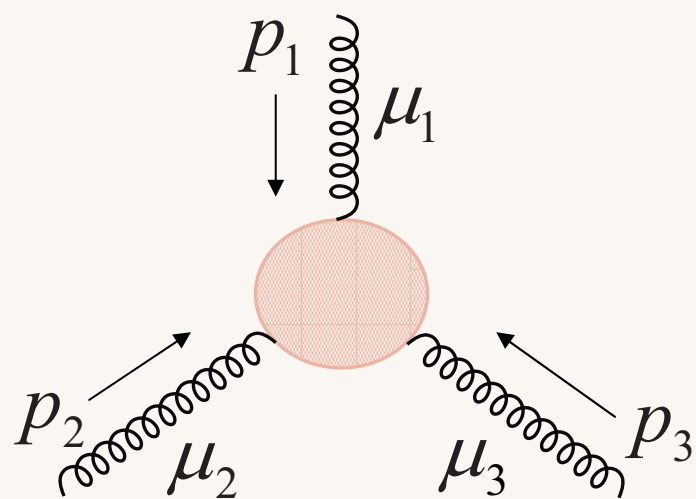


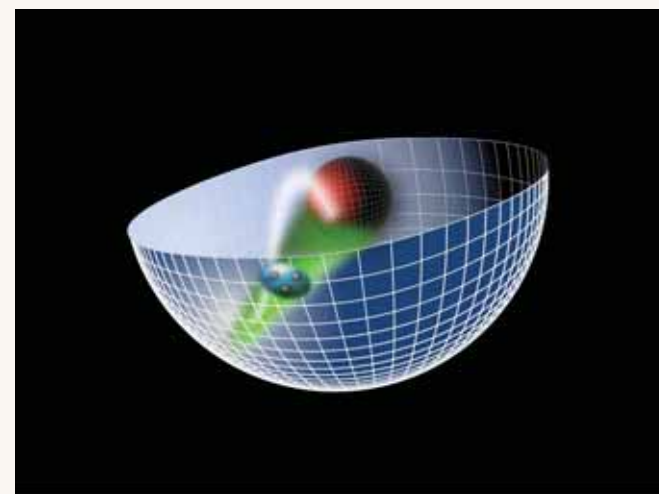
# The Conformal Template for QCD

**Quantum Field Theory and Beyond:  
Celebration of Mike Cornwall's 75th Birthday**

*UCLA November 14-15, 2009*



*Pinch Scheme*



*Stan Brodsky, SLAC National Accelerator Laboratory*

# Conformal symmetry: Template for QCD

- **Take conformal symmetry as initial approximation; then correct for non-zero beta function and quark masses**  
Frishman, Lepage, Mackenzie, Sachrajda, sjb, Gardi, Braun
- **Eigensolutions of ERBL evolution equation for distribution amplitudes**
- **Commensurate scale relations: relate observables at corresponding scales: Generalized Crewther Relation**  
Gardi, Grunberg, Rathsmann, Gabadadze, Kataev, Lepage, Lu, Mackenzie, sjb
- **Fix Renormalization Scale (BLM, Effective Charges)**
- **IR Fixed Point -- A Conformal Domain**
- **Use AdS/CFT**

# IR Fixed Point for QCD?

- *Effective Gluon Mass* *Cornwall*
- *Dyson-Schwinger Analysis: QCD coupling (mom scheme) has IR Fixed point!* *Alkofer, Fischer, von Smekal et al.*
- *Lattice Gauge Theory*
- Define coupling from observable, indications of IR fixed point for QCD effective charges
- Confined gluons and quarks: Decoupling of QCD vacuum polarization at small  $Q^2$
- Justifies application of AdS/CFT in strong-coupling conformal window

## Dynamical mass generation in continuum quantum chromodynamics

John M. Cornwall

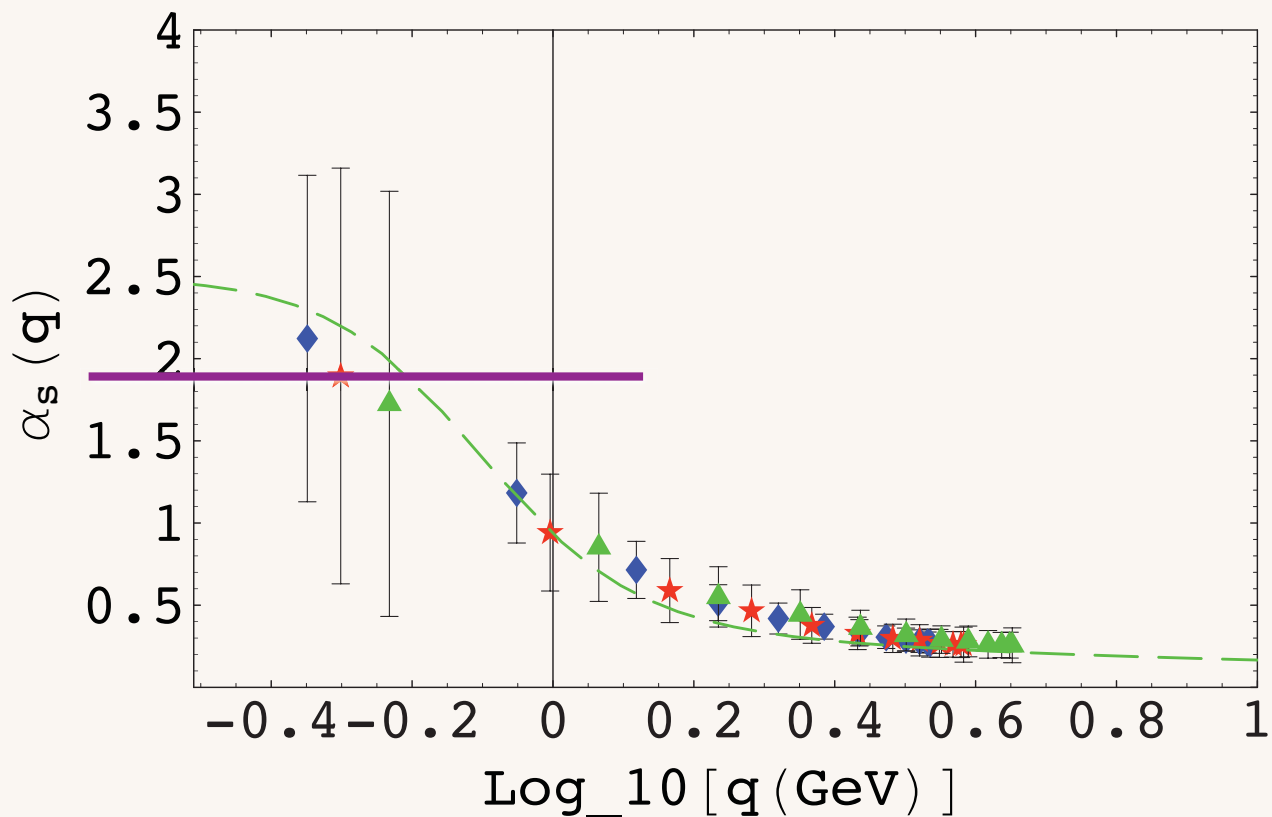
*Department of Physics, University of California, Los Angeles, California 90024*

(Received 30 April 1982)

We study the formation of a mass gap, or effective gluon mass (and consequent dimensional parameters such as the string tension, glueball mass,  $\langle \text{Tr} G_{\mu\nu}^2 \rangle$ , correlation lengths) in continuum QCD, using a special set of Schwinger-Dyson equations. These equations are derived from a resummation of the Feynman graphs which represent certain *gauge-invariant* color-singlet Green's functions, and are themselves essentially gauge invariant. This resummation is essential to the multiplicative renormalizability of QCD in the light-cone gauge, which we adopt for technical reasons. We close the dynamical equations by "solving" a Ward identity, a procedure which, while exact in the infrared regime, is subject to ambiguities and corrections in the ultraviolet regime which are beyond the scope of the present work. (These ambiguities are less prominent for QCD in three dimensions, which we discuss also.) As discussed in an earlier work, quark confinement arises from a vortex condensate supported by the mass gap. Numerical calculations of the mass gap are presented, suggesting an effective gluon mass of  $500 \pm 200$  MeV and a  $0^+$  glueball mass of about twice this value.

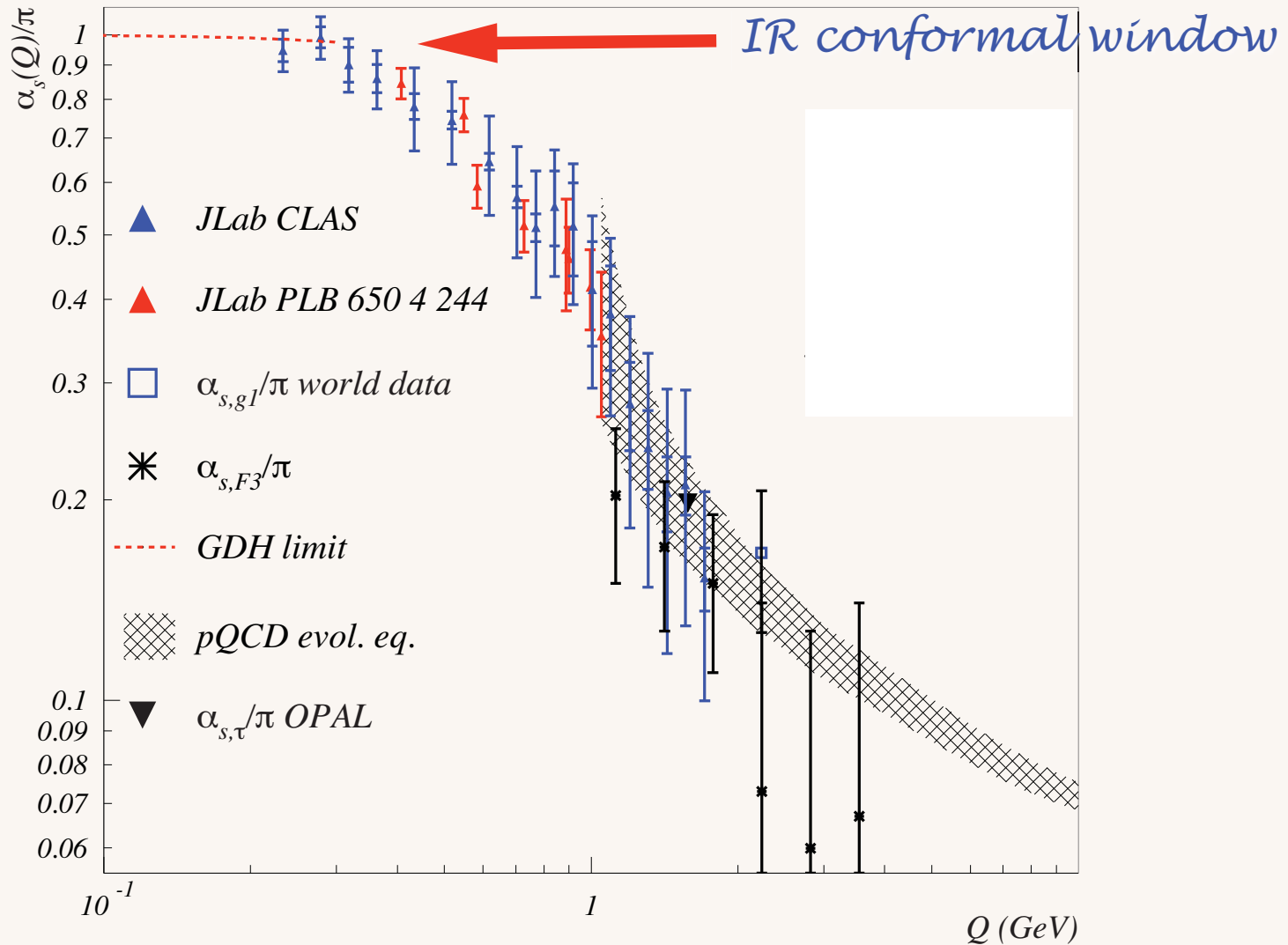
## • IR Fixed Point

- Does  $\alpha_s$  develop an IR fixed point? Dyson–Schwinger Equation [Alkofer, Fischer, LLanes-Estrada, Deur ...](#)
- Recent lattice simulations: evidence that  $\alpha_s$  becomes constant and is not small in the infrared [Furui and Nakajima, hep-lat/0612009](#) (Green dashed curve: DSE).



# Deur, Korsch, et al: Effective Charge from Bjorken Sum Rule

$$\Gamma_{bj}^{p-n}(Q^2) \equiv \frac{g_A}{6} \left[ 1 - \frac{\alpha_s^{g1}(Q^2)}{\pi} \right]$$



# Maximal Wavelength of Confined Fields

- **Colored fields confined to finite domain**

- **All perturbative calculations regulated in IR**  $(x - y)^2 < \Lambda_{QCD}^{-2}$

- **High momentum calculations unaffected**

- **Bound-state Dyson-Schwinger Equation**

- **Analogous to Bethe's Lamb Shift Calculation**

- **Similar in spirit to Cornwall's Effective Gluon mass**

*Quark and Gluon vacuum polarization insertions decouple: IR  
fixed Point*

**Shrock, sjb**

**A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).**

J. D. Bjorken,  
SLAC-PUB 1053  
Cargese Lectures 1989

# IR Conformal Window for QCD

- *Dyson-Schwinger Analysis:* **QCD gluon coupling has IR Fixed Point**

- *Evidence from Lattice Gauge Theory*

- *Stability of  $\Upsilon \rightarrow ggg$*

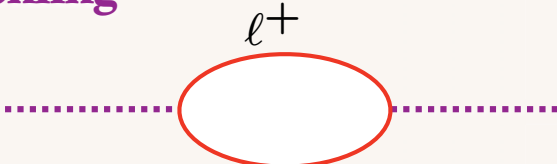
Shrock, sjb

- Define coupling from observable: **indications of IR fixed point for QCD effective charges**

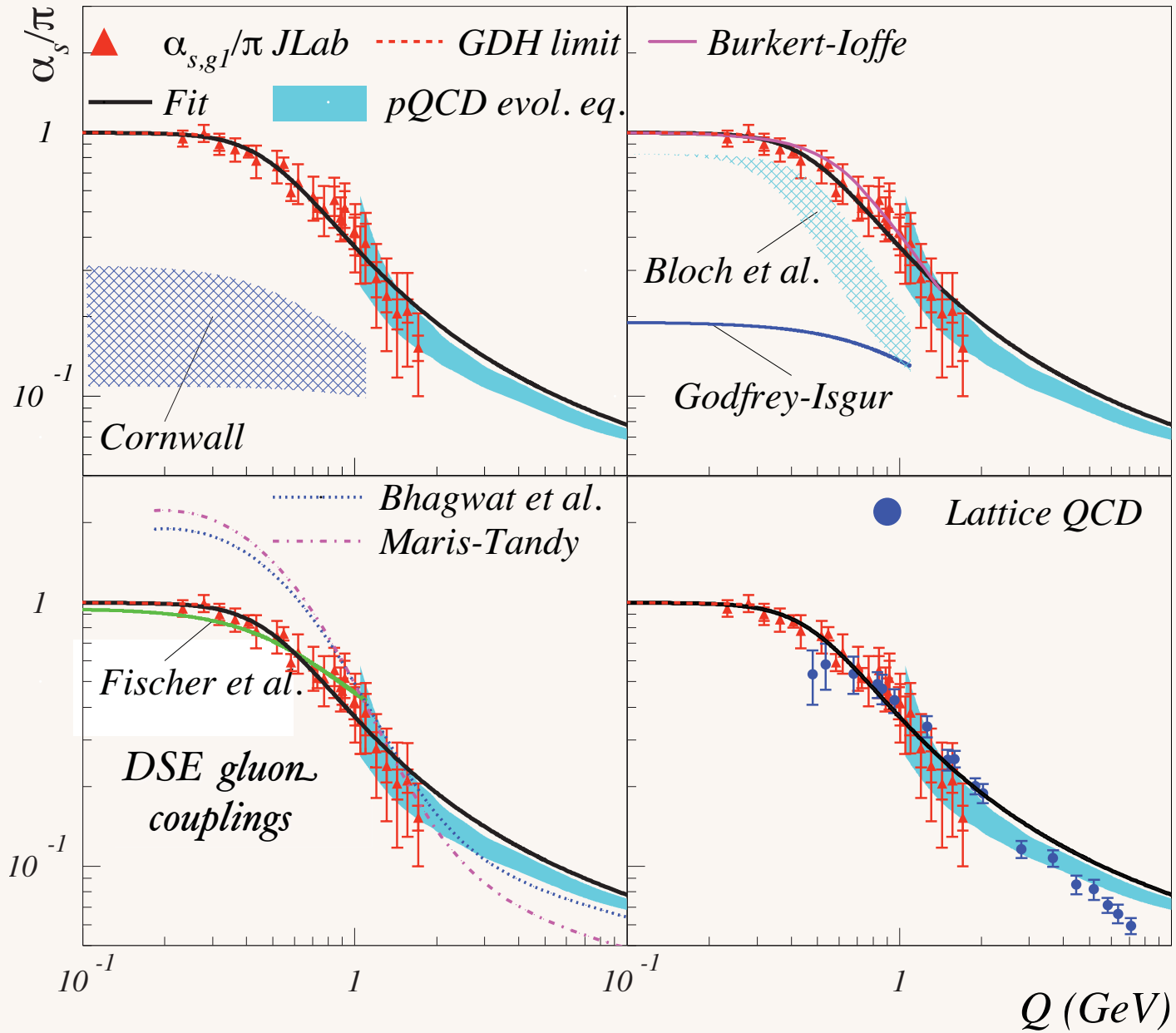
Deur, Chen, Burkert, Korsch,

- Confined gluons and quarks have maximum wavelength: **Decoupling of QCD vacuum polarization at small  $Q^2$**

Serber-Uehling

$$\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2$$


- **Justifies application of AdS/CFT in strong-coupling conformal window**



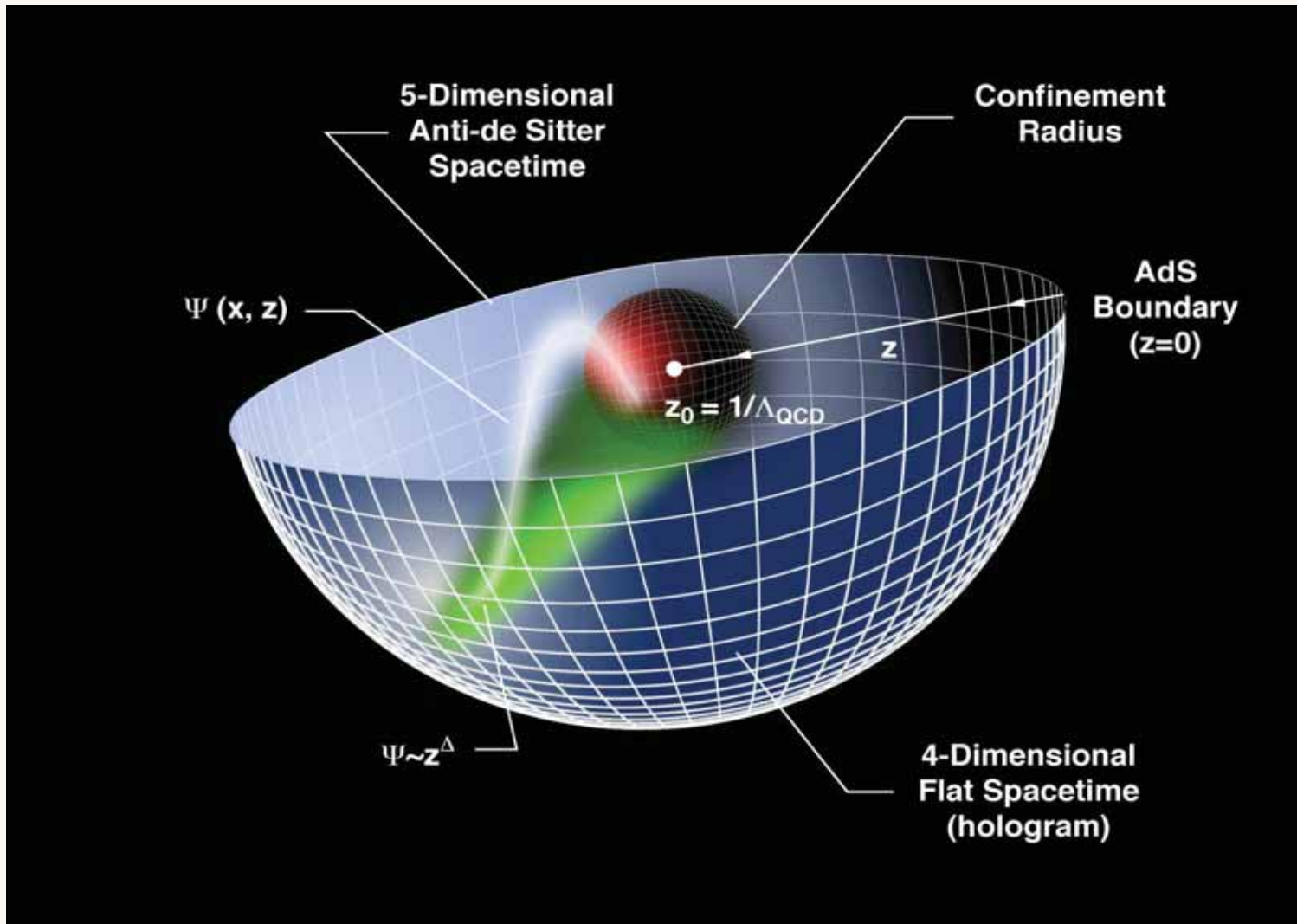
*Conformal Theories are invariant under the Poincare and conformal transformations with*

$$\mathbf{M}^{\mu\nu}, \mathbf{P}^{\mu}, \mathbf{D}, \mathbf{K}^{\mu},$$

*the generators of  $SO(4,2)$*

**$SO(4,2)$  has a mathematical representation on  $AdS_5$**

# Applications of AdS/CFT to QCD



*Changes in physical length scale mapped to evolution in the 5th dimension  $z$*

**in collaboration with Guy de Teramond**

# *AdS/CFT*: Anti-de Sitter Space / Conformal Field Theory

Maldacena:


Map  $AdS_5 \times S^5$  to conformal  $N=4$  SUSY

- **QCD is not conformal**; however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes
- **Conformal window:**  $\alpha_s(Q^2) \simeq \text{const}$  at small  $Q^2$
- **Use mathematical mapping of the conformal group  $SO(4,2)$  to  $AdS_5$  space**

## Scale Transformations

- Isomorphism of  $SO(4, 2)$  of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

*invariant measure* 

$x^\mu \rightarrow \lambda x^\mu, z \rightarrow \lambda z$ , maps scale transformations into the holographic coordinate  $z$ .

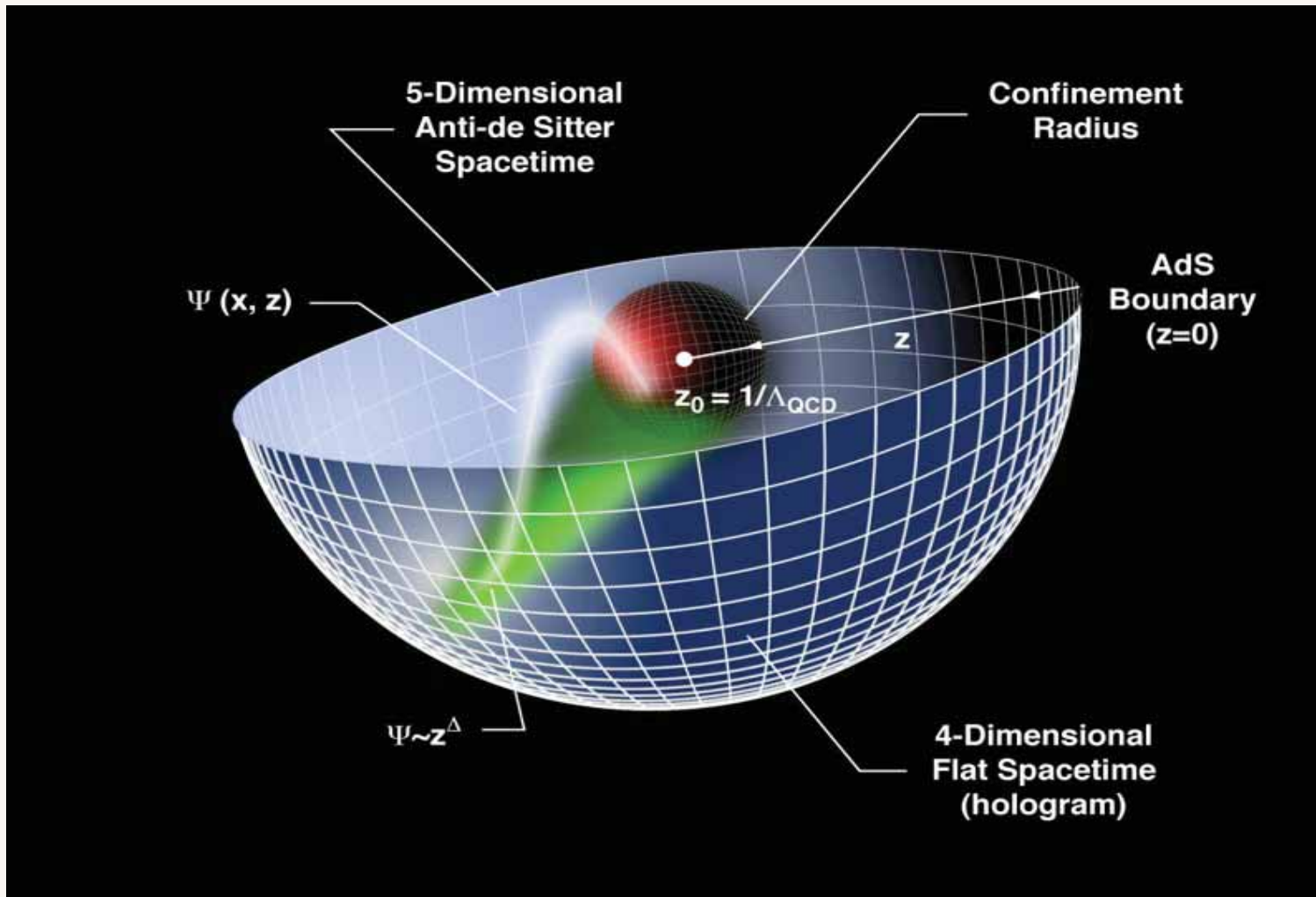
- AdS mode in  $z$  is the extension of the hadron wf into the fifth dimension.
- Different values of  $z$  correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$ : invariant separation between quarks

- The AdS boundary at  $z \rightarrow 0$  correspond to the  $Q \rightarrow \infty$ , UV zero separation limit.

# Applications of AdS/CFT to QCD



*Changes in physical length scale mapped to evolution in the 5th dimension  $z$*

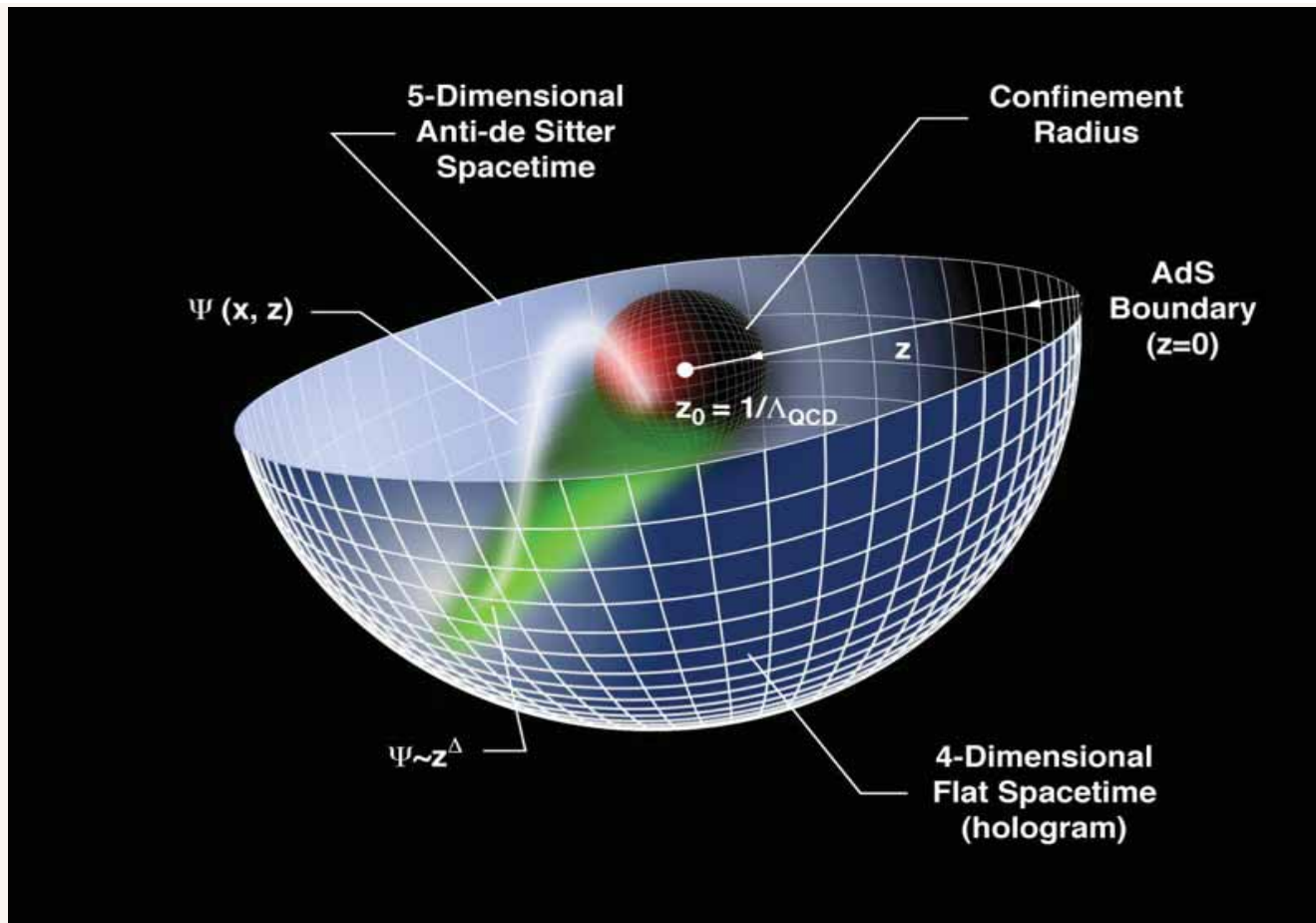
**String Theory**



**Bottom-Up**



**Top-Down**



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- Truncated AdS/CFT (Hard-Wall) model: cut-off at  $z_0 = 1/\Lambda_{\text{QCD}}$  breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) **Polchinski and Strassler (2001)**.
- Smooth cutoff: introduction of a background dilaton field  $\varphi(z)$  – usual linear Regge dependence can be obtained (Soft-Wall Model) **Karch, Katz, Son and Stephanov (2006)**.

# Goal:

- **Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances**
- **Analogous to the Schrödinger Theory for Atomic Physics**
- *AdS/QCD Light-Front Holography*
- *Hadronic Spectra and Light-Front Wavefunctions*

- **Polchinski & Strassler:** AdS/CFT builds in conformal symmetry at short distances; counting rules for form factors and hard exclusive processes; non-perturbative derivation
- **Goal:** Use AdS/CFT to provide an approximate model of hadron structure with confinement at large distances, conformal behavior at short distances
- **de Teramond, sjb: AdS/QCD Holographic Model:** Initial “semi-classical” approximation to QCD. Predict light-quark hadron spectroscopy, form factors.
- **Karch, Katz, Son, Stephanov:** Soft-Wall Model --Linear Confinement
- Mapping of AdS amplitudes to 3+1 Light-Front equations, wavefunctions!
- Use AdS/CFT wavefunctions as expansion basis for diagonalizing  $H^{\text{LF}}_{\text{QCD}}$ ; variational methods

# AdS/CFT

- Use mapping of conformal group  $SO(4,2)$  to  $AdS_5$
- Scale Transformations represented by wavefunction in 5th dimension  
$$x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z$$
- Match solutions at small  $z$  to conformal twist dimension of hadron wavefunction at short distances  
$$\psi(z) \sim z^\Delta \text{ at } z \rightarrow 0$$
- Hard wall model: Confinement at large distances and conformal symmetry in interior
- Truncated space simulates “bag” boundary conditions

$$0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}}$$

# Bosonic Solutions: Hard Wall Model

- Conformal metric:  $ds^2 = g_{\ell m} dx^\ell dx^m$ .  $x^\ell = (x^\mu, z)$ ,  $g_{\ell m} \rightarrow (R^2/z^2) \eta_{\ell m}$ .

- Action for massive scalar modes on  $\text{AdS}_{d+1}$ :

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along  $x^\mu$ -coordinates,  $\Phi_P(x, z) = e^{-iP \cdot x} \Phi(z)$ ,  $P_\mu P^\mu = \mathcal{M}^2$ :

$$\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution:  $\Phi(z) \rightarrow z^\Delta$  as  $z \rightarrow 0$ ,

$$\Phi(z) = C z^{d/2} J_{\Delta-d/2}(z\mathcal{M}) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4$$



## Soft-Wall Model

- Soft-wall model [Karch, Katz, Son and Stephanov (2006)] retain conformal AdS metrics but introduce a smooth cutoff - depends on the profile of a dilaton background field  $\varphi(z) = \pm \kappa^2 z^2$

$$S = \int d^d x dz \sqrt{g} e^{\varphi(z)} \mathcal{L},$$

- Equation of motion for scalar field  $\mathcal{L} = \frac{1}{2} (g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2)$

$$[z^2 \partial_z^2 - (d - 1 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0$$

with  $(\mu R)^2 \geq -4$ . See also [Metsaev (2002), Andreev (2006)]

- LH holography requires 'plus dilaton'  $\varphi = +\kappa^2 z^2$ . Lowest possible state  $(\mu R)^2 = -4$

$$\mathcal{M}^2 = 4\kappa^2 n, \quad \Phi_n(z) \sim z^2 e^{-\kappa^2 z^2} L_n(\kappa^2 z^2)$$

$\Phi_0(z)$  a chiral symmetric bound state of two massless quarks with scaling dimension 2: the pion

*Massless pion!*

*AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:*

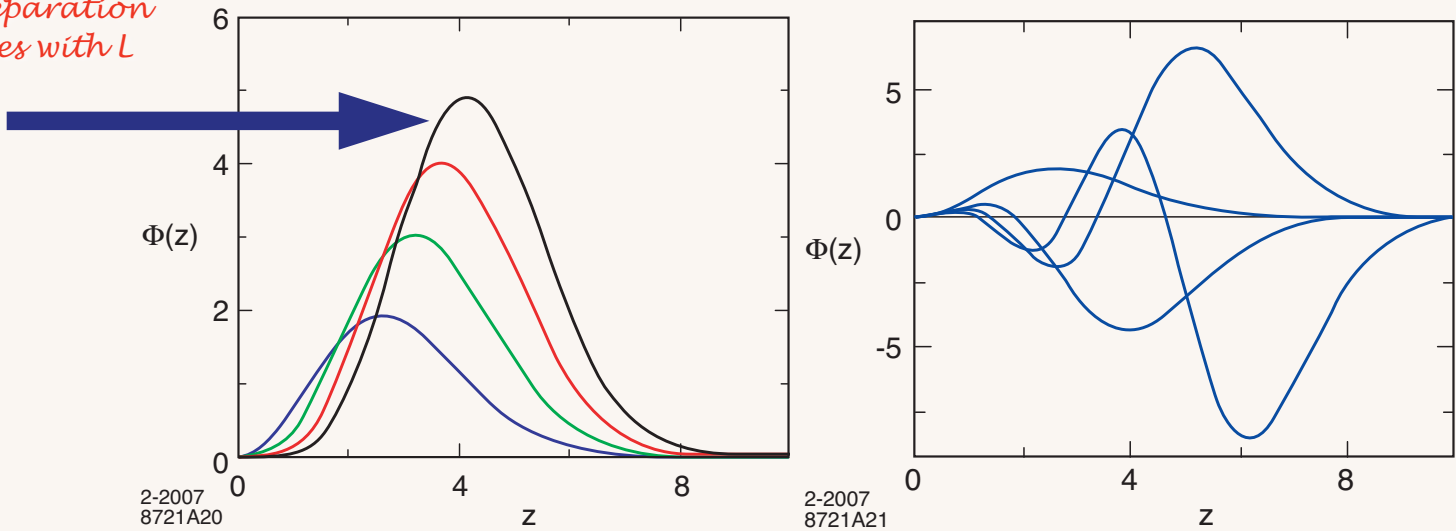
$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = \mathcal{M}^2 \phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

*Derived from variation of Action  
Dilaton-Modified AdS<sub>5</sub>*

$$e^{\Phi(z)} = e^{+\kappa^2 z^2}$$

Quark separation increases with  $L$

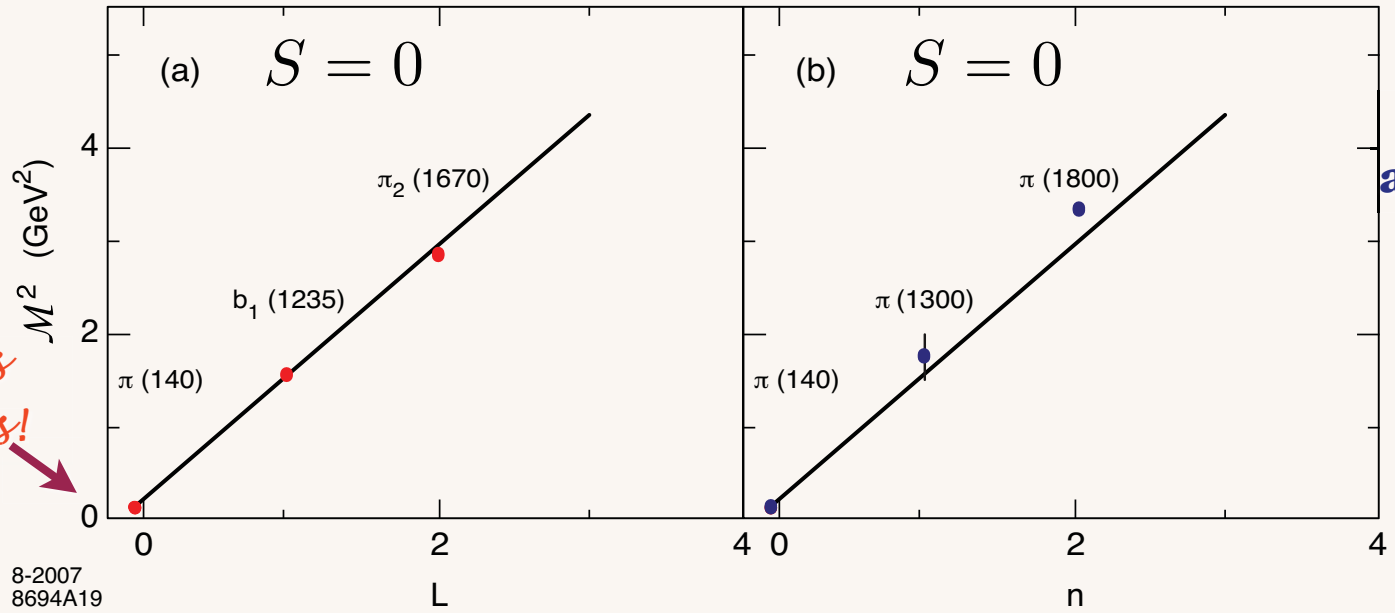


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Fig: Orbital and radial AdS modes in the soft wall model for  $\kappa = 0.6$  GeV .

Soft Wall Model



Pion has zero mass!

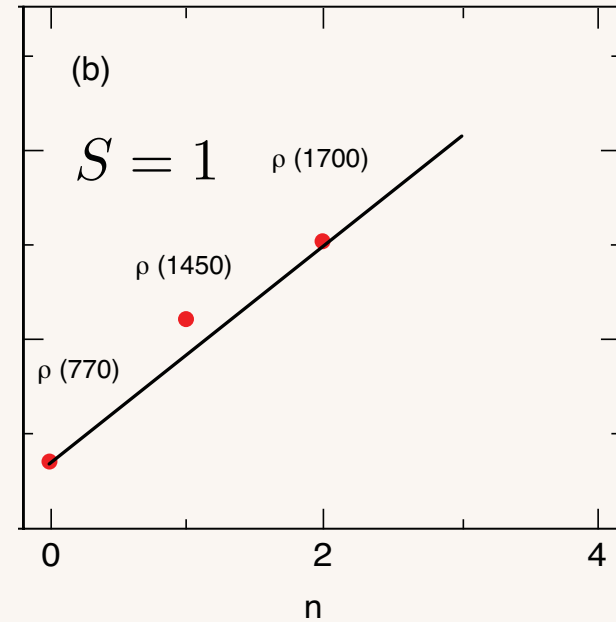
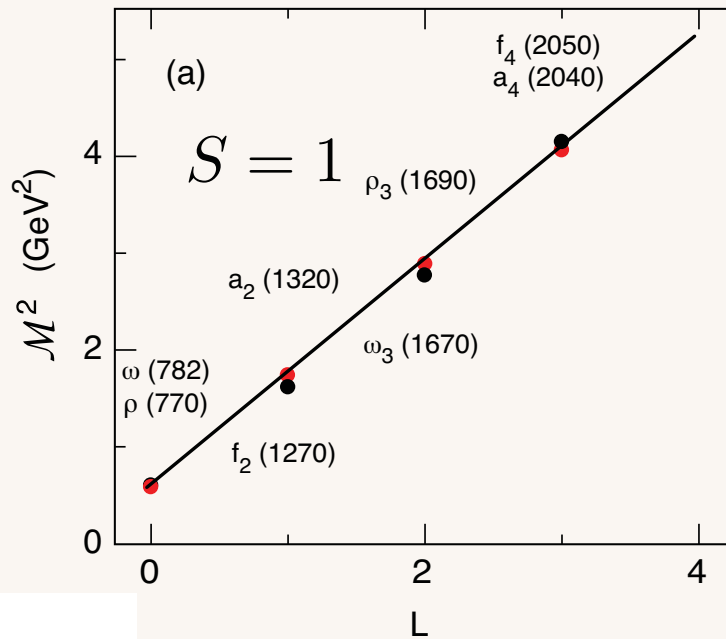
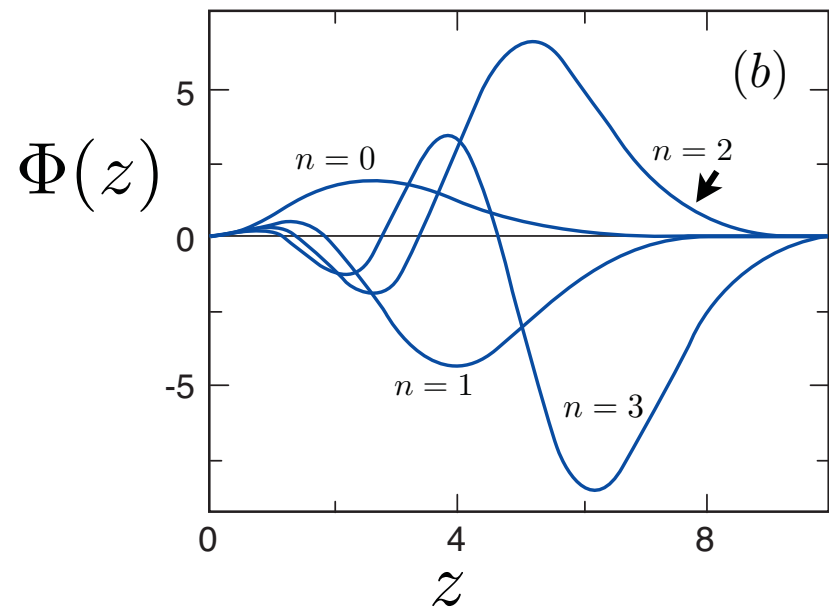
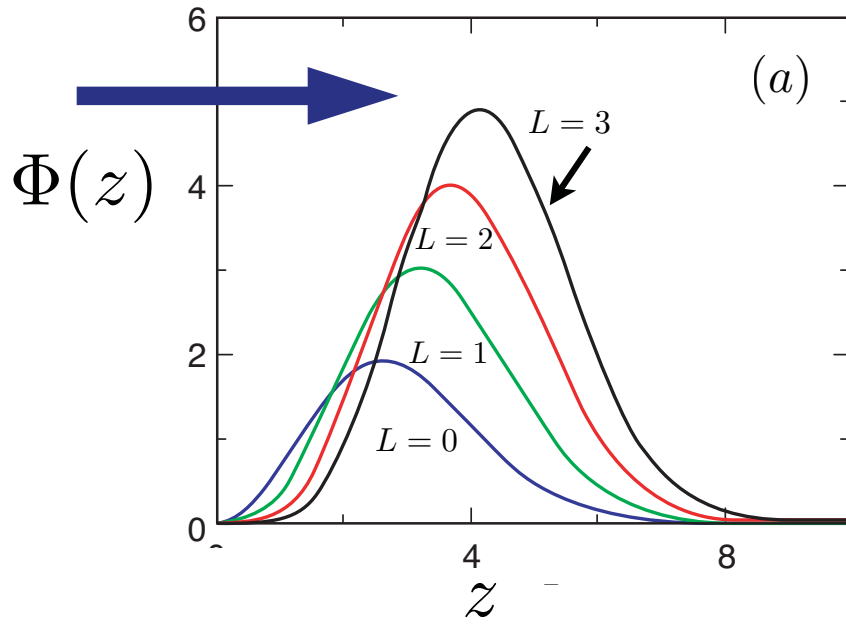
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Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for  $\kappa = 0.6$  GeV.

Quark separation increases with  $L$



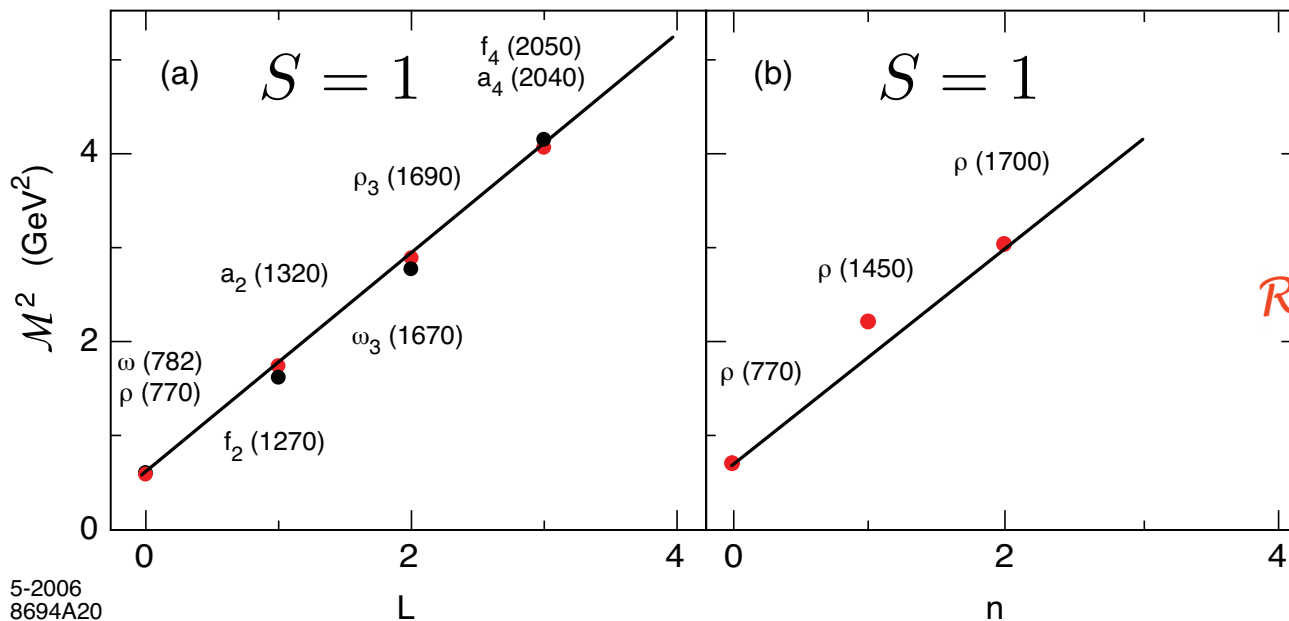
- Effective LF Schrödinger wave equation

$$\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2(L + S - 1) \right] \phi_S(z) = \mathcal{M}^2 \phi_S(z)$$

with eigenvalues  $\mathcal{M}^2 = 2\kappa^2(2n + 2L + S)$ .

*Same slope in n and L*

- Compare with Nambu string result (rotating flux tube):  $M_n^2(L) = 2\pi\sigma(n + L + 1/2)$ .

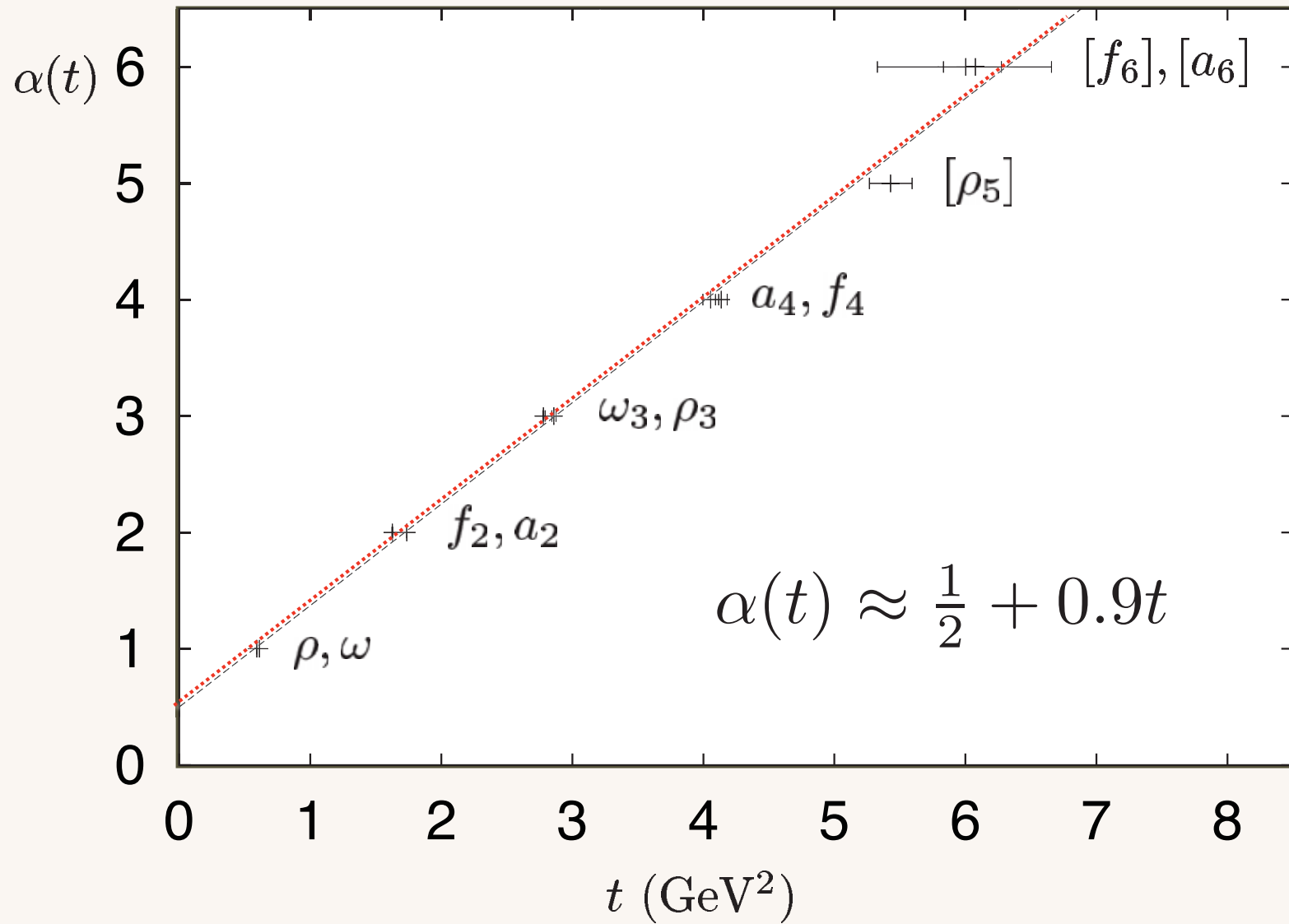


*Regge Trajectories*

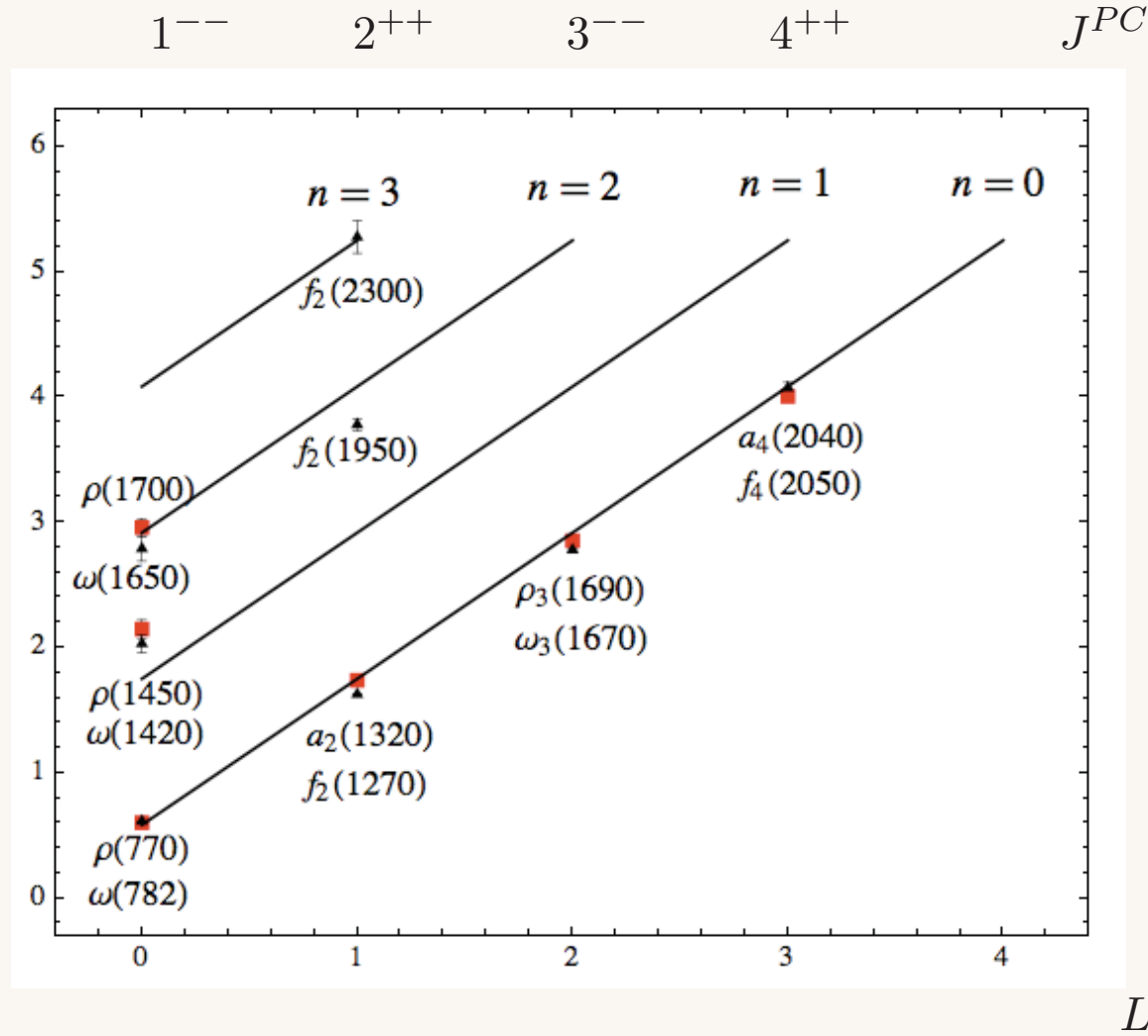
Vector mesons orbital (a) and radial (b) spectrum for  $\kappa = 0.54$  GeV.

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Fazio, Jugeau and Nicotri( 2007).

To do: compare with Ebert, Faustov, & Galkin, Plessas, et al



*AdS/QCD Soft Wall Model -- Reproduces Linear Regge Trajectories*

$\mathcal{M}^2$  $L$ 

Parent and daughter Regge trajectories for the  $I = 1$   $\rho$ -meson family (red)  
and the  $I = 0$   $\omega$ -meson family (black) for  $\kappa = 0.54$  GeV

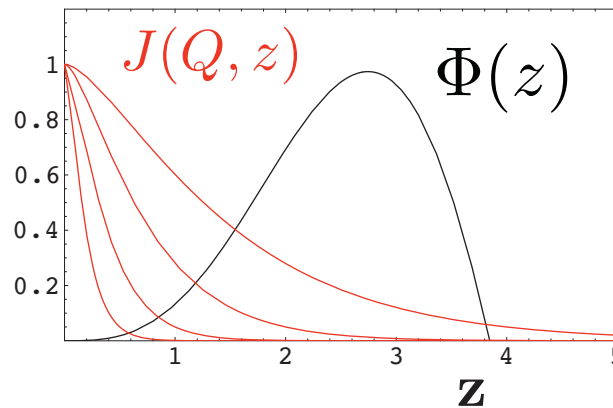
# Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

$$J(Q, z) = zQK_1(zQ)$$

$$F(Q^2)_{I \rightarrow F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z)$$

High  $Q^2$   
from  
small  $z \sim 1/Q$



**Polchinski, Strassler  
de Teramond, sjb**

Consider a specific AdS mode  $\Phi^{(n)}$  dual to an  $n$  partonic Fock state  $|n\rangle$ . At small  $z$ ,  $\Phi$  scales as  $\Phi^{(n)} \sim z^{\Delta_n}$ . Thus:

$$F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1},$$

**Dimensional Quark Counting Rules:  
General result from  
AdS/CFT and Conformal Invariance**

where  $\tau = \Delta_n - \sigma_n$ ,  $\sigma_n = \sum_{i=1}^n \sigma_i$ . The twist is equal to the number of partons,  $\tau = n$ .

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[ z^2 \partial_z^2 - z (1 + 2\kappa^2 z^2) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

where  $U(a, b, c)$  is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1+t)^{b-a-1} dt.$$

- Form factor in presence of the dilaton background  $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

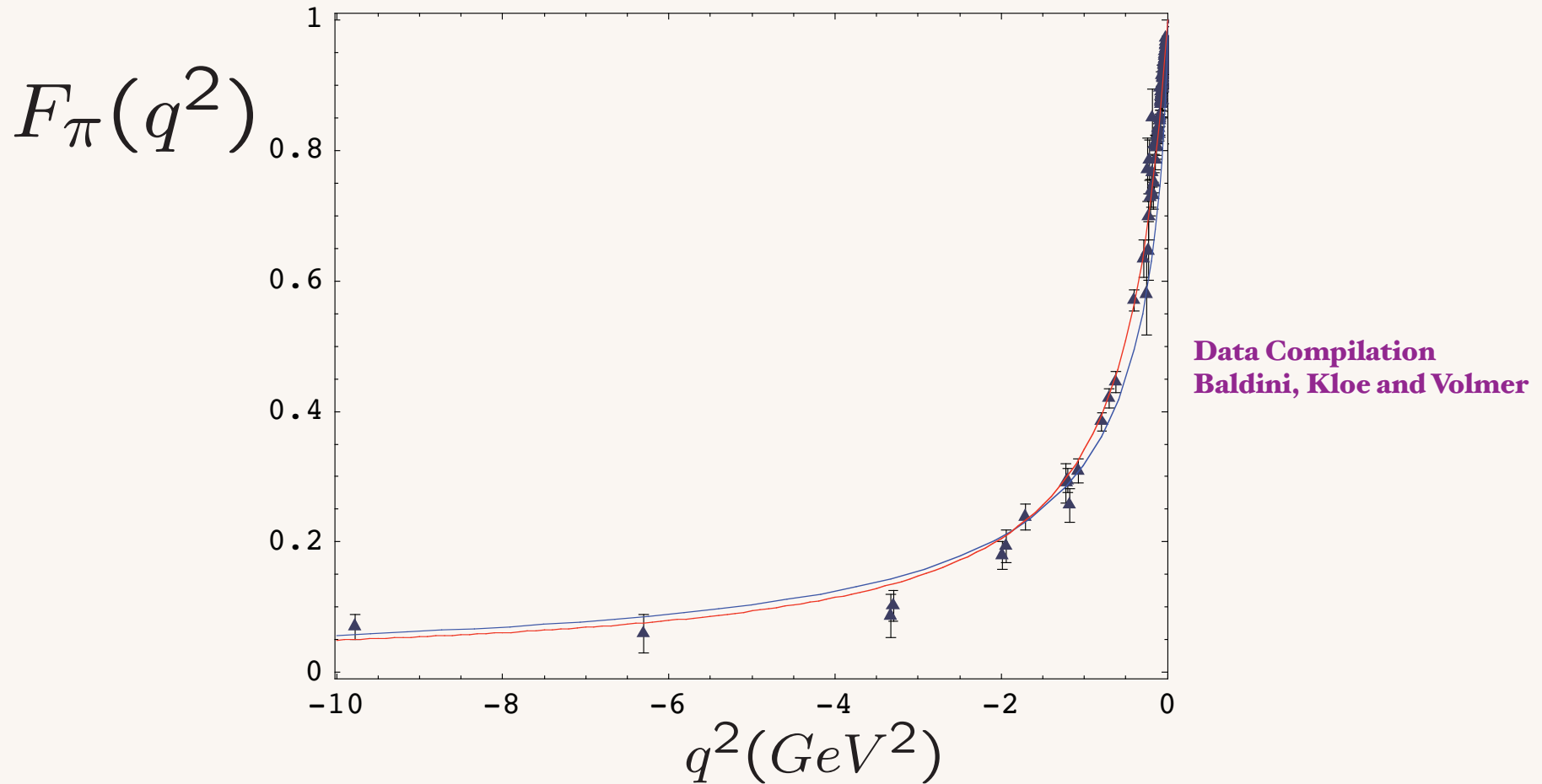
- For large  $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \rightarrow zQ K_1(zQ) = J(Q, z),$$

the external current decouples from the dilaton field.

*Soft Wall  
Model*

# Spacelike pion form factor from AdS/CFT



— Soft Wall: Harmonic Oscillator Confinement

— Hard Wall: Truncated Space Confinement

*One parameter - set by pion decay constant*

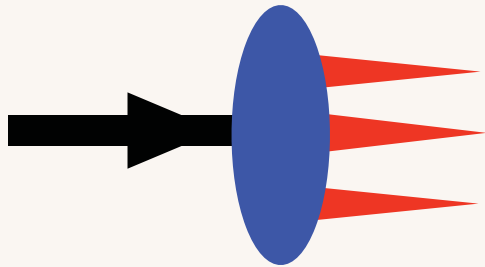
**de Teramond, sjb**  
**See also: Radyushkin**

# Light-Front Holography and Non-Perturbative QCD

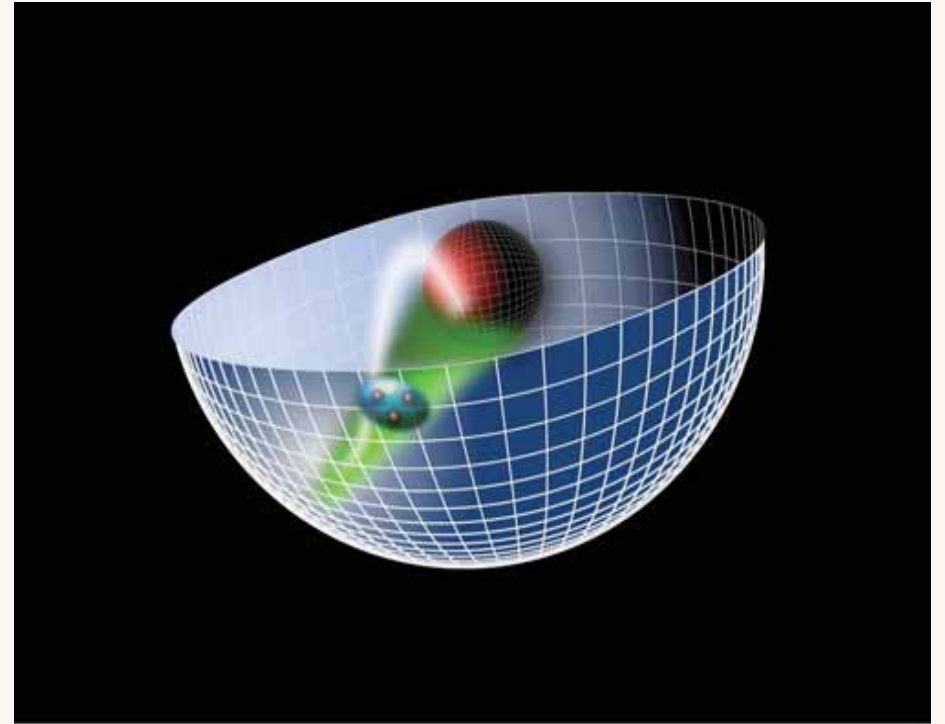
**Goal:**

**Use AdS/QCD duality to construct  
a first approximation to QCD**

*Hadron Spectrum  
Light-Front Wavefunctions,  
Form Factors, DVCS, etc*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

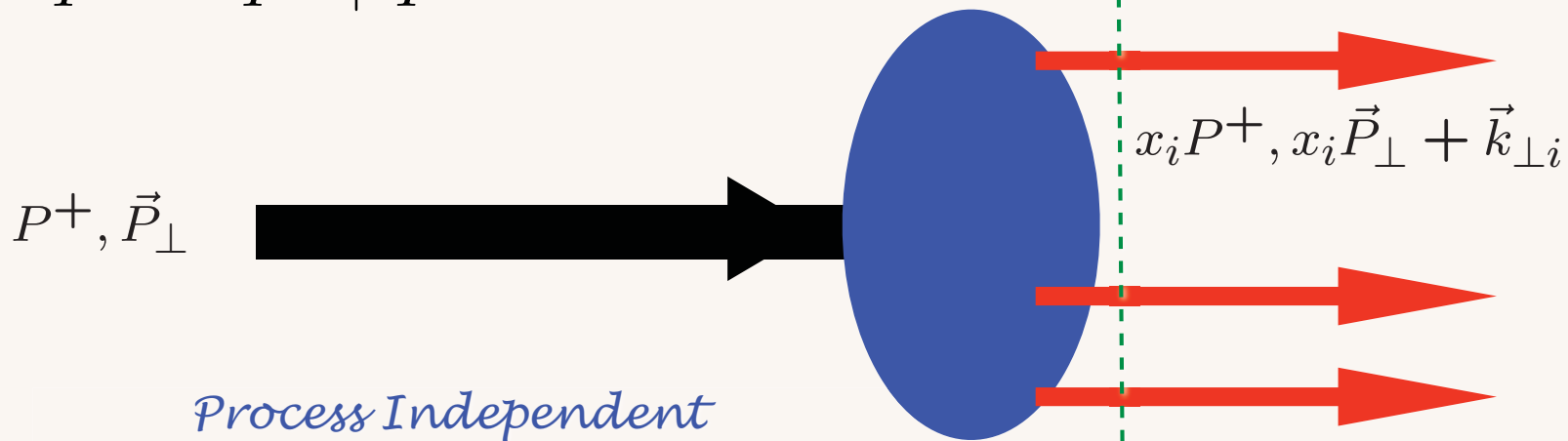


**in collaboration with  
Guy de Teramond**

# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed  $\tau = t + z/c$



*Process Independent  
Direct Link to QCD Lagrangian!*

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$\sum_i^n x_i = 1$$

$$\sum_i^n \vec{k}_{\perp i} = \vec{0}_{\perp}$$

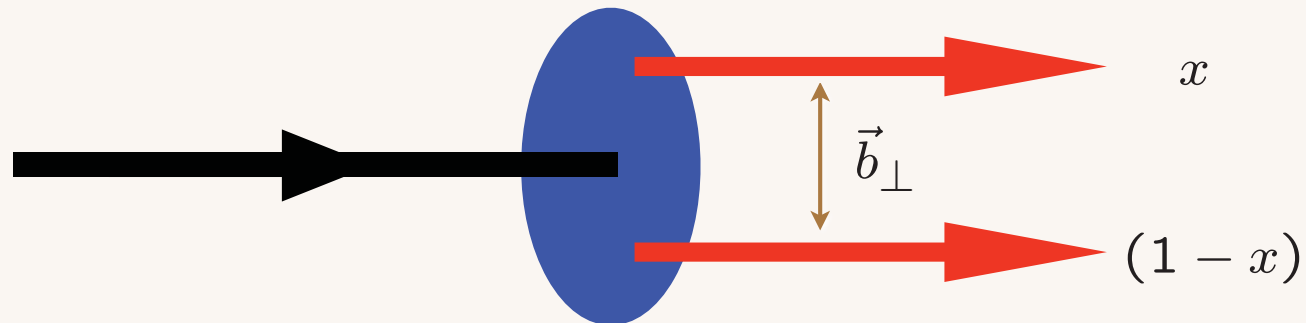
*Invariant under boosts! Independent of  $p^\mu$*

$LF(3+1)$

$AdS_5$

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

*Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements*

Consider the  $AdS_5$  metric:

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

$ds^2$  invariant if  $x^\mu \rightarrow \lambda x^\mu$ ,  $z \rightarrow \lambda z$ ,

Maps scale transformations to scale changes of the the holographic coordinate  $z$ .

We define light-front coordinates  $x^\pm = x^0 \pm x^3$ .

Then  $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0^2 - dx_3^2 - dx_\perp^2 = dx^+ dx^- - dx_\perp^2$

and

$$ds^2 = -\frac{R^2}{z^2} (dx_\perp^2 + dz^2) \text{ for } x^+ = 0.$$

## Light-Front/ $AdS_5$ Duality

- $ds^2$  is invariant if  $dx_\perp^2 \rightarrow \lambda^2 dx_\perp^2$ , and  $z \rightarrow \lambda z$ , at equal LF time.
- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate  $z$ .
- Holographic connection of  $AdS_5$  to the light-front.
- The effective wave equation in the two-dim transverse LF plane has the Casimir representation  $L^2$  corresponding to the  $SO(2)$  rotation group [The Casimir for  $SO(N) \sim S^{N-1}$  is  $L(L + N - 2)$ ].

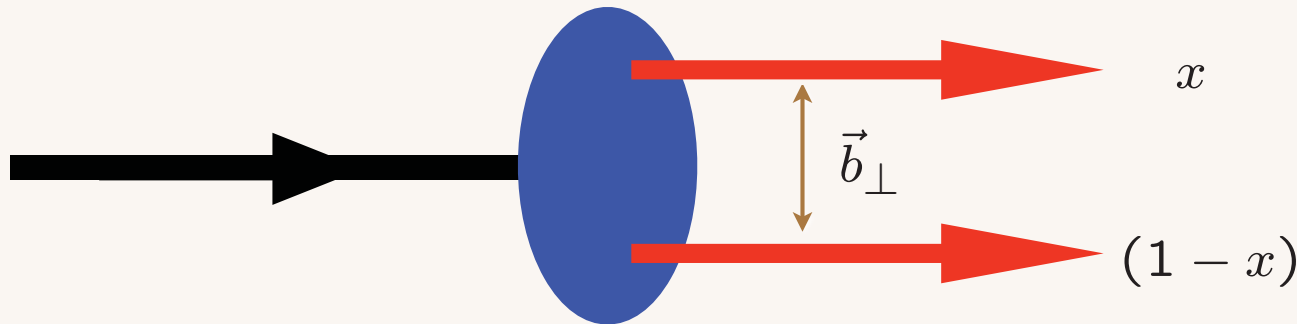
# Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

Frame Independent

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)$$

$$\zeta^2 = x(1-x)b_{\perp}^2.$$



$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

G. de Teramond, sjb

*soft wall  
confining potential:*

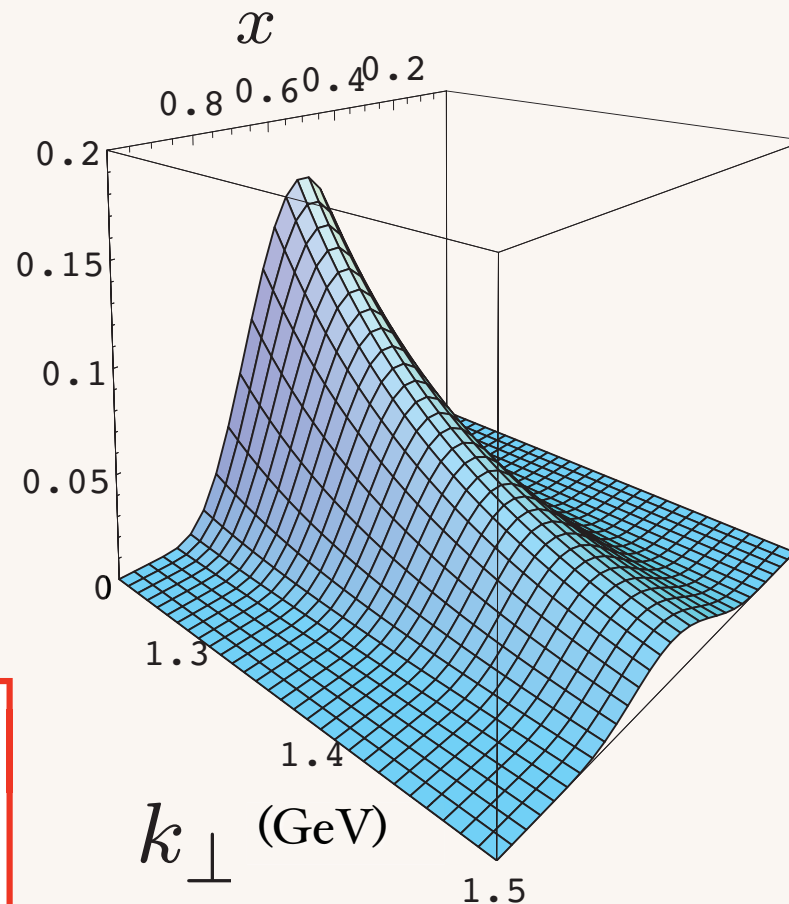
# Prediction from AdS/CFT: Meson LFWF

de Teramond, sjb

**“Soft Wall”  
model**

$\kappa = 0.375$  GeV  
massless quarks

$$\psi_M(x, k_{\perp}^2)$$



**Note coupling**

$$k_{\perp}^2, x$$

$$\psi_M(x, k_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_{\perp}^2}{2\kappa^2 x(1-x)}}$$

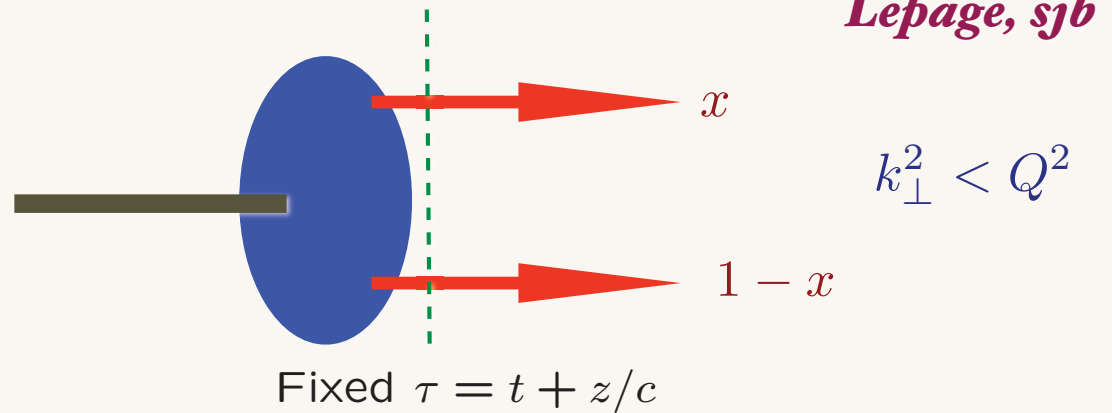
$$\phi_M(x, Q_0) \propto \sqrt{x(1-x)}$$

*Connection of Confinement to TMDs*

# Hadron Distribution Amplitudes

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$



- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

*Lepage, sjb*

*Efremov, Radyushkin*

*Sachrajda, Frishman Lepage, sjb*

*Braun, Gardi*

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \psi_{q\bar{q}}(x, \vec{k}_{\perp})$$

# Second Moment of Pion Distribution Amplitude

$$\langle \xi^2 \rangle = \int_{-1}^1 d\xi \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

$$\langle \xi^2 \rangle_{\pi} = 1/5 = 0.20 \quad \phi_{asympt} \propto x(1-x)$$

$$\langle \xi^2 \rangle_{\pi} = 1/4 = 0.25 \quad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

$$\text{Lattice (I)} \quad \langle \xi^2 \rangle_{\pi} = 0.28 \pm 0.03$$

Donnellan et al.

$$\text{Lattice (II)} \quad \langle \xi^2 \rangle_{\pi} = 0.269 \pm 0.039$$

Braun et al.

$H_{QED}$

*QED atoms: positronium and muonium*

$$(H_0 + H_{int}) |\Psi\rangle = E |\Psi\rangle$$

*Coupled Fock states*

$$\left[ -\frac{\Delta^2}{2m_{red}} + V_{eff}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r})$$

*Effective two-particle equation*

Includes Lamb Shift, quantum corrections

$$\left[ -\frac{1}{2m_{red}} \frac{d^2}{dr^2} + \frac{1}{2m_{red}} \frac{\ell(\ell+1)}{r^2} + V_{eff}(r, S, \ell) \right] \psi(r) = E \psi(r)$$

*Spherical Basis*  $r, \theta, \phi$

$$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

Bohr Spectrum

$$H_{QCD}^{LF}$$

QCD Meson Spectrum

$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

*Coupled Fock states*

$$\left[ \frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_\perp) = M^2 \psi_{LF}(x, \vec{k}_\perp)$$

*Effective two-particle equation*

$$\zeta^2 = x(1-x)b_\perp^2$$

$$\left[ -\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$$

*Azimuthal Basis*  $\zeta, \phi$

$$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$$

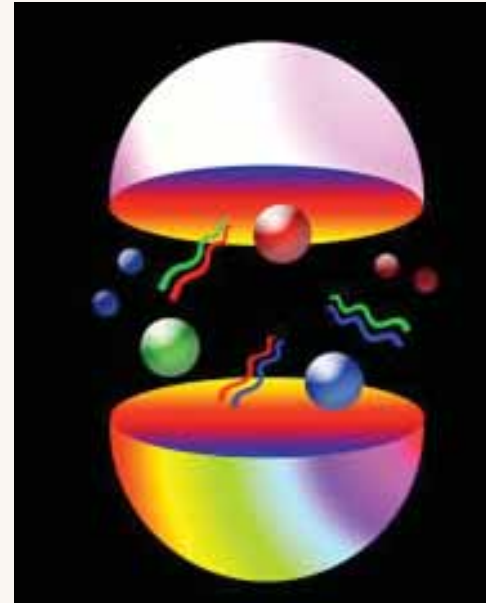
*Confining AdS/QCD potential*



- Baryons Spectrum in "bottom-up" holographic QCD

**GdT and Sjb** hep-th/0409074, hep-th/0501022.

## Baryons in AdS/CFT



- Action for massive fermionic modes on  $\text{AdS}_{d+1}$ :

$$S[\bar{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g} \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z).$$

- Equation of motion:  $(i\Gamma^\ell D_\ell - \mu) \Psi(x, z) = 0$

$$\left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0.$$

# Baryons

## Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin- $\frac{1}{2}$  modes  $\psi(\zeta)$  and spin- $\frac{3}{2}$  modes  $\psi_\mu(\zeta)$  are **two-component spinor** solutions of the Dirac light-front equation

$$\alpha\Pi(\zeta)\psi(\zeta) = \mathcal{M}\psi(\zeta),$$

where  $H_{LF} = \alpha\Pi$  and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint  $\Pi_L^\dagger(\zeta)$  satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$

- Note: in the Weyl representation ( $i\alpha = \gamma_5\beta$ )

$$i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

- Baryon: twist-dimension  $3 + L$  ( $\nu = L + 1$ )

$$\mathcal{O}_{3+L} = \psi D_{\{\ell_1 \dots D_{\ell_q} \psi D_{\ell_{q+1}} \dots D_{\ell_m}\}} \psi, \quad L = \sum_{i=1}^m \ell_i.$$

- Solution to Dirac eigenvalue equation with UV matching boundary conditions

$$\psi(\zeta) = C \sqrt{\zeta} [J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_-].$$

Baryonic modes propagating in AdS space have two components: orbital  $L$  and  $L + 1$ .

- Hadronic mass spectrum determined from IR boundary conditions

$$\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,$$

given by

$$\mathcal{M}_{\nu,k}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}_{\nu,k}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}},$$

with a scale independent mass ratio.

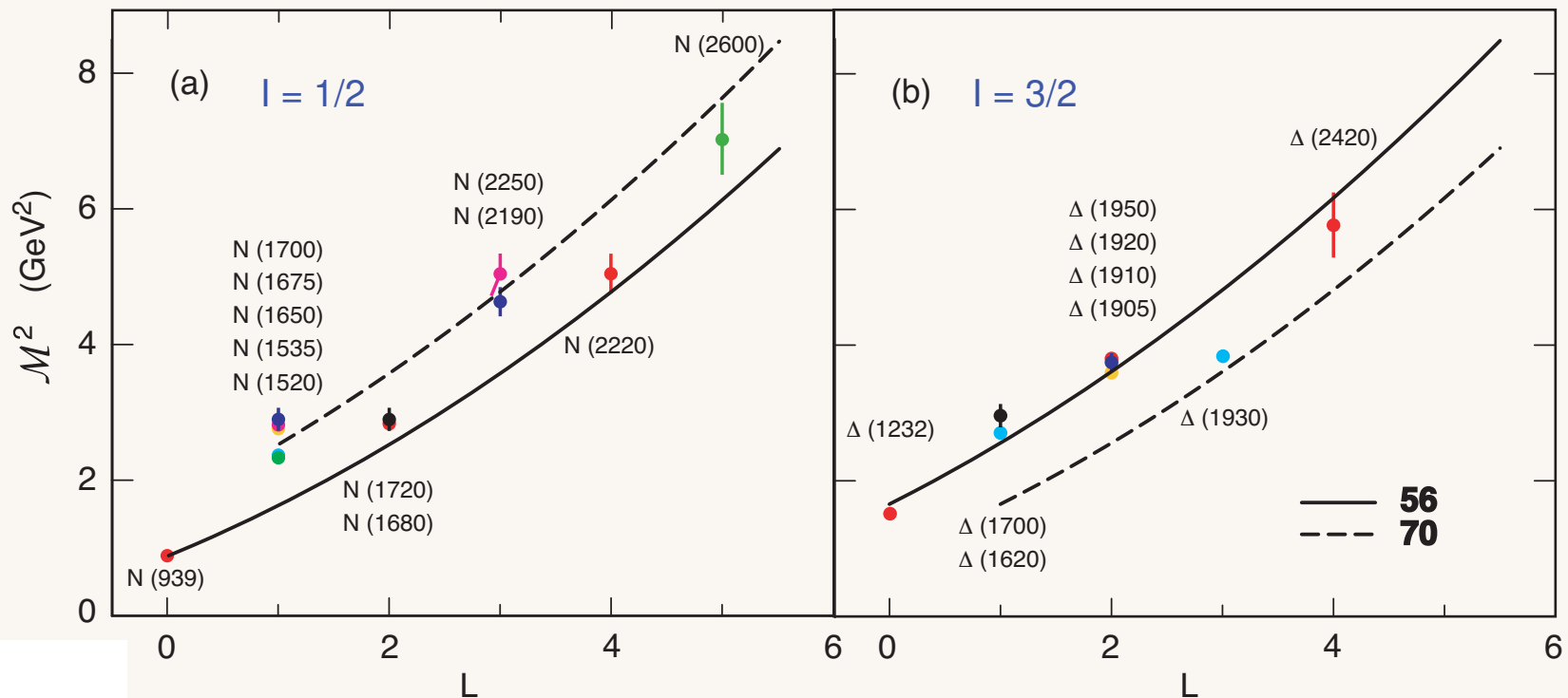


Fig: Light baryon orbital spectrum for  $\Lambda_{QCD} = 0.25$  GeV in the HW model. The **56** trajectory corresponds to  $L$  even  $P = +$  states, and the **70** to  $L$  odd  $P = -$  states.

# Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

$$(\alpha\Pi(\zeta) - \mathcal{M})\psi(\zeta) = 0,$$

in terms of the matrix-valued operator  $\Pi$

$$\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),$$

and its adjoint  $\Pi^\dagger$ , with commutation relations

$$\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.$$

- Solutions to the Dirac equation

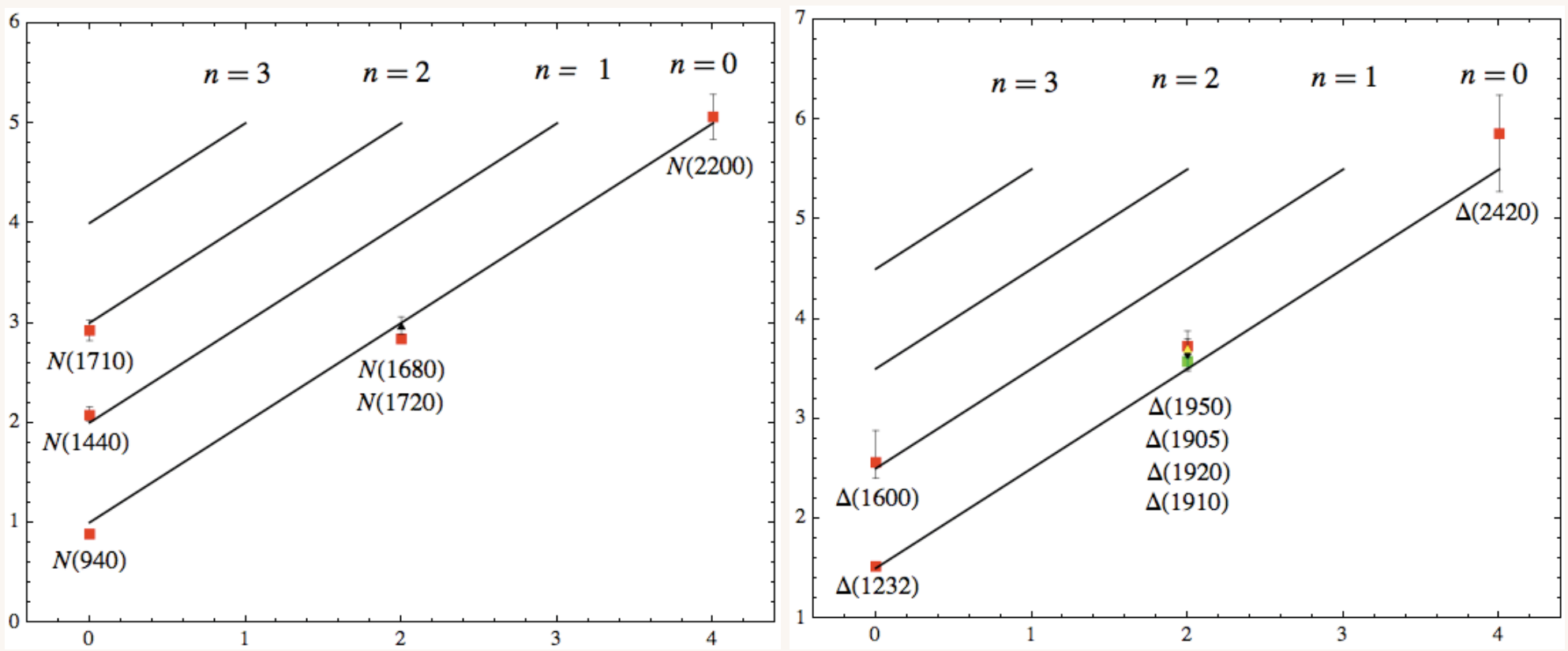
$$\begin{aligned} \psi_+(\zeta) &\sim z^{\frac{1}{2}+\nu} e^{-\kappa^2 \zeta^2/2} L_n^\nu(\kappa^2 \zeta^2), \\ \psi_-(\zeta) &\sim z^{\frac{3}{2}+\nu} e^{-\kappa^2 \zeta^2/2} L_n^{\nu+1}(\kappa^2 \zeta^2). \end{aligned}$$

- Eigenvalues

$$\mathcal{M}^2 = 4\kappa^2(n + \nu + 1).$$

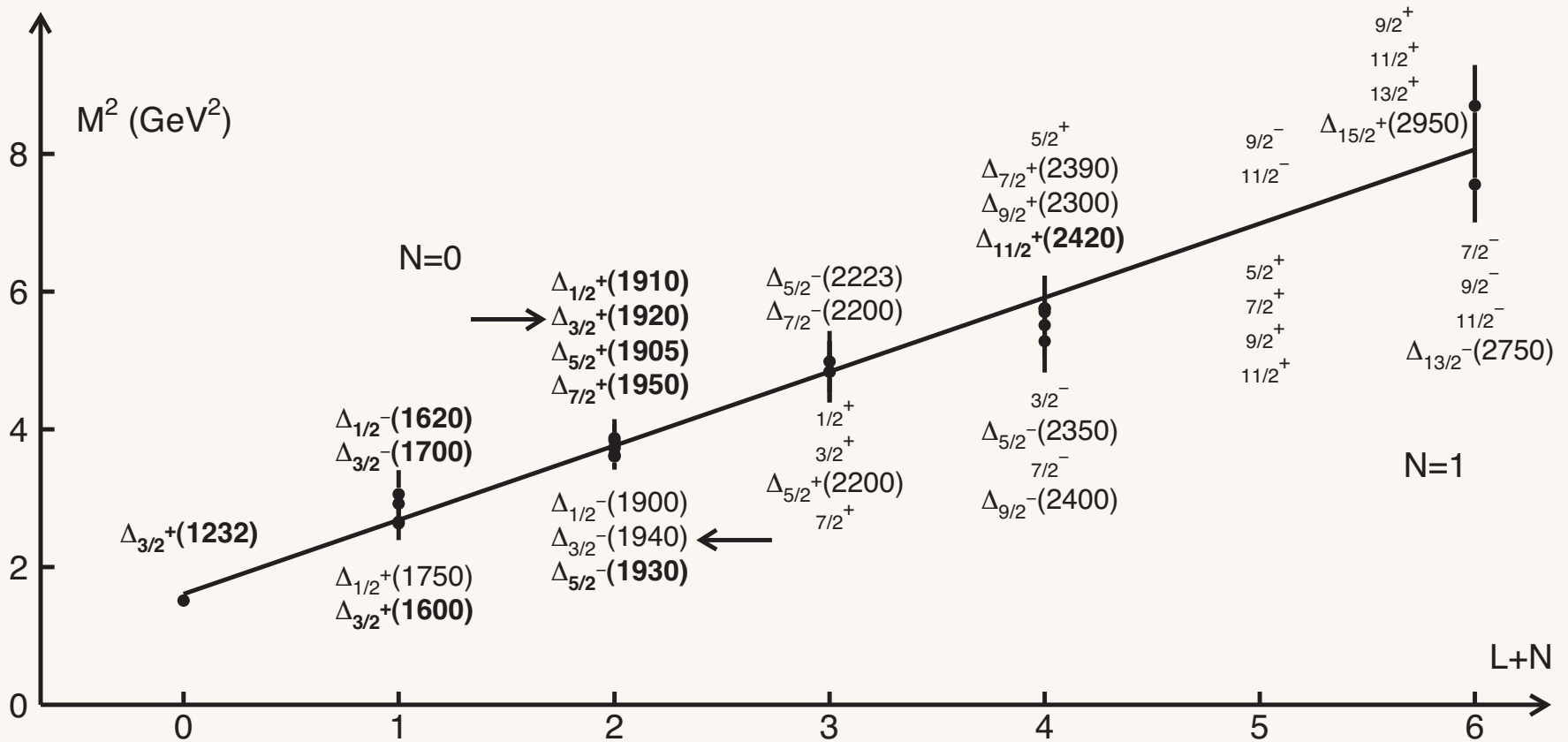
$4\kappa^2$  for  $\Delta n = 1$   
 $4\kappa^2$  for  $\Delta L = 1$   
 $2\kappa^2$  for  $\Delta S = 1$

$\mathcal{M}^2$



$L$

Parent and daughter **56** Regge trajectories for the  $N$  and  $\Delta$  baryon families for  $\kappa = 0.5$  GeV



## E. Klempt *et al.*: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

H. Forkel, M. Beyer and T. Frederico, *JHEP* **0707** (2007) 077.

H. Forkel, M. Beyer and T. Frederico, *Int. J. Mod. Phys. E* **16** (2007) 2794.

$SU(6)$	$S$	$L$	Baryon State
<b>56</b>	$\frac{1}{2}$	0	$N_{\frac{1}{2}}^{1+}$ (939)
	$\frac{3}{2}$	0	$\Delta_{\frac{3}{2}}^{3+}$ (1232)
<b>70</b>	$\frac{1}{2}$	1	$N_{\frac{1}{2}}^{1-}$ (1535) $N_{\frac{3}{2}}^{3-}$ (1520)
	$\frac{3}{2}$	1	$N_{\frac{1}{2}}^{1-}$ (1650) $N_{\frac{3}{2}}^{3-}$ (1700) $N_{\frac{5}{2}}^{5-}$ (1675)
	$\frac{1}{2}$	1	$\Delta_{\frac{1}{2}}^{1-}$ (1620) $\Delta_{\frac{3}{2}}^{3-}$ (1700)
<b>56</b>	$\frac{1}{2}$	2	$N_{\frac{3}{2}}^{3+}$ (1720) $N_{\frac{5}{2}}^{5+}$ (1680)
	$\frac{3}{2}$	2	$\Delta_{\frac{1}{2}}^{1+}$ (1910) $\Delta_{\frac{3}{2}}^{3+}$ (1920) $\Delta_{\frac{5}{2}}^{5+}$ (1905) $\Delta_{\frac{7}{2}}^{7+}$ (1950)
<b>70</b>	$\frac{1}{2}$	3	$N_{\frac{5}{2}}^{5-}$ $N_{\frac{7}{2}}^{7-}$
	$\frac{3}{2}$	3	$N_{\frac{3}{2}}^{3-}$ $N_{\frac{5}{2}}^{5-}$ $N_{\frac{7}{2}}^{7-}$ (2190) $N_{\frac{9}{2}}^{9-}$ (2250)
	$\frac{1}{2}$	3	$\Delta_{\frac{5}{2}}^{5-}$ (1930) $\Delta_{\frac{7}{2}}^{7-}$
<b>56</b>	$\frac{1}{2}$	4	$N_{\frac{7}{2}}^{7+}$ $N_{\frac{9}{2}}^{9+}$ (2220)
	$\frac{3}{2}$	4	$\Delta_{\frac{5}{2}}^{5+}$ $\Delta_{\frac{7}{2}}^{7+}$ $\Delta_{\frac{9}{2}}^{9+}$ $\Delta_{\frac{11}{2}}^{11+}$ (2420)
<b>70</b>	$\frac{1}{2}$	5	$N_{\frac{9}{2}}^{9-}$ $N_{\frac{11}{2}}^{11-}$ (2600)
	$\frac{3}{2}$	5	$N_{\frac{7}{2}}^{7-}$ $N_{\frac{9}{2}}^{9-}$ $N_{\frac{11}{2}}^{11-}$ $N_{\frac{13}{2}}^{13-}$