Factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions


The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.
8 leading-twist spin-$k_\perp$ dependent distribution functions

$h_q(x, k_T^2)$

$f_1^q(x, k_T^2)$

$f_{1T}^q(x, k_T^2)$

$h_{1T}^q(x, k_T^2)$

$\Psi_n(x, \vec{k}_{\perp i})$

$g^q_{1L}(x, k_T^2)$

$g_{1T}^q(x, k_T^2)$

$h_{1L}^q(x, k_T^2)$

$h_1^q(x, k_T^2)$

Courtesy of Aram Kotzinian
<table>
<thead>
<tr>
<th>Static</th>
<th>Dynamic</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Square of Target LFWFs</td>
<td>Modified by Rescattering: ISI &amp; FSI</td>
</tr>
<tr>
<td>• No Wilson Line</td>
<td>Contains Wilson Line, Phases</td>
</tr>
<tr>
<td>• Probability Distributions</td>
<td>No Probabilistic Interpretation</td>
</tr>
<tr>
<td>• Process-Independent</td>
<td>Process-Dependent - From Collision</td>
</tr>
<tr>
<td>• T-even Observables</td>
<td>T-Odd (Sivers, Boer-Mulders, etc.)</td>
</tr>
<tr>
<td>• No Shadowing, Anti-Shadowing</td>
<td>Shadowing, Anti-Shadowing, Saturation</td>
</tr>
<tr>
<td>• Sum Rules: Momentum and $J^z$</td>
<td>Sum Rules Not Proven</td>
</tr>
<tr>
<td>• DGLAP Evolution; mod. at large $x$</td>
<td>DGLAP Evolution</td>
</tr>
<tr>
<td>• No Diffractive DIS</td>
<td>Hard Pomeron and Odderon Diffractive DIS</td>
</tr>
</tbody>
</table>

$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)^2$
Remarkable observation at HERA

Fraction \( r \) of events with a large rapidity gap, \( \eta_{\text{max}} < 1.5 \), as a function of \( Q^2_{\text{DA}} \) for two ranges of \( x_{\text{DA}} \). No acceptance corrections have been applied.

**QCD Mechanism for Rapidity Gaps**

**Wilson Line:**

\[
\overline{\psi}(y) \int_0^y dx \ e^{i A(x) \cdot dx} \psi(0)
\]

\[
\psi(y) \int_0^y dx \ e^{i A(x) \cdot dx} \psi(0)
\]

- \( q^+ = 0 \)
- \( \gamma^* \)
- \( X_g \)
- \( \delta x \sim \frac{1}{v} \)
- \( (1-\beta)X_g \)
- \( \overline{\psi}(y) \int_0^y dx \ e^{i A(x) \cdot dx} \psi(0) \)

**Reproduces lab-frame color dipole approach**

Hoyer, Marchal, Peigne, Sannino, sjb

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Novel QCD Phenomena

Stan Brodsky, SLAC
Physics of Rescattering

- Sivers Asymmetry and Diffractive DIS: New Insights into Final State Interactions in QCD
- Origin of Hard Pomeron
- Structure Functions not Probability Distributions!
- T-odd SSAs, Shadowing, Antishadowing
- Diffractive dijets/ trijets, doubly diffractive Higgs
- Novel Effects: Color Transparency, Color Opaqueness, Intrinsic Charm, Odderon

Not square of LFWFs
Diffractive Structure Function $F_2^D$

Diffractive inclusive cross section

$$\frac{d^3\sigma_{\text{diff}}^{NC}}{dx_P d\beta dQ^2} \propto \frac{2\pi\alpha^2}{xQ^4} F_2^{D(3)}(x_P, \beta, Q^2)$$

$$F_2^D(x_P, \beta, Q^2) = f(x_P) \cdot F_2^{IP}(\beta, Q^2)$$

extract DPDF and $xg(x)$ from scaling violation

Large kinematic domain \(3 < Q^2 < 1600 \text{ GeV}^2\)

Precise measurements sys 5\%, stat 5–20\%
Final-State Interaction Produces Diffractive DIS

Quark Rescattering

Hoyer, Marchal, Peigne, Sannino, SJB
Enberg, Hoyer, Ingelman, SJB
Hwang, Schmidt, SJB

Low-Nussinov model of Pomeron

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QCD Mechanism for Rapidity Gaps

Wilson Line: \[ \overline{\psi}(y) \int_0^y dx \, e^{iA(x) \cdot dx} \, \psi(0) \]

Reproduces lab-frame color dipole approach

Hoyer, Marchal, Peigne, Sannino, sjb
Final State Interactions in QCD

\begin{align*}
&\text{Feynman Gauge} \quad \text{Light-Cone Gauge} \\
&\text{Result is Gauge Independent}
\end{align*}
Predict: Reduced DDIS/DIS for Heavy Quarks

\[ \sigma(DDIS) / \sigma(DIS) \approx \frac{\Lambda_{QCD}^2}{M_Q^2} \]

Reproduces lab-frame color dipole approach

See also: Bartels et al
Kopeliovitch, Schmidt, sjb

Novel QCD Phenomena
Stan Brodsky, SLAC
Novel QCD Phenomena


Anti-Shadowing

\[ Q^2 = 5 \text{ GeV}^2 \]

Stan Brodsky, SLAC
Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate T-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target
Nuclear Shadowing in QCD

Shadowing depends on understanding leading twist-diffraction in DIS

Nuclear Shadowing not included in nuclear LFWF!

Dynamical effect due to virtual photon interacting in nucleus
Integration over on-shell domain produces phase $i$

Need Imaginary Phase to Generate Pomeron

Need Imaginary Phase to Generate $T$-Odd Single-Spin Asymmetry

Physics of FSI not in Wavefunction of Target

Antishadowing (Reggeon exchange) is not universal!

Schmidt, Yang, sjb

Stan Brodsky, SLAC
\[ Q^2 = 5 \text{ GeV}^2 \]

Extrapolations from NuTeV

SLAC/NMC data

Scheinbein, Yu, Keppel, Morfin, Olness, Owens

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Origin of Regge Behavior of Deep Inelastic Structure Functions

Antiquark interacts with target nucleus at energy \( \hat{s} \propto \frac{1}{x_{bj}} \)

Regge contribution: \( \sigma_{\bar{q}N} \sim \hat{s}^{\alpha R - 1} \)

Nonsinglet Kuti-Weisskoff \( F_{2p} - F_{2n} \propto \sqrt{x_{bj}} \) at small \( x_{bj} \).

Shadowing of \( \sigma_{\bar{q}M} \) produces shadowing of nuclear structure function.
The one-step and two-step processes in DIS on a nucleus.

Coherence at small Bjorken $x_B$:
$$\frac{1}{M x_B} = \frac{2\nu}{Q^2} \geq L_A.$$  

If the scattering on nucleon $N_1$ is via pomeron exchange, the one-step and two-step amplitudes are opposite in phase, thus diminishing the $\bar{q}$ flux reaching $N_2$.

→ Shadowing of the DIS nuclear structure functions.

**Observed HERA DDIS produces nuclear shadowing**
Phase of two-step amplitude relative to one step:

\[
\frac{1}{\sqrt{2}}(1 - i) \times i = \frac{1}{\sqrt{2}}(i + 1)
\]

Constructive Interference

Depends on quark flavor!

Thus antishadowing is not universal

Different for couplings of $\gamma^*, Z^0, W^\pm$

**Critical test: Tagged Drell-Yan**
Non-singlet Reggeon Exchange

Kuti-Weisskopf behavior

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Shadowing and Antishadowing in Lepton-Nucleus Scattering

- **Shadowing**: Destructive Interference of Two-Step and One-Step Processes
  *Pomeron Exchange*

- **Antishadowing**: Constructive Interference of Two-Step and One-Step Processes!
  *Reggeon and Odderon Exchange*

- **Antishadowing is Not Universal**!
  Electromagnetic and weak currents: different nuclear effects!
  Potentially significant for NuTeV Anomaly
Nuclear Antishadowing not universal!
Shadowing and Antishadowing of DIS Structure Functions


Modifies NuTeV extraction of \( \sin^2 \theta_W \)

Test in flavor-tagged lepton-nucleus collisions

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Novel QCD Phenomena
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Predicted nuclear shadowing and antishadowing at $Q^2 = 1 \text{ GeV}^2$


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Novel QCD Phenomena

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$Q^2 = 5 \text{ GeV}^2$

Extrapolations from NuTeV

SLAC/NMC data

Scheinbein, Yu, Keppel, Morfin, Olness, Owens

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Exclusive Electroproduction

\[ ep \rightarrow e' \pi^+ n \]

Iterate kernel of LFWF to expose hard-scattering amplitude
QCD Factorization
Exclusive Electroproduction

\[ ep \to e' \pi^+ n \]

Universal distribution amplitudes. Renormalization Group Invariance:
The factorization scale \( \Lambda \) is arbitrary. The renormalization scale is unambiguous.
Proof from AdS/QCD: Polchinski and Strassler

$$\frac{d\sigma}{dt}(s, t) = \frac{F(\theta_{cm})}{s[n_{tot}^{-2}]} \quad s = E_{cm}^2$$

$$F_H(Q^2) \sim \left[ \frac{1}{Q^2} \right]^{n_H - 1}$$

QCD predicts leading-twist scaling behavior of fixed-CM angle exclusive amplitudes

$n_{tot} = n_A + n_B + n_C + n_D$

Fixed $t/s$ or $\cos \theta_{cm}$

Extension to soft pions: Strikman, Pobylitsa, Polyakov
• Phenomenological success of dimensional scaling laws for exclusive processes

\[ \frac{d\sigma}{dt} \sim \frac{1}{s^{n-2}}, \quad n = n_A + n_B + n_C + n_D, \]

implies QCD is a strongly coupled conformal theory at moderate but not asymptotic energies
Farrar and sjb (1973); Matveev et al. (1973).

• Derivation of counting rules for gauge theories with mass gap dual to string theories in warped space
Test of Scaling Laws

Constituent counting rules

\[ s^{n_{tot}} \frac{d\sigma}{dt}(A + B \rightarrow C + D) = F_{A+B\rightarrow C+D}(\theta_{CM}) \]

\[ s^7 \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) = F(\theta_{CM}) \]

\[ n_{tot} = 1 + 3 + 2 + 3 = 9 \]

Conformal invariance at high momentum transfers!
\[
\frac{d\sigma}{dt} (\gamma p \rightarrow MB) = \frac{F(\theta_{cm})}{s^7}
\]

\textbf{Counting Rules: } n=9
Deuteron Photodisintegration and Dimensional Counting

P. Rossi et al, P.R.L. 94, 012301 (2005)

\[ s^{n_{tot} - 2} \frac{d\sigma}{dt} (A + B \rightarrow C + D) = F_{A+B\rightarrow C+D}(\theta_{CM}) \]

\[ s^{11} \frac{d\sigma}{dt} (\gamma d \rightarrow np) = F(\theta_{CM}) \]

\[ n_{tot} - 2 = (1 + 6 + 3 + 3) - 2 = 11 \]

\[ \gamma d \rightarrow (uuddduds\bar{s}) \rightarrow np \]

at \( s \simeq 9 \text{ GeV}^2 \)

\[ \gamma d \rightarrow (uuddduc\bar{c}) \rightarrow np \]

at \( s \simeq 25 \text{ GeV}^2 \)
\[ \gamma d \rightarrow np \]
\[ \gamma d \rightarrow (uudduds\bar{s}) \rightarrow np \text{ at } s = 9 \text{ GeV}^2 \]

Fit of d\(\sigma/dt\) data for the central angles and \(P_T \geq 1.1\) GeV/c with \(A \propto s^{-11}\)

For all but two of the fits \(\chi^2 \leq 1.34\)

- Better \(\chi^2\) at 55° and 75° if different data sets are renormalized to each other
- No data at \(P_T \geq 1.1\) GeV/c at forward and backward angles
- Clear \(s^{-11}\) behaviour for last 3 points at 35°

Data consistent with CCR
• Remarkable Test of Quark Counting Rules

• Deuteron Photo-Disintegration $\gamma d \rightarrow np$

$$\frac{d\sigma}{dt} = \frac{F(t/s)}{s^{n_{tot}-2}}$$

$$n_{tot} = 1 + 6 + 3 + 3 = 13$$

Scaling characteristic of scale-invariant theory at short distances

Conformal symmetry

Hidden color: $$\frac{d\sigma}{dt}(\gamma d \rightarrow \Delta^{++} \Delta^{-}) \sim \frac{d\sigma}{dt}(\gamma d \rightarrow pn)$$

at high $p_T$ 

Ratio predicted to approach 2:5
Properties of Hard Exclusive Reactions

- Dimensional Counting Rules at fixed CM angle
- Hadron Helicity Conservation
- Color Transparency
- Hidden color
- \( s \gg -t \gg \Lambda_{\text{QCD}} \): Reggeons have negative-integer intercepts at large \(-t\)
- J=0 Fixed pole in DVCS
- Quark interchange
- Renormalization group invariance
- No renormalization scale ambiguity
- Exclusive inclusive connection with spectator counting rules
- Diffractive reactions from pomeron, Reggeon, odderon
Deuteron Light-Front Wavefunction

\[ P^+ = P^0 + P^z \]

Deuteron Light-Front Wavefunction

Fixed \( \tau = t + z/c \)

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i} \]

\[ \sum_i n x_i = 1 \]

\[ \sum_i k_{\perp i} = \vec{0}_\perp \]

Two color-singlet combinations of three 3C

\[ \psi_d(x_i, \vec{k}_{\perp i}) = \psi_d^{body} \times \psi_n \times \psi_p \]
Evolution of 5 color-singlet Fock states

\[ \psi_n^d(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ \sum_i^n k_{\perp i} = \vec{0}_{\perp} \]
\[ \sum_i^n x_i = 1 \]

\[ \Phi_n(x_i, Q) = \int k_{\perp i}^2 < Q^2 \prod' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j}) \]

5 x 5 Matrix Evolution Equation for deuteron distribution amplitude
Hidden Color in QCD

Lepage, Ji, sjb

- Deuteron six-quark wavefunction
- 5 color-singlet combinations of 6 color-triplets -- only one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

\[ \frac{d\sigma}{dt} (\gamma d \rightarrow \Delta^+\Delta^-) \approx \frac{d\sigma}{dt} (\gamma d \rightarrow pn) \text{ at high } Q^2 \]
Hidden Color of Deuteron

Deuteron six-quark state has five color-singlet configurations, only one of which is n-p.

Asymptotic Solution has Expansion

$$\psi_{[6]}^{[33]} = \left(\frac{1}{9}\right)^{1/2} \psi_{NN} + \left(\frac{4}{45}\right)^{1/2} \psi_{\Delta\Delta} + \left(\frac{4}{5}\right)^{1/2} \psi_{CC}$$

Look for strong transition to Delta-Delta
Test of Hidden Color in Deuteron Photodisintegration

\[ R = \frac{\frac{d \sigma}{d t}(\gamma d \rightarrow \Delta^{++} \Delta^{--})}{\frac{d \sigma}{d t}(\gamma d \rightarrow pn)} \]

Ratio predicted to approach 2:5

Ratio should grow with transverse momentum as the hidden color component of the deuteron grows in strength.

Possible contribution from pion charge exchange at small t.
Perturbative QCD Analysis of Structure Functions at $x \sim 1$

- Struck quark far off-shell at large $x$
  \[ k_F^2 \sim -\frac{k_\perp^2}{1-x} \]
- Lowest-order connected PQCD diagrams dominate
- Spectator counting rules
  \[ (1-x)^{2n_s-1} + 2\Delta S_z \]
- Helicity retention at large $x$
- Exclusive-Inclusive Connection
$q^+(x) \propto (1 - x)^3$

$q^-(x) \propto (1 - x)^5 \log^2(1 - x)$

*From nonzero orbital angular momentum*

**Avakian, sjb, Deur, Yuan**
Features of Hard Exclusive Processes in PQCD

Lepage, sjb; Duncan, Mueller

- Factorization of perturbative hard scattering subprocess amplitude and nonperturbative distribution amplitudes
- Dimensional counting rules reflect conformal invariance:
- Hadron helicity conservation: \( \sum_{\text{initial}} \lambda_i^H = \sum_{\text{final}} \lambda_j^H \)
- Color transparency  Mueller, sjb;
- Hidden color Ji, Lepage, sjb;
- Evolution of Distribution Amplitudes Lepage, sjb; Efremov, Radyushkin

\[ M = \int T_H \times \Pi \phi_i \]
\[ M \sim \frac{f(\theta_{CM})}{Q^{N_{tot}-4}} \]
Color Transparency

Bertsch, Gunion, Goldhaber, sjb
A. H. Mueller, sjb

- Fundamental test of gauge theory in hadron physics
- Small color dipole moments interact weakly in nuclei
- Complete coherence at high energies
- Clear Demonstration of CT from Diffractive Di-Jets
Theory:

Kopeliovich et al., PRC 65 (2002) 035201
Color Transparency Ratio

\[
T_{pp} = \frac{d\sigma/dt \text{ for } ^{12}\text{C}}{Z \cdot d\sigma/dt \text{ for } \text{proton}_1} \text{ proton}_3 \text{ proton}_4
\]

J. L. S. Aclander et al.,
“Nuclear transparency in \(\theta_{CM} = 90^0\)
quasielastic \(A(p, 2p)\) reactions,”
Color Transparency fails when $A_{nn}$ is large

Nuclear Transparency, $T_{pp}$

$P_{\text{eff}}$, Effective beam momentum [GeV/c]

$Q^2$

Mardor [1]
Leksanov [2]
Carroll-C [3]
Carroll-Al [3]
$1/R(s)$
Eva Experiment BNL

Bunce, Carroll, Heppelman...

Rapid Angular Variation!
Features of Light-Front Formalism

- **Hidden Color** Nuclear Wavefunction
- **Color Transparency, Opaqueness**
- **Intrinsic glue, sea quarks, intrinsic charm.**
- Simple proof of Factorization theorems for hard processes (Lepage, sjb)
- **Direct mapping to AdS/CFT** (de Teramond, sjb)
- New Effective LF Equations (de Teramond, sjb)
- Light-Front Amplitude Generator
Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Hadronic Observables, Constituent Counting Rules
- Insight into QCD Condensates
- Systematically improvable

*de Teramond, Deur, Shrock, Roberts, Tandy*
Light-Front formalism links dynamics to spectroscopy

Heisenberg Matrix Formulation

\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k_\perp^2}{x} \right] i + H^{int}_{LF} \]

\[ H^{int}_{LF}: \text{ Matrix in Fock Space} \]

\[ H^{QCD}_{LF} |\Psi_h > = \mathcal{M}^2_h |\Psi_h > \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Physical gauge: \( A^+ = 0 \)
Light-Front Wavefunctions

Dirac’s Front Form: Fixed $\tau = t + z/c$

$$\psi(x, k_\perp)$$

Invariant under boosts. Independent of $P^\mu$

$$H_{LF}^{QCD} |\psi >= M^2 |\psi >$$

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space
Light-Front Holography and Non-Perturbative QCD

**Goal:**
Use AdS/QCD duality to construct a first approximation to QCD

- **Hadron Spectrum**
- **Light-Front Wavefunctions,** Running coupling in IR

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

**Central problem for strongly-coupled gauge theories**

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Novel QCD Phenomena

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Applications of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension \( z \)

in collaboration with Guy de Teramond and Alexandre Deur
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu \nu}, P^\mu, D, K^\mu, \]

the generators of \( \text{SO}(4,2) \)

\( \text{SO}(4,2) \) has a mathematical representation on \( \text{AdS}_5 \)
**AdS/CFT**: Anti-de Sitter Space / Conformal Field Theory

**Maldacena:**

\[ \text{Map } AdS_5 \times S_5 \text{ to conformal } N=4 \text{ SUSY} \]

- **QCD is not conformal**: however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window**: \( \alpha_s(Q^2) \simeq \text{const} \) at small \( Q^2 \)

- Use mathematical mapping of the conformal group \( \text{SO}(4,2) \) to \( AdS_5 \) space
Deur, Korsch, et al.

\[ \frac{\alpha_{s,gl}}{\pi} \text{ JLab, GDH limit, Fit, pQCD evol. eq.} \]

\[ m_{\pi}^2, g_1, g_2 / m_{\pi}^2, \text{JLab, Fit, pQCD evol. eq.} \]

\[ \text{Bhagwat et al., Maris-Tandy, Burkert-Ioffe, \text{Bloch et al., Godfrey-Isgur, DSE gluon couplings, Alkofer, Lattice QCD}} \]

EIC-INT September 17, 2010 Novel QCD Phenomena 123 Stan Brodsky, SLAC
Nearly conformal QCD?

Define $\alpha_s$ from Björkén sum,

$$\Gamma_1^{p-n} \equiv \int_0^1 dx \left( g_1^p(x, Q^2) - g_1^n(x, Q^2) \right) = \frac{1}{6} g_A \left( 1 - \frac{\alpha_s g_1}{\pi} \right)$$

$g_1 = \text{spin dependent structure function}$

Recent JLab data from EG1(2008), CLAS, and Hall A

$s$ runs only modestly at small $Q^2$

Fig. from 0803.4119, Duer et al.
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[\zeta^2 = x(1 - x)b_\perp^2.\]

\[U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)\]

G. de Teramond, sjb

Soft wall confining potential:

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**Soft-Wall Model**

\[ S = \int d^4x \, dz \, \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2 \]

*Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field*

Karch, Katz, Son and Stephanov (2006)

- Equation of motion for scalar field \[ \mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_\ell \Phi - \mu^2 \Phi^2) \]

\[ [z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0 \]

with \((\mu R)^2 \geq -4\).

- LH holography requires ‘plus dilaton’ \(\varphi = +\kappa^2 z^2\). Lowest possible state \((\mu R)^2 = -4\)

\[ \mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2} \]

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

**Massless pion**
\[ ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2) \]

\[ ds^2 = e^{A(y)} (-dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2) + dy^2 \]
\[ ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 - dz^2) \]

\[ V(z) = mc^2 \sqrt{g_{00}} \]

\[ V(z) \sim \frac{R}{z} e^{+\kappa^2 z^2/2} \]

\[ V(z) \sim \frac{R}{z} e^{-\kappa^2 z^2/2} \]
AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

\[
\left[ - \frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[ U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \]

Derived from variation of Action Dilaton-Modified AdS₅

\[ e^{\Phi(z)} = e^{+\kappa^2 z^2} \]

Positive-sign dilaton
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$m_q = 0$
Quark separation increases with $L$.

(a) $\Phi(z)$ for different $L$ values:
- $L = 3$
- $L = 2$
- $L = 1$
- $L = 0$

(b) $\Phi(z)$ for different $n$ values:
- $n = 0$
- $n = 1$
- $n = 2$

3. $M^2$ vs. $L$ for $S = 1$:

- $\omega (782)$
- $\rho (770)$
- $f_2 (1270)$
- $a_2 (1320)$
- $\rho_3 (1690)$

(b) $M^2$ vs. $n$ for $S = 1$:

- $\rho (1450)$
- $\rho (1700)$
Parent and daughter Regge trajectories for the $I = 1$ $\rho$-meson family (red) and the $I = 0$ $\omega$-meson family (black) for $\kappa = 0.54$ GeV.
Baryons in AdS/CFT

- Action for massive fermionic modes on AdS$_5$:

$$ S[\Psi, \bar{\Psi}] = \int d^4x \, dz \, \sqrt{g} \bar{\Psi}(x, z) \left( i \Gamma^\ell D_\ell - \mu \right) \Psi(x, z) $$

- Equation of motion:

$$ \left( i \Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0 $$

$$ \left[ i \left( z \eta^{\ell m} \Gamma^\ell \partial_m + \frac{d}{2} \Gamma z \right) + \mu R \right] \Psi(x^\ell) = 0 $$

- Solution ($\mu R = \nu + 1/2$)

$$ \Psi(z) = C z^{5/2} \left[ J_\nu(z \mathcal{M}) u_+ + J_{\nu+1}(z \mathcal{M}) u_- \right] $$

- Hadron mass spectrum determined from IR boundary conditions $\psi_{\pm} (z = 1/\Lambda_{\text{QCD}}) = 0$

$$ \mathcal{M}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}} $$

with scale independent mass ratio

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}, J > \frac{1}{2}$, with all indices along 3+1 from $\Psi$ by shifting dimensions
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_P^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( \langle b = |\vec{b}_\perp| \rangle \):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix\vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \]

\[ \quad = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bq) \, |\tilde{\psi}(x, b)|^2, \]  

\[ \vec{q}_\perp^2 = Q^2 = -q^2 \]

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