\pi\sigma(n + L + 1/2).
\]

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \) GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
\[ \alpha(t) \approx \frac{1}{2} + 0.9t \]

**AdS/QCD Soft Wall Model** -- Reproduces Linear Regge Trajectories
Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQ K_1(zQ) \]

\[ F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

High \( Q^2 \) from small \( z \sim 1/Q \)

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi^{(n)} \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau - 1} \]

where \( \tau = \Delta_n - \sigma_n \), \( \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).
Current Matrix Elements in AdS Space (HW)

- Hadronic matrix element for EM coupling with string mode $\Phi(x^\ell), \ x^\ell = (x^\mu, z)$

\[ ig_5 \int d^4x \ dz \sqrt{g} \ A^\ell(x, z) \Phi^*_P(x, z) \left( \partial_\ell \Phi_P(x, z) \right). \]

- Electromagnetic probe polarized along Minkowski coordinates $(Q^2 = -q^2 > 0)$

\[ A(x, z)_\mu = \epsilon_\mu e^{-iQ \cdot x} J(Q, z), \quad A_z = 0. \]

- Propagation of external current inside AdS space described by the AdS wave equation

\[ \left[ z^2 \partial_z^2 - z \partial_z - z^2 Q^2 \right] J(Q, z) = 0, \]

subject to boundary conditions $J(Q = 0, z) = J(Q, z = 0) = 1$.

- Solution

\[ J(Q, z) = zQ K_1(zQ). \]

- Substitute hadronic modes $\Phi(x, z)$ in the AdS EM matrix element

\[ \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z), \quad \Phi(z) \to z^\Delta, \quad z \to 0. \]
Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[
[z^2 \partial^2_z - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2] J_\kappa(Q, z) = 0.
\]

- Solution bulk-to-boundary propagator

\[
J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),
\]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[
\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt}t^{a-1}(1 + t)^{b-a-1}dt.
\]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
\]

- For large \( Q^2 \gg 4\kappa^2 \)

\[
J_\kappa(Q, z) \rightarrow zQK_1(zQ) = J(Q, z),
\]

the external current decouples from the dilaton field.
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

- Soft Wall: Harmonic Oscillator Confinement
- Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

Data Compilation
Baldini, Kloe and Volmer

de Teramond, sjb
See also: Radyushkin

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Constituent Counting Rules

\[ \frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s[n_{tot} - 2]} \quad s = E_{cm}^2 \]

\[ F_H(Q^2) \sim \left[ \frac{1}{Q^2} \right]^{n_H - 1} \]

\[ n_{tot} = n_A + n_B + n_C + n_D \]

Fixed \( t/s \) or \( \cos \theta_{cm} \)

Conformal symmetry and PQCD predict leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

Characteristic scale of QCD: 300 MeV

Many new J-PARC, GSI, J-Lab, Belle, Babar tests

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Phenomenological success of dimensional scaling laws for exclusive processes

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^{n-2}}, \quad n = n_A + n_B + n_C + n_D, \]

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies

Farrar and sjb (1973); Matveev et al. (1973).

Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space

Conformal Invariance:

$$\frac{d\sigma}{dt} (\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7}$$
**Quark-Counting:**

\[ \frac{d\sigma}{dt}(pp \rightarrow pp) = \frac{F(\theta_{CM})}{s^{10}} \]

\[ n = 4 \times 3 - 2 = 10 \]

**Best Fit**

\[ n = 9.7 \pm 0.5 \]

Reflects underlying conformal scale-free interactions

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Deuteron Photodisintegration

\[ s^{n_{tot}} \frac{-2d\sigma}{dt} (A + B \rightarrow C + D) = F_{A+B\rightarrow C+D}(\theta_{CM}) \]

\[ s^{11} \frac{d\sigma}{dt} (\gamma d \rightarrow np) = F(\theta_{CM}) \]

\[ n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11 \]

Reflects conformal invariance
Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension $\tau$, $\Phi_\tau$ in the SW model

$$F(Q^2) = \frac{\Gamma \left( 1 + \frac{Q^2}{4\kappa^2} \right)}{\Gamma \left( \tau + \frac{Q^2}{4\kappa^2} \right)}.$$ 

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) \begin{cases} \frac{1}{1 + \frac{Q^2}{4\kappa^2}}, & N = 2, \\ \frac{2}{(1 + \frac{Q^2}{4\kappa^2})(2 + \frac{Q^2}{4\kappa^2})}, & N = 3, \\ \ldots \\ \frac{(N - 1)!}{(1 + \frac{Q^2}{4\kappa^2})(2 + \frac{Q^2}{4\kappa^2}) \ldots (N - 1 + \frac{Q^2}{4\kappa^2})}, & N. \end{cases}$$

- For large $Q^2$:

$$F(Q^2) \to (N - 1)! \left[ \frac{4\kappa^2}{Q^2} \right]^{(N-1)}.$$
• Analytical continuation to time-like region \( q^2 \rightarrow -q^2 \) \( M_\rho = 2\kappa = 750 \) MeV

• Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

Space and time-like pion form factor for \( \kappa = 0.375 \) GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 k_{\perp}}{16\pi^3} \psi^*_P(x, k_{\perp} - xq_{\perp}) \psi_P(x, k_{\perp}). \]

- Fourier transform to impact parameter space \( \vec{b}_{\perp} \)

\[ \psi(x, k_{\perp}) = \sqrt{4\pi} \int d^2 b_{\perp} e^{i\vec{b}_{\perp} \cdot \vec{k}_{\perp}} \tilde{\psi}(x, \vec{b}_{\perp}) \]

- Find \( b = |\vec{b}_{\perp}| \):

\[ F(q^2) = \int_0^1 dx \int d^2 b_{\perp} e^{ix\vec{b}_{\perp} \cdot \vec{q}_{\perp}} |\tilde{\psi}(x, b)|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^{\infty} b \, db \, J_0(bqx) \, |\tilde{\psi}(x, b)|^2, \]

\[ \text{Soper} \]
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[ F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta), \]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[ \zeta = \sqrt{\frac{x}{1-x}} \left| \sum_{j=1}^{n-1} x_j b_{\perp j} \right|. \]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[ \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q), \]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2, \]

where \( J(Q^2, z) = zQK_1(zQ) \).

Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0(\zeta Q \sqrt{\frac{1-x}{x}}) \]

Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0(\zeta Q \sqrt{\frac{1-x}{x}}) |\Phi_{\pi^+}(z)|^2 \]

Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4} \]

with \( \zeta = z, \ 0 \leq \zeta \leq \Lambda_{\text{QCD}} \)
Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements.
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

\[
\left[ -\frac{d^2}{d\zeta^2} + V(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta^2 = x(1-x)b^2_\perp.\]

Effective conformal potential:

\[V(\zeta) = -\frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2.\]

Confining potential:

G. de Teramond, sjb

Edinburgh
August 18, 2008

AdS/QCD & Novel Phenomena

Stan Brodsky
SLAC & IPPP
Prediction from AdS/CFT: Meson LFWF

\[
\psi_M(x, k^2_\perp)
\]

"Soft Wall" model

de Teramond, sjb

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AdS/QCD & Novel Phenomena

Stan Brodsky
SLAC & IPPP
Prediction from AdS/CFT: Meson LFWF

Note coupling $k_{\perp}^2, x$

$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

Connection of Confinement to TMDs

de Teramond, sjb

$k = 0.375$ GeV

massless quarks

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Hadron Distribution Amplitudes

\[ \phi_H(x_i, Q) \]

\[ \sum_i x_i = 1 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[ \phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp) \]

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Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman
Lepage, sjb
Braun, Gardi
\[ \phi_{\text{asympt}} \sim x(1 - x) \]

**AdS/CFT:**

\[ \phi(x, Q_0) \propto \sqrt{x(1 - x)} \]

Increases PQCD leading twist prediction $F_\pi(Q^2)$ by factor $16/9$
\[ F_\pi(Q^2) = \int_0^1 dx \phi_\pi(x) \int_0^1 dy \phi_\pi(y) \frac{16\pi C_F \alpha_V(Q_V)}{(1-x)(1-y)Q^2} \]

**AdS/CFT:**
Increases PQCD leading twist prediction for \( F_\pi(Q^2) \) by factor 16/9
Example: Pion LFWF

- Two parton LFWF bound state:

\[
\tilde{\psi}_{\bar{q}q/\pi}^{HW}(x, b_\perp) = \frac{\Lambda_{\text{QCD}} \sqrt{x(1-x)}}{\sqrt{\pi} J_{1+L}(\beta_L, \kappa)} J_L \left( \sqrt{x(1-x)} |b_\perp| \beta_L, \kappa \Lambda_{\text{QCD}} \right) \theta \left( b_\perp^2 \leq \frac{\Lambda_{\text{QCD}}^2}{x(1-x)} \right),
\]

\[
\tilde{\psi}_{\bar{q}q/\pi}^{SW}(x, b_\perp) = \kappa^{L+1} \sqrt{\frac{2n!}{(n+L)!}} \left[ x(1-x) \right]^{\frac{1}{2}+L} |b_\perp|^L e^{-\frac{1}{2} \kappa^2 \frac{1}{2} x(1-x)} b_\perp^2 L_n^L \left( \kappa^2 x(1-x) b_\perp^2 \right).
\]

Fig: Ground state pion LFWF in impact space. (a) HW model \( \Lambda_{\text{QCD}} = 0.32 \) GeV, (b) SW model \( \kappa = 0.375 \) GeV.